PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

A Holographic Quantum Hall Ferromagnet

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Razieh Pourhasan

with

Charlotte Kristjansen (NBI)

Gordon Semenoff (UBC)

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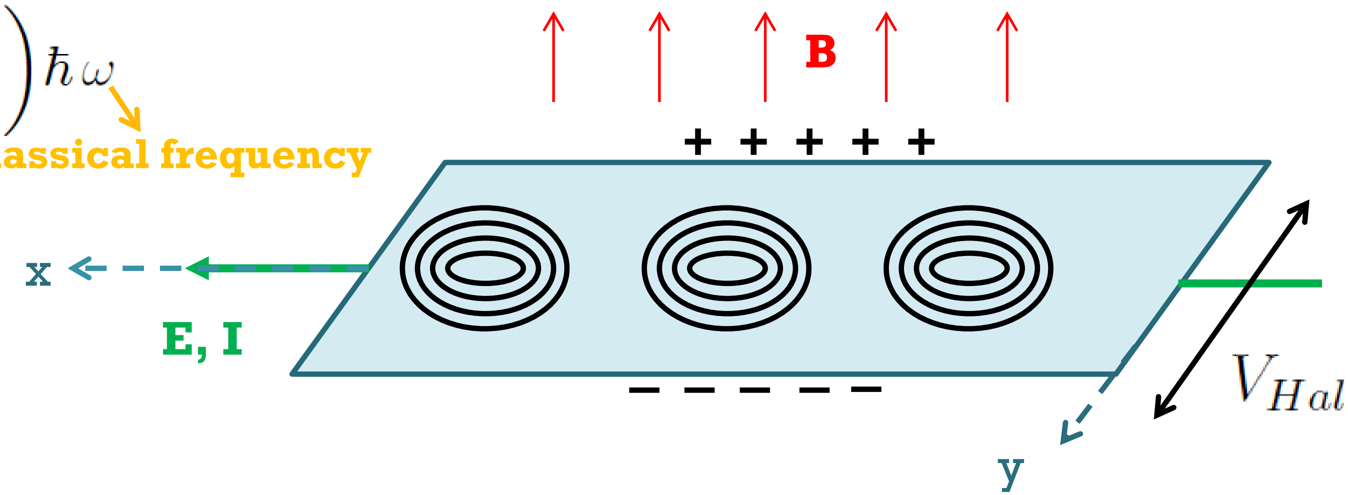
Quantum Hall effect

Consider 2D electron system in x − y plane with magnetic filed B

$$E_n = \left(n + \frac{1}{2}\right)\hbar\,\omega$$

classical frequency

Landau level



In 2D block we define conductance matrix

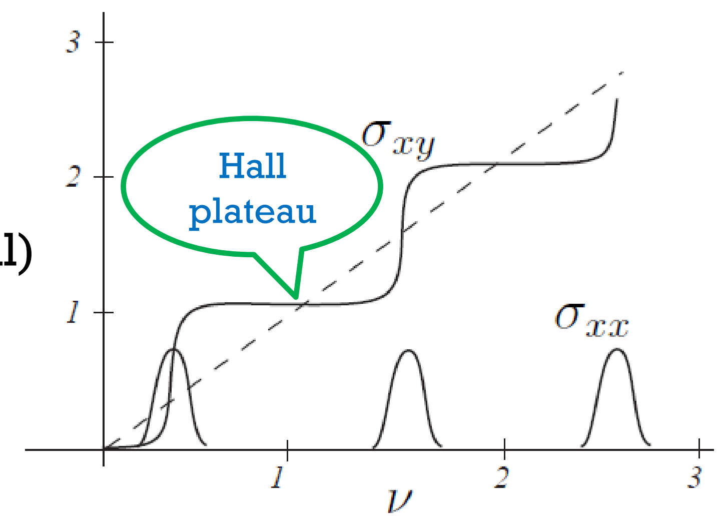
$$I_i \equiv \sum_j \sigma_{ij} V_j$$

The quantum Hall states (integer or fractional) are characterized by the property that(T=0):

$$\sigma_{xx} = 0, \sigma_{xy} = \text{const.}$$

Longitudinal conductivity

Hall conductivity



filling fraction ≡ charge density/B

- The non-zero width of plateaus indicates that a QHS is an incompressible gapped state. It forms around an integer # of filling fraction in IQHE or a rational # in FQHE.

- Away from these values, the state is un-gapped, the longitudinal conductivity is non-zero and the Hall conductivity varies.

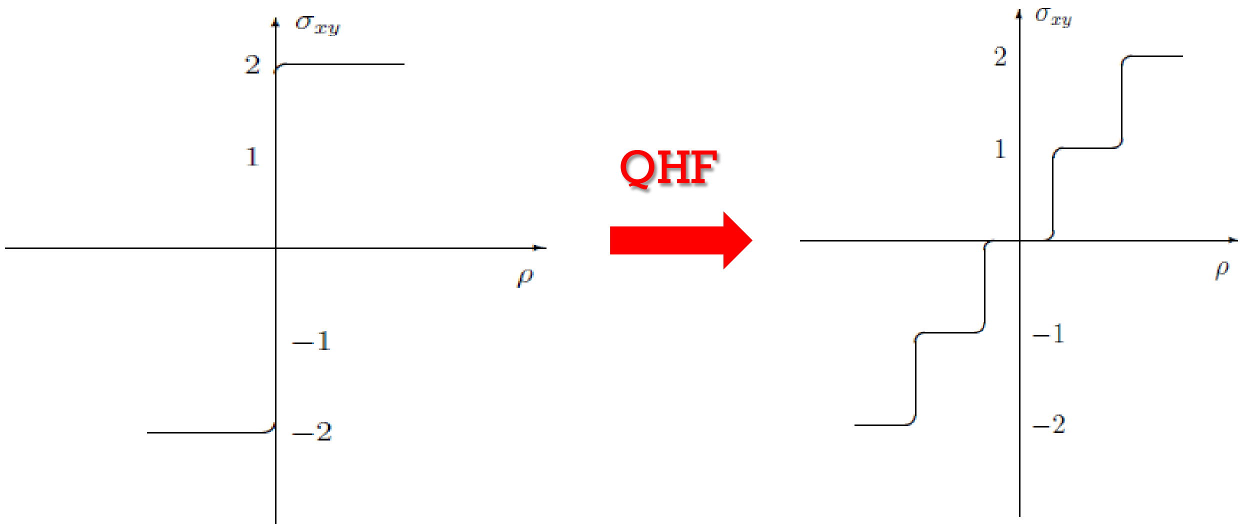
- The formation of gapped state is very well understood in terms of the Landau states(IQHE) or the Laughlin states(FQHE).

- Also, there are good theoretical arguments for the persistence of the IQHE in the presence of *electron-electron interactions*, at least when the ***interactions are weak*** enough that pert. theory can be applied.

TO EXPLORE:

Whether any feature of the IQHE can persist when the *coupling is strong* beyond the reach of perturbation theory?

An example: Graphene



- ✓ These features were observed in the initial experiments soon after the discovery of graphene in 2004.

- ✓ Later, with stronger magnetic field and cleaner sample, the formation of new plateaus were discovered.

- ✓ The mechanism for formation of these additional plateaus is thought to be SSB and has an elegant explanation at weak coupling: when electrons are non-interacting a partially filled Landau level is a highly deg. state. Any interaction no matter how weak will split the degeneracy.

But the Coulomb interaction in graphene is strong!

AdS/CFT:

Gauge theory		Gravity theory
(3+1)-dim $\mathscr{N}=4$ SYM + infinite flat (2+1)-dim SC defect	$\Longleftrightarrow$	N coincident D3-branes + N5/N7 coincident D5/D7-branes

N5 & N7 << N

The geometric set-up:

We embed D5 and D7 probe branes in the asymptotically $AdS_5 \times S^5$ black hole bgr:

$$ds^2 = \sqrt{\lambda}\alpha' \left[\overbrace{r^2(-h(r)dt^2 + dx^2 + dy^2 + dz^2) + \frac{dr^2}{h(r)r^2}}^{AdS_5} + \right.$$

$$\left. + d\psi^2 + \underbrace{\sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)}_{S^2} + \underbrace{\cos^2\psi(d\tilde{\theta}^2 + \sin^2\tilde{\theta}d\tilde{\phi}^2)}_{\tilde{S}^2} \right]$$

$\psi \in [0, \pi/2]$

$$h(r) = 1 - \frac{r_h^4}{r^4} \qquad T = r_h/\pi$$

RR 4-form of IIB

$$\boxed{C^{(4)} = \lambda\alpha'^2 \left[h(r)r^4 dt \wedge dx \wedge dy \wedge dz + \frac{c(\psi)}{2} d\cos\theta \wedge d\phi \wedge d\cos\tilde{\theta} \wedge d\tilde{\phi} \right]}$$

worldvolume coordinates

	t	x	y	z	r	ψ	θ	ϕ	$\tilde{\theta}$	$\tilde{\phi}$
$D3$	$\times$	$\times$	$\times$	$\times$						
$D5$	$\times$	$\times$	$\times$		$\times$		$\times$	$\times$		
$D7$	$\times$	$\times$	$\times$		$\times$		$\times$	$\times$	$\times$	$\times$

Probe D5-branes (N5<<N)

$$ds^2 = \sqrt{\lambda}\alpha' \left[r^2(-h(r)dt^2 + dx^2 + dy^2) + \overbrace{\sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)}^{S^2} + \frac{dr^2}{h(r)r^2}\left(1 + h(r)\left(r\frac{d\psi}{dr}\right)^2\right)\right]$$

asymptotically $AdS_4 \times S^2$

Parameters

$$\boxed{B = \frac{\sqrt{\lambda}}{2\pi}\, b \qquad f = \frac{2\pi}{\sqrt{\lambda}} N_5 \qquad \nu = \frac{2\pi}{N}\frac{\rho}{B}}$$

Probe D7-branes (N7=1)

$$ds^2 = \sqrt{\lambda}\alpha' \left[r^2(-h(r)dt^2 + dx^2 + dy^2) + \frac{dr^2}{h(r)r^2}\left(1 + h(r)\left(r\frac{d\psi}{dr}\right)^2\right) + \right.$$
$$\left. + \underbrace{\sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)}_{S^2} + \underbrace{\cos^2\psi(d\tilde{\theta}^2 + \sin^2\tilde{\theta} d\tilde{\phi}^2)}_{\tilde{S}^2} \right],$$

$f \longrightarrow$ N5 Dirac monopoles

Asymptotic behavior of the solutions:

- $r \to \infty \Longrightarrow \psi(r) = \frac{\pi}{2} + \frac{c_1}{r} + \frac{c_2}{r^2} + \ldots.$ **for both D5 & D7**

In the dual FT on the defect:

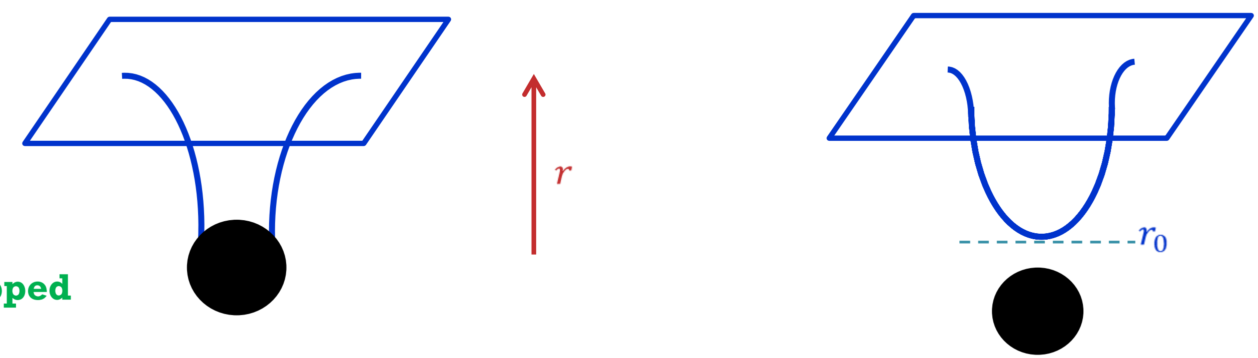
0

$c_1 \propto$ bare mass of the fields & $c_2 \propto$ chiral condensate

Two types of solutions: $c_2 = 0 \longrightarrow$ chiral sym. (constant sol.) **D5**

$c_2 \neq 0 \longrightarrow$ SChSB (r-dependent sol.) **D5/D7**

- $r \to r_h \Longrightarrow \psi = \pi/2,\ \psi = \psi_0 \in [0, \pi/2)$ **for both D5 & D7(ungapped sols.)**

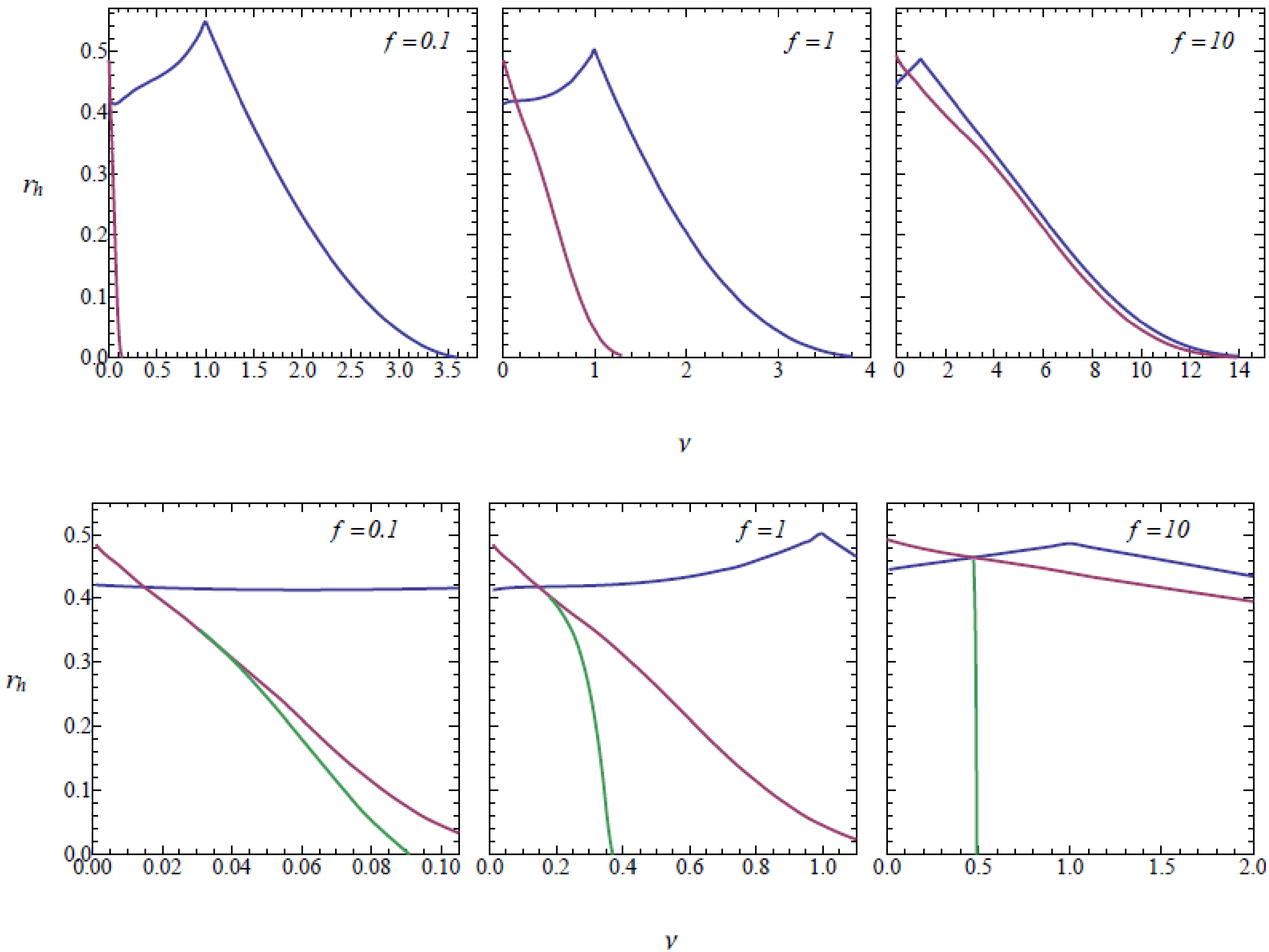


ungapped **gapped**

However, for $\nu = 1$ the D7 EOM *diverges* at the horizon.

We can still find a solution if $\psi = 0$ at some $r_0 > r_h$.
This is called a Minkowski embedding and corresponds to a ***gapped*** solution.

Numerical results (comparing free energies):



Composite systems ($\nu > 1$):

$N5$ D5-branes $(f, n < \nu < n+1)$ $\longrightarrow$ a single D7-brane (f, ν)

$n5$ D5-brane $(f_0, \nu - \nu_0)$ $\downarrow$ n gapped D7-brane $(f_0, \nu = 1)$

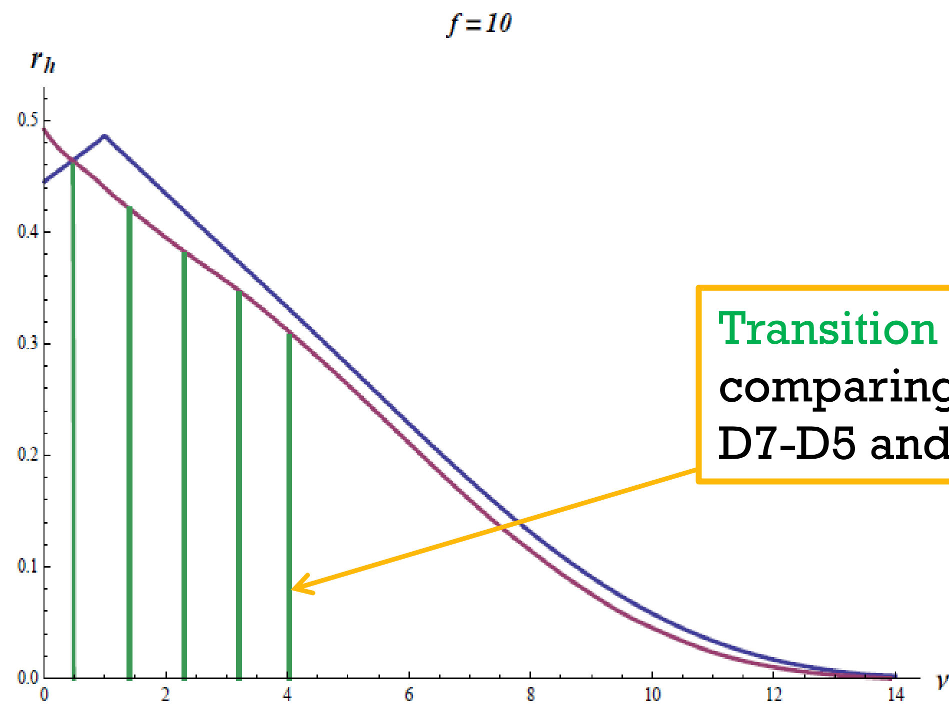
+

D7-D5

$(N5 - n5)$ D5-brane $(f - f_0, 0 \le \nu_0 < 1)$ $\downarrow$

+ a single ungapped D7-brane $(f - f_0, \nu_0)$

D7-D7



Transition line in each interval comparing the free energy of D7-D5 and D7-D7

Conclusion:

- ✓ For $\nu < 1$ there are three solutions: D5 Ch. Sym. solution, D5&D7 SChSB solutions competing and the one with lower free energy is stable.

- ✓ D7-brane solutions are ungapped for all values of filling fraction except for $\nu = 1$ which is an incompressible gapped solution.

- ✓ For $\nu > 1$ there are also composite systems (D5-D7 or D7-D7) competing with the above mentioned solutions.

- ✓ Phase diagrams showing the stability regions of all solutions have been obtained numerically.

- ✓ Our holographic model resembles some of the observed features in graphene which has strong Coulomb interactions.

Probe D5-branes (N5<<N):

$$S_5 = \frac{T_5}{g_s} N_5 \int d^6\sigma \left[-\sqrt{-\det(g + 2\pi\alpha' \mathcal{F}_5)} + 2\pi\alpha' C^{(4)} \wedge \mathcal{F}_5 \right]$$

$$C^{(4)} = \lambda\alpha'^2 \left[h(r) r^4 dt \wedge dx \wedge dy \wedge dz + \frac{c(\psi)}{2} d\cos\theta \wedge d\phi \wedge d\cos\tilde{\theta} \wedge d\tilde{\phi} \right]$$

$$2\pi\alpha' \mathcal{F}_5 = \sqrt{\lambda}\alpha' \left[\frac{d}{dr} a(r) dr \wedge dt + b dx \wedge dy \right]$$

$$\rho = \frac{1}{V_{2+1}} \frac{2\pi}{\sqrt{\lambda}} \frac{\delta S_5}{\delta \frac{d}{dr} a(r)}$$

parameters

$$\left\{ \begin{aligned} B &= \frac{\sqrt{\lambda}}{2\pi} b \\ f &= \frac{2\pi}{\sqrt{\lambda}} N_5 \\ \nu &= \frac{2\pi}{N} \frac{\rho}{B} \end{aligned} \right.$$

$$\boxed{\mathcal{L}_5 = \sqrt{4\sin^4\psi f^2 (1 + r^4) + (\pi\nu)^2} \sqrt{1 + h(r) \left(r \frac{d\psi}{dr} \right)^2}}$$

Probe D7-branes (N7=1):

$$S = \frac{T_7}{g_s}\int d^8\sigma\left[-\sqrt{-\det(g+2\pi\alpha'\mathcal{F}_7)}+\frac{(2\pi\alpha')^2}{2}C^{(4)}\wedge\mathcal{F}_7\wedge\mathcal{F}_7\right]$$

$$2\pi\alpha'\mathcal{F}_7 = \sqrt{\lambda}\alpha'\left(\frac{d}{dr}a(r)dr\wedge dt + b dx\wedge dy + \frac{f}{2}d\cos\tilde{\theta}\wedge d\tilde{\phi}\right)$$

$$c(\psi) = \psi - \frac{1}{4}\sin 4\psi - \frac{\pi}{2}$$

$$\boxed{\mathcal{L}_7 = \sqrt{4\sin^4\psi(f^2+4\cos^4\psi)(1+r^4)+(\pi(\nu-1)+2\psi-\frac{1}{2}\sin 4\psi)^2}\ \times \sqrt{1+h(r)\left(r\frac{d\psi}{dr}\right)^2},}$$