Thermodynamics of holographic models for QCD in the Veneziano limit

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Outline

- The model
- Thermodynamics
- Phenomenological improvement of the hadron gas
- Conclusions

Motivation

Often in holography, the quenched approximation $N_f \ll N_c$, is used. In contrast, the Veneziano limit $N_f \sim N_c$, $\frac{N_f}{N_c}$ finite allows access to

- The QCD phase diagram and thermodynamics as a function of N_f
- finite baryon density
- and more...

Veneziano QCD

Veneziano QCD is a YM theory with N_c colors and N_f fermion flavors, at the limit $N_c, N_f \to \infty$ but $x_f \equiv \frac{N_f}{N_c}$ constant.

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A holographic string-inspired bottom-up model:

- start with gravity + dilaton
- dilaton potential related to the beta function of the field theory
- Add a tachyonic scalar and a DBI -action for it.
- A U(1) gauge field in the DBI action is dual to net quark density $q^\dagger q$.
- Scalar potentials not uniquely fixed

[Järvinen, Kiritsis, arXiv:1112:1261
TA, Järvinen, Kajantie, Kiritsis, Tuominen arXiv:1210.4516
TA, Järvinen, Kajantie, Kiritsis, Rosen, Tuominen arXiv:1312.5199
Arean, Iatrakis, Järvinen, Kiritsis arXiv:1309.2286
]

VQCD Action

The full action is

$$S = \frac{1}{16\pi G_5} \int d^5x \, \mathcal{L}, \tag{1}$$

where

$$\mathcal{L} = \left[\sqrt{-g} \left(R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right) \right. \tag{2}$$
$$\left. - V_f(\lambda, \tau) \sqrt{\det \left(g_{ab} + \kappa(\lambda, \tau)(D_a T)^* (D_b T) + \omega(\lambda, \tau) F_{ab} \right)} \right] .$$

The metric Ansatz is

$$ds^2 = b^2(r) \left[-f(r) dt^2 + d\mathbf{x}^2 + \frac{dr^2}{f(r)} \right], \qquad b(r) = \frac{\mathcal{L}_{UV}}{r} \text{ in the UV}, \tag{3}$$

and the two scalar functions, $1/\lambda$ sourcing F^2 and τ sourcing $\bar{q}q$, are

$$\lambda = \lambda(r) = e^{\phi(r)} \sim N_c g^2, \quad \tau = \tau(r), \text{where } T = \tau \mathbb{1}. \tag{4}$$

Potentials

We need to choose V_g, V_f, κ and ω. String inspired $V_f(\lambda,\tau) = e^{-a(\lambda)\tau^2} V_{f0}(\lambda)$, and

- When $\tau \equiv 0$, simplifies to gravity-dilaton with $V_g - V_{f0}$ as the dilaton potential. Fix to perturbative β-function, as a function of x_f.
- κ asymptotes to $\lambda^{-4/3}$ to have correct tachyon divergence in the IR, should be a power series in UV. An extra logarithmic factor gives linear meson trajectories.
- One ansatz: $\kappa(\lambda) = \frac{1}{(1+\kappa_0\lambda)^{4/3}\sqrt{1+\log(1+\lambda)}}$.
- Setting $\omega = \kappa$ would be simplest, but ω should vanish slower than κ to give the same trajectory for the vector and axial vector mesons

Chiral symmetry breaking

Quark mass m_q and chiral condensate σ:

$$\tau(r) = m_q r \log(r)^a + \sigma r^3 \log(r)^{-a} + \dots \tag{5}$$

Consider $m_q = 0$ solutions in this talk:

- $\tau \equiv 0$ corresponds to a chirally symmetric phase
- $\tau \neq 0$ gives chiral symmetry breaking

Finding solutions

We need to find all regular, $m_q = 0$ solutions to the e.o.m.'s, and order the solutions corresponding to each T, μ according to pressure:

- Two vacuum solutions, $\tau \equiv 0$ and $\tau \neq 0$, fixed by $m_q = 0$
- BH solutions, two branches, $\tau_h \equiv \tau(r_h)$ either zero, or again fixed by $m_q = 0$
- BH initial conditions for numerics from a near-horizon expansion
- two free parameters in both branches: $(\lambda_h, \tilde{n})$, where $\tilde{n} \propto \frac{n}{s}$
- $\tilde{n} = 0$ gives $\mu = 0$

Thermodynamics

Any BH solution to the equations of motion, corresponding to a pair $(\tilde{n}, \lambda_h)$ and a choice of tachyon or no tachyon, gives

$$
\begin{aligned}
T &= -\tfrac{1}{4\pi} f'(r_h; \tilde{n}, \lambda_h) & s &= \frac{1}{4G_5} b^3(\tilde{n}, \lambda_h) \\
\mu &= \lim_{r\to 0} A_0(r; \tilde{n}, \lambda_h) & n &= \frac{\mathcal{L}_A^2}{4\pi} b^3(\tilde{n}, \lambda_h)\tilde{n}.
\end{aligned}
\tag{6}
$$

In addition, both vacuum solutions can be compactified to any T, μ. Of these, $\tau \neq 0$ thermodynamically preferred when $x_f < x_c \Rightarrow$ chiral symmetry breaking

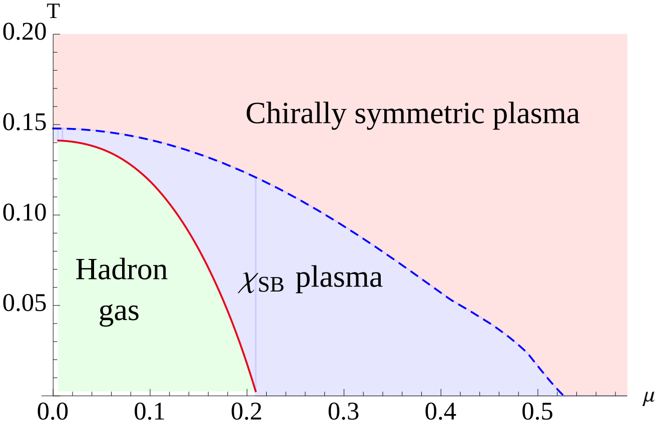
It is now simple in principle to compute phase diagrams and extract thermodynamic observables:

- Compute numerically a number of solutions for various values of $\tilde{n}, \lambda_h$
- At each value of (μ, T), order the corresponding solutions according free energy
- Compute observables from the thermodynamically favored solution

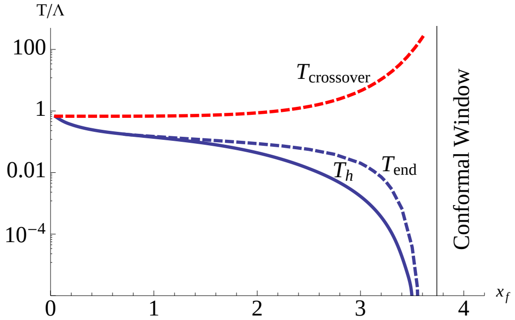
However, lots of bookkeeping and other technical details. Mathematica code for automating this is available at

github.com/timoalho/VQCDThermo

Phase diagram, $x_f = 1$



$\mu = 0$ as a function of x_f



- A conformal window at $x_c < x_f < 5.5$. Generically and independent of the potential $x_c \sim 4$; here $x_c = 3.8$
- Miranski scaling when approaching x_c
- A deconfinement transition T_h, followed by a chiral symmetry restoring transition T_{end}
- At higher T, a crossover related to walking behavior (very weak at small x_f).

$T = 0$, extremal black holes

For finite $\mu, T = 0$, need to look at extremal solutions, $f'(r_h) = 0$. In the chirally symmetric phase:

- near-horizon geometry is $AdS_2 \times R^3$ as expected
- but this is essentially just one solution after fixing all scales
- We need a family of solutions corresponding to all values of μ
- need a more general power series near the horizon, with fractional powers of $(r - r_h)$
- single independent coefficient in the expanded series parametrizes μ
- the full solution can then be obtained numerically

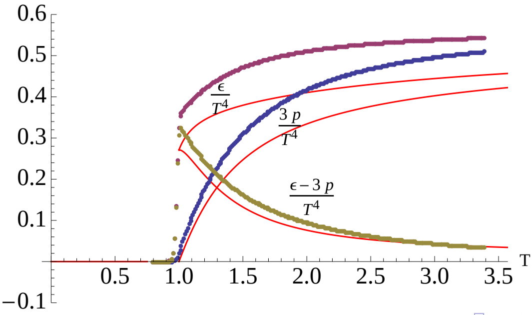
However, the $T = 0$ chiral symmetry breaking phase is still under investigation.

Finetuning the potentials

Exact form of the potentials need to be fitted:

- two parameter fit to lattice done in IHQCD, $x_f = 0$, works well
- at finite x_f, the same fit does not work
- need to add non-analytic terms

$$V_g \sim \ldots + 36 e^{-\frac{1}{2\lambda}} (2\lambda)^{4/3} \times \text{analytic} \tag{7}$$



The hadron gas phase

The hadron gas phase seems problematic:

- nothing depends on temperature
- therefore, $p_{HG} = 0$
- hadron gas in perturbation theory has $p_{low} \sim N_f^2$
- Stefan-Boltzmann limit: $p_{high} \sim 2N_c^2 + \frac{7}{2}N_f N_c$.
- at $x_f = 1$, $p_{low}/p_{high} \sim \frac{2}{11}$, not too bad
- However $p_{low}/p_{high} \to 1$ as $x_f \to 4$.

need a better model for the hadron gas phase

Phenomenological hadron gas model

Try to model the hadron gas dynamics based on the particle spectrum:

- there should be N_f^2 massless Goldstone bosons
- also, meson states computed in arXiv:1309.2286
- the computed meson states are just the few lowest lying states from infinite towers
- approximate the meson towers by a Hagedorn spectrum
- minimum mass from the meson spectrum, or as a free parameter

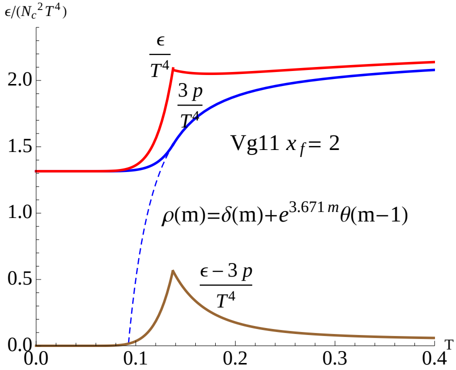
Ansatz for the spectral function:

$$\rho(m) = \frac{\pi^2}{90} x_f^2 \left\{ \delta(m) + \theta(m - m_{\min}) \exp(bm) \right\} \tag{8}$$

Parameters

The ansatz has two free parameters: m_{min} and b.

- m_{min} could be set to equal the minimum mass in the computed meson spectrum (just a fit below, though)
- adjust b to get second order deconfinement transition
- not trivial that this is possible, but seems to work this far



Consistency

The HG model is very much a work in progress still, but some thoughts

- we've considered free particles, but interactions could change the picture
- might be possible to compute as 1-loop corrections to the gravity dual
- would at least need to estimate 1-loop corrections to the BH phase for consistency

Conclusions

- We can compute the full T, μ phase diagram of VQCD, given the potentials
- Some dependence on model specifics, although many features are generic.
- Computing the thermodynamic backgrounds is now fully automated, code available at github.com/timoalho/VQCDThermo

Outlook

- Finish constraining the potentials by matching to QCD
- Mapping out the finite T, μ phase diagram as a function of x_f.
- More thermodynamical observables.
- $m_q > 0$ in detail (Järvinen will talk about this at $T = 0$)
- Finding the extremal tachyonic solutions
- Nature of the dense matter at large μ, $T = 0$.

That's all, folks! Thank you!