Far-from-equilibrium energy flow in quantum critical systems

Koenraad Schalm

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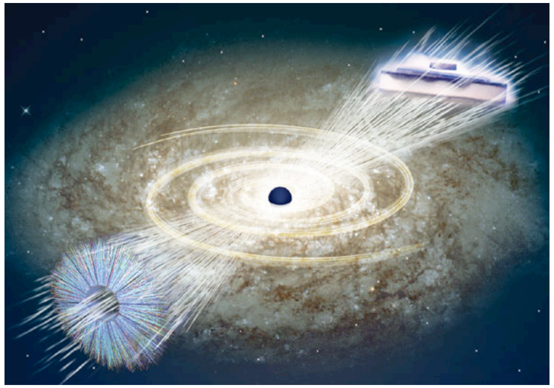


M.J. Bhaseen, B. Doyon, A. Lucas

Holograv, Reykjavik
Aug 2014

Netherlands Organisation for Scientific Research

Far-from-equilibrium energy flow in quantum critical systems

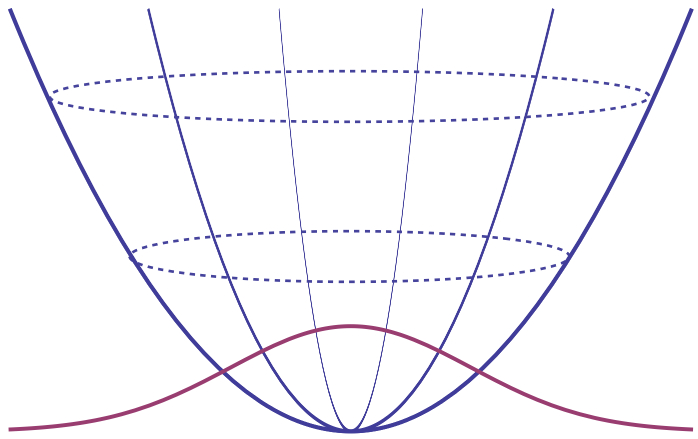
Joe Bhaseen

Benjamin Doyon

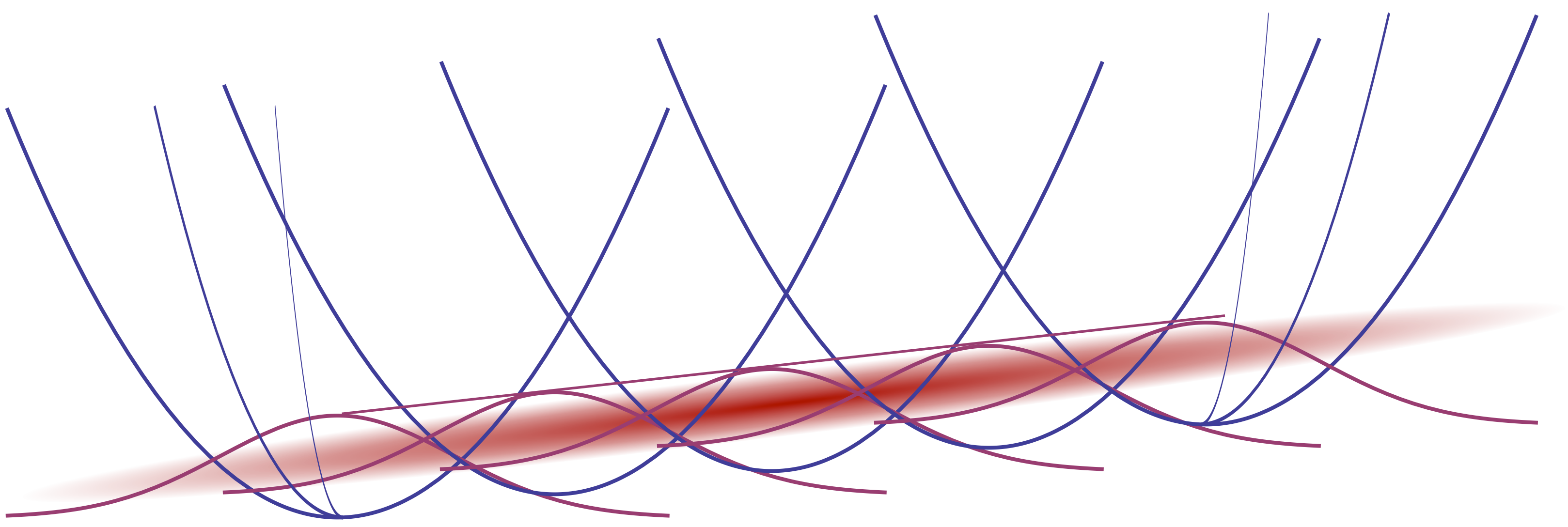
Andrew Lucas

- Atomic Bose Einstein Condensates

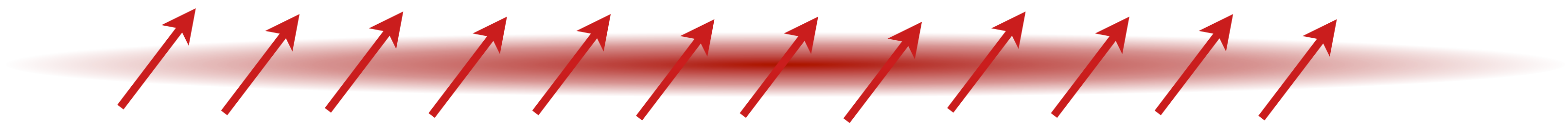
Cornell, Wieman, Ketterle



- Atomic Bose Einstein Condensates
  - Many technical breakthroughs since then

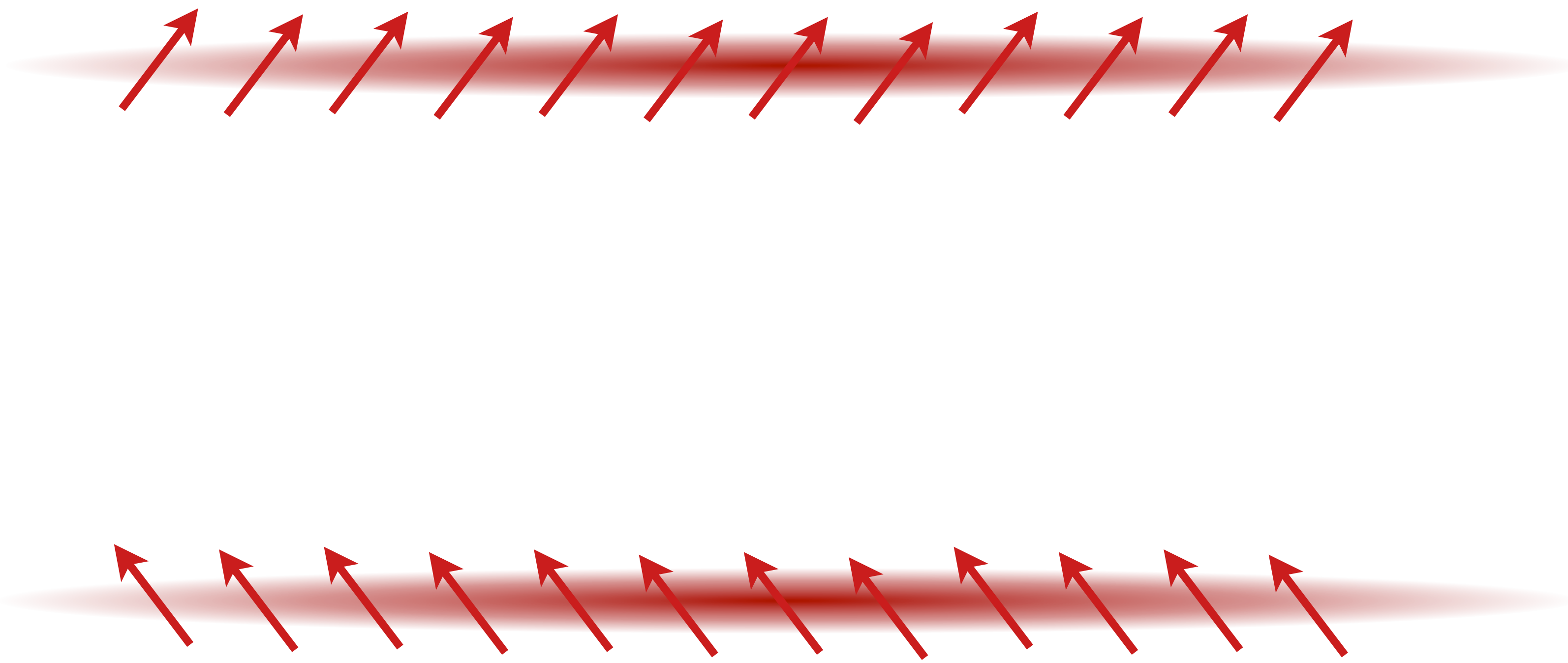


- BEC: spontaneously broken U(1)



- Phase quenches in a BEC

Dalla Torre, Demler, Polkovnikov
Betz et al, Schmiedmayer (Vienna)
...



$$S_0 = \int dxdt \left[-(\partial\phi_1)^2 - (\partial\phi_2)^2 \right]$$

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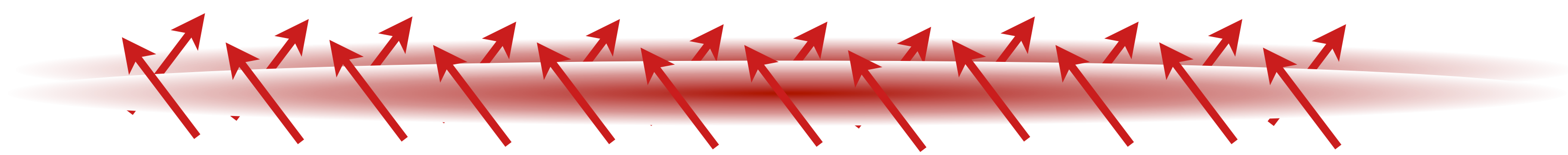

quench

$$S_{int} = \int dxdt \left[-(\partial\phi_1)^2 - (\partial\phi_2)^2 - g(t)\cos(\phi_1 - \phi_2) \right] \qquad g(t) = \theta(t)$$

- Phase quenches in a BEC

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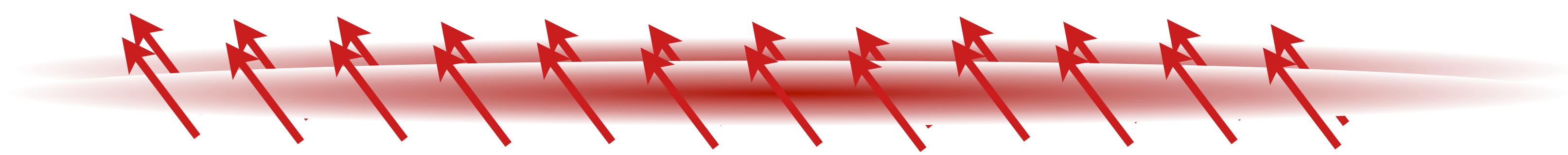

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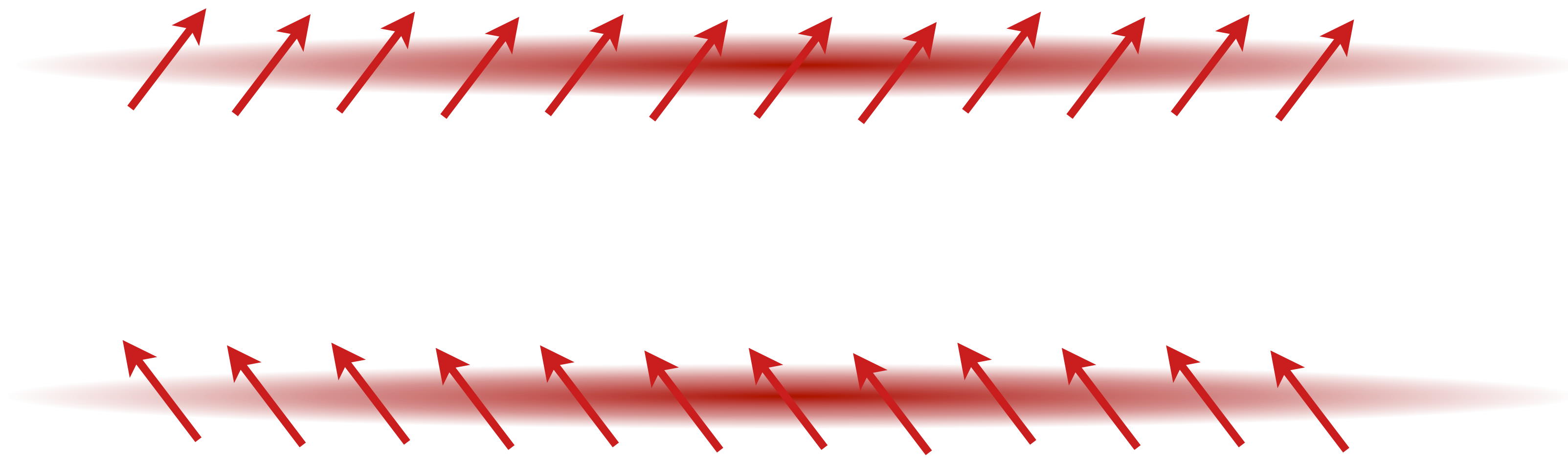
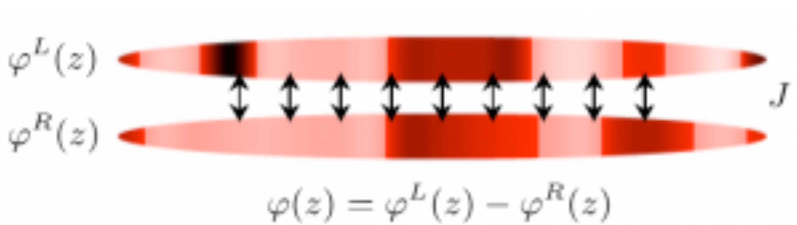
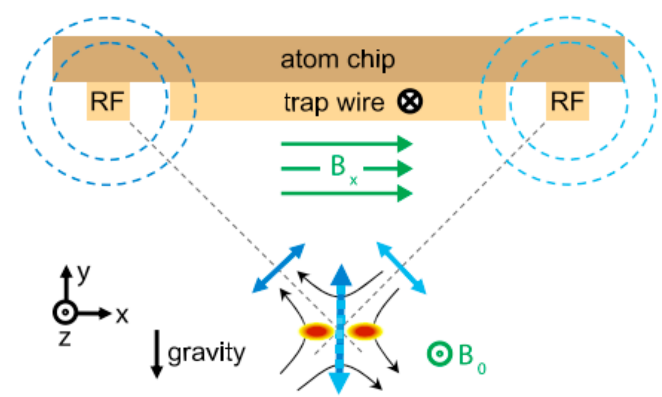
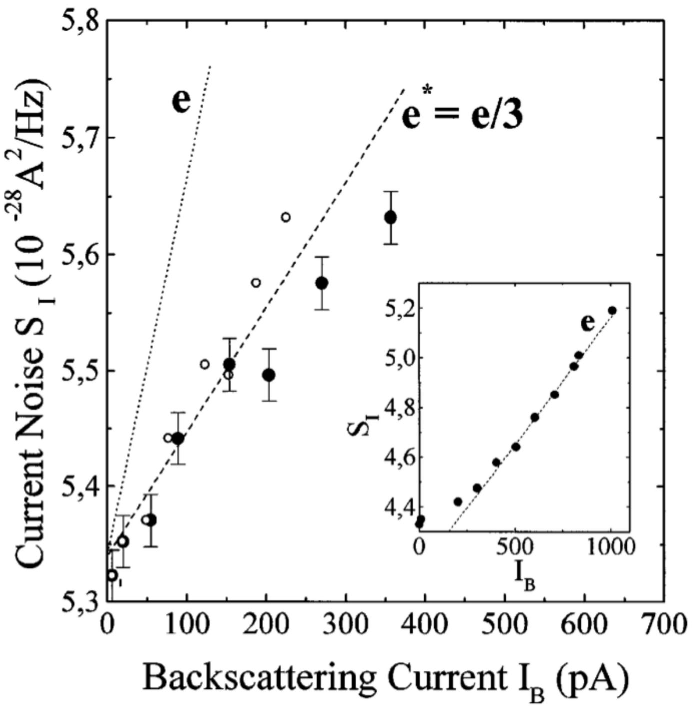


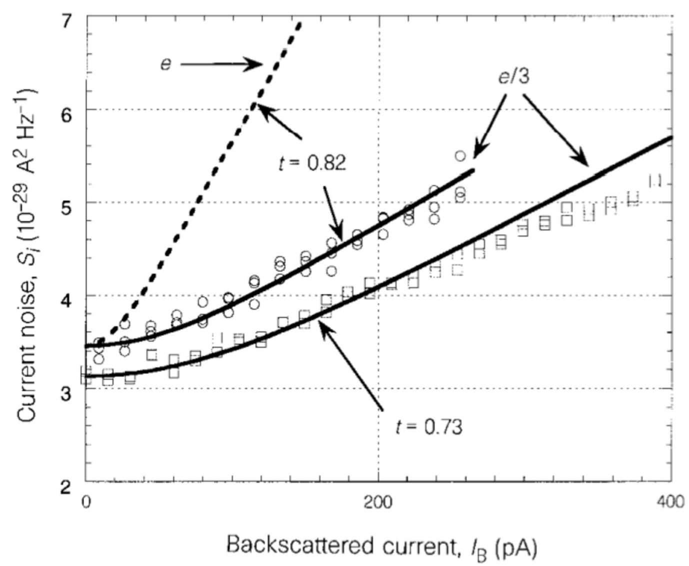




- A new window on (quantum) non-equilibrium physics

- Fractional Charge determination from noise





Saminadayar et al '97 de Picciotto et al '97

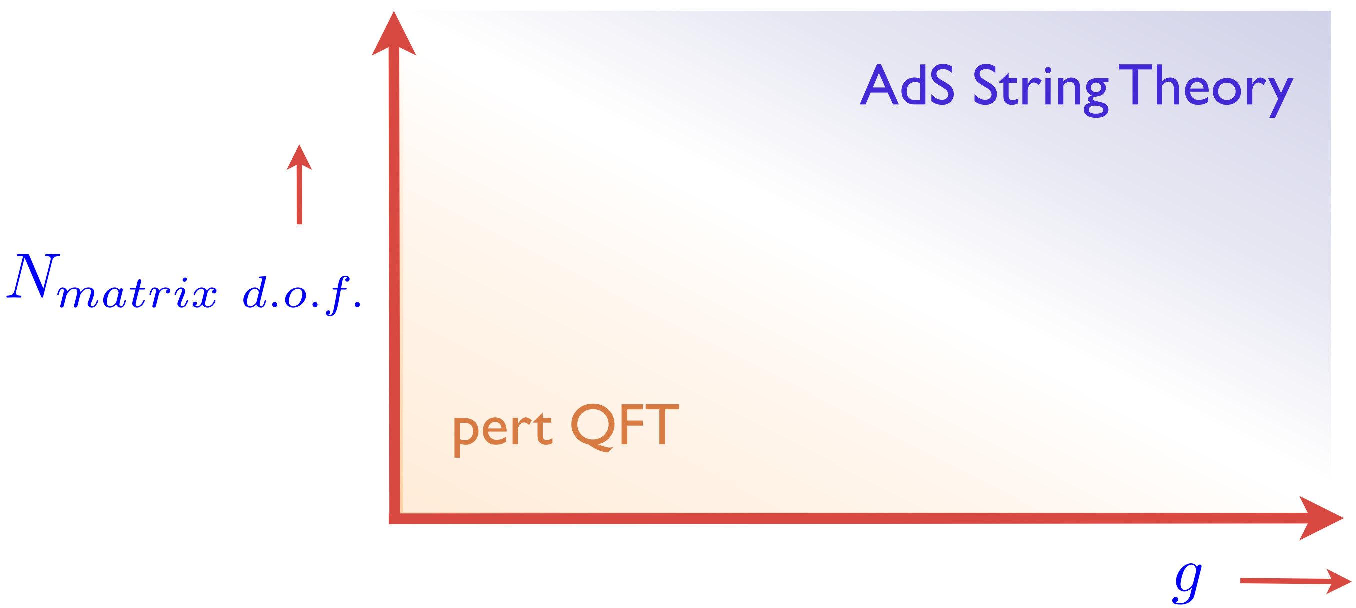
- A new window on (quantum) non-equilibrium physics

- The theory of quantum non-equilibrium physics...

- The AdS/CFT correspondence

Maldacena; Witten;
Gubser, Klebanov, Polyakov

$$Z_{CFT}(J;g,N) = \exp iS_{AdS}^{\mathrm{on-shell}}(\phi(\phi_{\partial AdS} = J))$$



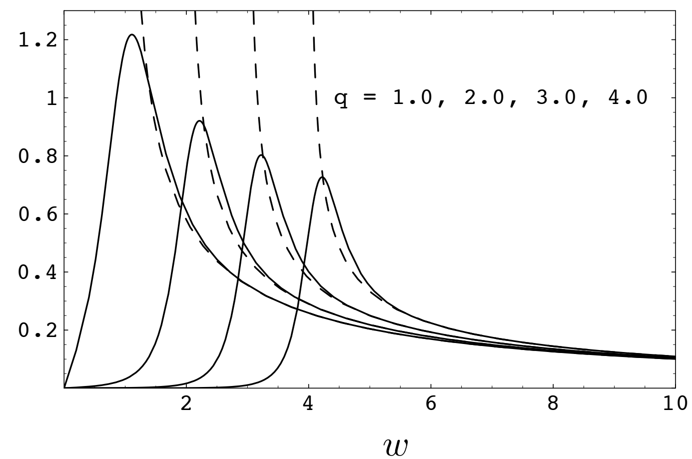
Use AdS/CFT as a tool to generate strongly coupled critical theories

Real time responses

- A remarkable ability of AdS/CFT
  - Direct crossover to hydrodynamics

Son, Starinets
Herzog, Son
Herzog, Kovtun, Sachdev, Son

$$\frac{1}{q^2}\mathrm{Im}\langle J^0 J^0\rangle_R$$



$$q = 3k/4\pi T$$

$$w = 3\omega/4\pi T$$

Real time responses

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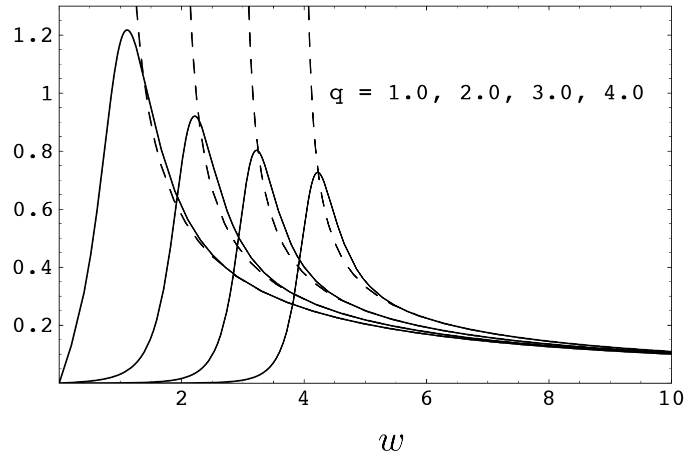

$$q = 3k/4\pi T$$
$$w = 3\omega/4\pi T$$

- *Real-time dynamics:*

Combine analytical with numerical data

Real time responses

- A remarkable ability of AdS/CFT
  - Direct crossover to hydrodynamics

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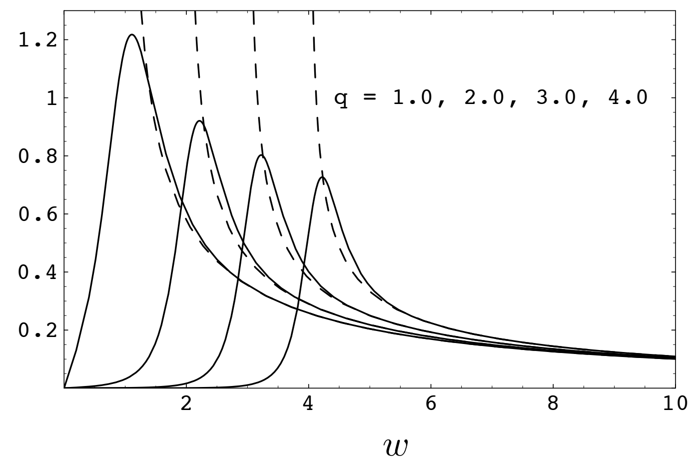
$$\frac{1}{q^2}\mathrm{Im}\langle J^0 J^0\rangle_R$$



$$q = 3k/4\pi T$$
$$w = 3\omega/4\pi T$$

- *Real-time dynamics*
- *Full non-equilibrium and transition to hydro*

beyond linear response

Real time responses

- A remarkable ability of AdS/CFT
  - Direct crossover to hydrodynamics

Son, Starinets
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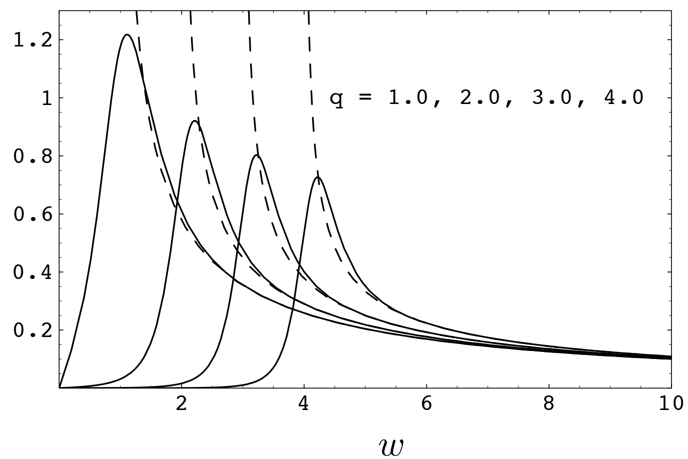
$$\frac{1}{q^2}\mathrm{Im}\langle J^0 J^0\rangle_R$$



$$q = 3k/4\pi T$$
$$w = 3\omega/4\pi T$$

- *Real-time dynamics*
- *Full non-equilibrium and transition to hydro*
- *Strongly coupled systems, especially critical theories*

Real time responses

- A remarkable ability of AdS/CFT
  - Direct crossover to hydrodynamics

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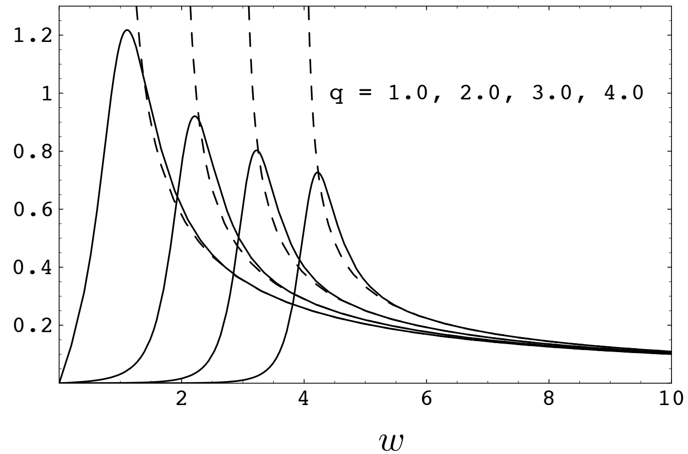


$$q = 3k/4\pi T$$
$$w = 3\omega/4\pi T$$

- *Real-time dynamics*
- *Full non-equilibrium and transition to hydro*
- *Strongly coupled systems, especially critical theories*

New organizing principles out of equilibrium

- Driven Steady State?
  - Non-thermal distributions

- Universality in Non-equilibrium dynamics?
  - Kibble-Zurek scaling
  - Kolmogorov scaling

Chesler, Yaffe; de Boer, Kesko-Vakkuri +9; Bhaseen, Gauntlett, Simons, Sonner, Wiseman; Basu, Das, Nishioka Takanayagi; Albash, Johnson; Abajo-Arrastia, Aparicio, Lopez; Ebrahim, Headrick; Bhattacharyya, Minwalla;.... Buchel, Lehner, Myers, van Niekerk; Das, Galante, Myers, ...

Motivation: unique ability of holography

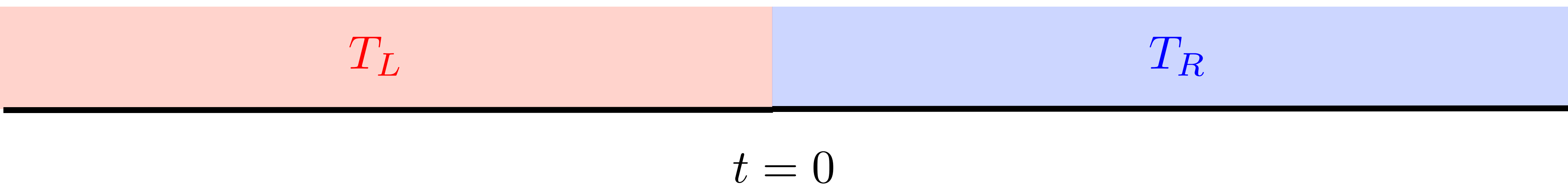
Actual: Combination of holography, hydrodynamics and QFT

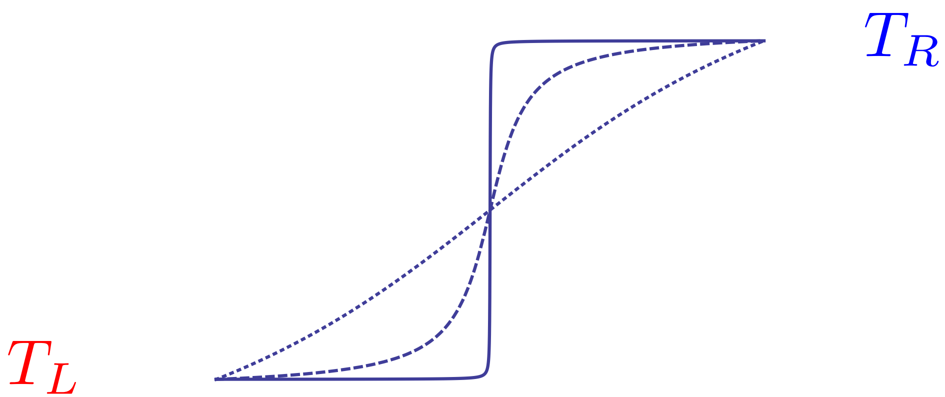
- Thermal Quench in 1+1 CFTs

Bernard, Doyon

- Thermal Quench in 1+1 CFTs

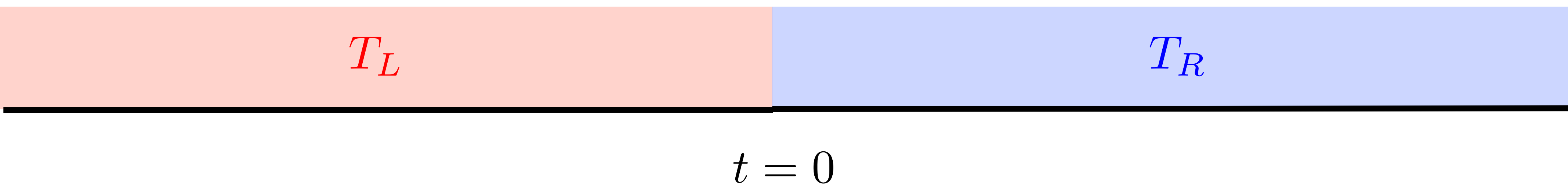
Bernard, Doyon



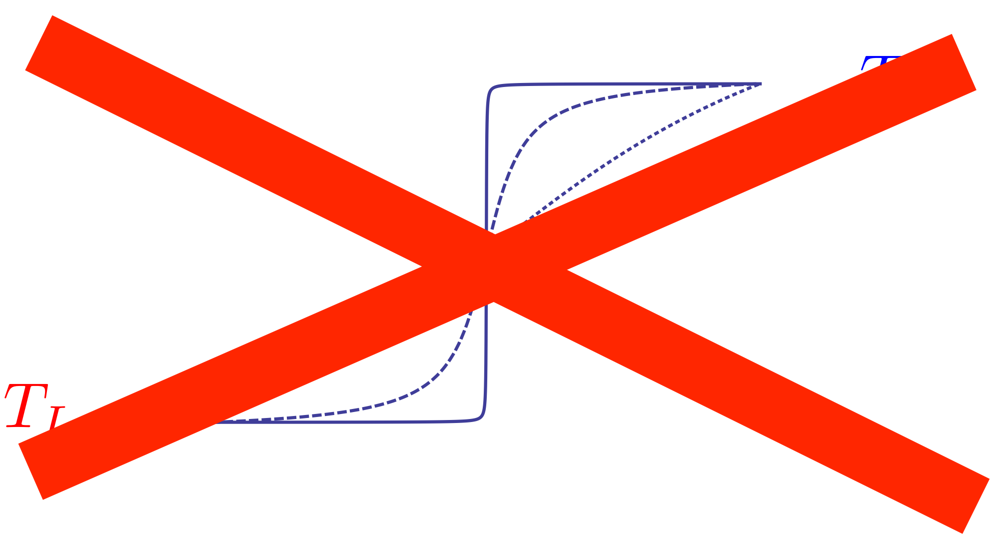


- Thermal Quench in 1+1 CFTs

Bernard, Doyon

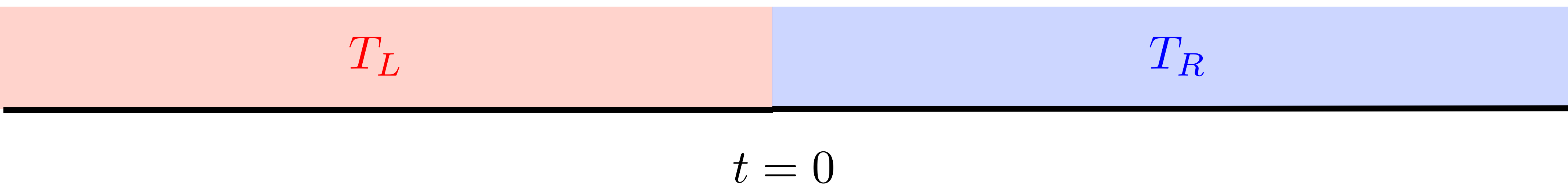


- Intuitive expectation

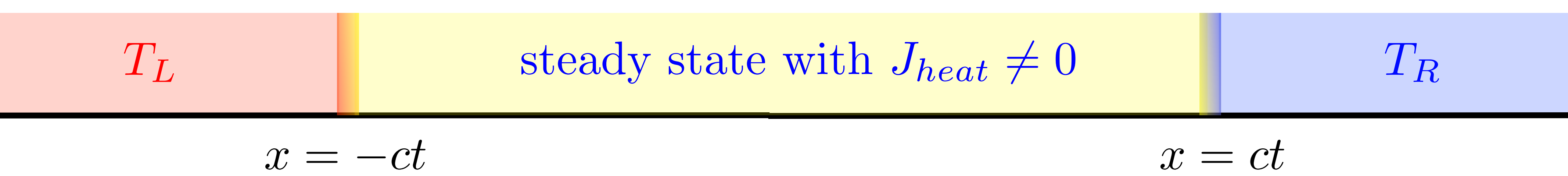


- Thermal Quench in 1+1 CFTs

Bernard, Doyon

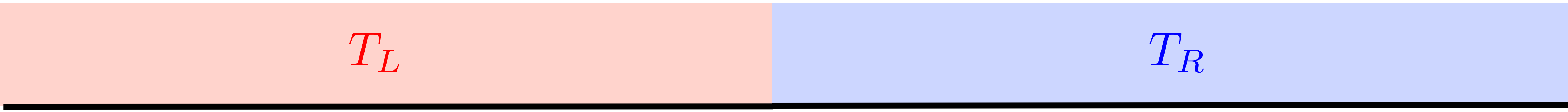


- Instead

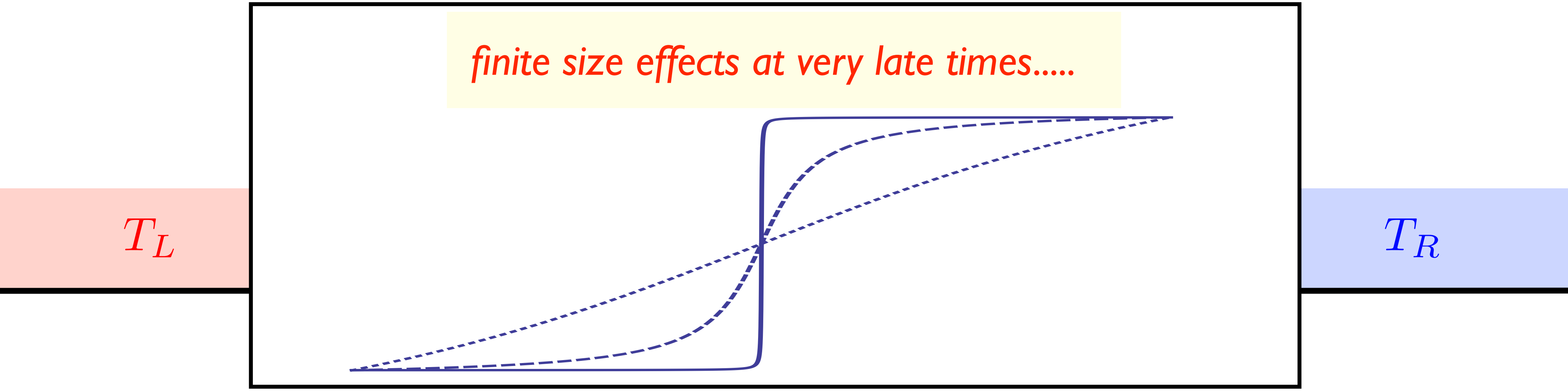


- Thermal Quench in 1+1 CFTs

Bernard, Doyon

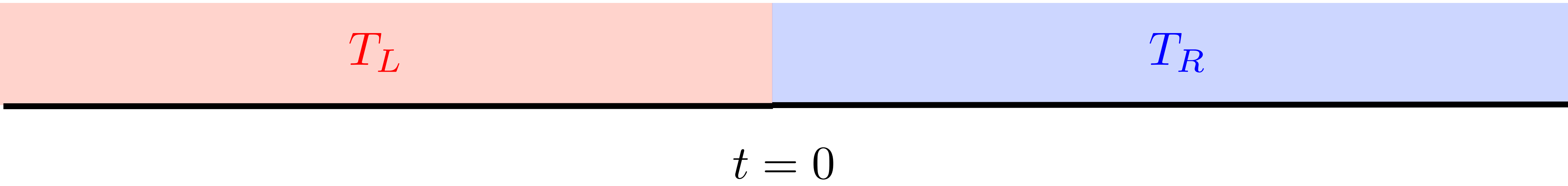


$t = 0$



- Thermal Quench in 1+1 CFTs

Bernard, Doyon



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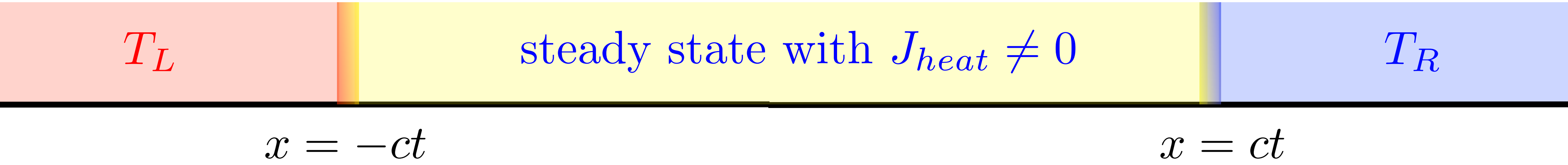


- Thermal Quench in 1+1 CFTs

Bernard, Doyon

T_L | steady state with $J_{heat} \neq 0$ | T_R

$x = -ct$ $\qquad$ $x = ct$

$$\langle J \rangle = \frac{c\pi}{12}(T_L^2 - T_R^2)$$

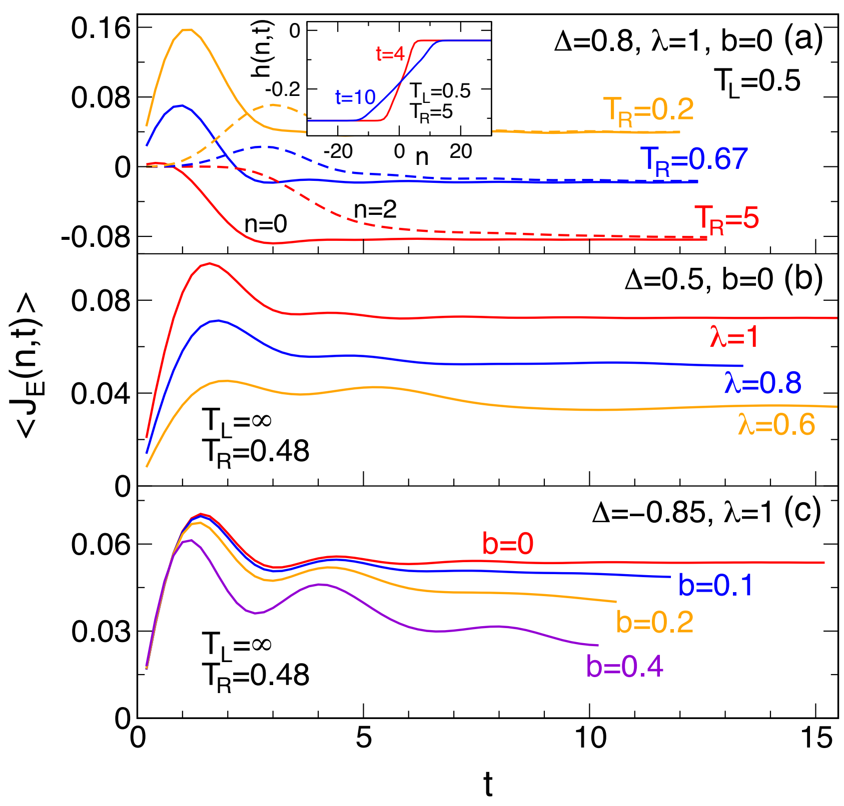
- Call $\langle J \rangle = J(\beta_L, \beta_R)$ with $\beta_L = 1/T_L,\ \beta_R = 1/T_R$

$$\langle J^{n+1} \rangle = \frac{d^n}{d\mu^n}\ J(\beta_L - \mu, \beta_R + \mu)\Big|_{\mu=0}$$

- What is this steady state?
  - Not an obvious driven state

Karrasch, Ilan, Moore

- Time dependent DMRG (density matrix renormalization group)
  - XXZ Hamiltonian

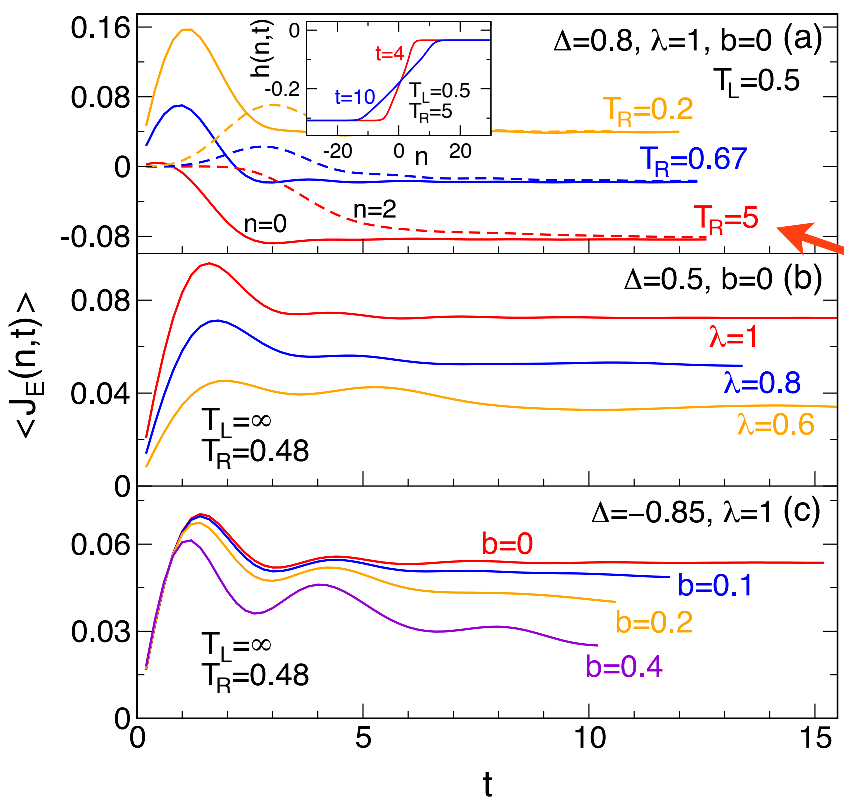


$$h_n = J_n\left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta_n S_n^z S_{n+1}^z\right) + b_n\left(S_n^z - S_{n+1}^z\right)$$

$$J_n = \begin{cases} 1 & n \text{ odd} \\ \lambda & n \text{ even} \end{cases} , \quad \Delta_n = \Delta , \quad b_n = \frac{(-1)^n b}{2}$$

Karrasch, Ilan, Moore

- Time dependent DMRG
  - XXZ Hamiltonian



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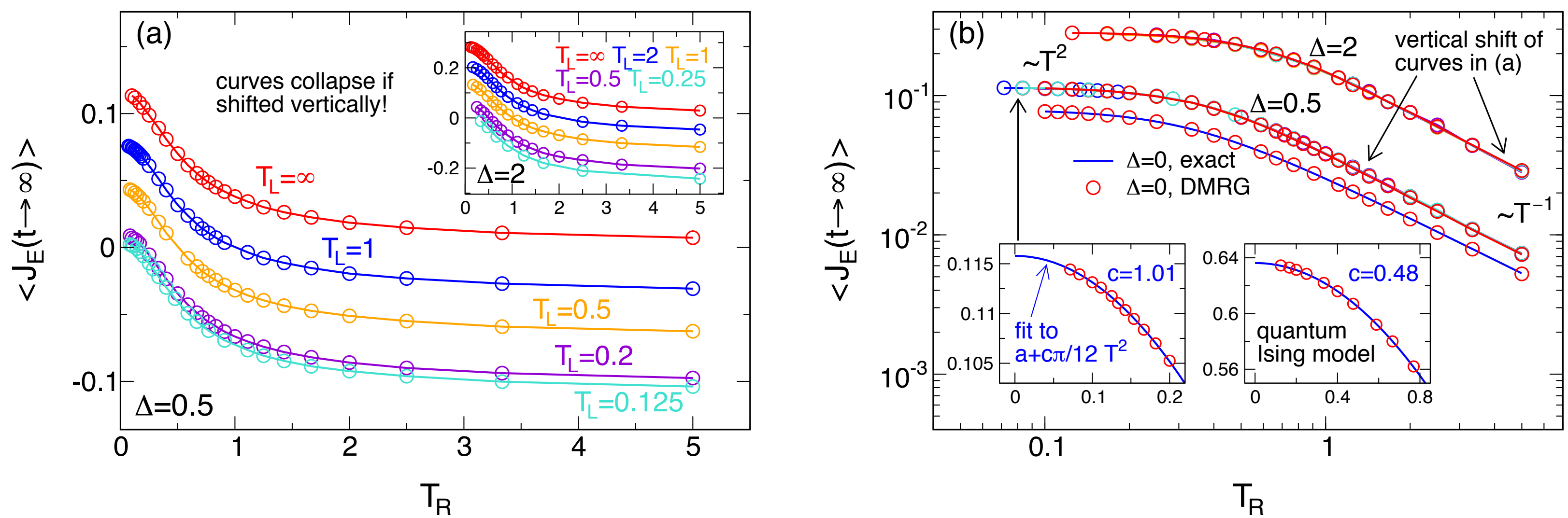
homogeneous constant heat flow

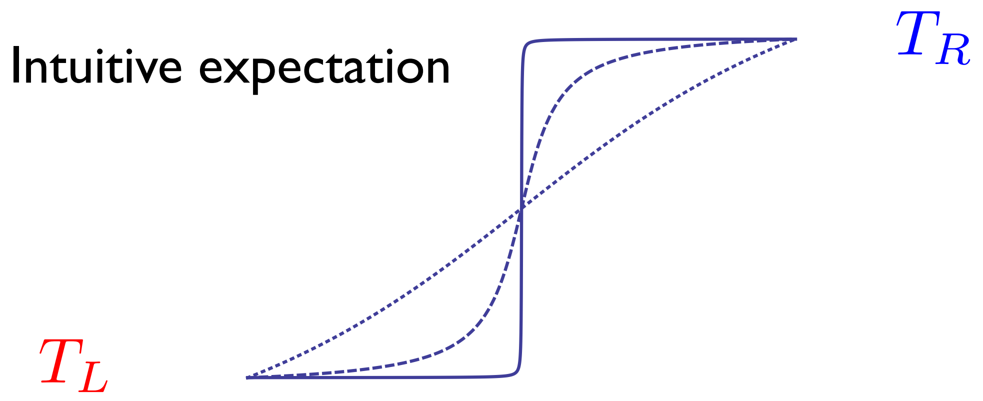
Karrasch, Ilan, Moore

- Time dependent DMRG
  - XXZ Hamiltonian



$$\langle J\rangle \sim f_L(T_L) + f(T_R)$$

- How to understand this state?
  - Constant Heat flow vs Temperature relaxation



$$T^{00} = -aT(x)^2$$

EM Conservation plus CFT equation of state.

$$\partial_0 T^{0x} = -\partial_x T^{xx}$$

$$T^{xx} = -T^{00}$$

$$\partial_0 T^{0x} = 0 \quad \Rightarrow \quad T(x) = T$$

- How to understand this state?
  - holomorphic factorization (integrability)
  - In a 1+1 dim CFT left and right movers do not interact

Left of the interface

$$J_{p>0} \sim T_L^2 \ , \quad J_{p<0} \sim T_L^2$$

Right of the interface

$$J_{p>0} \sim T_R^2 \ , \quad J_{p<0} \sim T_R^2$$

At the interface $t = 0$

$$J_{p>0} \sim T_L^2 \ , \quad J_{p<0} \sim T_R^2$$

very special to 1+1 D

- How to understand this state?
  - Constant and homogeneous
  - Scaling plus linear response

$$[J] = 2$$

Odd under $T_L \leftrightarrow T_R$

$$J = a(T_L - T_R)(T_L + T_R)$$

Linear response fixes $a = \dfrac{c\pi}{12}$

Holography

- Holography
  - Only Heat, i.e. pure AdS-gravity

$$S = \int \sqrt{-g}(R - 2\Lambda)$$

  - aAdS Solution to Einstein with a constant unsourced Heat current

$$ds^2 = \frac{L^2}{r^2} dr^2 + g^{(0)}_{ij}(r) dx^i dx^j$$

$$g^{(0)}_{ij} = \frac{r^2}{L^2} + \ldots + \frac{1}{r^d} \langle T_{ij} \rangle + \ldots$$

$$\langle T_{ij} \rangle = \begin{pmatrix} -\rho & J \\ J & \rho \end{pmatrix}$$

- Holography
  - Unique solution: boosted BTZ black hole

$$ds^2 = -\frac{r^2}{L^2}(1 - \frac{M^2\cosh^2(\eta)}{r^2})dt^2 + \frac{r^2 + M^2\sinh^2(\eta)}{L^2}dx^2 + \frac{M^2}{L^2}\sinh(2\eta)dxdt + \frac{L^2}{r^2}\frac{dr^2}{(1-\frac{M^2}{r^2})}$$

  - This is dual to a state with constant heat current

$$\langle T^{0x}\rangle = \frac{c\pi}{6}T_{BH}\sinh(2\eta)$$

...,
...,
eg Figueras, Wiseman
Fischetti, Marolf

- Holography
  - Unique solution: boosted BTZ black hole

$$ds^2 = -\frac{r^2}{L^2}(1 - \frac{M^2\cosh^2(\eta)}{r^2})dt^2 + \frac{r^2 + M^2\sinh^2(\eta)}{L^2}dx^2 + \frac{M^2}{L^2}\sinh(2\eta)dxdt + \frac{L^2}{r^2}\frac{dr^2}{(1-\frac{M^2}{r^2})}$$

  - This is dual to a state with constant heat current

$$\langle T^{0x}\rangle = \frac{c\pi}{6}T_{BH}\sinh(2\eta)$$

The novel steady state coincides with the boosted equilibrium state identifying

$$T_L = Te^{\eta},$$

$$T_R = Te^{-\eta}$$

Bhaseen, Doyon, Lucas, KS

- “Boosts” to understand real transport
  - Classical Hall effect

Bhaseen, Green, Sondhi;
Hartnoll, Kovtun

rest frame

$$E = 0$$

$$J = 0$$

boosted frame

$$E = -v \times B$$

$$J = \rho v$$

$$J_i = \sigma_{ij} E_j \quad \Rightarrow \quad \sigma_{xy} = \frac{\rho}{B_z}$$

- Checking with a free boson $m = 0$

$$E_{equilibrium} = \int \frac{dp}{(2\pi)} \frac{E_p}{1 - e^{-\beta E_p}} \qquad E_p = |p|$$

  - Boost

$$J = \int \frac{dp}{(2\pi)} \frac{p}{1 - e^{-\beta(\cosh\eta E - \sinh\eta p)}}$$
$$= \frac{1}{2}\sinh(2\eta) E_{equilibrium}$$

$$E \sim T^2 \quad \to J \sim T_L^2 - T_R^2 \qquad T_L = e^{\eta} T$$
$$T_R = e^{-\eta} T$$

- Cumulants of density of states

$$Z(\beta,\mu) = \mathrm{Tr} e^{-\beta H - \mu P}$$

$$H = \int dx T^{00} \ , \quad P = \int dx T^{0x}$$

$$\beta + \mu = 1/T_R \ \ , \quad \beta - \mu = 1/T_L$$

$$\langle J^{n+1} \rangle = \frac{d^n}{d\mu^n} \ J(\beta_L - \mu, \beta_R + \mu)\Big|_{\mu=0}$$

Exact Dual Solution

- 1+1 CFT/AdS3 is special

Bhaseen, Doyon, Lucas, KS

  - Factorization: can solve the full quench exactly

$$ds^2_{FG} = \frac{L^2}{r^2}\left[\mathrm{d}r^2 + \widetilde{g}_{\mu\nu}(r,t,x)\mathrm{d}x^\mu \mathrm{d}x^\nu\right].$$

$$\begin{aligned}
\widetilde{g}_{tt} &= -\left(1 - \frac{r^2}{L^2}(f_\mathrm{R}(x-t) + f_\mathrm{L}(x+t))\right)^2 + \left(\frac{r^2}{L^2}(f_\mathrm{R}(x-t) - f_\mathrm{L}(x+t))\right)^2, \\
\widetilde{g}_{tx} &= -2\frac{r^2}{L^2}(f_\mathrm{R}(x-t) - f_\mathrm{L}(x+t)), \\
\widetilde{g}_{xx} &= \left(1 + \frac{r^2}{L^2}(f_\mathrm{R}(x-t) + f_\mathrm{L}(x+t))\right)^2 - \left(\frac{r^2}{L^2}(f_\mathrm{R}(x-t) - f_\mathrm{L}(x+t))\right)^2.
\end{aligned}$$

$$\langle T^{tx}\rangle = \frac{c}{6\pi L^2}\left(f_\mathrm{R}(x-t) - f_\mathrm{L}(x+t)\right),$$

$$f_\mathrm{L}(x) = f_\mathrm{R}(x) = \frac{\pi^2}{2L^2}\left(T_\mathrm{L}^2 + \left(T_\mathrm{R}^2 - T_\mathrm{L}^2\right)\Theta(x)\right)$$

- Is there an equivalent phenomenon in d+1 dimensional systems?

Free $QFT_{1+1} \quad = \quad CFT_{1+1}$

Int. QFT_{1+1}

Many papers...

Bernard, Doyon,
Many papers...

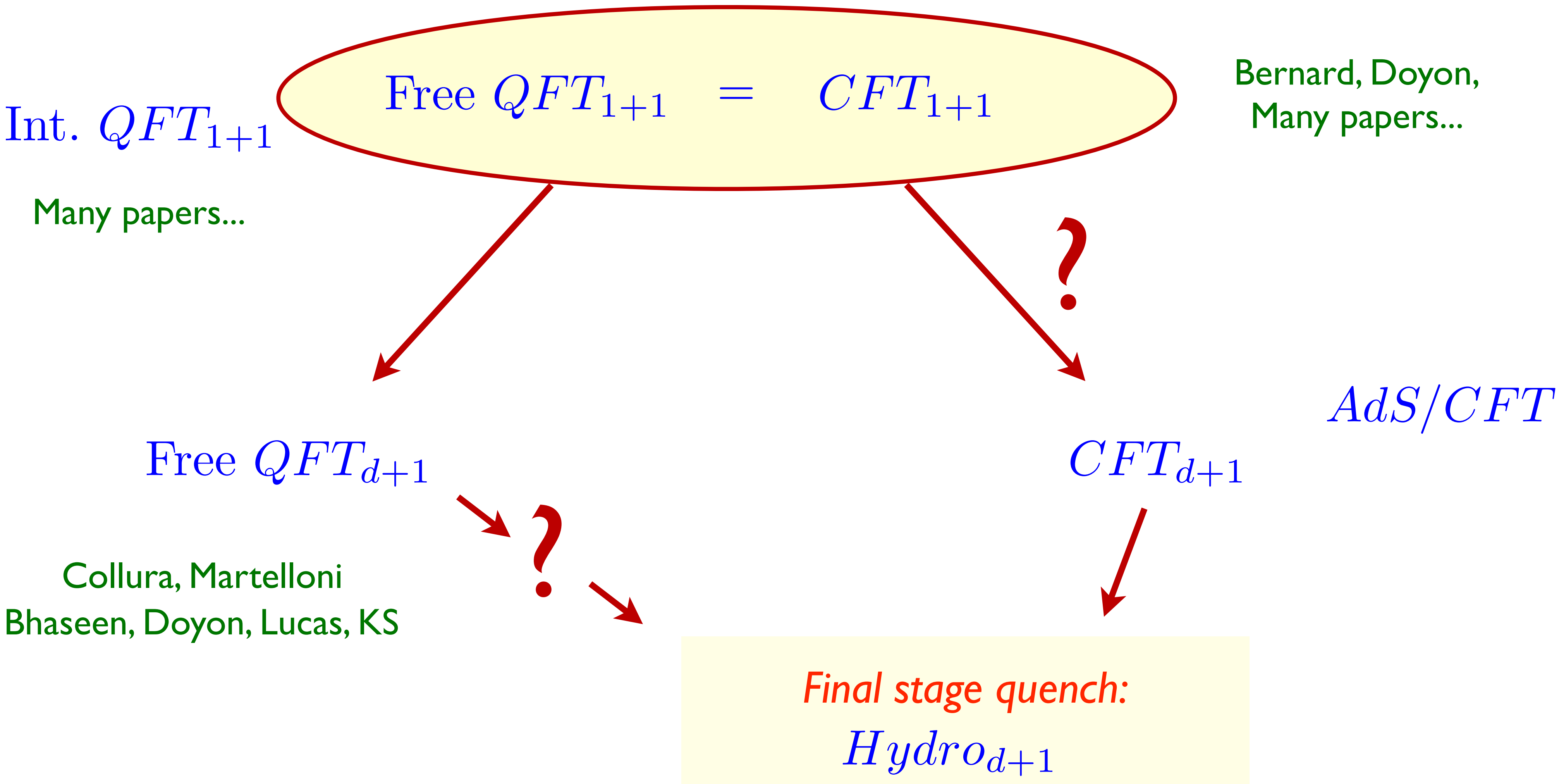
?

Free QFT_{d+1}

CFT_{d+1}

Collura, Martelloni
Bhaseen, Doyon, Lucas, KS

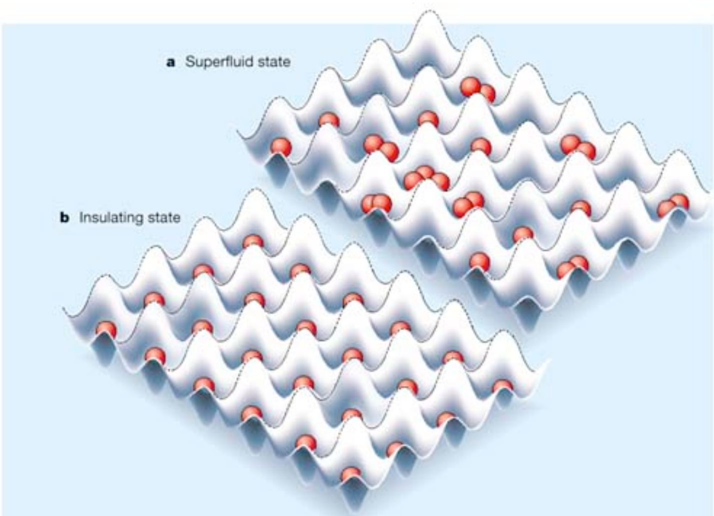
- Is there an equivalent phenomenon in d+1 dimensional systems?

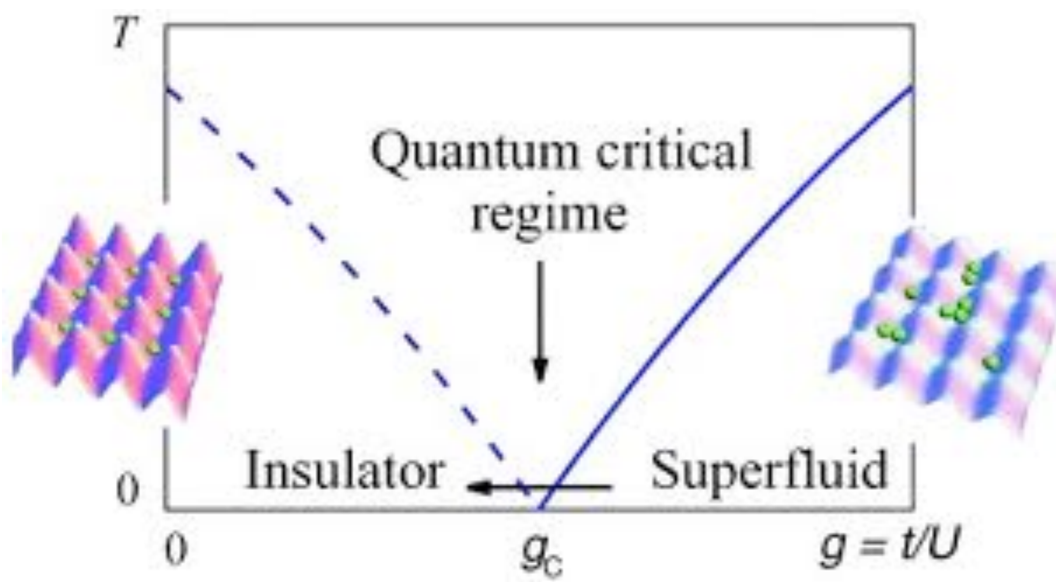


- Is there an equivalent phenomenon in d+1 dimensional systems (CFTs)?

- 2+1d quantum critical system in cold atoms

Greiner, Mandel, Esslinger, Haensch, Bloch





- Is there an equivalent phenomenon in d+1 dimensional systems (CFTs)?

- Is there an equivalent phenomenon in d+1 dimensional systems (CFTs)?
  - If such a steady state exists, what does it look like

    (no holomorphic factorization in higher d; no integrability)

AdS/CFT: The unique **non-singular** *stationary gravity solution dual to a state with homogeneous constant heat flow is the boosted black brane.*

  - This state is the boosted equilibrium state
  - Does this state in fact occur after a thermal quench?

From holography to hydrodynamics

- Boosted equilibrium suggests hydro applies
  - 1+1 Conformal Hydro

$$\langle T_{ij} \rangle = \begin{pmatrix} -\rho & J \\ J & \rho \end{pmatrix}$$

  - 1+1 Conformal = Integrable = no dissipation

$$\partial_t \rho = \partial_x J \quad , \quad \partial_t J = \partial_x \rho$$

- Boosted equilibrium suggests hydro applies
  - 1+1 Conformal Hydro

$$\langle T_{ij} \rangle = \begin{pmatrix} -\rho & J \\ J & \rho \end{pmatrix}$$

  - 1+1 Conformal = Integrable = no dissipation

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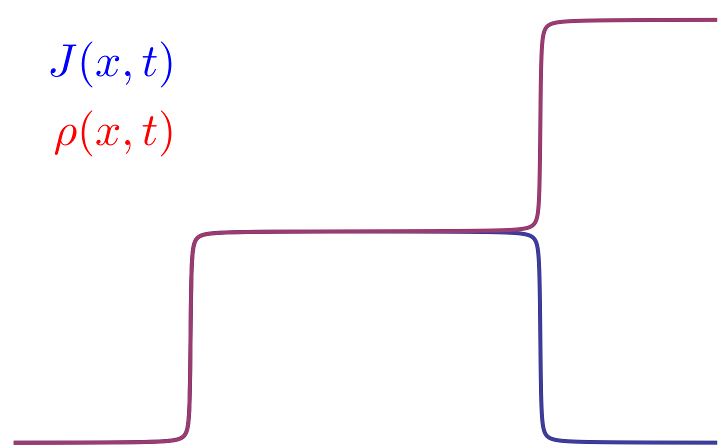

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instantaneous thermalization

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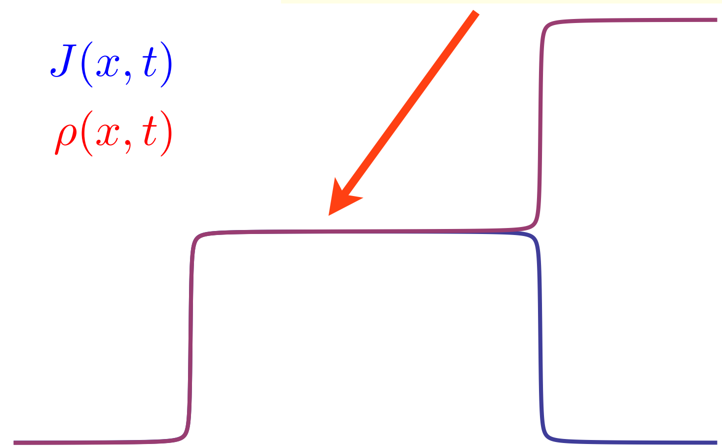

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    Effective dimensional reduction to 1+1 dimension

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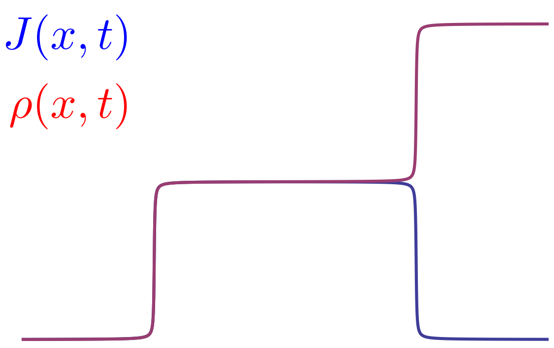

sound fronts

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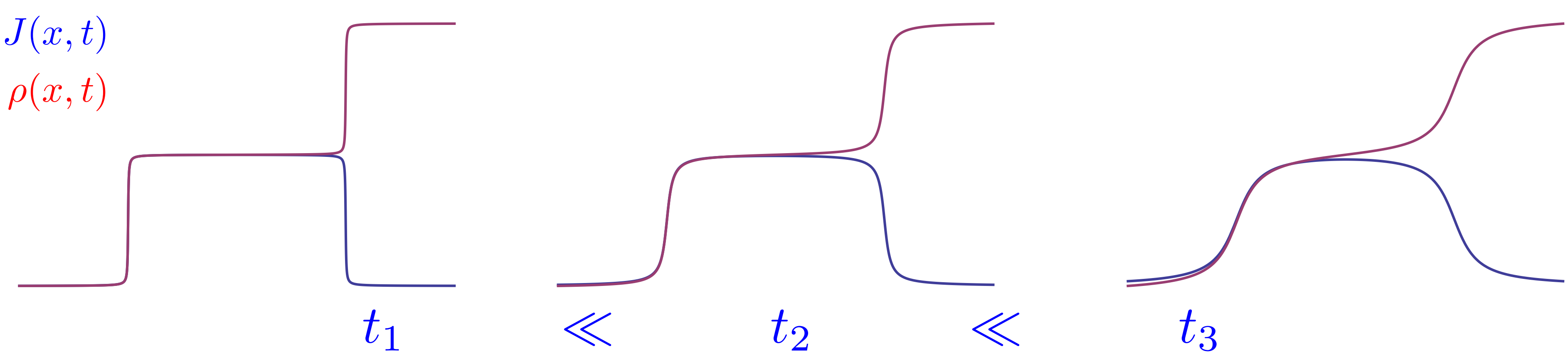

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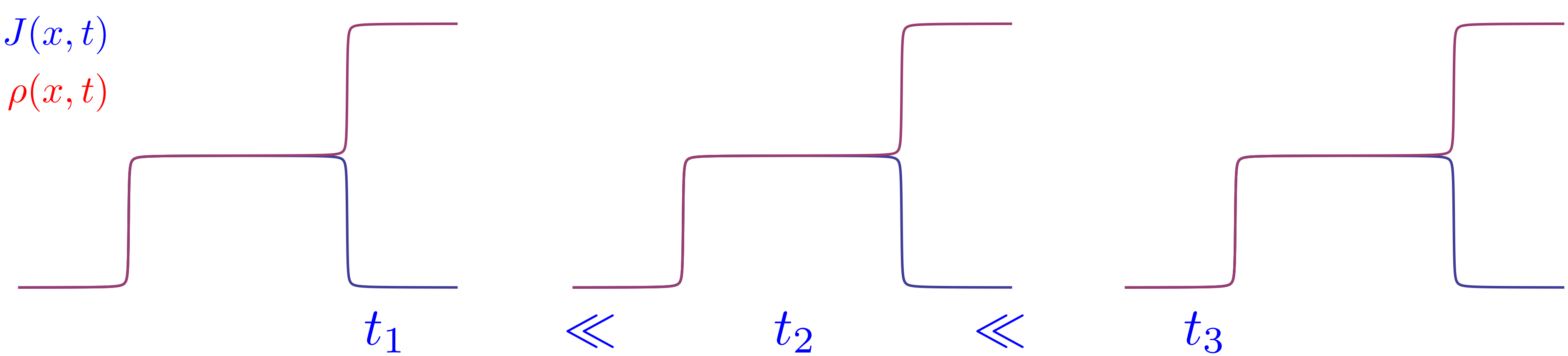
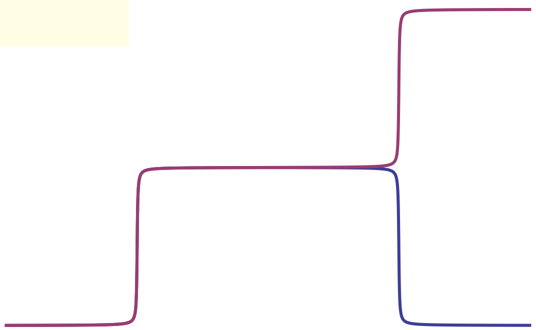


- Assume intermediate homogeneous steady state exists
  - Holography: the state must be boosted thermal state
  - Two-shock solution (Riemann problem)

knowns T_L, T_R

unknowns $T_{\rm ss}, \eta_{ss}, u_L, u_R$

boosted state: otherwise 4 parameters
ρ, p_L, p_T, J



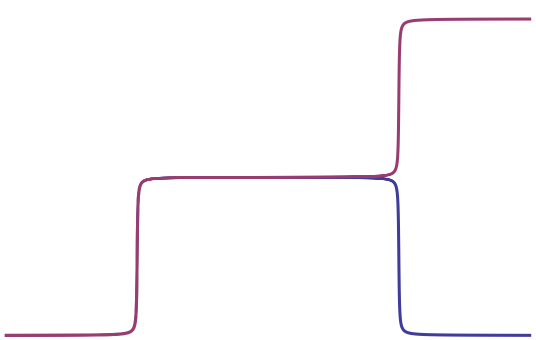
$x = -u_L t$ $\quad$ $x = u_R t$

- Matching across shocks

$$T_{\mu\nu} = T_{\mu\nu}(x + u_L t) + T_{\mu\nu}(x - u_R t)$$

$$\int_{shock} \partial_\mu T^{\mu\nu} = \int_{shock} \partial_x T^{x\mu} + u_{shock} \partial_x T^{0\mu} = 0$$

4 equations for 4 unknowns



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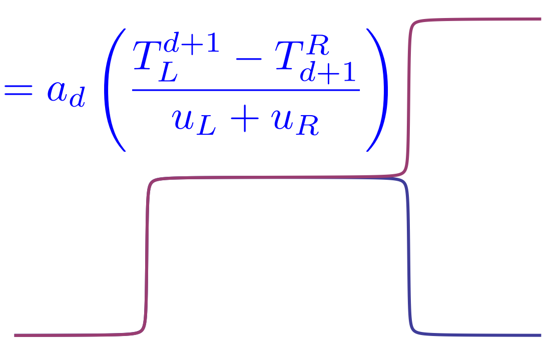
4 equations for 4 unknowns

$$T_{ss} = \sqrt{T_L T_R} \qquad u_L = \frac{1}{d}\sqrt{\frac{\chi + d}{\chi + d^{-1}}} \qquad u_R = \sqrt{\frac{\chi + d^{-1}}{\chi + d}} \qquad \eta_{ss} = \frac{\chi - 1}{\sqrt{(\chi + d^{-1})(\chi + d)}}$$

$$\langle T^{tx} \rangle = a_d \left(\frac{T_L^{d+1} - T^R_{d+1}}{u_L + u_R} \right)$$

$$\chi = \left(\frac{T_L}{T_R} \right)^{\frac{d+1}{2}}$$



$$x = -u_L t \qquad x = u_R t$$

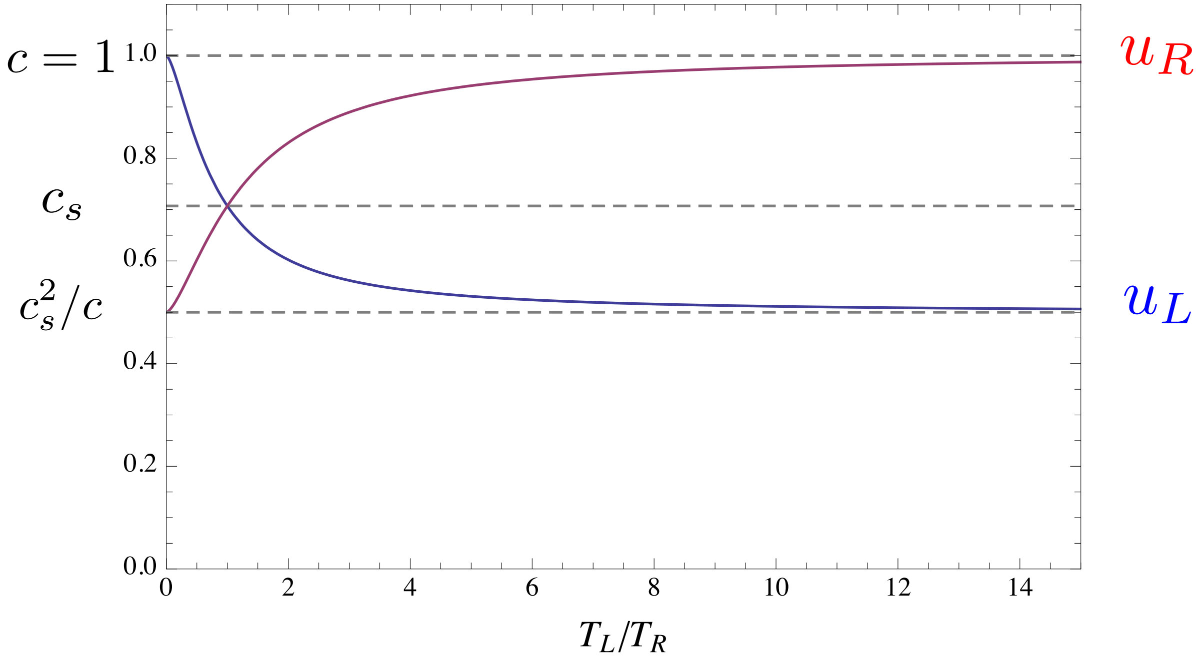
- Shocks are non-linear sound waves

$$u_L = \frac{1}{d}\sqrt{\frac{\chi + d}{\chi + d^{-1}}} \qquad u_R = \sqrt{\frac{\chi + d^{-1}}{\chi + d}}$$

asymmetric

$$\chi = \left(\frac{T_L}{T_R}\right)^{\frac{d+1}{2}}$$

$$u_L u_R = c_s^2 = \frac{1}{d}$$



- Confirming with numerical (ideal) hydro

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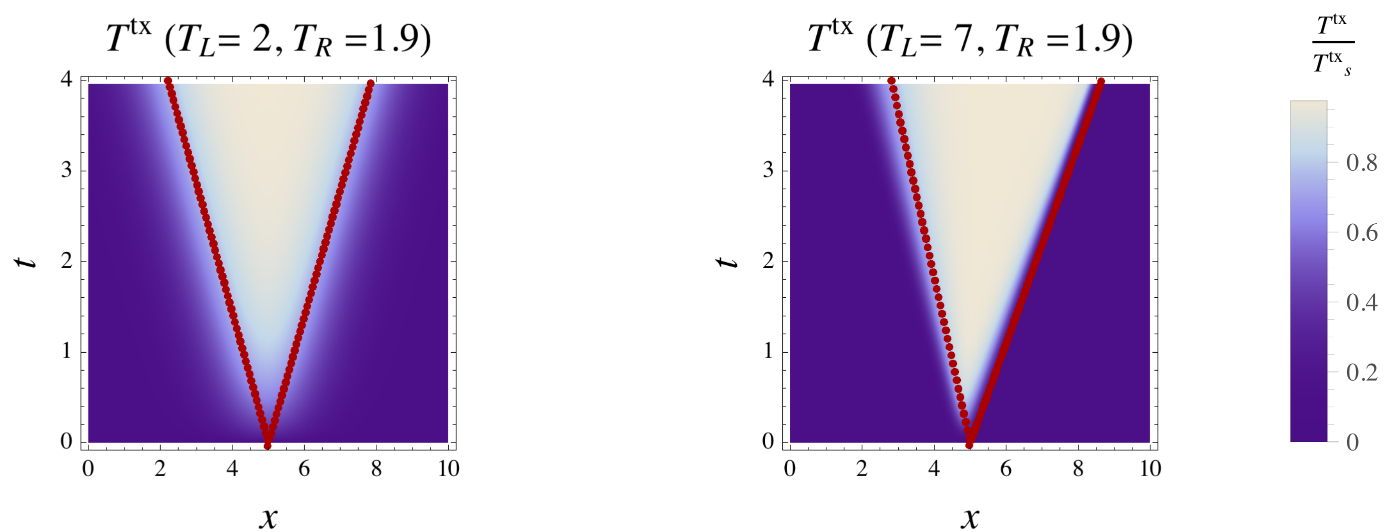

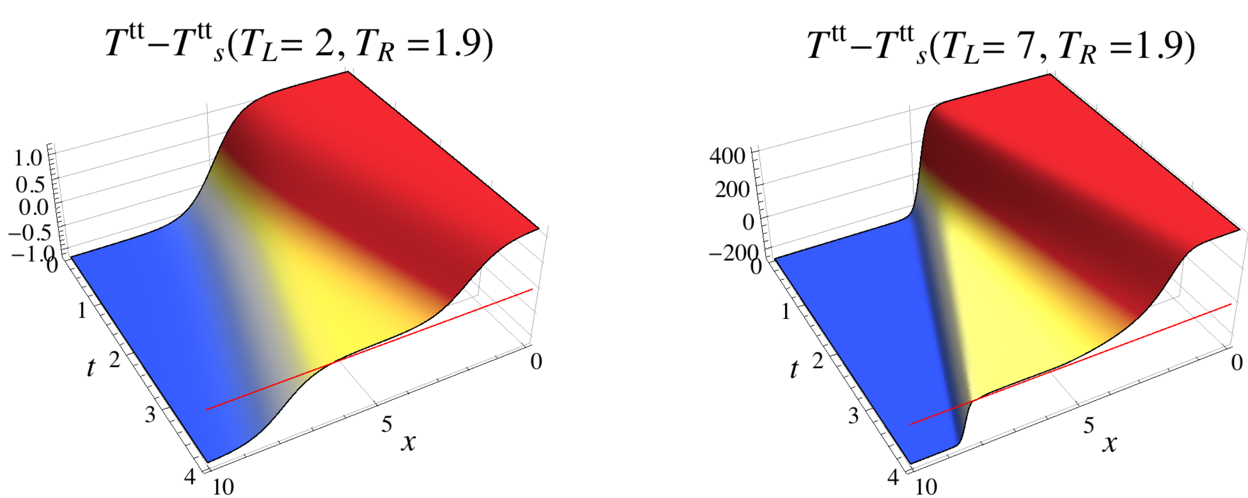

formation of steady state

This is not input!

- Dissipative corrections should not change this
  - Small shocks can be traced in linear response

$$T(x,t) = T_L + \frac{T_R - T_L}{4}\left[2 + \mathrm{erf}\frac{x - t/\sqrt{d}}{4D_{\|}t} + \mathrm{erf}\frac{x + t/\sqrt{d}}{4D_{\|}t}\right]$$

width $\sqrt{D_{\|}t}$ *is smaller than the distance* $t/\sqrt{d}$

Numerically confirmed by Chang, Karch, Yarom

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Numerically confirmed by Chang, Karch, Yarom

- Turbulence?
  - Assumption: completely smooth *T* discontinuity
  - Allows reduction to eff 1+1 dim system

From holography and hydrodynamics to QFT/CFT

- The Fluctuation Spectrum
  - So far we have looked at xpv $\langle T_{\mu\nu} \rangle$
  - Cumulants of the current at the interface

$$c_n \equiv \langle J^n(x=0) \rangle$$

Extended Fluctuation Relation

$$\langle J^{n+1} \rangle = \frac{d^n}{d\mu^n} \, J(\beta_L - \mu, \beta_R + \mu)\Big|_{\mu=0}$$

- The Fluctuation Spectrum
  - So far we have looked at xpv $\langle T_{\mu\nu} \rangle$ (=hydro)
  - Cumulants of the current at the interface

$$c_n \equiv \langle J^n(x=0) \rangle$$

Extended Fluctuation Relation

$$F(z) = \sum \frac{1}{n!} z^n c_n$$

$$\frac{dF(z)}{dz} = J(\beta_L - z, \beta_R + z)$$

holds in any d !

- # Proof of the Extended Fluctuation Relation

For any operator $\mathcal{O}(t) \equiv e^{iHt}\mathcal{O}e^{-iHt}$

$$\left(\frac{\partial}{\partial\beta_L} - \frac{\partial}{\partial\beta_R}\right)\langle\mathcal{O}(t)\rho_{t=0}\rangle = \langle(H_L - H_R)\mathcal{O}(t)\rho_{t=0}\rangle$$ where $\rho_{t=0} \equiv e^{-\beta_L H_L - \beta_R H_R}$

At late times, in the steady state

$$\left(\frac{\partial}{\partial\beta_L} - \frac{\partial}{\partial\beta_R}\right)\langle\mathcal{O}\rangle_{\rm ss} \simeq \langle(H_L(-t) - H_R(-t))\mathcal{O}\rangle_{\rm ss}$$

<u>Energy conservation</u> $\frac{dH_R(t)}{dt} = -\frac{dH_L(t)}{dt} = J(t)$ and PT reversal

$$H_L(t) - H_R(t) - H_L(-t) + H_R(-t) = 2\int_{-t}^{t} dt' J(t')$$

EFR follows

$$\left(\frac{\partial}{\partial\beta_L} - \frac{\partial}{\partial\beta_R}\right)\langle\mathcal{O}(t)\rangle_{\rm ss} = \int_{-t}^{t} dt' \langle J(t')\mathcal{O}(t)\rangle_{\rm ss}$$

- Summary
  - 1+1 dim CFTs show novel steady states with homogeneous heat flow after a thermal quench.
  - This steady state can be identified as a boosted thermal equilibrium state.
  - *The same happens in d+1 CFTs. An intermediate steady state appears. Holography determines that is a boosted equilibrium state.*

Warning!

This is a collective effect. This cannot be seen in free field theory.

(Many 1+1 CFTs have a free field representation, but this is not so in d+1 CFTs.)

- Summary
  - 1+1 dim CFTs show novel steady states with homogeneous heat flow after a thermal quench.
  - This steady state can be identified as a boosted thermal equilibrium state.
  - *The same happens in d+1 CFTs. An intermediate steady state appears. Holography determines that is a boosted equilibrium state.*
  - Knowing this, hydrodynamics applies. Two non-linear sound shock Riemann problem can be solved analytically (ideal).
  - *All higher order moments follow from energy conservation. This is QFT information beyond hydrodynamics.*

- Outlook
  - Direct Momentum relaxation vs dissipation.
  - Effects of turbulence.

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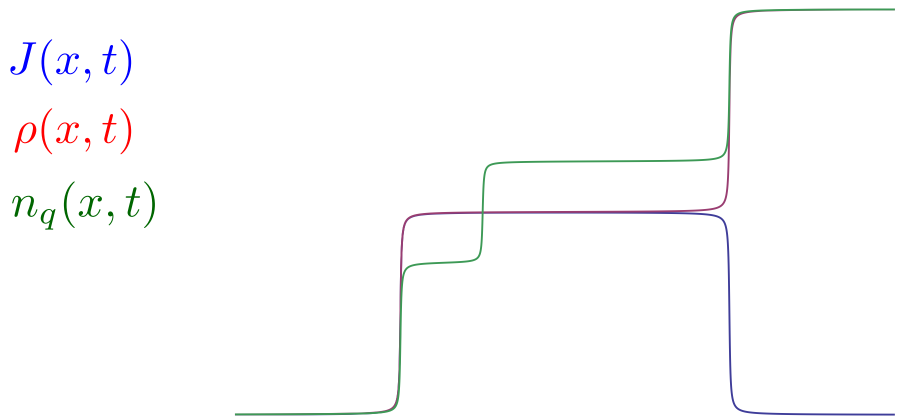
  - Direct Momentum relaxation vs dissipation.
  - Effects of turbulence.

  - Including charge discontinuity in the quench.



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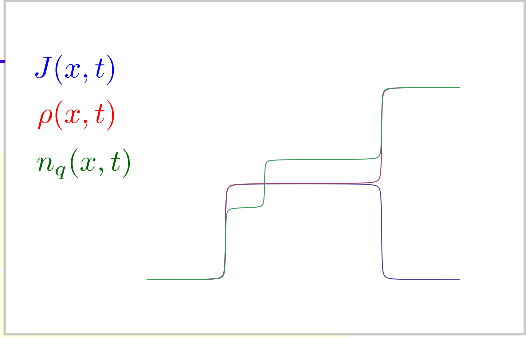

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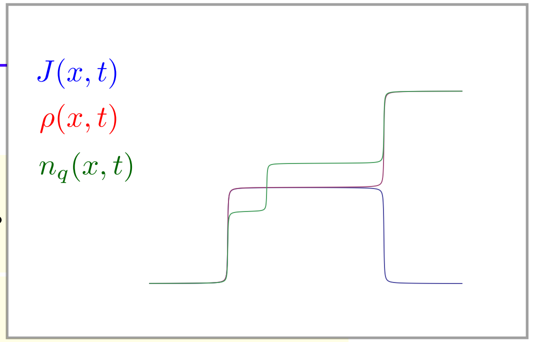

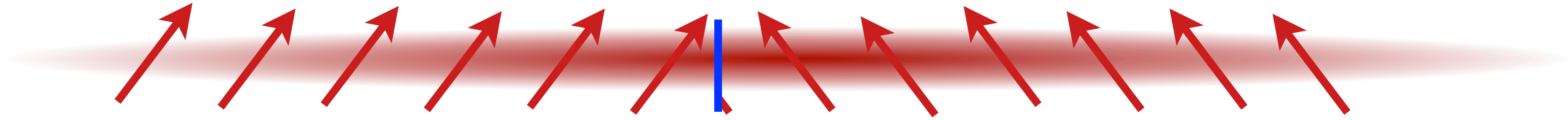

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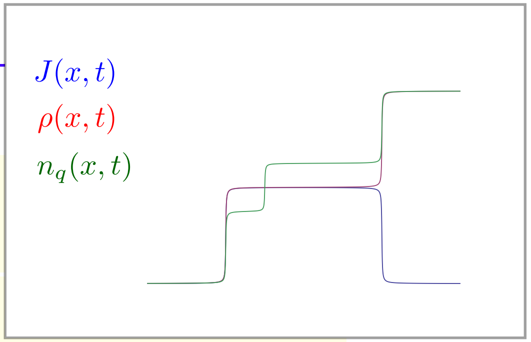

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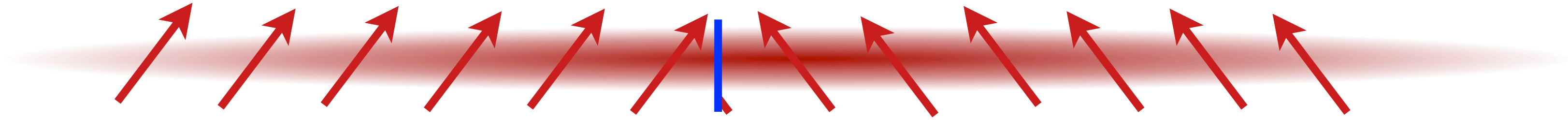

  - Direct connection to cold atom experimental set-ups

Thank you