HOLOGRAV 2014

HOLOGRAPHIC CHECKERBOARDS

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STAG RESEARCH CENTRE

FINITE CHARGE DENSITY

Interested in modelling charged phases of matter with 2 spatial dimensions. A minimal set of holographic ingredients:

$$S = \int d^4x \sqrt{-g}\left(R + 6 - \frac{1}{4}F^2\right)$$

Turn on chemical potential μ
1-parameter family of AdS-RN black brane solutions

at T=0 these solutions interpolate between UV AdS_4 and and IR $\mathrm{AdS}_2 \times \mathrm{R}^2$ solution.

Adding more ingredients (e.g. scalars) results in instabilities, which can be diagnosed using the AdS_2 factor in the IR. e.g. superfluids [Hartnoll et. al.] spatial modulation [Donos, Gauntlett]

At finite Tc they stabilise. This point marks a potential new branch of solutions emerging at that T, and a potential new contribution to the phase diagram.

MODULATED INSTABILITIES

Just like in condensed matter, there can be phases in holography where translational invariance is spontaneously broken. Holographic examples:

- Maxwell-axion probe brane construction [Bergman, Jokela, Lifschytz, Lippert]
- D=4 Bulk pseudo-scalar [Donos, Gauntlett]

$$S=\int d^4x\sqrt{-g}\left(R-\frac{1}{2}(\partial\phi)^2-\frac{\tau(\phi)}{4}F^2-V(\phi)\right)-\int\frac{\vartheta(\phi)}{2}F\wedge F.$$

one indicator is AdS_2-BF bound violations for finite k.

(will use this model)

- D=5 Helical [Domokos, Harvey][Nakamura,Ooguri,Park] whose black holes can be constructed in full using ODEs [Donos, Gauntlett] see also [Donos, Gauntlett, Pantelidou]

MODULATED INSTABILITIES

Spatial modulation can be added manually using boundary conditions. e.g. [Horowitz, Santos, Tong].

Can exploit design freedom to simplify/gain analytic control, e.g.
—5d helical [Donos, Hartnoll]
—4D Q-lattices / linear massless axions [Donos,Gauntlett], [Andrade,BW] **talks by Gouteraux, Gauntlett, Donos** (similar approach in mass. grav. [Vegh][Davison][Blake,Tong])

For the spontaneous case this freedom doesn't exist: PDEs.
Striped case now well studied.
But in D=4 there are more symmetries to break, here: checkerboards.

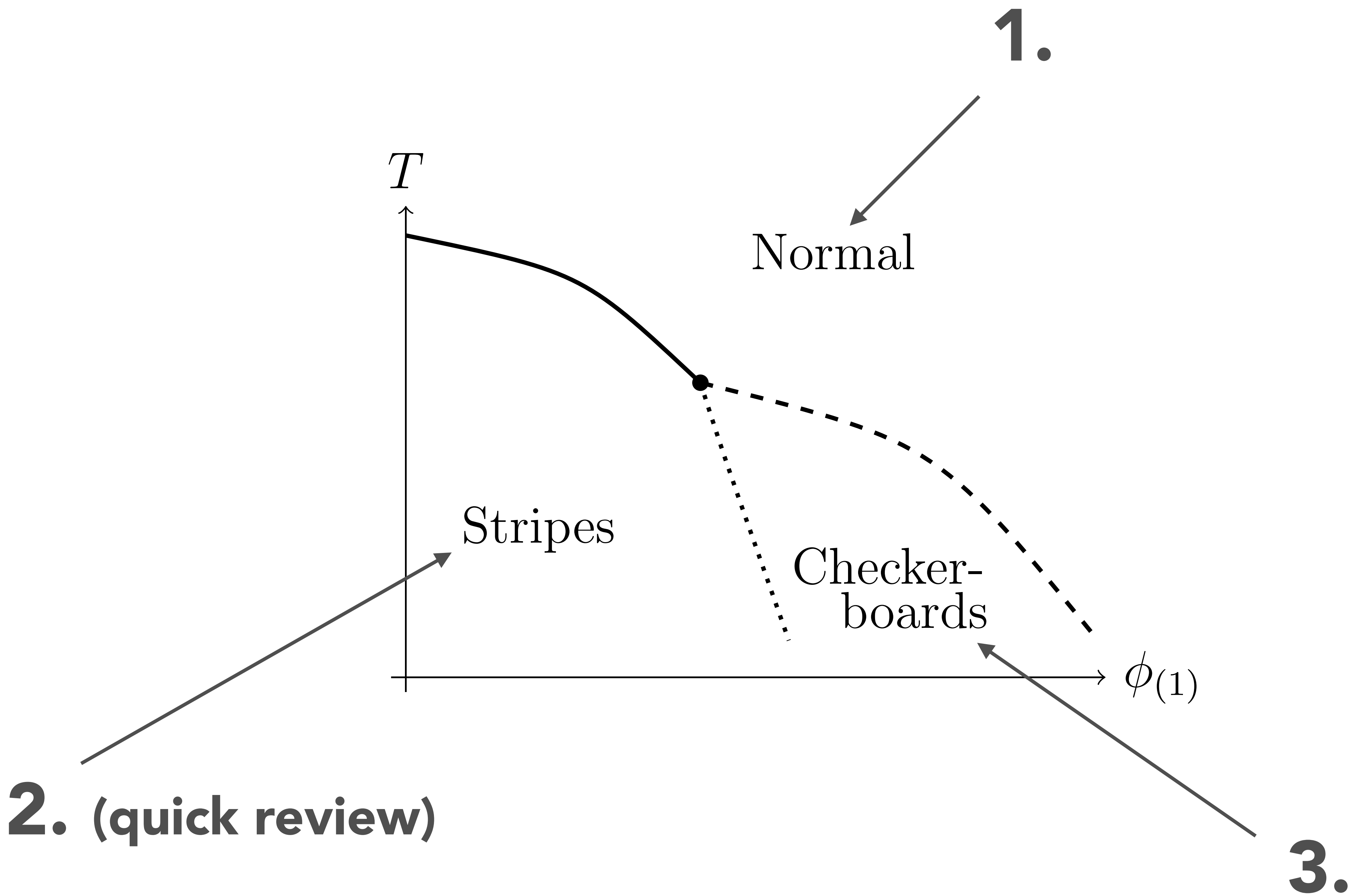
MOTIVATIONS

Possible that such instabilities are generic in holography at finite density at low enough temperatures. Can't (easily) control the outcome. Natural to seek phases with no surviving continuous spatial symmetries.

modulation observed — what is possible holographically?
— what is dominant?

They're a step towards asking what the ground states of these systems are.

PLAN OF THIS TALK



MODEL

4D Gravity + U(1) gauge field + pseudoscalar

$$S = \int d^4x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{\tau(\phi)}{4}F^2 - V(\phi)\right) - \int \frac{\vartheta(\phi)}{2}F \wedge F.$$

with functions

$$\tau(\phi) = \mathrm{sech}\frac{\sqrt{n}}{2\sqrt{3}}\phi, \quad V(\phi) = -6\cosh\frac{\phi}{\sqrt{3}}, \quad \vartheta(\phi) = \frac{c_1}{6\sqrt{2}}\tanh\sqrt{3}\phi$$

two model parameters n, c_1

Why these choices?

The case $n = 36, c_1 = 6\sqrt{2}$ arises in a consistent truncation from M-theory [Gauntlett et. al.]. This motivates the functional form, with the parameters n, c_1 making contact with a more general linear parameterisation [Donos, Gauntlett]

NORMAL PHASE

Look for electrically charged black branes.

ϕ has a mass $m^2 = -2$

$$\phi(z, x^\mu) = \phi^{(1)} z + \phi^{(2)}(x^\mu) z^2 + O(z)^3$$

for $\phi^{(1)} = 0$ with a chem. pot. normal phase is simply RN

but we'll also consider deforming by operator dual to ϕ, by turning on a spatially constant $\phi^{(1)}$

Constructing the normal phase is a standard ODE shooting problem enforcing horizon regularity and UV normalisablity with sources.

In particular for homogeneous ansatz:

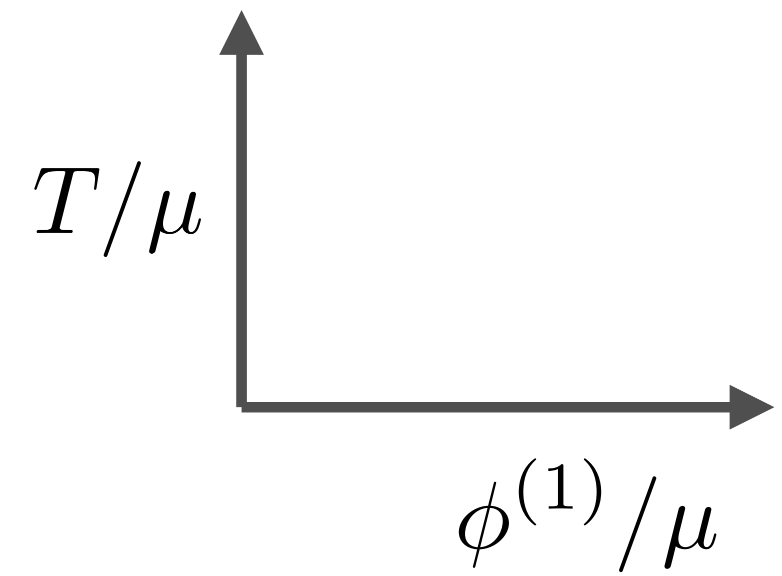
$$ds^2 = g_{MN} dx^M dx^N = \frac{1}{z^2}\left(-f(z)T(z)dt^2 + \frac{Z(z)}{f(z)}dz^2 + dx^2 + dy^2\right),$$

$$A(z, x^i) = a(z)dt, \qquad \phi(z, x^i) = \Phi(z)$$

counting data: black branes exist in two-parameter families.

NORMAL PHASE

natural parameter choices are:



- $\phi^{(1)}/\mu = 0$ corresponds to RN (IR AdS_2 factor at T=0)
- $\phi^{(1)}/\mu \neq 0$ solutions have IR HSV factor at T=0 in the cases studied

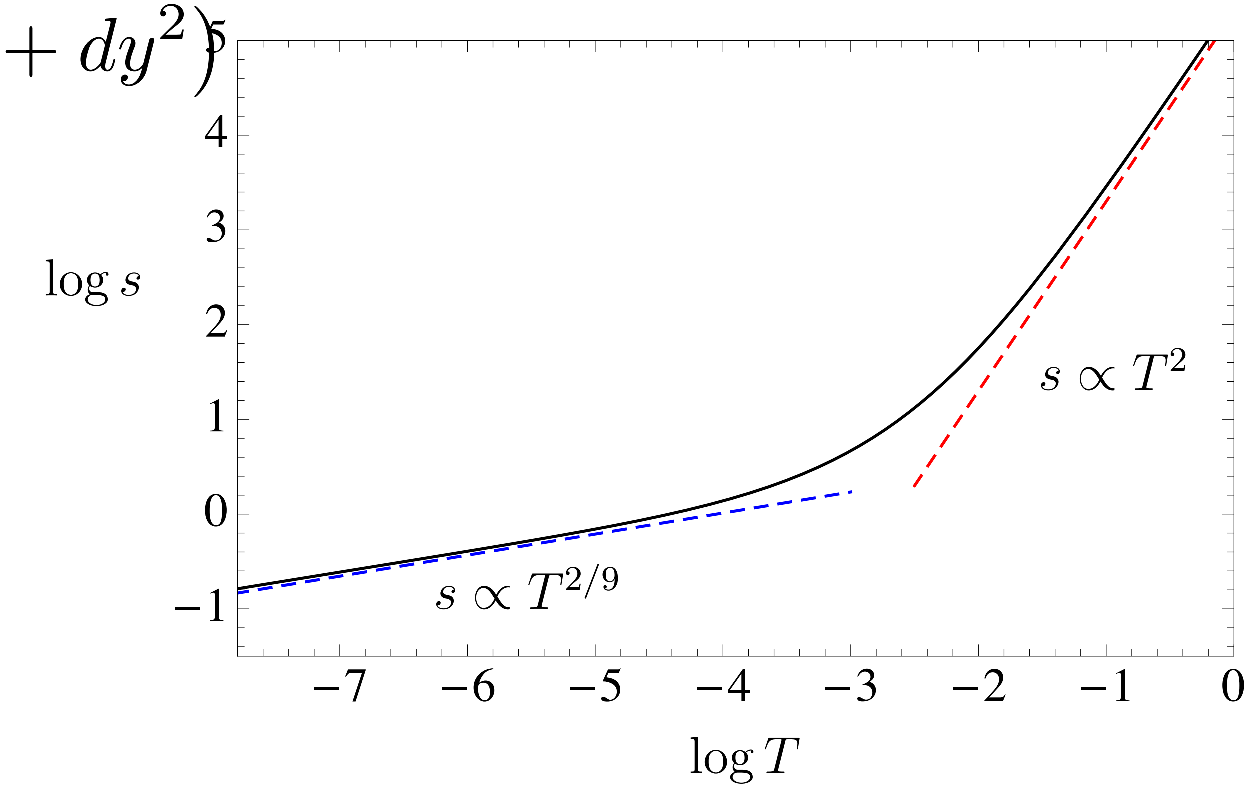
An example: n=0 model. Has HSV solutions:

$$ds^2 = -z^{11/2}dt^2 + \frac{11e^{\frac{\sigma}{\sqrt{3}}}}{4}\frac{dz^2}{z} + \sqrt{z}\left(dx^2 + dy^2\right)$$

$$\phi = \sqrt{3}\log z + \sigma \qquad A = 2\sqrt{\frac{5}{11}}z^{11/4}dt$$

expect e.g. entropy scaling

$$s \propto T^{2/9}$$



NORMAL PHASE - INSTABILITIES

about RN ($\phi = 0$)

$$\delta\phi = \lambda(z)\cos kx \quad \delta g_{ty} = h_{ty}(z)\sin kx \quad \delta A_y = a_y(z)\sin kx$$

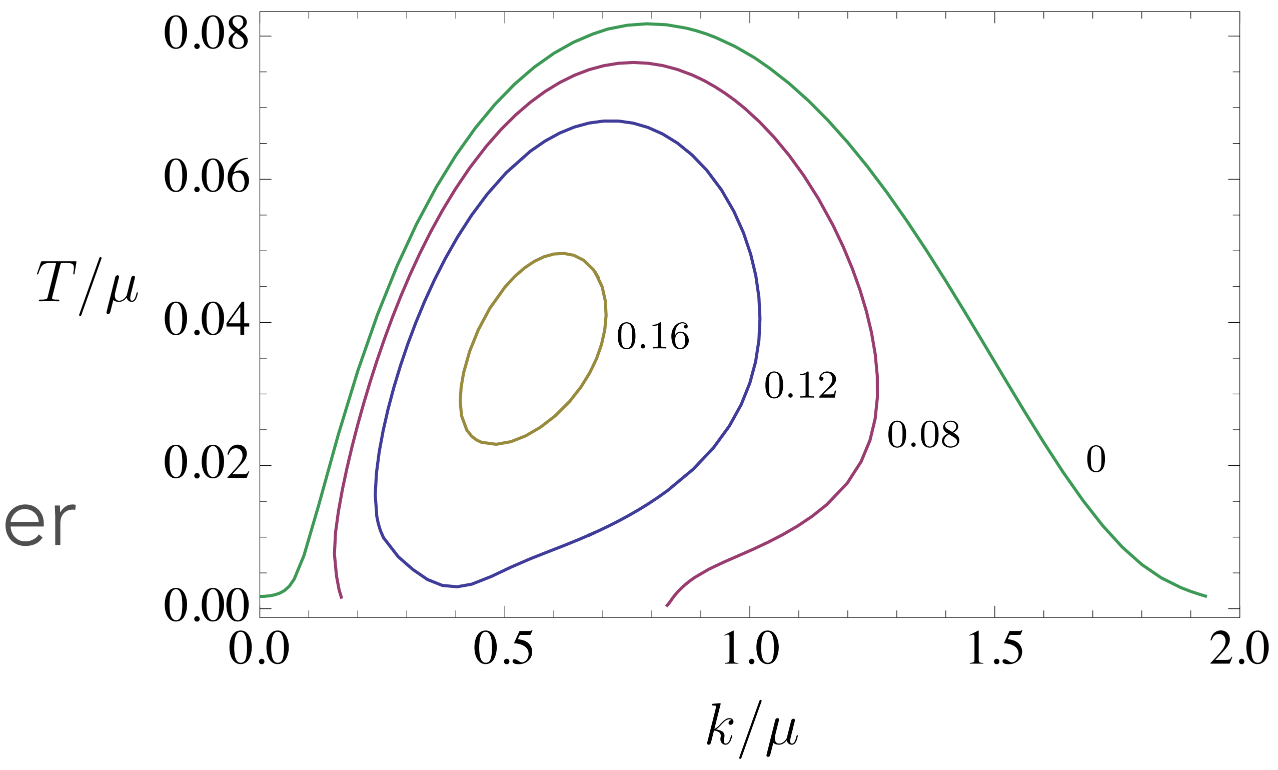
[Donos, Gauntlett]

with A_t modulated at higher orders in this expansion.

Has current in translationally invariant direction.

about the numerical $\phi \neq 0$ backgrounds a consistent set of fluctuations now include also:

$$\delta A_t(z,x) = a_t(z)\cos kx$$
$$\delta g_{ii}(z,x) = h_{ii}(z)\cos kx$$
$$\delta g_{zz}(z,x) = h_{zz}(z)\cos kx.$$

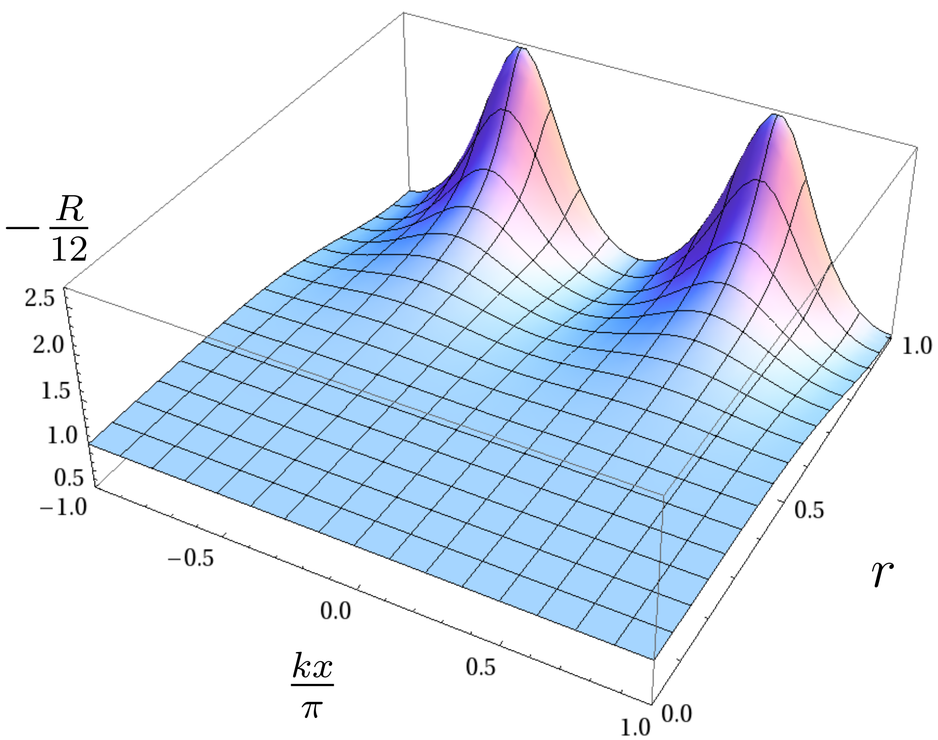


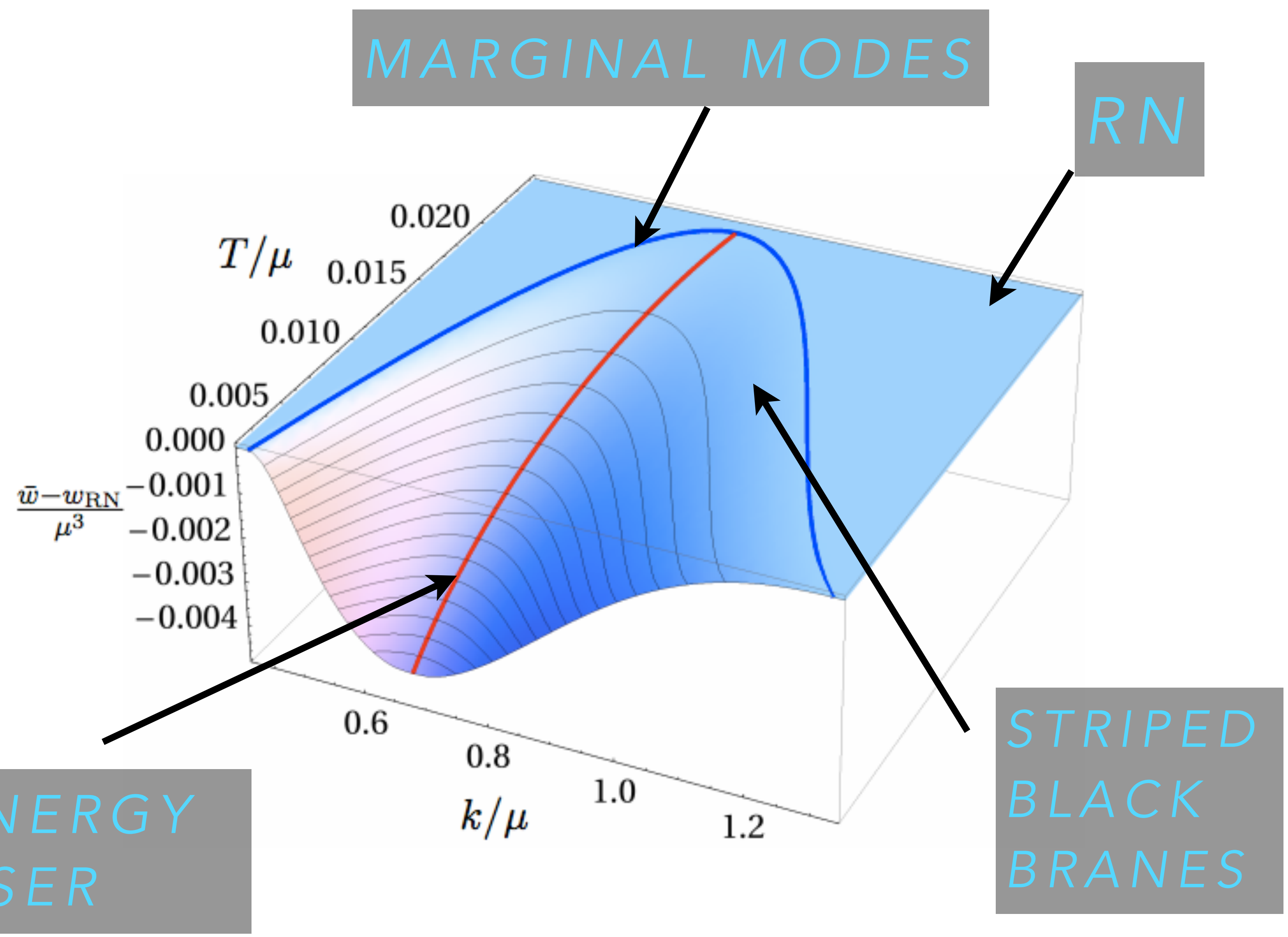
counting indicates two parameter families, e.g. k/μ $\phi^{(1)}/\mu$

STRIPES

The marginal modes indicate branches of spatially modulated black branes. e.g. A black brane branch which continuously connects to a single marginal mode with momentum k.

can construct by solving 2d PDEs [Rozali et. al][Donos][B.W.]

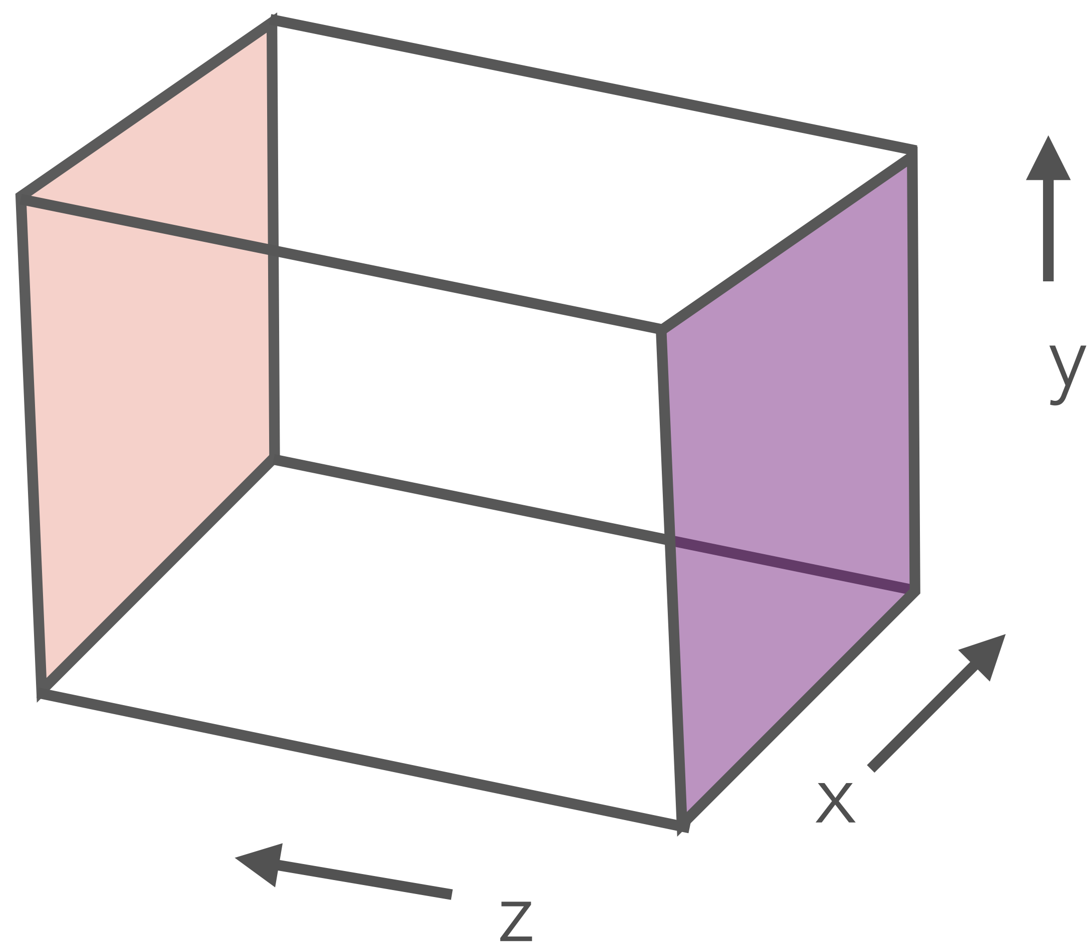




CHECKERBOARDS

stationary problem of Einstein equations in a box:

Regular horizon



UV Dirichlet conditions: normalisable + constant sources ($\phi^{(1)}, \mu$)

Periodic boundary conditions in x,y

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Use the `Harmonic Einstein' / DeTurck approach

[Headrick, Kitchen, Wiseman]

$$R_{MN} \to R^H_{MN} = R_{MN} - \nabla_{(M}\xi_{N)}$$
$$\xi^M = g^{NP}(\Gamma^M_{NP} - \tilde{\Gamma}^M_{NP})$$

in addition stop a U(1) gauge redundancy;

$$\nabla_M F^M_{\ N} \to \left(\nabla_M F^M_{\ N}\right)^H = \nabla_a F^M_{\ N} + \partial_N \psi$$
$$\psi = \tilde{g}^{MN}\tilde{\nabla}_M(A_N - \tilde{A}_N)$$

work with this (modified) system and checking that ξ^M, ψ vanish on any solution.

with this we have 15 fields, F_I, (all metric + gauge field components, plus the scalar) and equations $E_I(F_J) = 0$

Discritise and iteratively solve using Newton-Raphson method.

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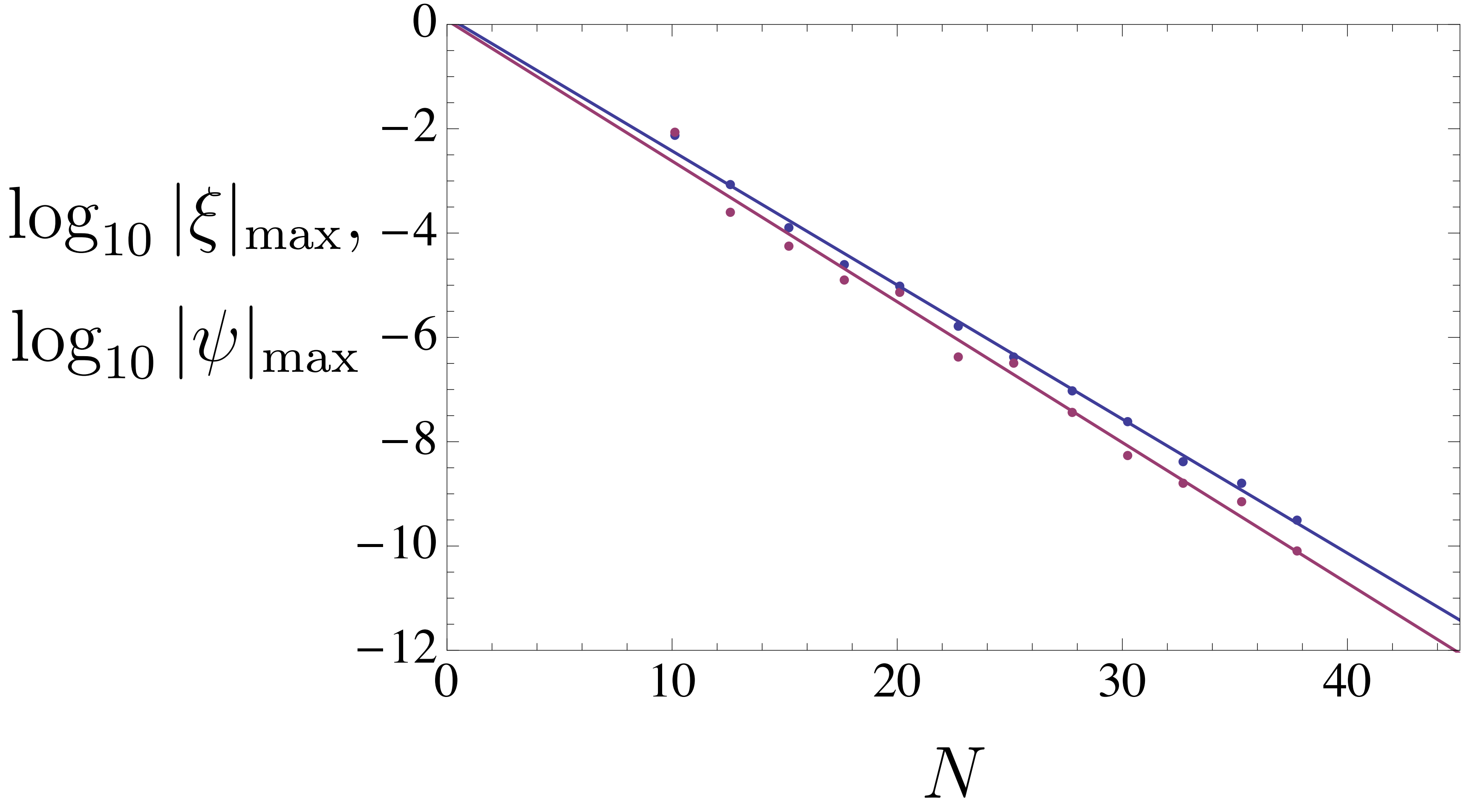
numerical convergence of gauge variables:

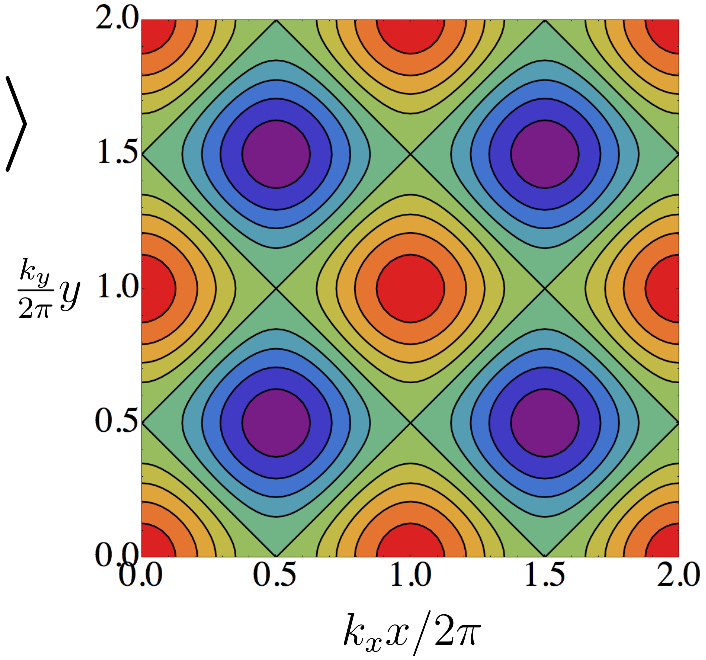


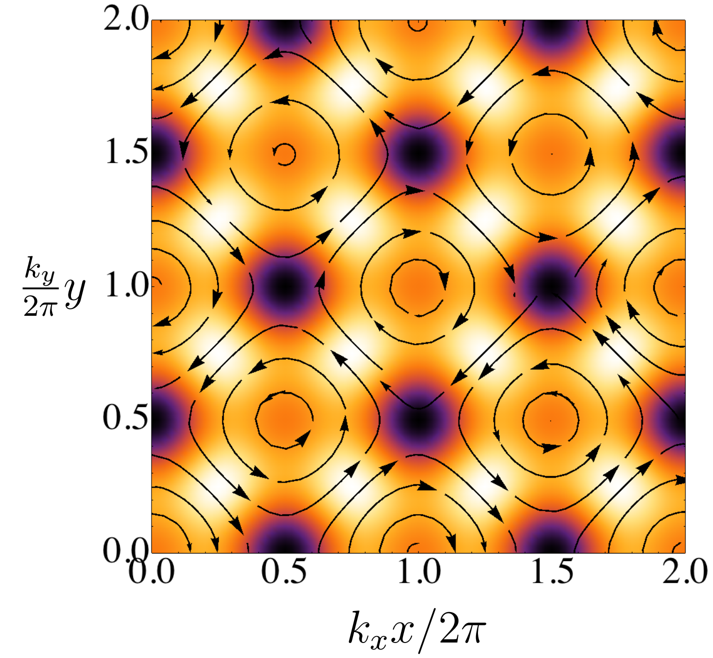
CHECKERBOARDS

black brane branch which continuously connects to two marginal modes with momentum k_x k_y

$k_x = k_y$ case (40^3, spectral, T=0.55Tc, $\phi^{(1)}/\mu = 0$)

$\langle O_\phi \rangle$

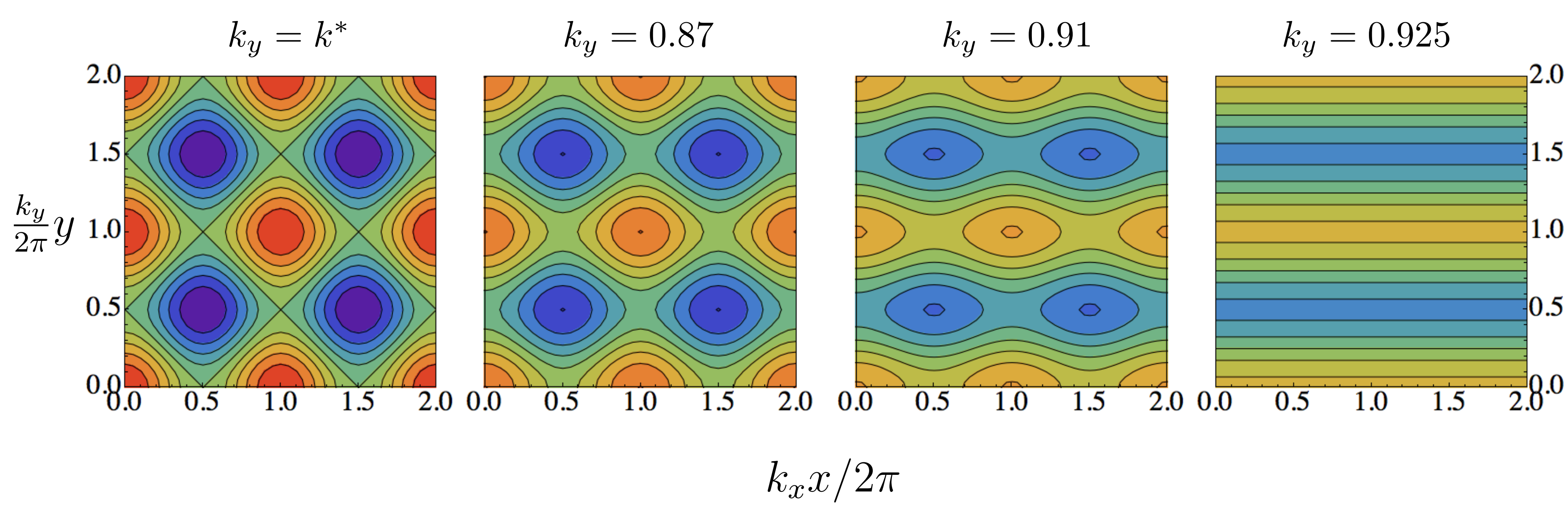




$\langle J_t \rangle$

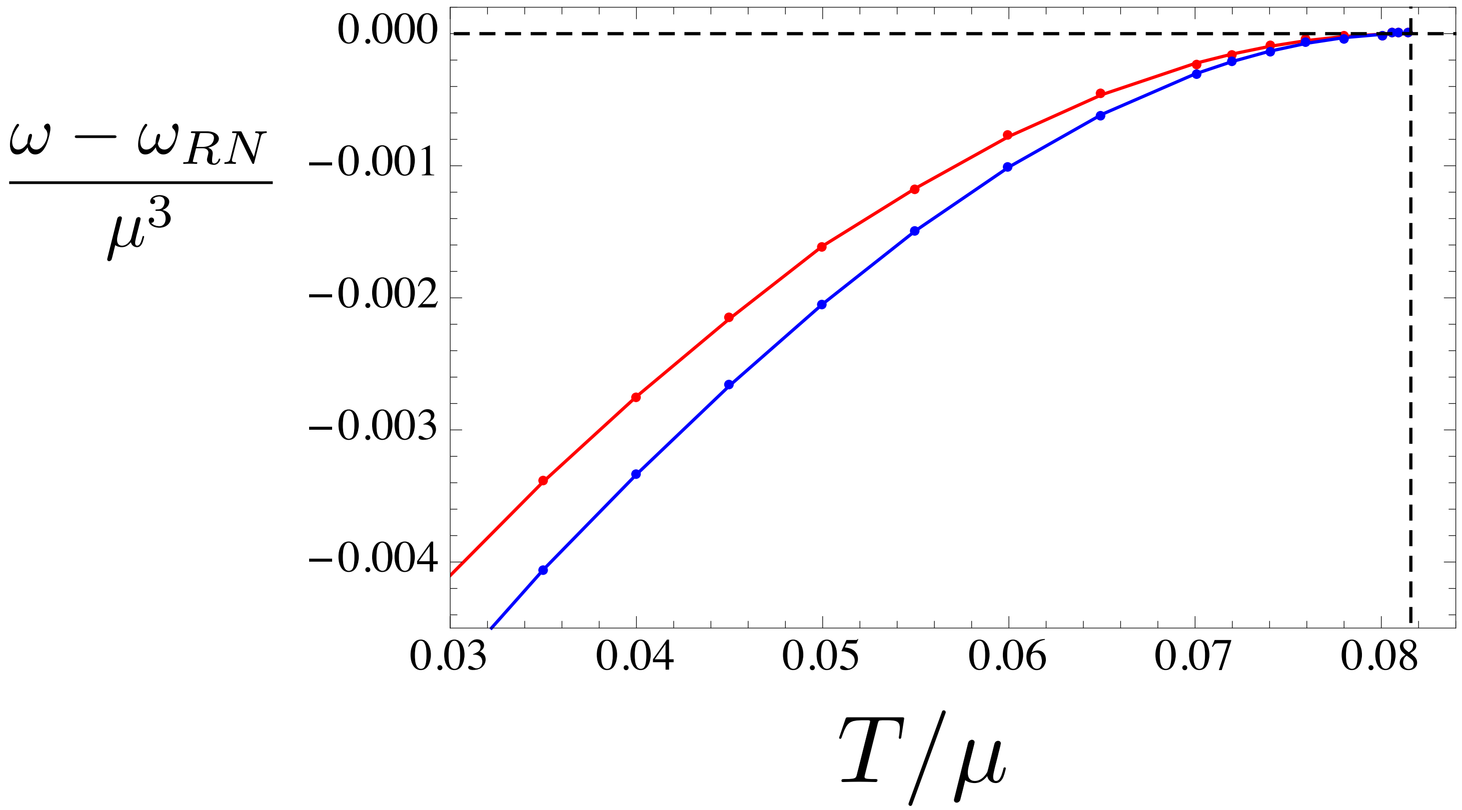
integral curves of $\langle J_i \rangle$

a large space of solutions, e.g.

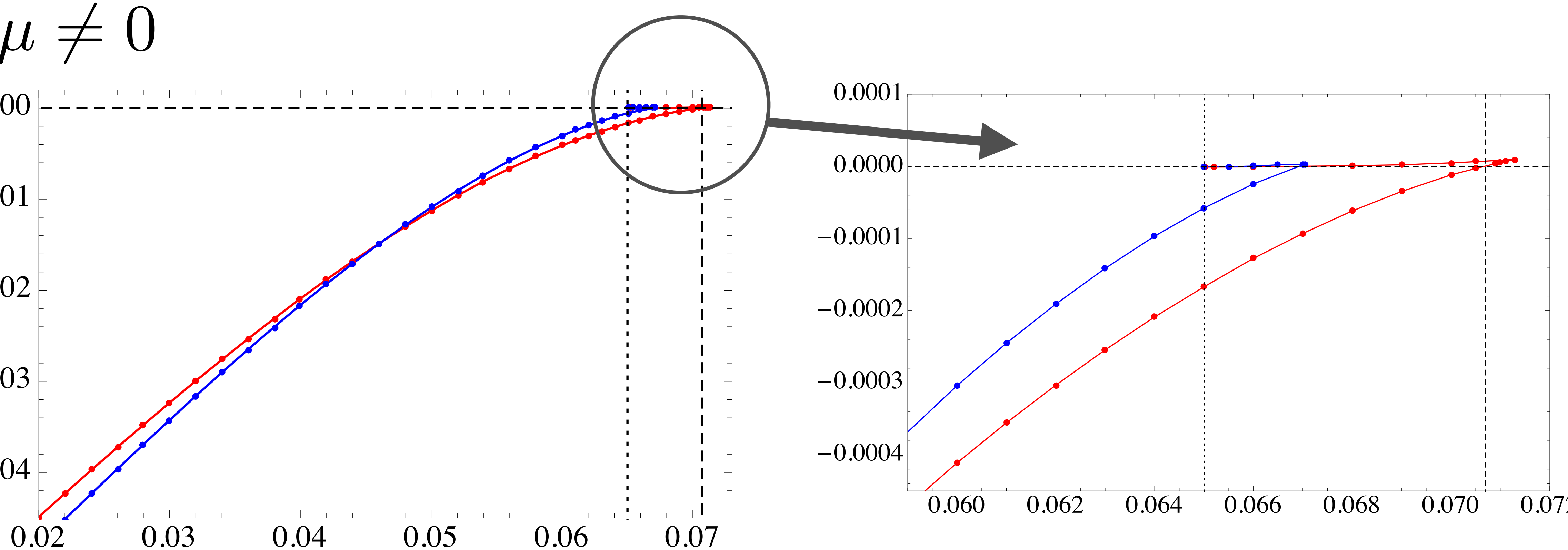


minimise free energy w.r.t. $k_x = k_y$ at each T

$\phi^{(1)}/\mu = 0$

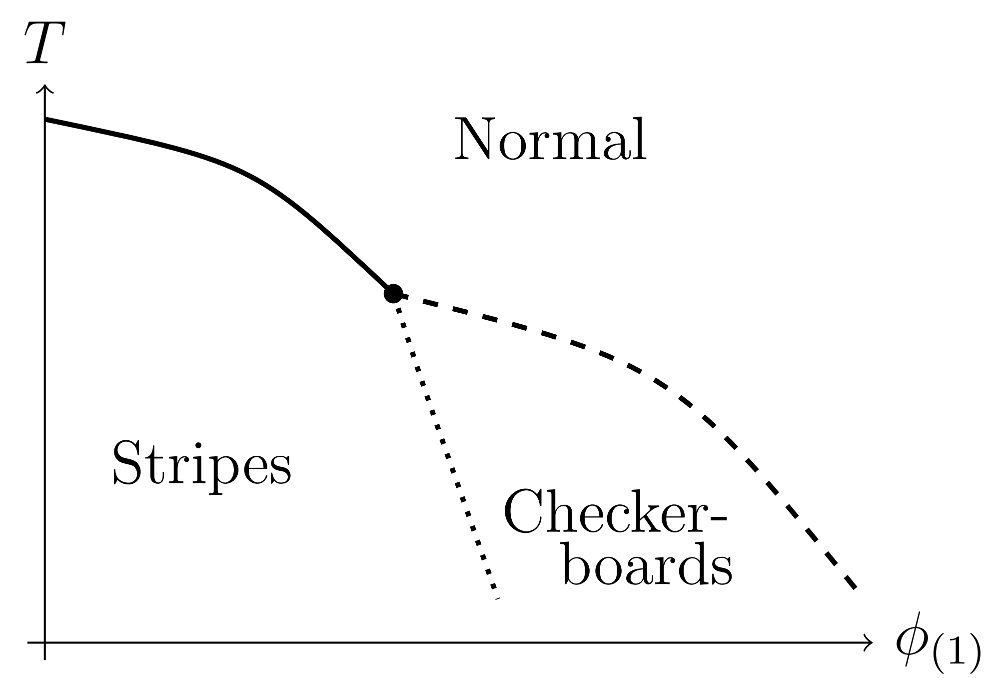


$\phi^{(1)}/\mu \neq 0$



SUMMARY

- Spontaneously modulated phases appear in holography, natural to seek those which break all continuous spatial symmetries.
- Constructed stationary, cohomogeneity-three black holes describing checkerboard phases
- Stripe-to-checkerboard first order phase transitions. Schematic phase diagram:



- Deformations extending known modulated instabilities to black holes without AdS_2 factors in the IR at T=0.
- Questions:
  zero temperature. triangular lattices. lattice symmetry breaking

THANK YOU