

Multiboundary wormholes and holographic entanglement

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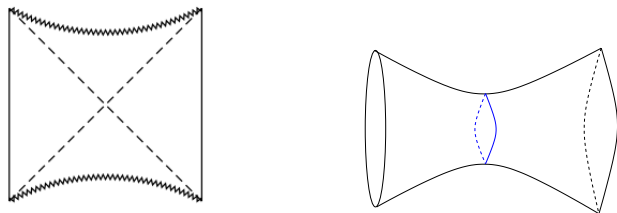
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Entanglement and wormholes

Entanglement in CFT related to wormholes/connectedness in the bulk.

First example: Eternal black hole

Maldacena



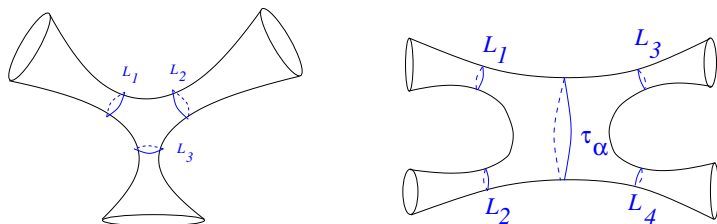
State dual to bulk $t = 0$ surface defined by path integral over half of Euclidean black hole: TFD state

$$|\psi\rangle = \frac{1}{\sqrt{\mathcal{Z}}} \sum_i e^{-\beta E_i/2} |E_i\rangle_1 \otimes |E_i\rangle_2$$

Entangled state explains why $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \neq 0$.

Entanglement and wormholes

Generalize this to multiboundary wormholes:



- Additional explicit examples (in 2+1)
- Σ a Riemann surface with n boundaries (for simplicity, no handles). Moduli minimal geodesic lengths L_a , internal moduli τ_α for $n > 3$.
- Explore multipartite entanglement

Entanglement and wormholes

Multipartite entanglement:

- Entanglement \neq Bell pairs.
- E.g., GHZ state: $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle)$. Entangled pure state.
- Trace over any one spin gives $\rho = \frac{1}{2}(|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\downarrow\downarrow\rangle\langle\downarrow\downarrow|)$.
Unentangled. \Rightarrow Entanglement intrinsically involves all three spins.
- Does this type of entanglement play a role in holography?

Entanglement and wormholes

Key results:

- Nature of entanglement depends on moduli
 - ▶ Purely bipartite for some range of moduli
 - ▶ No entanglement on $< n/2$ boundaries for generic moduli
 - ▶ At least some n -party ($(n - 1)$ -party) entanglement for generic moduli
- Bulk phase transitions as functions of moduli
- Random state model captures overall entanglement structure

Study in $2 + 1$ bulk: quotients of AdS_3 by a discrete group

$$\Gamma \subset SL(2, \mathbb{R}) \subset SL(2, \mathbb{R}) \times SL(2, \mathbb{R}).$$

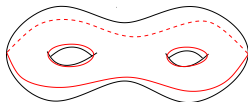
- Acts within $t = 0$ surface, $\Sigma = H^2/\Gamma$.
- Minimal geodesics L_a are bifurcation surfaces for event horizons; outside geometry is BTZ.
- Geometry has no global timelike Killing vector, but time-reflection symmetry about $t = 0$ implies it has a Euclidean continuation.

Bulk phase transitions

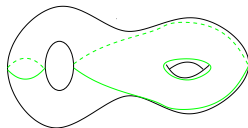
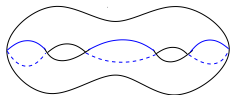
Corresponding action on Euclidean AdS_3 :

$$ds^2 = d\tau^2 + \cosh^2 \tau d\Sigma^2.$$

Euclidean boundary is two copies of Σ . E.g., for $n = 3$, a genus 2 surface.



Bulk is a handlebody filling in the boundary Riemann surface. \exists different bulks for same boundary conditions:

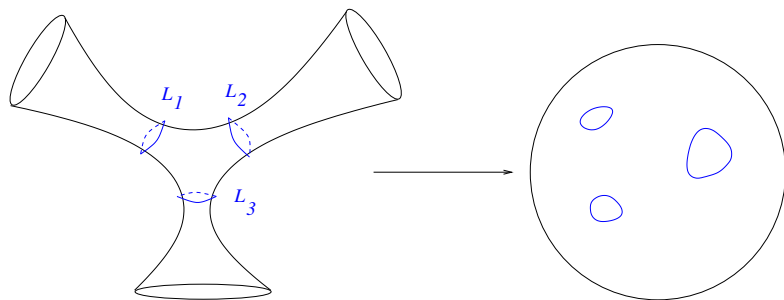


Phase transitions analogous to Hawking-Page. Can restrict possibilities by choosing a spin structure incompatible with making boundary circles contractible.

On Euclidean AdS_3 , boundary is two copies of Σ .

\Rightarrow state $|\Sigma\rangle$ on $t = 0$ surface given by CFT path integral on Σ .

Map to sphere with holes: $|\Sigma\rangle$ related to n -point functions.



Dual CFT description

In the puncture limit $L_a \rightarrow 0$, can approximately control this map. E.g.,

$$|\Sigma_{123}\rangle = \sum_{ijk} C_{ijk} e^{-\frac{1}{2}\tilde{\beta}_1 H_1 - \frac{1}{2}\tilde{\beta}_2 H_2 - \frac{1}{2}\tilde{\beta}_3 H_3} |i\rangle_1 |j\rangle_2 |k\rangle_3,$$

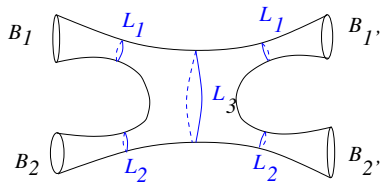
where C_{ijk} are OPE coeffs, encoding dependence on the CFT considered, and $\tilde{\beta}_i$ are roughly inverse temperatures of the BTZ regions, encoding moduli dependence.

- Separation of CFT dependence, moduli dependence
- Boltzmann-like suppression of high energy contributions
- State has tripartite entanglement, but vacuum state contribution (e.g. $i = \mathbb{I}$) gives a bipartite component
- In this $L_a \rightarrow 0$ limit “thermal AdS” is dominant bulk saddle if it’s permissible.

Entanglement structure

Consider partial traces to diagnose entanglement structure.

For $n = 3$, $\rho_{12} = \text{Tr}_3(|\Sigma_{123}\rangle\langle\Sigma_{123}|)$ given by four-boundary Riemann surface \sim four-point function.



- $L_3 \rightarrow 0$ s-channel limit: $\rho_{12} \approx |\Sigma_{12}\rangle\langle\Sigma_{12}|$
- $L_3 \rightarrow \infty$ t-channel limit: $\rho_{12} \approx \rho_1 \otimes \rho_2$. Implies $|\Sigma_{123}\rangle$ just entangles 1 with 3, 2 with 3. Purely bipartite in this limit!

Entanglement structure from holography

- Compute entanglement entropies at leading order in central charge from minimal surfaces in bulk.
- Ryu-Takayanagi prescription:

$$S_A = \min_{\gamma} A_{\gamma}/4G,$$

where the bulk surface γ is homologous to the region A .

- Since we consider A to be the whole of one or more boundaries, γ is a closed surface in the bulk. Natural candidate is some combination of the L_a 's.

Entanglement structure from holography

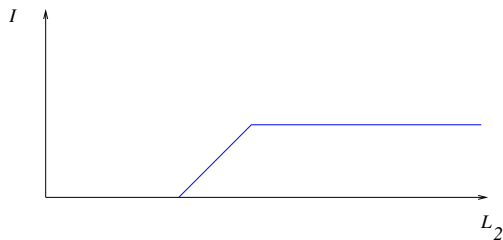
E.g. $n = 3$:

$$S(\rho_{12}) = \frac{\pi}{2G}(L_1 + L_2) \text{ if } L_1 + L_2 < L_3,$$

$$S(\rho_{12}) = \frac{\pi}{2G}(L_3) \text{ if } L_1 + L_2 > L_3, \text{ so}$$

$$I(B_1 : B_2) = S(\rho_1) + S(\rho_2) - S(\rho_{12})$$

vanishes if $L_1 + L_2 < L_3$.



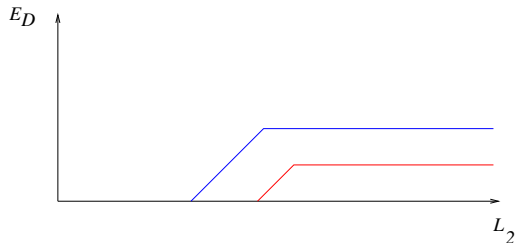
Entanglement measures

$I = 0 \Rightarrow$ no entanglement, but $I \neq 0$ does not imply entanglement; could be classical correlation.

A useful measure is **distillable** entanglement E_D

- Number of Bell pairs which can be obtained from the state ρ_{12}
- Bounded by calculable quantities:
 - ▶ $E_D \leq \min(S(\rho_1), S(\rho_2), I(B_1 : B_2))$
 - ▶ $E_D \geq S(\rho_1) - S(\rho_{12})$.

Holographically,



More boundaries

- If $L_a \approx L$, $S(B_a \cup B_b) = S(B_a) + S(B_b)$, so $I(B_a : B_b) = 0$: no bipartite entanglement.
- Regions of purely bipartite entanglement when say $L_1 \gg L_a$.
- Diagnose multiparty entanglement by considering $E_D(X : Y)$ where X and Y are each some collection of boundaries. (Note no good intrinsically multiparty entanglement measures). For generic moduli, $E_D(X : Y) > 0$ if $X \cup Y > 1/2$ boundaries.
- For generic moduli, $\exists n$ -party entanglement for n even, $n - 1$ -party entanglement for n odd.

Discussion

- Multiboundary wormholes provide interesting new examples of relation of entanglement to connectedness of geometry
- Rich dependence on moduli: regions of purely bipartite entanglement, generic behaviour involves multiparty entanglement
- Future directions:
 - ▶ Entanglement between subregions of the boundary
 - ▶ Understanding relation to n -point functions away from the puncture limit
 - ▶ Improving our characterisation of multiparty entanglement in general.