Comments on finite counter-terms in holography

based on:

- A.Amoretti,A.Braggio,N.Maggiore, N.Magnoli,DM (1406.4134,1407.0306,1409.....)
- D.Forcella,A.Mezzalira,DM (1404.4048)
- R.Argurio,DM,D.Redigolo (1409.....)

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Outline

- **Motivation**
- **Scheme dependence**
- **Holographic Renormalization**
- **Examples**
  - **Thermoelectric transport *(in momentum-dissipating systems)***
  - **Holographic General Gauge Mediation**
  - **Optical properties of AdS/CFT media**
- **Conclusion**

Motivation

Finite counter-terms are a ubiquitous and *physically relevant* feature of holographic models

- **Correct physical behavior (*i.e. the proper definition of a model*) may require to tune finite counter-terms**
- **Relation with the symmetries of the model and the UV structure (*e.g. SUSY*)**
  [M.Bianchi,D.Freedman,K.Skenderis 0105276]
- **Finite counter-terms as parameters of a model?**
- **Relation with the total energy problem in general relativity systems** [S.Lau 9903038]

Scheme dependence

Physical observables should NOT depend on the renormalization scheme

- **Green functions, couplings, running parameters,... are scheme dependent**
- **At *weak coupling*, in perturbation theory, truncation at finite order introduces scheme dependence** [e.g. T.Muta 1998; M.Peskin,D.Schroeder 1995]
- **What about the *strongly-coupled* picture?**

Holographic Renormalization

… or renormalization in the dual, semi-classical gravity picture

[M.Bianchi,D.Freedman,K.Skenderis 0105276,0112119; K.Skenderis 0209067]

$$S_{\text{renormalized}} = \lim_{\epsilon \to 0} \left[S^{(\epsilon)}_{\text{on shell}} + S^{(\epsilon)}_{\text{counter-terms}} \right]$$

- **Getting rid of divergent contributions (*i.e. making sense of the computational results*)**
- **Minimal subtraction**
- **Beyond...**

Counter-terms

Local terms defined on the boundary which MUST

- **have the same divergent behavior of the regularized action**
- **respect all the symmetries of the boundary theory**
- **lead to a well-defined variational bulk problem**
- **do not introduce new dimensionful parameters (*power-counting*)**
- ***lead to meaningful physical behavior!***

[A.Amoretti,A.Braggio,N.Maggiore,N.Magnoli,DM 1406.4134]

Massive gravity

Holographic models are usually translational invariant; if charged they have divergent DC conductivity

- **Momentum dissipation (e.g. impurities, disorder, lattices,...) usually difficult to treat (from ODE to PDE)**
- **Q-lattices, helical lattices and *massive gravity* are simple alternatives**

[Horowitz,Santos,Hartnoll,Tong,Donos,Gauntlett,Arean,Salazar Landea,Scardicchio,Zayas,Farahi,Davison,Blake,Vegh, Zaanen,Schalm,Gouteraux,Kiritsis,Amoretti,Braggio,Magnoli, Maggiore,DM,...]

Breaking spatial diffeomorphism in the bulk (by means of mass terms for the graviton) corresponds to relax momentum conservation in the boundary theory

Radical bottom-up

Massive gravity in itself is problematic (ghosts, unclear UV origin,...)

[e.g. *bigravity* E.Kiritsis, V.Niarchos 0805.4234;0808.3410]

- **Holographic bottom-up approach still viable**

  [D.Vegh 1301.0537; R.Davison,K.Schalm,J.Zaanen 1311.2451; R.Davison 1306.5792; M.Blake,D.Tong,D.Vegh 1310.3832]

- **Accurate and complete set of tests is however required**
- **Holographic renormalization and thermodynamics of massive gravity systems** [M.Blake,D.Tong 1308.4970]
- **Thermo-electric transport in massive gravity systems**

  [A.Amoretti,A.Braggio,N.Maggiore,N.Magnoli,DM 1406.4134,1407.0306]

Less symmetry, more counter-terms

As usual (in 2+1 boundary):

- Gibbons-Hawking (*definition of the variational problem*) [e.g. R.Wald 1984]
- boundary cosmological constant
- vanishing boundary scalar curvature
- no AA term (*gauge invariance*)
- no FF term (*power counting*)

Novelty:

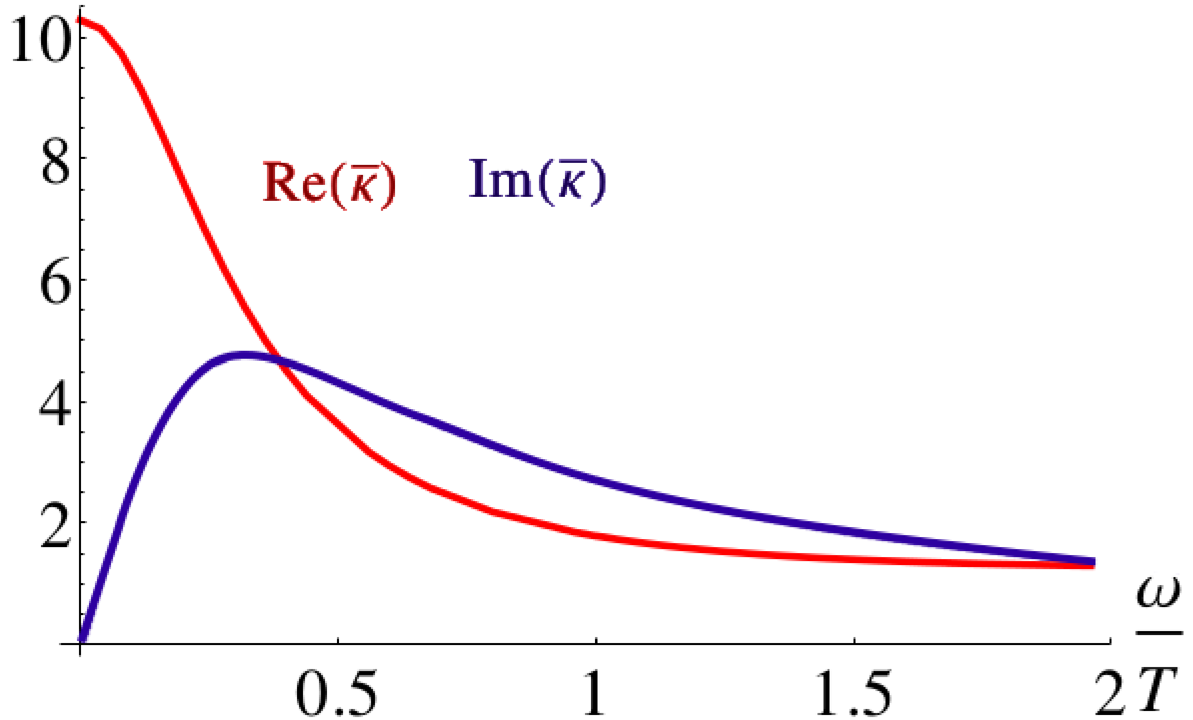
- *terms involving mixed fluctuations tx*
- and derivatives thereof (*though not radial*)

$$S^{(\text{fin})}_{\text{c.t.}}(a) = \frac{a}{2}\,\mathcal{E}\int_{z=z_{UV}} d^3x\;\frac{z}{L}\sqrt{-g_{b\,tt}}\,g^{tt}g^{xx}\,h_{tx}h_{tx}$$

the coefficient a is *a priori* arbitrary...

Curing unphysical features

- **with generic a → DC delta function in the thermal conductivity!!**
- **momentum-dissipating transport with perfectly efficient heat transport? (*two fluid components?*)**
- ***tuning a*** **→ non-divergent, Drude-like thermal transport**



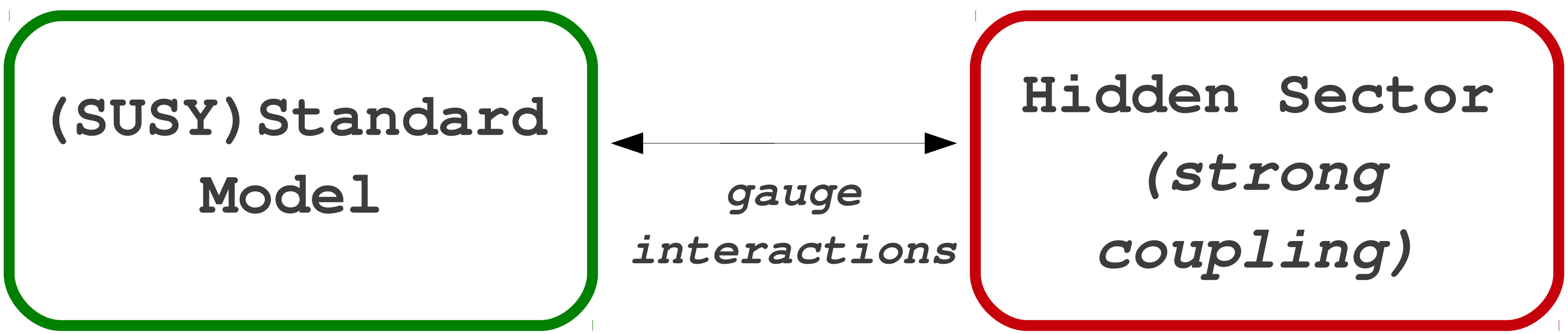
Moral

the reduced amount of symmetry leads to a larger set of possible *(finite) counter-terms*

↓

physical soundness fixes the extra freedom leading to unambiguous phenomenology

Holographic General Gauge Mediation



- **Hidden sector modeled in holography**
- **Hidden global symmetry weakly gauged corresponding to visible gauge symmetry**
- **Phenomenological info encoded in hidden correlators *(masses of SUSY partners,...)***

[P.Meade,N.Seiberg,D.Shih,P.McGuirk,K.Skenderis,M.Taylor, K.Intrilligator,M.Sudano,M.Bertolini,R.Argurio,D.Redigolo, F.Porri,L.DiPietro,DM,...]

A sketch of the bulk models

- **N=2 SUGRA truncation in the bulk**
- **non-trivial scalars (*breaking explicitly/spontaneously conformal/SUSY/R symmetries*)**
- **Universal + other multiplets**

$$\eta = z(\eta_0 + \tilde{\eta}_2 z^2 + \ldots)$$

- **Graviton + *graviphoton* + scalars + …**

[M.Bertolini,R.Argurio,D.Redigolo,F.Porri,L.DiPietro,DM,...]

Graviphoton

$$R^i = R^i_0 + z^2 \tilde{R}^i_2 + \frac{1}{2}(k^2 + 12\eta_0^2) z^2 \log(z) R^i_0 + \ldots$$

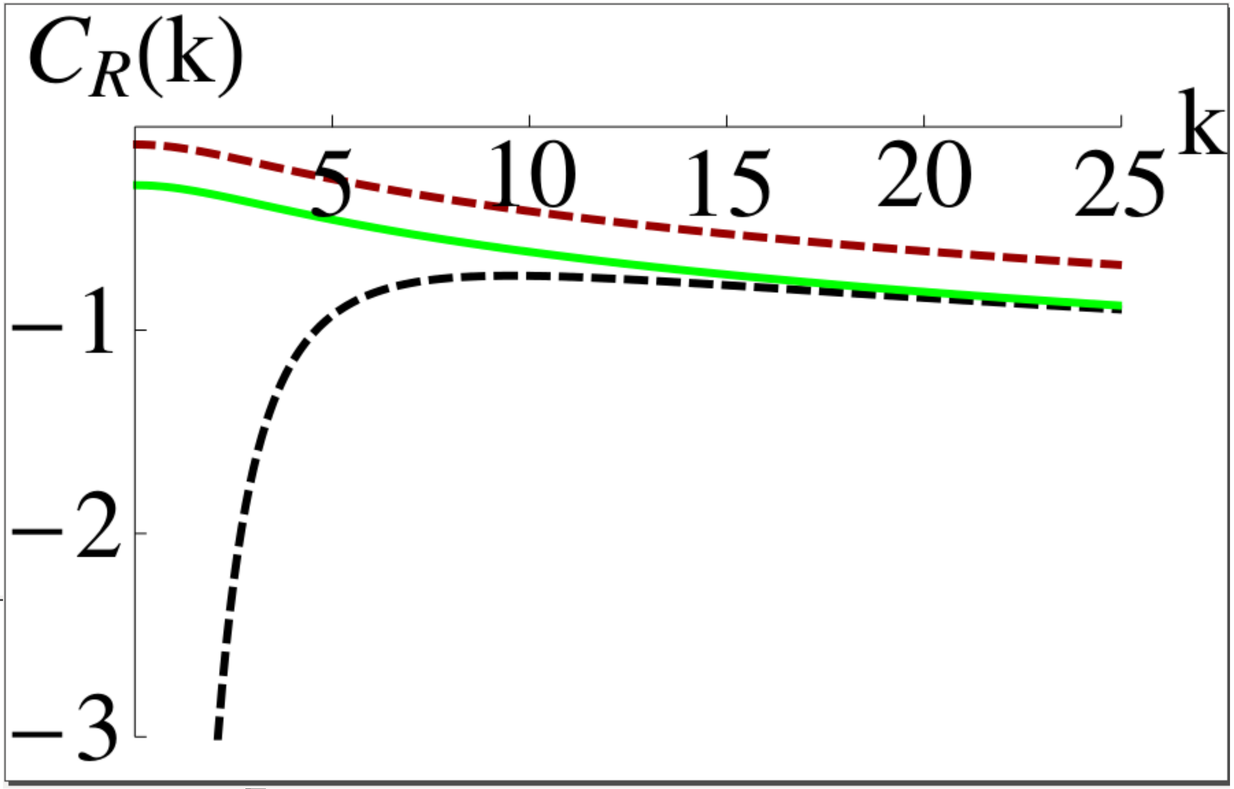
- **transverse sector in the presence of explicit SUSY, R and conformal breakings**

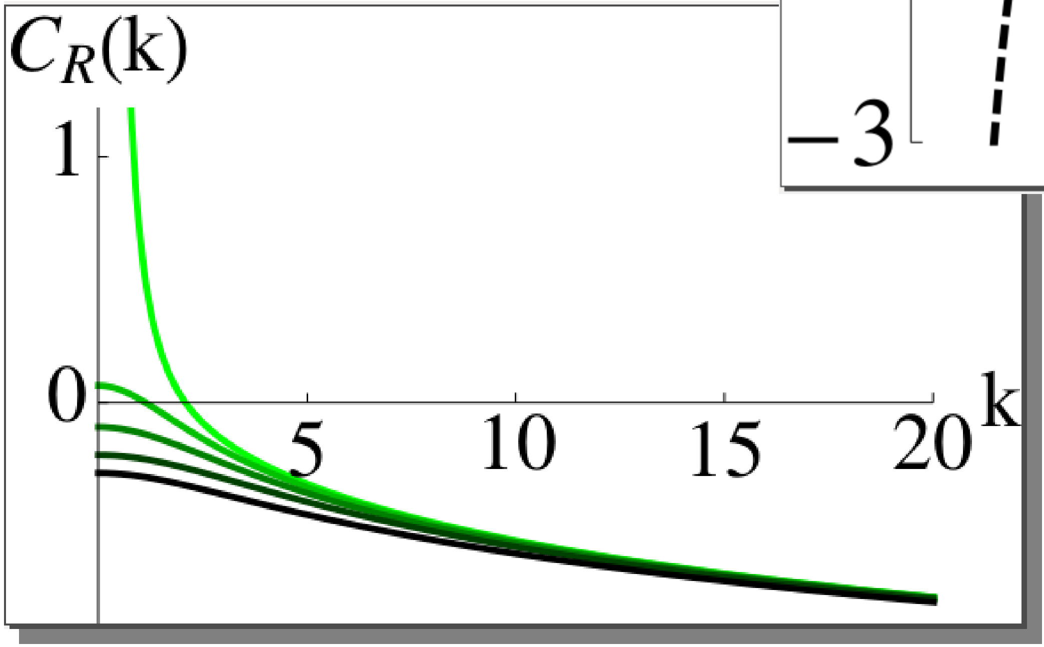
$$S_{\rm ren} = \frac{N^2}{4\pi^2} \int dk^4 \Big[- R^i_0 \tilde{R}^i_2 - R^i_0 \left(\frac{k^2}{4} + 3\eta_0^2 \right) R^i_0 \Big]$$

$$\Delta S_{\rm ren} = \int dx^4 \sqrt{\gamma}\, \left[\alpha\, F^{ij} F_{ij} + \beta\, \eta^2 R^i R_i\right]$$

Curing unphysical features 2

- UV & IR behaviors fix the finite counter-terms
- IR gapped
- UV conformality





C is the form factor

$\boxed{\texttt{C = 2-point corr. / k^2}}$

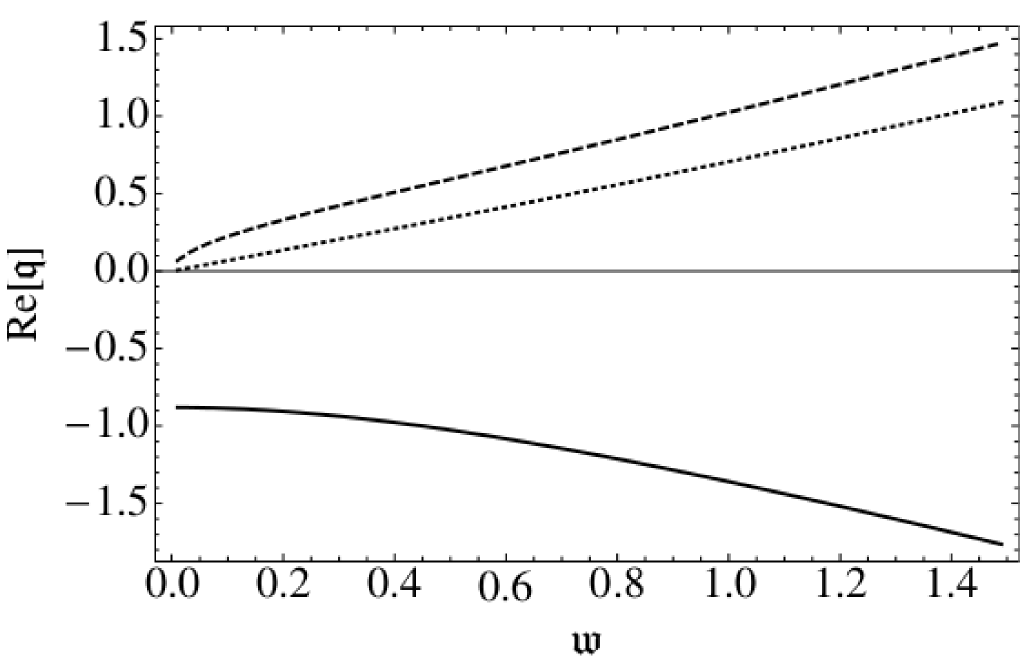
Optics and AdS/CFT

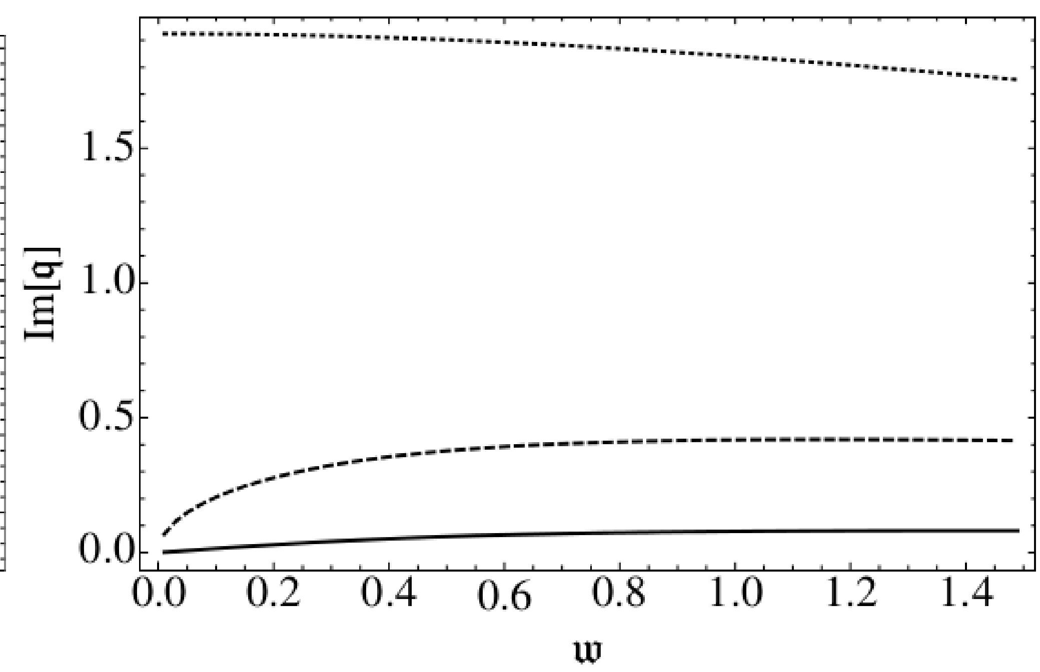
- **simplest holographic model (AdS-Schwarzschild ↔ stress-energy tensor + conserved current)**
- **current-current correlator → response of the medium to external e.m. sources**
- **polarizability of the medium**
- **refraction index**

[G.Policastro,A.Amariti,A.Mariotti,D.Forcella,
A.Mezzalira,DM,...]

Light-waves in a medium

- **wave equation (from Maxwell)**
- **dispersion relation of the various light-wave modes (not to be confused with the QNM!)**
- **systematic study of the optical properties (ALW, *negative refraction*,...)**





Finite counter-term

- **background: AdS-Schwarzschild**

$$\phi = \phi_0 + u\,\phi_1 + u\ln(u)\,\tilde{\phi}_1 + u^2\phi_2 + u^2\ln(u)\,\tilde{\phi}_2 + \ldots$$

$$S_{\rm reg} = \frac{(NT)^2}{16}\int_{u=\epsilon}\frac{d\omega d^3q}{(2\pi)^4}\;\phi_0\left[\phi_1 - \phi_0(\mathfrak{w}^2 - \mathfrak{q}^2)(1+\ln(\epsilon))\right]$$

$$S_{\rm c.t.} = -\frac{N^2}{16(2\pi)^2}\int_{u=\epsilon}\frac{d\omega d^3q}{(2\pi)^4}\;\sqrt{-\gamma}\;\frac{1}{2}(\ln(\epsilon)+\tilde{c})F_{ij}F^{ij}$$

$$G^{(c)}(\mathfrak{w},\mathfrak{q}) = -\frac{(NT)^2}{16}\;\left[\frac{\phi_1}{\phi_0} - c(\mathfrak{w}^2 - \mathfrak{q}^2)\right]$$

How to interpret it?

- **no symmetry requirement**
- **no "physical" requirement**

is it a parameter of the boundary model?

though not phenomenologically crucial... optical response (included the exotic features) is robust against the ambiguity fixing. One can take a minimal subtraction scheme.

Conclusion

- **finite counter-terms are ubiquitous and physically relevant in holography**
- **symmetry and physical soundness usually fix the ambiguities**
- **exceptional cases? Bottom-up hunger for UV data?**

THANKS !