Higher Spin Theories and Vector Models in various dimensions

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Massless higher spins

- Consistent interactions of *massless* higher spin fields (gauge fields!) are highly constrained
- In flat space, no consistent theory of interacting massless higher spin fields of spin $s > 2$ (Coleman-Mandula, Weinberg…)
- However, with non-zero cosmological constant, Vasiliev explicitly constructed ('89-'92) consistent fully non-linear theories of interacting massless higher spin fields (in arbitrary dimensions). No smooth flat space limit.
- These theories involve infinite towers of higher spin fields, including in particular the *graviton (s=2).* Hence, they are in particular theories of gravity.

Higher spins in AdS

- Vasiliev wrote down a set of consistent gauge invariant equations of motion. They admit a vacuum solution which is AdS space (or dS if the cosmological constant is positive. Will focus on AdS case in this talk).
- In the simplest bosonic 4d theory, the linearized spectrum around the AdS vacuum consists of an infinite tower of higher spin fields plus a scalar

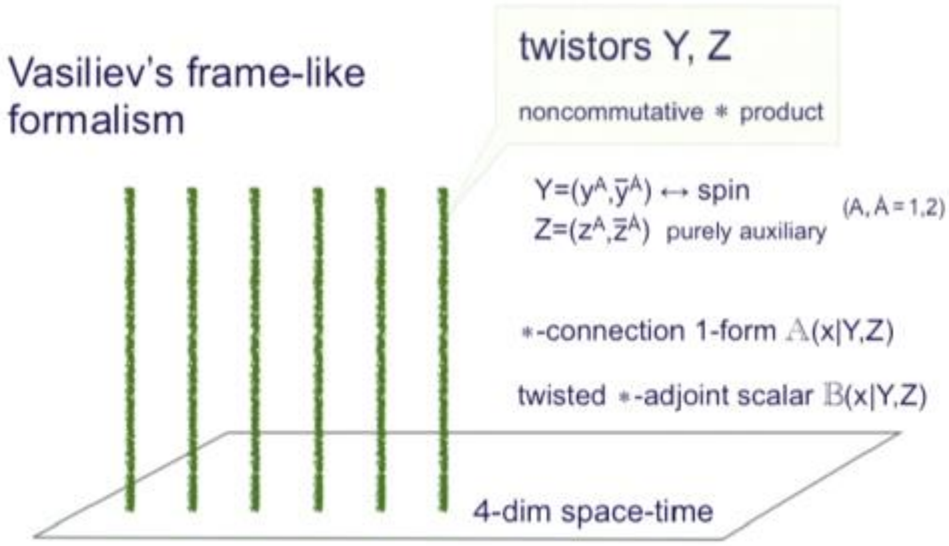
$$\begin{array}{lll} \text{Spectrum :} & s = 1, 2, 3, \ldots, \infty & \text{gauge fields} \\ & s = 0, \quad m^2 = -2/\ell_{AdS}^2 & \text{scalar} \end{array}$$

Vasiliev equations

- The Vasiliev equations in 4d

$$d\mathcal{A} + \mathcal{A} * \mathcal{A} = e^{i\theta_0} B * \kappa \, dz^\alpha dz_\alpha + e^{-i\theta_0} B * \bar{\kappa} \, d\bar{z}^{\dot{\alpha}} d\bar{z}_{\dot{\alpha}}$$

$$dB + \mathcal{A} * B - B * \pi(\mathcal{A}) = 0$$



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$$dB + \mathcal{A} * B - B * \pi(\mathcal{A}) = 0$$

- Two distinct parity preserving theories: ``Type A'': $\theta_0 = 0$. ``Type B'': $\theta_0 = \pi/2$. General θ_0: parity breaking HS theory

- Physical fields: essentially, the master field $\mathcal{A}$ contains the metric and all other higher spin fields (more precisely, $\mathcal{A}$ contains "vielbein" and "spin connections"), and *B* contains the scalar field and the curvatures (generalized Weyl tensors) of the HS fields.

Vasiliev equations

$$dA + A * A = e^{i\theta_0} B * \kappa \, dz^{\alpha} dz_{\alpha} + e^{-i\theta_0} B * \bar{\kappa} \, d\bar{z}^{\dot{\alpha}} d\bar{z}_{\dot{\alpha}}$$
$$dB + A * B - B * \pi(A) = 0$$

- AdS vacuum solution: flat connection A

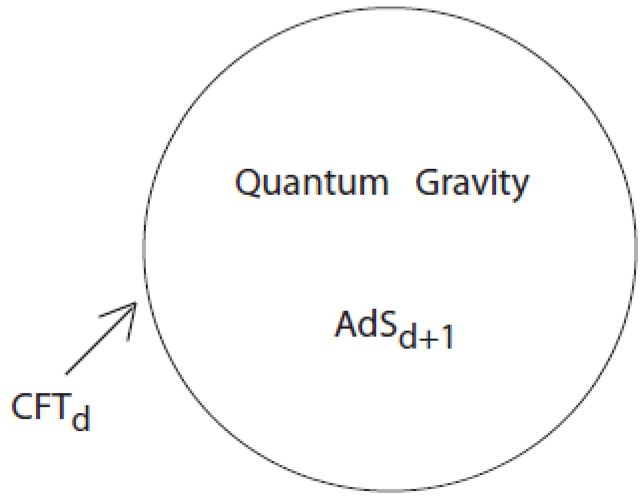
$$A = (e_0)_{\alpha\dot{\beta}} y^{\alpha} \bar{y}^{\dot{\beta}} + (\omega_0)_{\alpha\beta} y^{\alpha} y^{\beta} + (\omega_0)_{\dot{\alpha}\dot{\beta}} \bar{y}^{\dot{\alpha}} \bar{y}^{\dot{\beta}}, \quad B = 0$$

- Linearizing the equations around this vacuum, one finds the standard equations for a scalar (with mass m^2=-2), linearized graviton and free massless HS fields (Fronsdal).

$$\left(\nabla^2 - \kappa^2\right) \varphi_{\mu_1\mu_2\ldots\mu_s} = 0 \,, \qquad \kappa^2 = (s-2)(s+d-3) - 2$$
$$\nabla^{\mu} \varphi_{\mu\mu_2\ldots\mu_s} = 0 \,, \qquad \varphi^{\mu}_{\mu\mu_3\ldots\mu_s} = 0$$

Higher spins and AdS/CFT

- From the point of view of AdS/CFT correspondence, it is not too surprising that such theories exist.



- The AdS/CFT correspondence is an *exact equivalence*, or *duality*, between quantum gravity in AdS and a conformal quantum field theory that can be thought of as living at the boundary of AdS

Higher spins and AdS/CFT

- If Vasiliev theory defines a consistent quantum gravity theory in AdS, what is its CFT dual?
- Consider a free theory of N free complex scalar fields in 3d

$$S = \frac{1}{2}\int d^3x \partial_\mu \phi_i^* \partial^\mu \phi^i \,, \qquad i = 1, \ldots, N$$

- It has a U(N) global symmetry under which the scalar transforms as a *vector*. (This is different from familiar examples of AdS/CFT, where the CFT side is usually a gauge theory with matrix (adjoint) type fields).

Higher spins and AdS/CFT

- By virtue of being free, it is easy to see that this theory has an infinite tower of conserved HS currents

$$J_{\mu_1\cdots\mu_s} = \phi_i^* \partial_{(\mu_1} \cdots \partial_{\mu_s)} \phi^i + \ldots$$
$$\partial^\mu J_{\mu\mu_2\cdots\mu_s} = 0\,, \qquad \Delta(J_s) = s+1$$

- If we consider U(N) invariant operators (*singlet sector*), these currents, together with the scalar operator

$$J_0 = \phi_i^* \phi^i\,, \qquad \Delta = 1$$

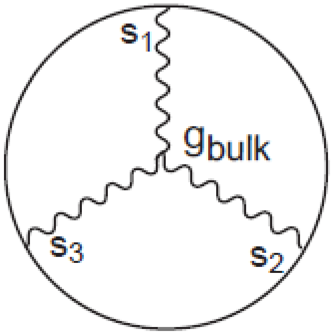
are all the "single trace" primaries of the CFT.

Higher spins and AdS/CFT

- By the usual AdS/CFT dictionary, single trace primary operators in the CFT are dual to single particle states in AdS.
- *Conserved* currents are dual to *gauge* fields (massless HS fields)
- A scalar operator is dual to a bulk scalar with $\Delta(\Delta - d) = m^2$
- This precisely matches the spectrum of Vasiliev's bosonic theory in AdS_4.

Higher spins and AdS/CFT

- The dual higher spin fields are necessarily interacting in order to reproduce the non-vanishing correlation functions of HS currents in the CFT



$$g_{\text{bulk}} \sim \frac{1}{\sqrt{N}}$$

$$G_N \sim \frac{1}{N}$$

- The large N limit corresponds as usual to weak interactions in the bulk.
- So from AdS/CFT point of view we conclude that consistent theories of interacting massless higher spins in AdS indeed have to exist, as they should provide AdS duals to free CFT's.

Higher spins and AdS/CFT

- We could also consider a theory of N free real scalars, and look at the O(N) singlet sector
- The spectrum of single trace primaries now includes a scalar plus all the *even* spin HS currents
- On Vasiliev's theory side, this corresponds to a consistent truncation of the equations which retains only the even spin gauge fields ("Minimal bosonic HS theory").

$$\begin{aligned} \text{Spectrum}: \quad & s = 2, 4, 6, \ldots, \infty \quad \text{gauge fields} \\ & s = 0, \quad m^2 = -2/\ell_{AdS}^2 \quad \text{scalar} \end{aligned}$$

The interacting O(N) model

- The conjecture that the (singlet sector of) the O(N)/U(N) vector model is dual to the bosonic Vasiliev theory was made by Klebanov and Polyakov (2002).
- A crucial observation is that one can also consider the Wilson-Fisher IR fixed point reached by a relevant “double trace” deformation of the free theory.

$$S = \int d^3x \left[\frac{1}{2} \left(\partial_\mu \phi^i \right)^2 + \frac{\lambda}{4} (\phi^i \phi^i)^2 \right]$$

- The IR fixed point can be studied in d=4-ε in the framework of ε–expansion.

The interacting O(N) model

- At large N, the model can be solved by introducing the Hubbard-Stratonovich auxiliary field

$$S = \int d^d x \left(\frac{1}{2}(\partial\phi^i)^2 + \frac{1}{2}\sigma\phi^i\phi^i - \frac{\sigma^2}{4\lambda} \right)$$

- The fixed point ("critical vector model") is an *interacting* CFT whose single trace spectrum has a scalar operator of dimension $\Delta_{\phi^2} = 2 + O(1/N)$ and slightly broken higher spin currents of dimension $\Delta = s + 1 + O(1/N)$
- The 1/N corrections to operator dimensions can be computed explicitly as a function of dimension *d* .
- The $\frac{\lambda}{4}(\phi^i\phi^i)^2$ interaction can be viewed as an example of ``double-trace" deformation, which is well understood in AdS/CFT. The dual to the IR fixed point should be the *same* Vasiliev theory dual to the free vector model, with the difference that the $m^2=-2$ bulk scalar is assigned the alternate $\Delta=2$ boundary condition.

Fermionic vector model

- Similarly we can consider a free fermionic vector model

$$S = \frac{1}{2}\int d^3x \bar{\psi}_i \gamma^\mu \partial_\mu \psi^i$$

- The single trace spectrum is similar to the scalar case, there is an infinite tower of conserved HS current, plus a parity odd scalar $\bar{\psi}_i \psi^i$ of dimension 2.
- It turns out that there is indeed a Vasiliev's theory with such a single particle spectrum: at linearized level, the only difference is that the bulk scalar is now a pseudo-scalar. This is the "type B" theory, while the theory dual to scalars is the "type A" theory.
- At non-linear level "type A" and "type B" theories have different interactions (for instance, the graviton cubic coupling is different: fermion and boson CFT's have different <TTT>).

The critical fermionic theory

- One can also consider the UV fixed point of the Gross-Neveu model with $(\bar{\psi}_i \psi^i)^2$ interaction. Similarly to scalar case, at large N it corresponds to alternate boundary conditions in the bulk.
- The UV fixed point of the GN model can be studied in the 2+ε expansion. Or at large N for any *d* (in particular *d*=3).
- There is an alternative description as conventional IR fixed point of the ``Gross-Neveu-Yukawa" model in d=4-ε

(Hasenfratz et al; Zinn Justin)

$$S(\bar{\psi}, \psi, \sigma) = \int d^d x \left[-\bar{\psi}^i \left(\partial\!\!\!/ + g_1 \sigma \right) \psi^i + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{g_2}{24} \sigma^4 \right]$$

- RG analysis shows a IR stable fixed point (for any N). This IR CFT is equivalent to UV fixed point of GN model (anomalous dimensions of operators can be explicitly matched).

Higher spin/vector model dualities

- Summary of higher spin/vector model AdS_4/CFT_3 dualities

	HS A-type	HS B-type
Δ=1 scalar b.c.	Free U(N)/O(N) scalar	Critical U(N)/O(N) fermion
Δ=2 scalar b.c.	Critical U(N)/O(N) scalar	Free U(N)/O(N) fermion

- Modifying the bulk spin 1 gauge field b.c., one can also describe critical QED_3 with many flavors or CP^N models in d=3.
- There is a rich AdS_3/CFT_2 story that I will not discuss in this talk (Gaberdiel-Gopakumar: HS_{3d} ←→ W_N model coset CFT)

Some comments

- Pure HS gauge theories have exactly the right spectrum to be dual to *vector models* (adjoint theories have many more single trace operators)
- The restriction to singlet sector can be implemented by *gauging* the U(N)/O(N) symmetry, and taking the limit of zero gauge coupling. In 3d, we can do this with Chern-Simons gauge theory.
- In the large N limit with $\lambda = N/k$ fixed (k is the CS level), the singlet sector of the free (critical) theories correspond to the limit $\lambda \to 0$.

HS/Chern-Simons vector model dualities

- At finite λ , and large N, it can be shown that these interacting Chern-Simons vector models still have approximate HS symmetry (*SG et al, Aharony et al; Maldacena-Zhiboedov*)

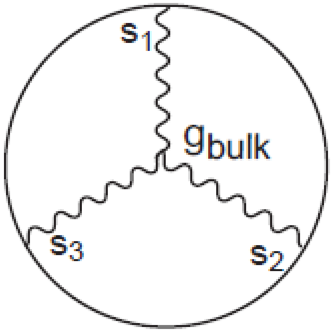
- They were conjectured to be dual to parity breaking versions of Vasiliev's theory in AdS_4 , with $\theta_0 = \theta_0(\lambda)$.

CS+vector model ←→ parity breaking HS theory

- On the CFT side, this has suggested a novel bose-fermi duality relating theories of bosons coupled to CS to theories of fermions coupled to CS, with N $<\!\!-\!\!>$ k (generalizes level-rank duality).

Tests of the duality

- These HS_{4d}/3d-vector model dualities have been explicitly tested at the level of 3-point correlation functions (SG-Yin, Maldacena-Zhiboedov, Didenko-Skorvstov…)



$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle = \cos^2\theta_0 \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{\rm sc} + \sin^2\theta_0 \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{\rm fer} + \cos\theta_0 \sin\theta_0 \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{\rm odd}$$

Sphere free energies

- Recently we obtained new tests of these dualities based on a different observable of the CFT: the partition function of the free energy on a round sphere S^3, $F=-\log Z$. (*SG, Klebanov; SG, Klebanov, Safdi*)
- It is an interesting quantity that for any RG flow satisfies $F_{UV} > F_{IR}$. ("F-theorem").
- For a CFT, it is also related to the entanglement entropy across a circle (this relation was used by Casini-Huerta to prove the 3d F-theorem)

Free energy on S^3

- In the CFT, it is simply defined as the log of the partition function of the theory on S^3 (generalization to S^d is straightforward)

$$F = -\log Z \qquad\qquad Z = \int D\phi e^{-S}$$

$$S = \int d^3x \sqrt{g} \left(\partial_\mu \phi^i \partial^\mu \phi^i + \frac{R}{8} \phi^i \phi^i \right)$$

- This is easy to compute in the free theory: need to evaluate the determinant of the kinetic operator

$$F = \frac{1}{2} \log\det\left(-\nabla^2 + 3/4\right)$$

Free energy on S^3

- The explicit computation gives (*Klebanov, Pufu, Safdi*)

$$F = \frac{N}{2} \sum_{n=0}^{\infty} (n+1)^2 \log[(n+1/2)(n+3/2)] = N \left(\frac{\log 2}{8} - \frac{3\zeta_3}{16\pi^2} \right)$$

for N real scalars, and twice this result for N complex scalars. Free theory: trivial N dependence.

- One can also perform the calculation in the critical theory (in a large N expansion), with the result

$$F^{\text{critical}} = F^{\text{free}} - \frac{\zeta_3}{8\pi^2} + O(1/N)$$

Free energy on S^3 from the bulk

- The challenge is: can we reproduce these results from the bulk? In particular, can we see the vanishing of the subleading corrections in the large N expansion of the free energy from the HS dual to the free theory?
- How do we compute F from the bulk?

$$Z_{\rm CFT} = Z_{\rm bulk}$$

- We "simply" have to compute the partition function of the Vasiliev's theory on the Euclidean AdS_4 vacuum

$$ds^2 = d\rho^2 + \sinh^2\rho \, d\Omega_3$$

Free energy on S^3 from the bulk

- In practice, we should compute the path integral of the bulk theory, where we expand the metric around AdS_4 and integrate over all quantum fluctuations

$$
\begin{aligned}
Z_{\text{bulk}} &= \int D\varphi_{(0)} D g_{\mu\nu} D\varphi_{(s)} e^{-S[g=g_0+h,\varphi_{(0)},\varphi_{(s)}]} \\
&= e^{-\frac{1}{G_N}F^{(0)} - F^{(1)} - G_N F^{(2)} + \ldots} = e^{-F_{\text{bulk}}}
\end{aligned}
$$

- Here G_N is Newton's constant, which scales as $1/G_N \sim N$.

Free energy on S^3 from the bulk

- The explicit bulk action is not well understood (proposals by *Douroud, Smolin*; *Boulanger, Sundell*), but in terms of physical fields is expected to take a form

$$S \sim \frac{1}{G_N} \int d^4x \sqrt{g} \left(R + \Lambda + R^3 + R^4 + \ldots \right.$$
$$\left. + \varphi_{(s)} \Delta_s \varphi_{(s)} + \sum C_{s_1 s_2 s_3} \partial^{k_1} \varphi_{s_1} \partial^{k_2} \varphi_{s_2} \partial^{k_3} \varphi_{s_3} + \ldots \right)$$

- The leading term $\frac{1}{G_N} F^{(0)}$ in the bulk free energy corresponds to evaluating this action on the AdS_4 background metric, with all other fields set to zero.
- This is already very non-trivial, as it requires to know the form of all the higher derivative corrections in the metric sector (we know they are non-trivial from knowledge of correlation functions).

Free energy on S^3 from the bulk

- One would like to show that

$$S_{\text{classical}}[g_{\mu\nu} = AdS_4, \varphi_{(s)} = 0] = \frac{1}{G_N} F^{(0)} = N\left(\frac{\log 2}{4} - \frac{3\zeta_3}{8\pi^2}\right)$$

- Alternatively, one would need a generalization of Ryu-Takayanagi prescription to compute holographic entanglement entropy in 4d HS theory.
- This is one of the outstanding open problems in testing HS/vector model dualities.
- While we cannot show this yet, we can start by something simpler, namely assume that this tree level piece works, and compute the one-loop contribution $F^{(1)}$ to the bulk free energy.

The one-loop piece

- Let us now concentrate on the calculation of the one-loop contribution $F^{(1)}$ to the bulk free energy

$$e^{-\frac{1}{G_N}F^{(0)} - F^{(1)} - G_N F^{(2)} + \dots} = e^{-F_{\text{bulk}}}$$

- Even if we don't know the full action, we know that the linearized equations correspond to standard kinetic terms for all the higher spin fields, so we assume a canonical quadratic action

$$S_{(2)} = \int d^4x \sqrt{g} \left(\varphi_{(0)}(-\nabla^2 - 2)\varphi_{(0)} + \sum_{s=1,2,\dots} \varphi_{(s)} \Delta_s \varphi_{(s)} \right)$$

The one-loop piece

- One can introduce spin s-1 ghosts, then after decomposing physical and ghost fields into their irreducible parts, the contribution to the one-loop free energy of each HS field is the ratio of determinants on symmetric traceless transverse (STT) fields (Gaberdiel et al, Grumiller, Gupta-Lal…)

$$\frac{\left[\det_{s-1}^{STT}\left(-\nabla^2+s^2-1\right)\right]^{\frac{1}{2}}}{\left[\det_{s}^{STT}\left(-\nabla^2+s(s-2)-2\right)\right]^{\frac{1}{2}}}$$

One-loop free energy

- We have to compute

$$F_{1-\text{loop}} = \frac{1}{2}\log\det\left(-\nabla^2 - 2\right) + \frac{1}{2}\sum_{s=1}^{\infty}\left[\log\det{}_s\left(-\nabla^2 - 2 + s(s-2)\right) - \log\det{}_{s-1}\left(-\nabla^2 + s^2 - 1\right)\right]$$

- These determinants can be computed using heat kernel methods.
- The spectral zeta function (Mellin transform of the heat kernel) for $-\nabla^2 + \kappa^2$ operators acting on STT fields of arbitrary spin in hyperbolic space is known (*Camporesi, Higuchi, '90's*)

AdS Spectral zeta function

- The explicit spectral zeta function in AdS is

$$\zeta_{(\Delta,s)}(z) = \left(\frac{\int \mathrm{vol}_{AdS_{d+1}}}{\int \mathrm{vol}_{S^d}}\right) \frac{2^{d-1}}{\pi} g(s) \int_0^\infty du\, \frac{\mu_s(u)}{\left[u^2 + \left(\Delta - \frac{d}{2}\right)^2\right]^z}$$

with $\Delta(\Delta - d) - s = \kappa^2$

- In the present case of d=3

$$\mathrm{vol}_{AdS_4} = \frac{4}{3}\pi^2\,, \qquad \mathrm{vol}_{S^3} = 2\pi^2$$

$$\mu_s(u) = \frac{\pi u}{16}\left[u^2 + \left(s + \frac{1}{2}\right)^2\right] \tanh \pi u\,, \qquad g(s) = 2s + 1$$

AdS Spectral zeta function

- In terms of the spectral zeta function, the contribution to the one-loop free energy is then obtained as

$$F^{(1)}_{(\Delta,s)} = -\frac{1}{2}\zeta'_{(\Delta,s)}(0) - \frac{1}{2}\zeta_{(\Delta,s)}(0)\log\left(\ell^2\Lambda^2\right)$$

- Importantly, in every even dimensional bulk spacetime, there is a logarithmic divergence proportional to the value of the spectral zeta function at z=0.

UV finiteness

- For the duality to be exact and Vasiliev theory to be “UV complete”, this divergence should not be present in the full HS theory: the bulk theory should be finite.
- While each spin contributes a log divergence, can the divergence cancel in the sum over the infinite tower of fields?

$$
\begin{aligned}
F^{(1)}\Big|_{\log-\mathrm{div}} &= -\frac{1}{2}\left(\zeta_{(1,0)}(0)+\sum_{s=1}^{\infty}\left(\zeta_{(s+1,s)}(0)-\zeta_{(s+2,s-1)}(0)\right)\right)\log\left(\ell^2\Lambda^2\right) \\
&= \left(\frac{1}{360}+\sum_{s=1}^{\infty}\left(\frac{1}{180}-\frac{s^2}{24}+\frac{5s^4}{24}\right)\right)\log\left(\ell^2\Lambda^2\right)
\end{aligned}
$$

UV finiteness

- Regulating the sum with the usual Riemann zeta function regularization, and recalling that ζ(0)=-1/2, and ζ(-2)=ζ(-4)=0:

$$\frac{1}{360}+\sum_{s=1}^{\infty}\left(\frac{1}{180}-\frac{s^2}{24}+\frac{5s^4}{24}\right)=0$$

- So Vasiliev's theory is one-loop finite.
- Regularization can be understood as analytic continuation of spectral zeta function in the spectral parameter z (summing over all spins first). Regulator should be singled out by consistency with HS symmetry.
- The same result holds for the theory with even spins only, and regardless of boundary conditions on the scalar.
- This is similar to the cancellation of UV divergences in N>4 SUGRA in AdS_4 , but here we have a purely bosonic theory (with an *infinite* number of fields).

The finite part

- Having shown that the log divergence cancels, we can move on to the computation of the finite contribution.
- This is considerably more involved. Computing the derivative of the spectral zeta-function, the result is expressed as

$$F^{(1)} = -\frac{1}{2}\mathcal{I}(-1/2,0) - \frac{1}{2}\sum_{s=1}^{\infty}\left[\mathcal{I}(s-1/2,s) - \mathcal{I}(s+1/2,s-1)\right]$$

with:

$$\mathcal{I}(\nu,s) = \frac{1}{3}(2s+1)\int_0^{\nu} dx \left[\left(s+\frac{1}{2}\right)^2 x - x^3\right]\psi(x+\frac{1}{2})$$

The finite part

- Summing up all fields we find that the infinite tower of spins precisely cancels the Δ=1 scalar field contribution! We conclude that the one-loop bulk free energy in Vasiliev's type A theory with boundary condition for the scalar is exactly zero

$$F^{(1)} = 0$$

- This is precisely consistent with the fact that in the dual free CFT the large N expansion should be trivial.
- In the case of Δ=2 boundary condition on the bulk scalar, one instead find

$$F^{(1)} = -\frac{\zeta_3}{8\pi^2}$$

which is the expected non-trivial $O(N^0)$ correction in the interacting IR fixed point.

The minimal HS theory

- We can repeat the same calculation in the minimal theory, with contain even spins only, which should be dual to the O(N) vector model.
- Here there is an interesting twist. The total one loop free energy is *not* zero, but it is equal to

$$F^{(1)}_{\rm min} = \frac{\log 2}{8} - \frac{3\zeta(3)}{16\pi^2}$$

- This is precisely equal to the value of the S^3 free energy of a single real conformal scalar field…Why?

The minimal HS theory

- So far we have always assumed that Newton's constant is given by $G_N^{-1} = cN$. But there can in principle be subleading corrections in the map between G_N and N.
- Because the one-loop piece is precisely proportional to the expected classical piece, this suggests that the result can be consistent with the duality if we assume a shift N->N-1, i.e. $G_N^{-1} = c(N-1)$, so that the classical piece is

$$\frac{1}{G_N} F^{(0)}_{\rm min} = (N-1)\left(\frac{\log 2}{8} - \frac{3\zeta(3)}{16\pi^2}\right)$$

combined with the one-loop piece would give the expected result for F.

One-loop shift

- This effect may perhaps be thought as a finite "one-loop renormalization" of the bare coupling constant in Vasiliev's theory, somewhat similar to the one-loop shift of the level in Chern-Simons gauge theory
- The fact that the shift is simply an integer is consistent with the idea that the coupling constant in Vasiliev's theory should be quantized (*Maldacena-Zhiboedov*).

General dimensions

- There is a formulation of Vasiliev's theory in arbitrary dimensions. The equations of motion have a AdS_{d+1} vacuum solution, and the linearized spectrum around this background is

$$\text{Spectrum}: \qquad s = 1, 2, 3, \ldots, \infty \qquad \text{gauge fields}$$
$$s = 0, \quad m^2 = -2(d-2) \quad \text{scalar}$$

- This spectrum is in one-to-one correspondence with the single trace primaries of a free scalar vector model in dimension d (the scalar bilinear has dimension Δ=d-2). This generalizes the type A theory to all *d*.
- The spectral zeta functions for HS fields in general dimensions are also known (*Camporesi-Higuchi*)

One loop tests in general dimensions

- It is then natural to repeat the one loop calculations in general dimensions (*SG, Klebanov, Safdi*).
- In all odd d (even dim. AdS), there are UV logarithmic divergences spin by spin. Summing over all spins, the UV divergence always vanishes. Vasiliev theory is one-loop UV finite in *any* dimension.
- Finite part of $F^{(1)}$ is consistent with AdS/CFT in all dimensions (for minimal theories with even spins only, this requires the shift N->N-1 as found in AdS_4).

Interacting 5d O(N) model

- Can we have interacting vector models in higher dimensions?
- The possible b.c.'s for bulk scalar in Vasiliev theory on AdS_{d+1} are Δ=d-2 (free theory) and Δ=2. The latter is not unitary if d>6.
- The scalar large N model with $\frac{\lambda}{4}(\phi^i\phi^i)^2$ interaction is IR trivial in d>4, but it has unitary large N UV fixed points for 4<d<6.
- In d=5, this large N UV fixed point should be dual to Vasiliev theory in AdS_6 with Δ=2 boundary condition.

Interacting 5d O(N) model

- The d>4 UV fixed point can also be seen formally in the Wilson-Fisher ε-expansion in d=4+ε. But it requires a negative coupling constant.
- Is there an alternative description of this interacting scalar CFT as the IR fixed point of another theory?
- Proposal: look for IR fixed points in the cubic O(N) symmetric theory in d=6-ε. (*Fei, SG, Klebanov*)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{g_1}{2}\sigma\phi^i\phi^i + \frac{g_2}{6}\sigma^3$$

Perturbative fixed points in d=6-ε

- The one-loop beta functions in d=6-ε (*Fei, SG, Klebanov)*

$$\beta_1 = -\frac{\epsilon g_1}{2} + \frac{(N-8)g_1^3 - 12g_1^2 g_2 + g_1 g_2^2}{12(4\pi)^3}$$
$$\beta_2 = -\frac{\epsilon g_2}{2} + \frac{-4N g_1^3 + N g_1^2 g_2 - 3g_2^3}{4(4\pi)^3}$$

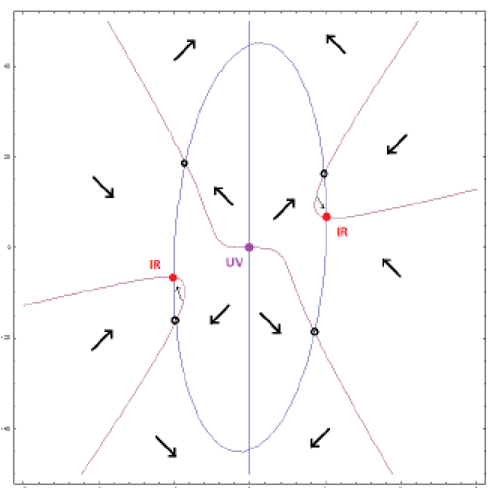
- At large N, one finds a unitary, IR stable fixed point at *real* couplings

$$g_1^* = \sqrt{\frac{6\epsilon(4\pi)^3}{N}}\left(1 + \frac{22}{N} + \dots\right), \qquad g_2^* = 6\sqrt{\frac{6\epsilon(4\pi)^3}{N}}\left(1 + \frac{162}{N} + \dots\right)$$

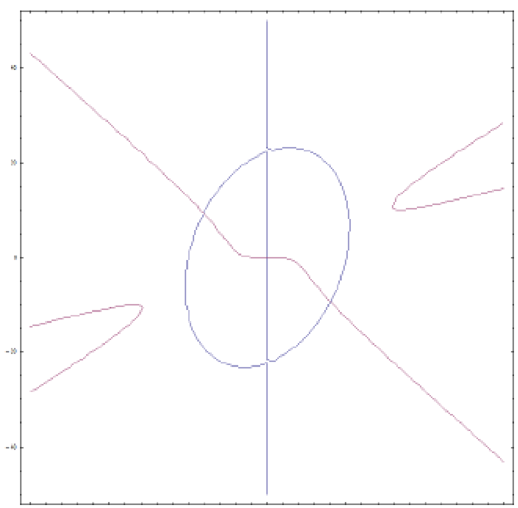
- The conformal dimensions of operators at the fixed point can be computed to any order in the 1/N expansion. They indeed precisely match all the known results in the large N $\frac{\lambda}{4}(\phi^i\phi^i)^2$ model (*Vasiliev et al; Petkou; Lang-Ruhl*) continued to 4<d<6.

Critical N

- For N=0, it is known that the cubic model has non-unitary IR fixed point at imaginary coupling (*M. Fisher '78*). Do the large N real fixed points persist at finite N?
- Suprising result: the IR stable unitary fixed point only exist for N>1038! (They become complex for smaller N).



N=2000



N=500

- Large N expansion breaks down very early for these models.

Critical N

- This critical value of N comes from a one-loop calculation near d=6. Can the actual value be lower in d=5?
- A 3-loop calculation in d=6-ε (*Fei, SG, Klebanov, Tarnopolsky, in progress*) indicates that

$$N_{\text{crit}} = 1038 - 610\epsilon - 364\epsilon^2 + O(\epsilon^3)$$

- So the critical N appears to significantly decrease as we approach d=5.
- The 4+ε expansion, and large N methods in d=5, suggest a critical N of order $N_{crit} \sim$ 10-30.
- Nevertheless, the existence of a critical N seems to be a real effect which is a peculiarity of the 4<d<6 fixed points.
- It should have interesting consequences for the stability/unitarity of higher spin gravity in AdS_6 (with alternate b.c.) at the non-perturbative level. No obvious problems are seen in 1/N perturbation theory.

Conclusion and summary

- Consistent interacting theories of massless higher spins can be constructed if the cosmological constant is non-zero. They involve infinite towers of fields of all spins.
- The Vasiliev theory in AdS was conjectured to be exactly dual to simple vector model CFT's.
- We recently obtained new simple tests of higher spin/vector model dualities, by comparing partition functions on both sides of the duality.
- Vasiliev theory appears to be *one-loop finite* in all dimensions. A ``UV complete" model of quantum gravity?

Conclusion and summary

- Vasiliev theories provide exact AdS dual not only to free theories, but also to interesting interacting theories such as the critical O(N) model, the Gross-Neveu model, CP^N model, theories involving Chern-Simons gauge fields...
- The large N UV fixed point of the scalar O(N) model in d=5 should be dual to Vasiliev theory in AdS_6 with suitable b.c. We found a new description of the UV fixed points in $4<d<6$ as IR fixed points of a cubic theory with N+1 fields. Are these theories well defined/stable nonperturbatively?

Conclusion and summary

- More to be done in higher spin AdS/CFT: higher point/higher loop correlation functions; understand action principle for Vasiliev equations; find a prescription for entanglement entropy; study non-trivial solution of the theory (black holes; mass deformations of the CFT…); relation to string theory;…