Towards a Field Theory over Tensor Network States

Andrew G. Green

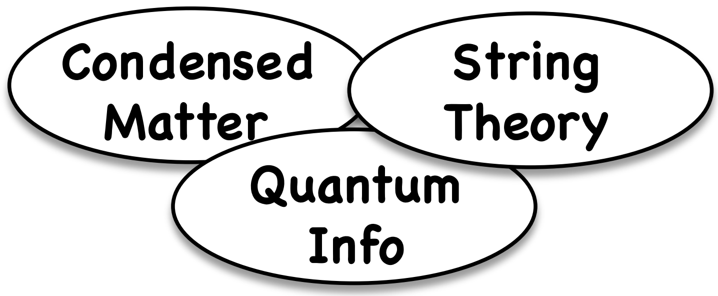
Steve Simon[1]
Chris Hooley[2]
Jonathan Keeling[2]

Philip Crowley
Tanja Duric
Vid Stojevic

[1]University of Oxford, [2]University of St Andrews

Background:

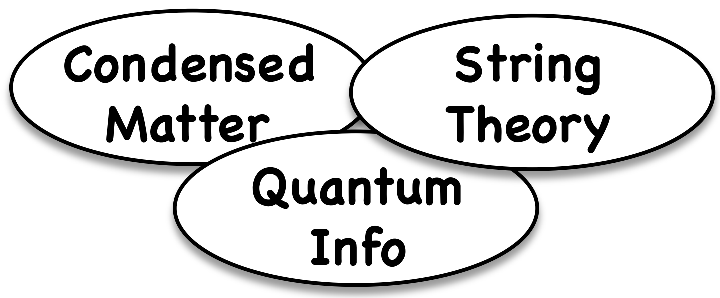
- Convergence of ideas



- Suggestive links
  - Holography vs hierarchical tensor networks
  - Is Holography an entanglement ansatz?
  - Can we capture new strongly-correlated phases?
- Ideas formulated in very different language

Background:

- Convergence of ideas



- Suggestive links

  Holography vs hierarchical tensor networks

  Is Holography an entanglement ansatz?

Need to develop a common Language

Import insights of tensor networks to field theory

Outline:

- Background

    Variational States in Condensed Matter
    Tensor Networks
    RG and Hierarchical Tensor Networks

- Towards a Field Theory Over Tensor Networks

    Goal and Key steps
    Formulating the Field Theory
    Potential Applications
    Extensions to Higher Dimensions
    Extension to Critical Systems

- Conclusions

I Background:

Variational States in Condensed Matter
Tensor Networks and Strong Correlation
Tensor Networks and AdS/CFT

I Background:

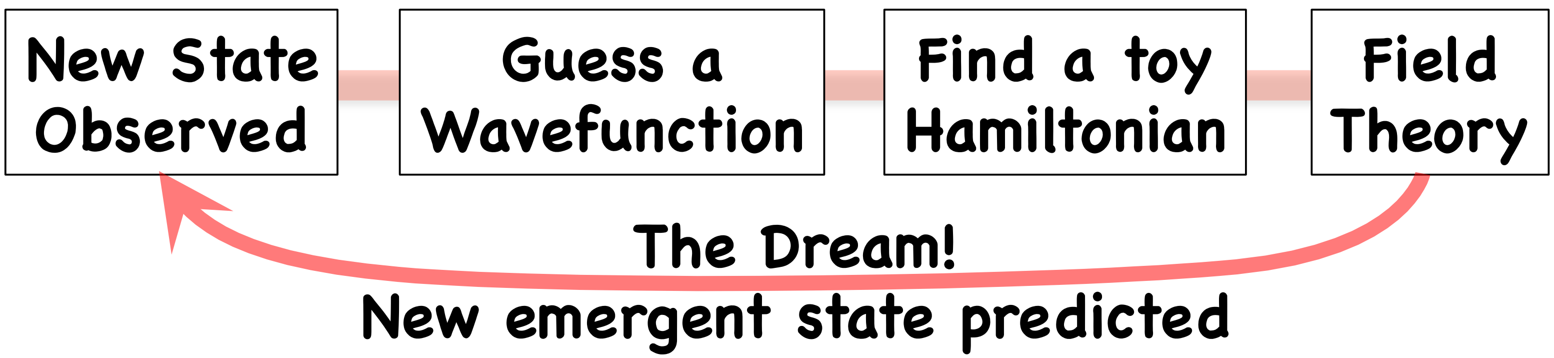
Variational States in Condensed Matter

Tensor Networks and Strong Correlation

Tensor Networks and AdS/CFT

Background

Role of Variational States in Condensed Matter



Examples:

FQHE: Expt[DPvK]->Exact Diag->Wavefunction[Laughlin]->Hamiltonian [Haldane]
->Composite Fermion Picture[Jain/Read]->Field Theory[Lopez/Fradkin]

BCS: Experiment -> Toy Hamiltonian -> Guess wavefunction -> Field Theory

Exceptions:

Haldane Conjecture -> Field Theory -> Expt

Topological Insulator -> Toy Hamiltonian -> wavefunction -> experiment

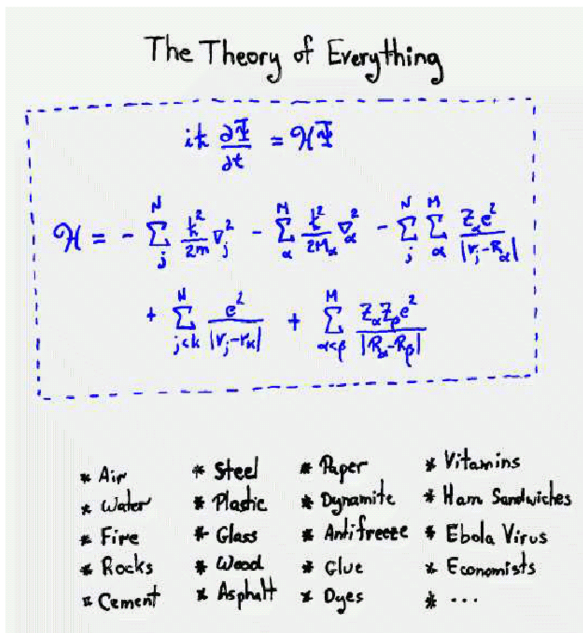
Bethe Ansatz: variational wavefunction -> tremendous power in 1d

Conformal Field Theory, Renormalisation Group... etc. etc.

Background

Ab Initio Ideal/Myth

Ab Initio Hamiltonian — **Big Computer** — **Experimental Prediction**



I Background:

Variational States in Condensed Matter

Tensor Networks and Strong Correlation

Tensor Networks and AdS/CFT

Background

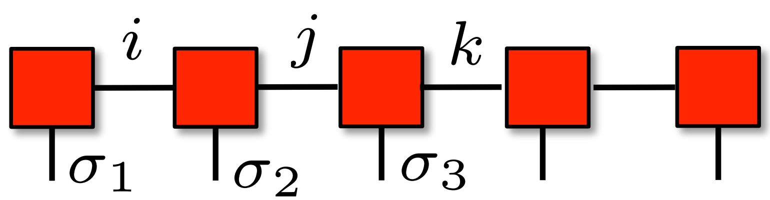
UCL

Tensor Networks

- Class of variational wavefunctions
- Embody insights about entanglement structure
- Describe groundstates of local Hamiltonians efficiently
- Exact for some model H (AKLT, Majumdar-Ghosh, etc)

- Matrix Product States (1d tensor network)

$$|\phi\rangle = \sum_{\{\sigma\}} A_i^{\sigma_1} A_{ij}^{\sigma_2} A_{jk}^{\sigma_3} A_{kl}^{\sigma_4} \ldots |\sigma_1, \sigma_2, \sigma_3, \sigma_4, \ldots\rangle$$



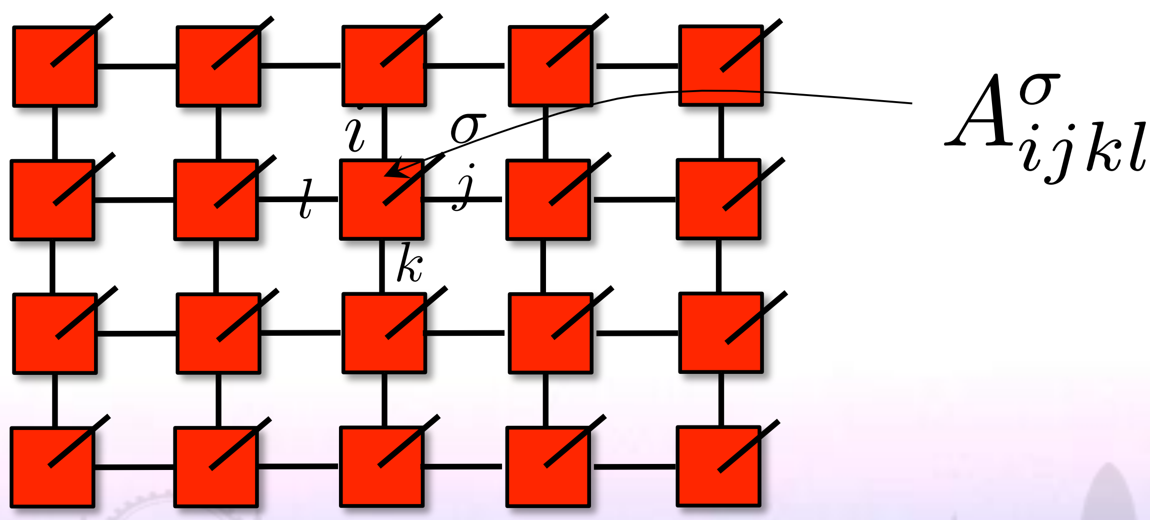
- MPS is a restricted sum of product states
- MPS dense on Hilbert space

Background

Tensor Networks

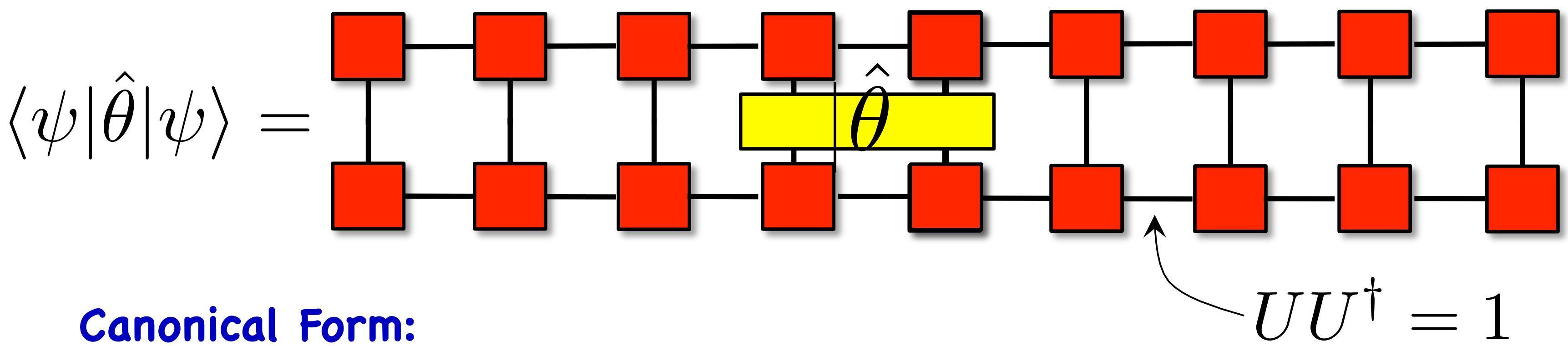
- Class of variational wavefunctions
- Embody insights about entanglement structure
- Describe groundstates of local Hamiltonians efficiently
- Exact for some model H (AKLT, Majumdar-Ghosh, etc)

- PEPS (projected entangled pair state - 2d tensor network)

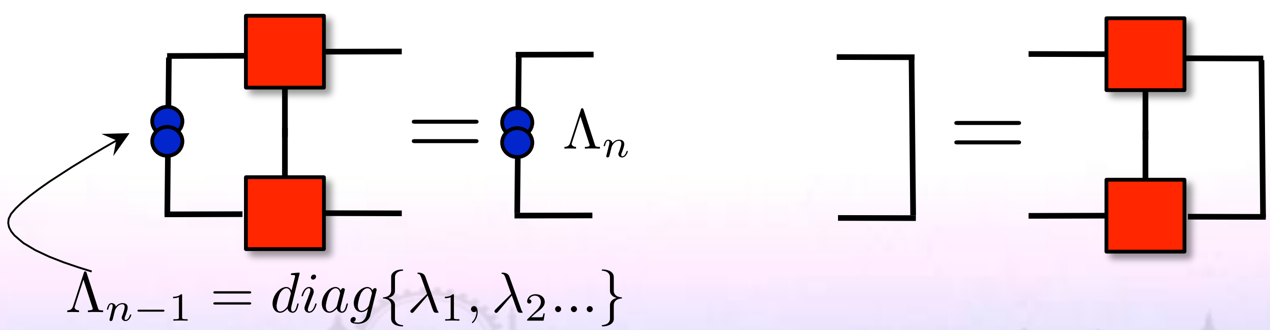


Tensor Networks – Locality and Gauge Fixing

- **Q.** Is the expectation of local operators local in terms of As?
- **A.** Not in general. In 1d can always gauge fix to make it so.



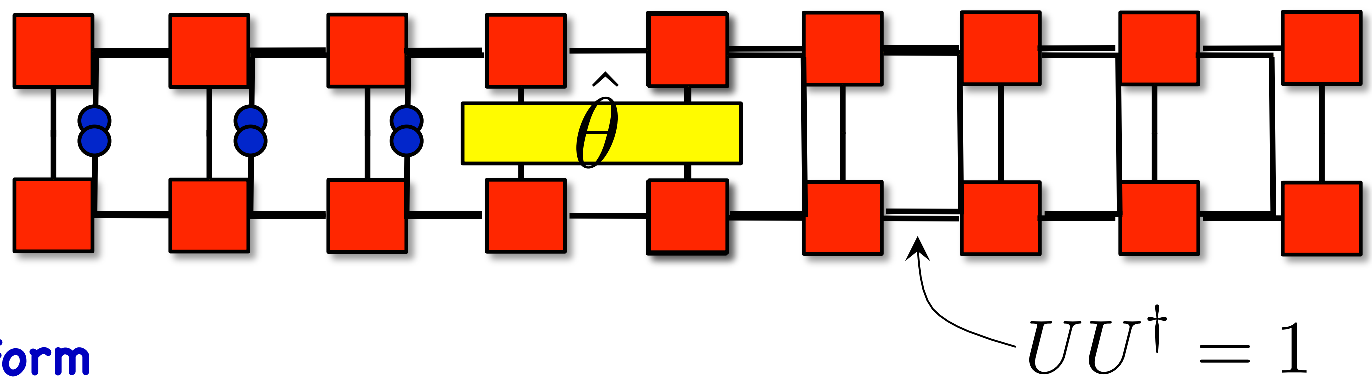
Canonical Form:



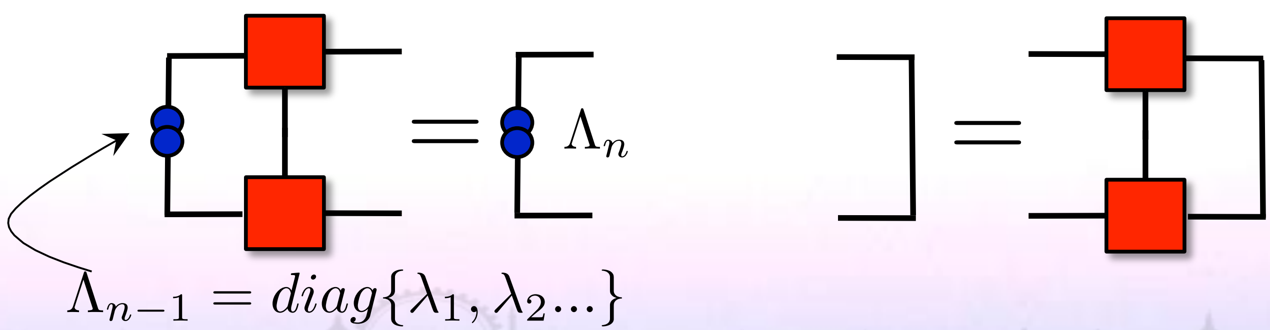
Tensor Networks - Locality and Gauge Fixing

- **Q.** Is the expectation of local operators local in terms of As?
- **A.** Not in general. In 1d can always gauge fix to make it so.

$$\langle\psi|\hat{\theta}|\psi\rangle =$$



Canonical Form



Tensor Networks – Locality and Gauge Fixing

- **Q.** Is the expectation of local operators local in terms of As?
- **A.** Not in general. In 1d can always gauge fix to make it so.

Canonical Form:

- In canonical form, MPS is a Schmidt decomposition $|\phi\rangle = \sum_{\alpha} \lambda_\alpha |\phi_L^\alpha\rangle|\phi_R^\alpha\rangle$ of each bond [Vidal,PRL91,147902,(2003)]

$$|\phi\rangle = \sum_{\{\sigma\}} A_i^{\sigma_1} A_{ij}^{\sigma_2} A_{jk}^{\sigma_3} A_{kl}^{\sigma_4} \ldots |\sigma_1, \sigma_2, \sigma_3, \sigma_4, \ldots\rangle$$

vs

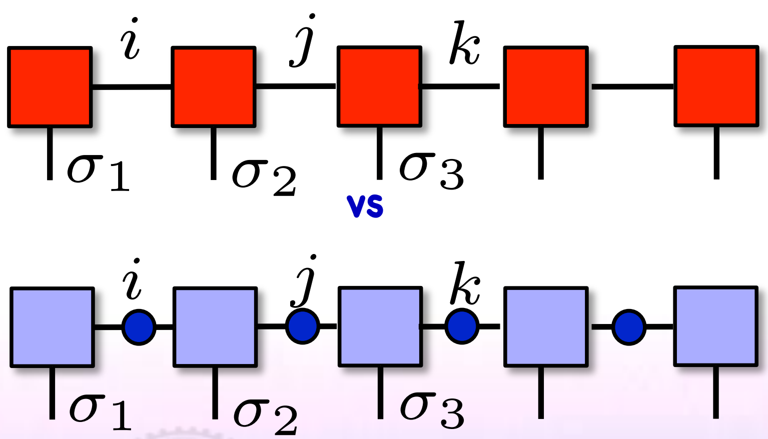
$$|\phi\rangle = \sum_{\{\sigma\}} \Gamma_i^{\sigma_1} \lambda_i^1 \Gamma_{ij}^{\sigma_2} \lambda_i^2 \Gamma_{jk}^{\sigma_3} \lambda_i^3 \Gamma_{kl}^{\sigma_4} \lambda_i^4 \ldots |\sigma_1, \sigma_2, \sigma_3, \sigma_4, \ldots\rangle$$

Tensor Networks – Locality and Gauge Fixing

- **Q.** Is the expectation of local operators local in terms of As?
- **A.** Not in general. In 1d can always gauge fix to make it so.

Canonical Form:

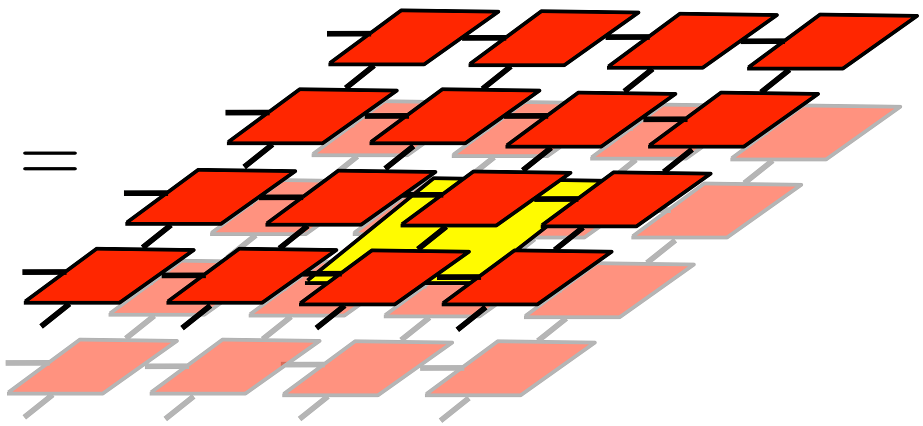
- In canonical form, MPS is a Schmidt decomposition $|\phi\rangle = \sum_\alpha \lambda_\alpha |\phi_L^\alpha\rangle |\phi_R^\alpha\rangle$ of each bond [Vidal,PRL91,147902,(2003)]



Tensor Networks – Locality and Gauge Fixing

- Q. Is the expectation of local operators local in terms of As?
- A. Not in general. In 2d (unlike 1d) there is no exact local form

$$\langle\psi|\hat{\theta}|\psi\rangle =$$



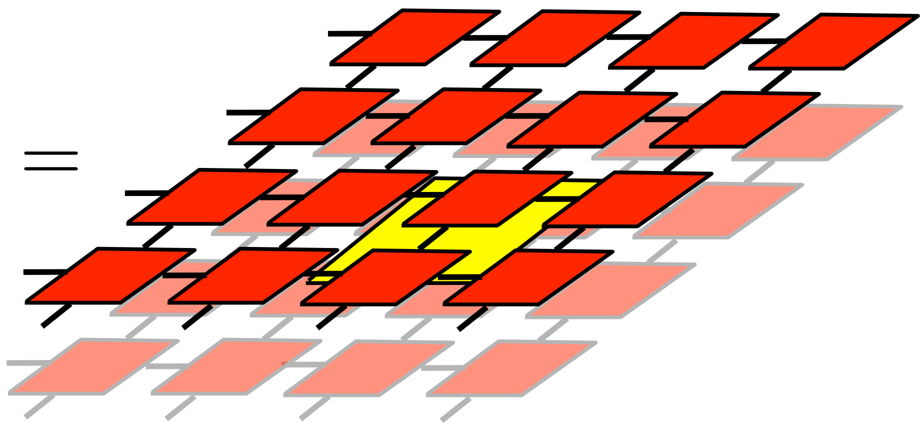
- Physically, expect entanglement to decay, so should be local
- Obey area laws by construction
- Numerically, various approximation schemes seem to work
- Similar to maximally localised Wanier orbitals? [Mazari et al Rev Mod Phys (2012)]

Background

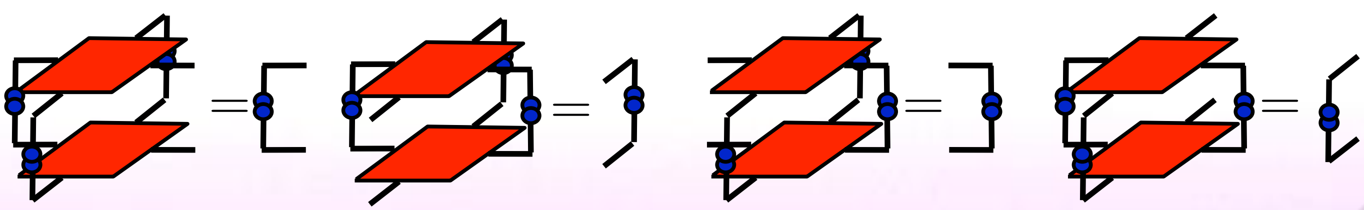
Tensor Networks – Locality and Gauge Fixing

- **Q.** Is the expectation of local operators local in terms of As?
- **A.** Not in general. In 2d (unlike 1d) there is no exact local form

$$\langle\psi|\hat{\theta}|\psi\rangle =$$



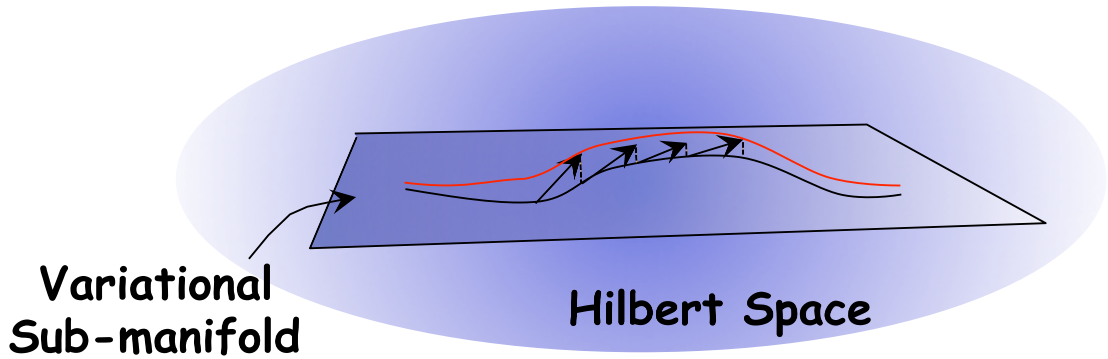
Quasi-Canonical Form:



Background

Time dependence of Tensor Network States

- Fixed bond order – restricted sub-manifold of Hilbert space
- Higher bond order – higher dimension sub-manifold



Time Dependent Variational Principle:

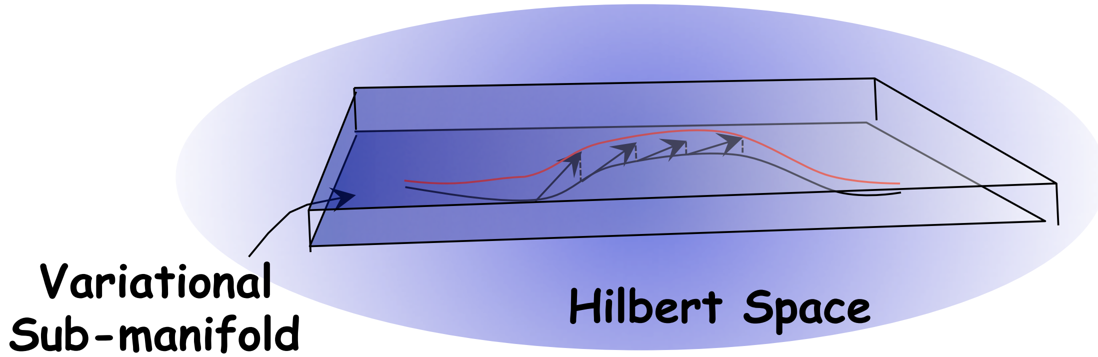
- Bond-order grows under Hamiltonian evolution
- Continually Project back to fixed bond order

$$\langle \partial_{\bar{A}_i} \psi | \partial_{A_j} \psi \rangle \dot{A}_j = \langle \partial_{\bar{A}_i} \psi | \hat{\mathcal{H}} | \psi \rangle$$

Background

Time dependence of Tensor Network States

- Fixed bond order – restricted sub-manifold of Hilbert space
- Higher bond order – higher dimension sub-manifold



Time Dependent Variational Principle:

- Bond-order grows under Hamiltonian evolution
- Continually Project back to fixed bond order

$$\langle \partial_{\bar{A}_i} \psi | \partial_{A_j} \psi \rangle \dot{A}_j = \langle \partial_{\bar{A}_i} \psi | \hat{\mathcal{H}} | \psi \rangle$$

I Background:

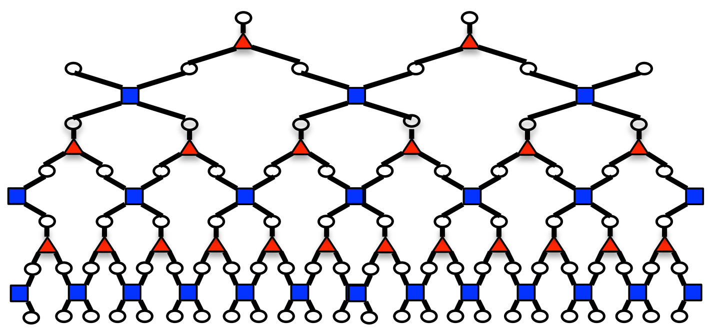
Variational States in Condensed Matter
Tensor Networks and Strong Correlation
Tensor Networks and AdS/CFT

Background

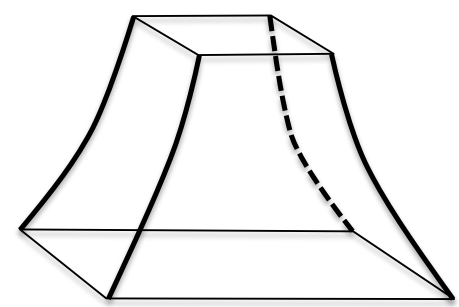
RG and Hierarchical Tensor Networks

- Critical systems don't obey area laws
- Exponentially large bond order required
- Scaling suggests a more efficient way to encode

MERA (multi-scale entanglement renormalisation ansatz)
[Vidal Phys Rev Lett 101, 110501 (2008)]



vs



Similarity to AdS [Swingle Phys Rev D86, 065007 (2012); ArXiv1209.3304]

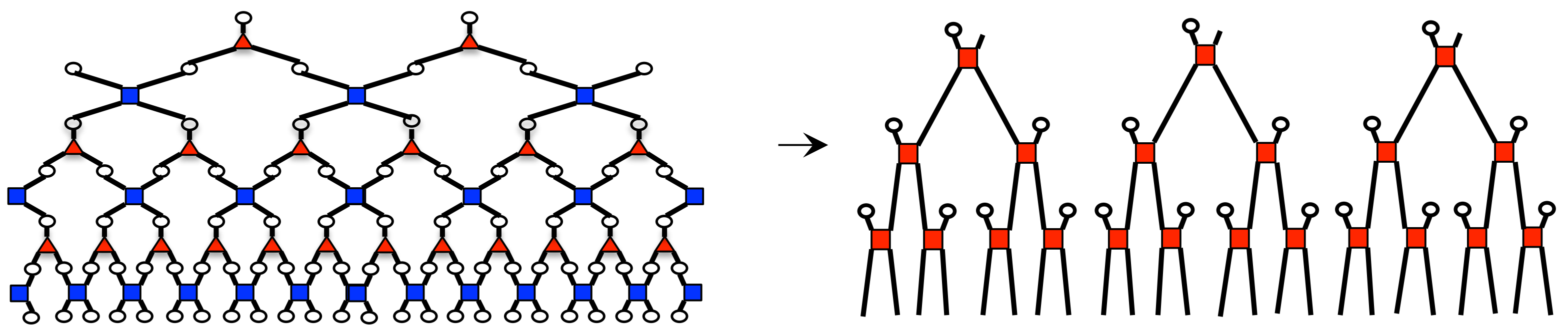
- Extends 1d-2d
- Extra dimension - entanglement RG scale
- Entanglement minimal surface [Ryu, Takayanagi PRL 96, 181602 (2006)]
- Finite T -> finite extent

Background

RG and Hierarchical Tensor Networks

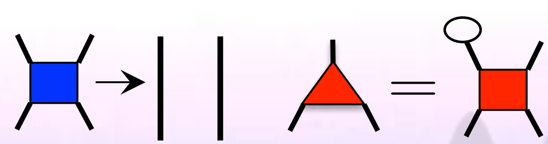
- Critical systems don't obey area laws
- Exponentially large bond order required
- Scaling suggests a more efficient way to encode

MERA (multi-scale entanglement renormalisation ansatz)
[Vidal Phys Rev Lett 101, 110501 (2008)]

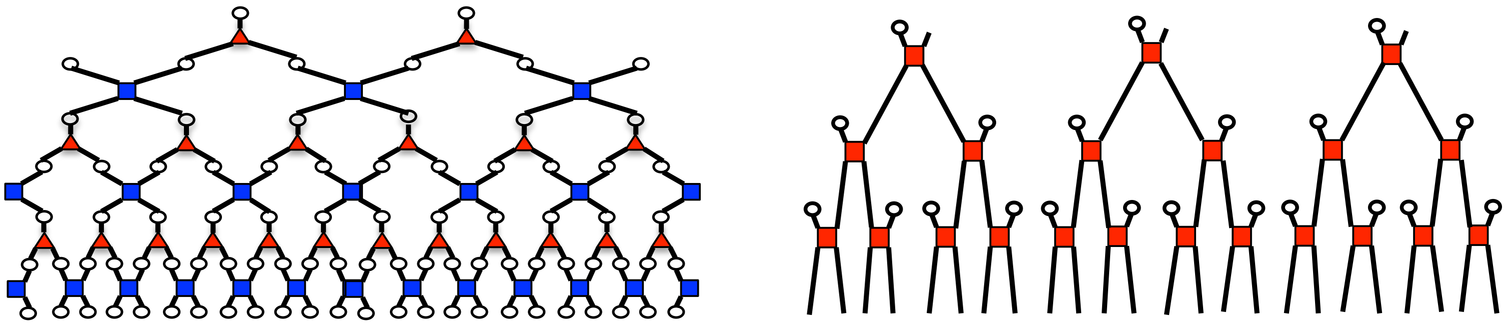


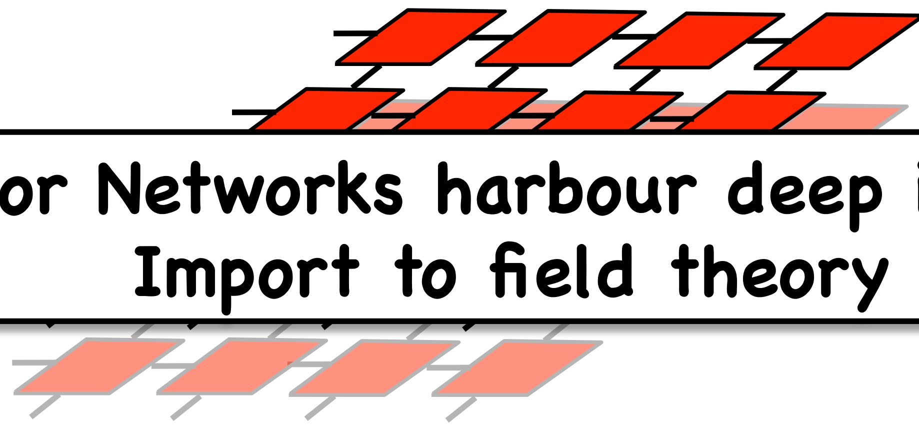
Exact Holographic mapping [Xiao-Liang Qi arXiv:1309.6282]

- Unitary transformation to disk
- Wavelet transform on Cayley tree
- Residual entanglement -> metric

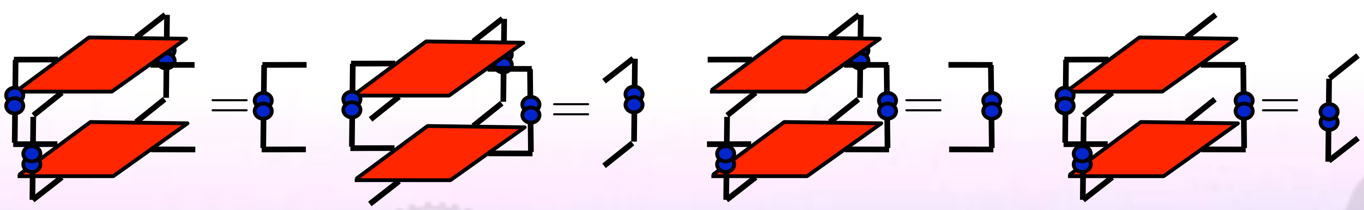


Background





Tensor Networks harbour deep insights
Import to field theory



II Towards a Field Theory over Tensor Network States:

Goal and Key Steps
Formulating the Field Theory
Special Cases
Potential Applications
Extensions to Higher Dimensions and Critical Systems

II Towards a Field Theory over Tensor Network States:

Goal and Key Steps

Formulating the Field Theory

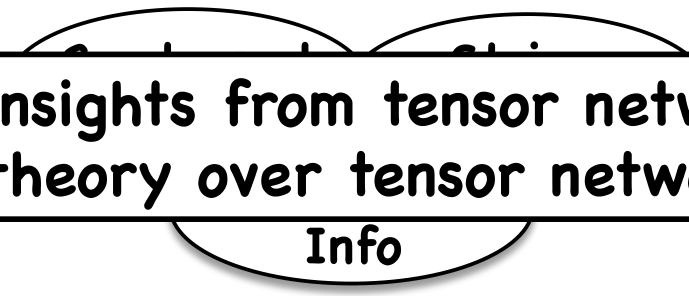
Special Cases

Potential Applications

Extensions to Higher Dimensions and Critical Systems

Towards a Field Theory over MPS

Goal



Import insights from tensor networks into a Field theory over tensor network states

$$\mathcal{Z} = Tr\ e^{-\beta \mathcal{H}} = \int [DA] e^{-\mathcal{S}[A]}$$

MPS data

Advantages

- Saddle points=> TDVP (time-dependent variational principle)
- Fluctuations – expansion about saddle or increase bond order
- Field theory treatment of gauge freedoms in MPS?
- Extension to higher dimensions?
- Various potential applications…

Key Steps

Import insights from tensor networks into a Field theory over tensor network states

$$\mathcal{Z} = Tr\ e^{-\beta \mathcal{H}}$$

$$\mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}$$

$$\mathbb{1} = \int D\psi |\psi\rangle\langle\psi|$$

$$= \int D\psi e^{\int d\tau \left[\langle\psi|\partial_\tau \psi\rangle - \langle\psi|\hat{\mathcal{H}}|\psi\rangle\right]}$$

- **Insert resolutions of identity over over-complete set**
- **Usually $|\psi\rangle$ product states**
- **Can we do the same with matrix product states?**

Towards a Field Theory over MPS

Key Steps

Import insights from tensor networks into a Field theory over tensor network states

$$\mathcal{Z} = Tr\, e^{-\beta \mathcal{H}}$$

$$\mathbb{1} \;\mathbb{1} \;\mathbb{1} \;\mathbb{1} \;\mathbb{1} \;\mathbb{1} \;\mathbb{1} \;\mathbb{1}$$

$$\mathbb{1} = \int DA |A\rangle\langle A|$$

$$= \int DA\, e^{\int d\tau \left[\langle A|\partial_\tau A\rangle - \langle A|\hat{\mathcal{H}}|A\rangle \right]}$$

- **Insert resolutions of identity over over-complete set**
- **Usually $|\psi\rangle$ product states**
- **Can we do the same with matrix product states?**

Key Steps

Import insights from tensor networks into a Field theory over tensor network states

$$\mathcal{Z} = Tr\, e^{-\beta\mathcal{H}}$$

$$\mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}$$

$$\mathbb{1} = \int DA |A\rangle\langle A|$$

$$= \int DA\, e^{\int d\tau \left[\langle A|\partial_\tau A\rangle - \langle A|\hat{\mathcal{H}}|A\rangle\right]}$$

- Insert ... set
- Usually ...
- Can we ...

Q. What is the Measure?

Q. What is the Berry Phase?

Q. Is the theory local?

II Towards a Field Theory over Tensor Network States:

Goal and Key Steps

Formulating the Field Theory

Special Cases

Potential Applications

Extensions to Higher Dimensions and Critical Systems

Gauge Fixing, Locality and General Parameterization

Local Field Theory => Gauge Fix to Canonical Form

- Canonical Equations

$$\sum_\sigma A^{\sigma\dagger} \Lambda_{n-1} A^\sigma = \Lambda_n \qquad \sum_\sigma A^\sigma A^{\sigma\dagger} = 1$$

- General Parameterization

$$A^\sigma = N^\sigma U^\sigma, \quad \sum_\sigma U^{\sigma\dagger} N^{\sigma\dagger} \Lambda_{n-1} N^\sigma U^\sigma = \Lambda_n$$

Diagonal matrix of spin coherent state spinors (N^σ)

SU(N)/DU(N) (U^σ)

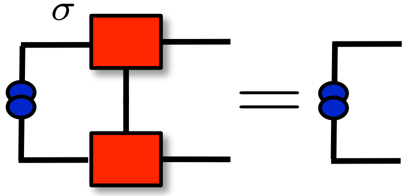
Residual canonical equations

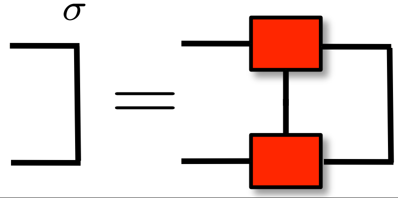
Gauge Fixing, Locality and General Parameterization

Local Field Theory => Gauge Fix to Canonical Form

- **Canonical Equations**

$$\sum_\sigma A^{\sigma\dagger}\Lambda_{n-1}A^\sigma = \Lambda_n \qquad \sum_\sigma A^\sigma A^{\sigma\dagger} = 1$$





- **Bond Order 2 Parameterization**

$$N^\sigma = \begin{pmatrix} n_1^\sigma & 0 \\ 0 & n_2^\sigma \end{pmatrix}$$

$$U^\sigma = \cos\theta^\sigma/2 + i\boldsymbol{\tau}.\mathbf{u}\sin\theta^\sigma/2$$

$$\mathbf{u} = (\cos\phi, \sin\phi)$$

$$d\Lambda_n = \sum_\sigma \Gamma^\sigma \cos\theta_\sigma$$

$$0 = \sum_\sigma \Gamma^\sigma \sin\theta_\sigma$$

$$\Lambda_n = (\mathbb{1} + \tau_z d\Lambda_n)/2$$

$$2\Gamma_n = [d\Lambda_{n-1}(1+\sigma\bar{n}^z) + \sigma\Delta n^z/2]$$

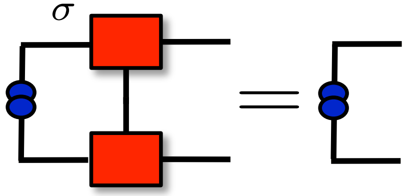
Towards a Field Theory over MPS

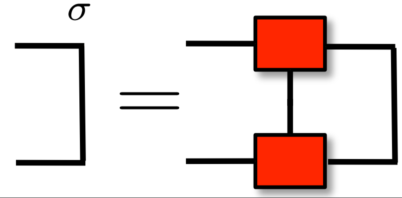
Gauge Fixing, Locality and General Parameterization

Local Field Theory => Gauge Fix to Canonical Form

- **Canonical Equations**

$$\sum_\sigma A^{\sigma\dagger}\Lambda_{n-1}A^\sigma = \Lambda_n \qquad \sum_\sigma A^\sigma A^{\sigma\dagger} = 1$$





- **General Bond Order – split into SU(2) subgroups**

$$N^\sigma = \mathrm{diag}(n_1^\sigma, n_2^\sigma, \ldots)$$

$$U^\sigma = \prod_l e^{i\theta^\sigma \mathbf{v}_l^\sigma . \tau/2}$$

$$d\Lambda_n^i = \sum_\sigma \Gamma^{\sigma,j}\left(\overrightarrow{\prod_l} e^{\theta^\sigma \mathbf{v}_l^\sigma . \mathbf{f}}\right)_{j,i}$$

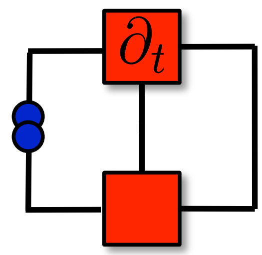
$$\Gamma^{\sigma,j} = Tr\left[\tau_j \Lambda_{n-1} N^{\sigma\dagger} N^\sigma\right] \qquad [\tau_i, \tau_j] = 2if_{ijk}\tau_k \qquad \Lambda_n = \frac{1 + \sum'_{\beta_d} d\Lambda_n^{\beta_d}\tau_{\beta_d}}{2}$$

Berry Phase

- **Contribution from n[th] site in chain**

$$\langle\psi|\partial_t\psi\rangle_n = \sum_\sigma Tr\left[A_n^{\sigma\dagger}\Lambda_{n-1}\partial_t A_n^\sigma\right]$$

$$=$$



$$= \sum_\sigma Tr\left[U_n^{\sigma\dagger}N_n^{\sigma\dagger}\Lambda_{n-1}\partial_t\left(N_n^\sigma U_n^\sigma\right)\right]$$

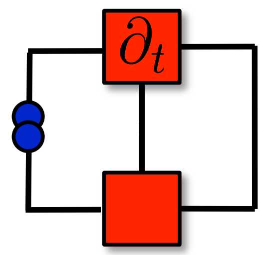
$$= \sum_\sigma Tr\left[\Lambda_{n-1}N_n^{\sigma\dagger}\partial_t N_n^\sigma\right] + \sum_\sigma Tr\left[U_n^{\sigma\dagger}\left(\Lambda_{n-1}N_n^{\sigma\dagger}N_n^\sigma\right)\partial_t U_n^\sigma\right]$$

Berry Phase

- **Contribution from nth site in chain**

$$\langle\psi|\partial_t\psi\rangle_n = \sum_\sigma Tr\left[A_n^{\sigma\,\dagger}\Lambda_{n-1}\partial_t A_n^\sigma\right]$$

$$=$$



Bond Order 2

$$= \sum_n \left[\sum_{\alpha=1}^{\chi}\Lambda_{n-1}^\alpha\langle\mathbf{n}^\alpha|\partial_t\mathbf{n}^\alpha\rangle + \partial_t\phi(d\Lambda_n - d\Lambda_{n-1})\right]$$

General

$$= \sum_n \left[\sum_{\alpha=1}^{\chi}\Lambda_{n-1}^\alpha\langle\mathbf{n}^\alpha|\partial_t\mathbf{n}^\alpha\rangle + \sum_{i=1}^{\chi(\chi-1)}\partial_t\phi_i u_\gamma^i(d\Lambda_n^\gamma - d\Lambda_{n-1}^\gamma)\right]$$

γ **Labels diag generators**
i **Labels SU(2) subgroups**

$$\Lambda_n = \frac{1}{2}\left(\mathbb{1} + \sum_{i=1}^{\chi-1}\tau_i d\Lambda_n^i\right) \qquad u_i^\gamma = \frac{1}{2}Tr[\tau_\gamma^z, \tau_i]$$

Gauge Fixing

- **Canonical constraints**

$$d\Lambda_n = \sum_\sigma \Gamma^\sigma \cos\theta_\sigma \ , \qquad 0 = \sum_\sigma \Gamma^\sigma \sin\theta_\sigma$$

Iteratively defines $d\Lambda_n$, (together with def'n of Γ^σ)

Fixes $\theta^\downarrow$, given $\theta^\uparrow$

$$d\Lambda_n \equiv d\Lambda_n(\{\mathbf{n}_1, \mathbf{n}_2, \theta^\uparrow\}) \quad \theta^\downarrow \equiv \theta^\downarrow(\{\mathbf{n}_1, \mathbf{n}_2, \theta^\uparrow\})$$

- **Gauge fixing**

$$\mathbb{1} = \int D\Lambda D\theta^\downarrow \delta[\theta^\downarrow - \theta^\downarrow(\{\mathbf{n}_1, \mathbf{n}_2, \theta^\uparrow\})]\delta[d\Lambda_n - d\Lambda_n(\{\mathbf{n}_1, \mathbf{n}_2, \theta^\uparrow\})]$$

$$D\mathbf{n}D\chi D\theta^\sigma D\psi \Longleftrightarrow D\mathbf{n}D\chi D\theta^\uparrow D\psi \Longleftrightarrow D\mathbf{n}D\chi D\Lambda D\psi$$

II Towards a Field Theory over Tensor Network States:

Goal and Key Steps

Formulating the Field Theory

Special Cases

Potential Applications

Extensions to Higher Dimensions and Critical Systems

Interesting Special Cases

- **Maximally Entangled States** $\Lambda \propto \mathbb{1} \quad \Rightarrow \quad A^\sigma = N^\sigma U$

$$\langle \psi | \partial_t \psi \rangle = \frac{1}{\chi} \sum_n \sum_{\alpha=1}^{\chi} \langle \mathbf{n}^\alpha | \partial_t \mathbf{n}^\alpha \rangle$$

$$\langle \psi | \hat{\mathcal{H}} | \psi \rangle = \langle \frac{1}{2} \sum_n J \sigma_n . \sigma_{n+1} \rangle = \frac{1}{2} \sum_n J |U_{\alpha,\beta}|^2 \mathbf{n}_n^\alpha \mathbf{n}_{n+1}^\beta$$

- **Spatially Uniform**

$$\langle \psi | \partial_t \psi \rangle \quad = \quad \sum_n \sum_{\alpha=1}^{\chi} \Lambda^\alpha \langle \mathbf{n}^\alpha | \partial_t \mathbf{n}^\alpha \rangle$$

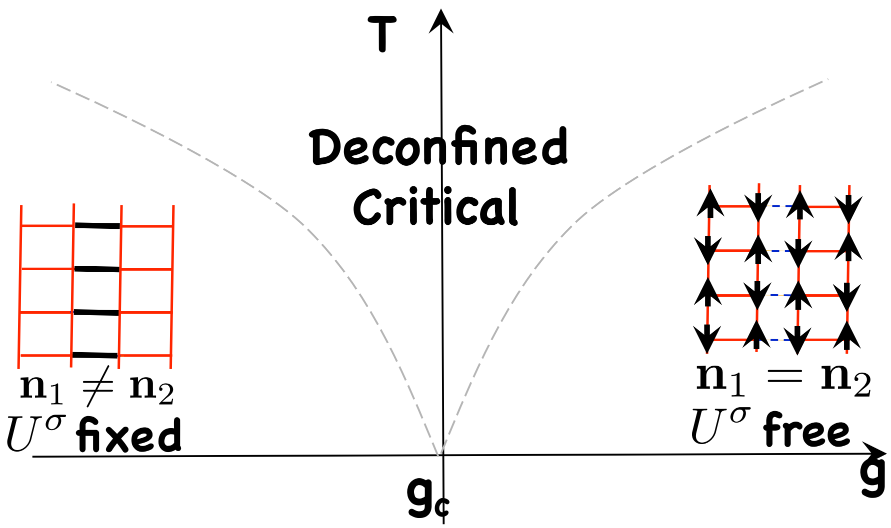
- **In both Cases, Effectively χ replicas of system**
- **glued together with $SU(\chi)$ field**
- **No intrinsic dynamics for U**
  - **Behaves as fancy Lagrange multiplier (maximally entangled)**
  - **Inherited through those of N^σ (spatially uniform case)**

II Towards a Field Theory over Tensor Network States:

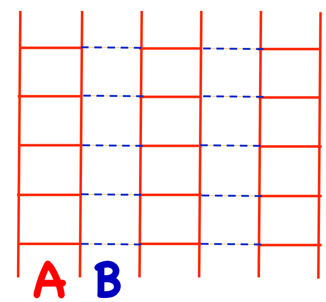
Goal and Key Steps
Formulating the Field Theory
Special Cases
Potential Applications
Extensions to Higher Dimensions
and Critical Systems

Towards a Field Theory over MPS

Deconfined Criticality



$$\mathcal{H} = g \sum_{\langle ij \rangle}^{A} J \hat{\sigma}_i . \hat{\sigma}_j + \sum_{\langle ij \rangle}^{B} J \hat{\sigma}_i . \hat{\sigma}_j$$



- [Senthil et al, Science 303, 1490 (2004)]
- Critical theory not described by order parameter fluctuations
- Gauge fields/Lagrange multipliers determine critical behaviour
- MPS states may characterize both sides at low bond order
- Certain MPS degrees of freedom soften at transition

Q. Emergent fluctuations in Connection?

Potential Applications

Extended Truncated Wigner Approximation

- Propagates density matrix using saddle-point approx to Keldysh

$$\hat{\rho}(t) = \int D\psi_+ \psi_- \hat{\rho}(0) e^{-i\mathcal{S}_+ + i\mathcal{S}_-}$$

- All entanglement contained in $\hat{\rho}(0) = \sum_\alpha \lambda_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|$
- Propagate $|\psi_\alpha\rangle$ independently
- Perturbative corrections increase Schmidt rank [Polkovnikov PRA68, 053604 (2003)]

Alternative

- Propagate with saddle points of MPS Keldysh
  i. Decompose $\hat{\rho}(0)$ over product states – propagate with MPS saddle point
  =>Allows entanglement to grow to some limit
  ii. Decompose $\hat{\rho}(0)$ over MPS – propagate with MPS saddle point
  =>Allows restructuring of entanglement
- Quench action formalism – decompose $\hat{\rho}(0)$ over Bethe states and propagate
  [Caux and Essler, PRL110, 257203, (2013)]

Potential Applications

Extended Truncated Wigner Approximation

- Propagates density matrix using saddle-point approx to Keldysh

$$\hat{\rho}(t) = \int D\psi_+\psi_- \hat{\rho}(0) e^{-i\mathcal{S}_+ + i\mathcal{S}_-}$$

- All entanglement contained in $\hat{\rho}(0) = \sum_\alpha \lambda_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|$
- Propagate $|\psi_\alpha\rangle$ independently
- Perturbative corrections increase Schmidt rank [Polkovnikov PRA68, 053604 (2003)]

Alternative

- Propagate with saddle points of MPS Keldysh
  i. Decompose $\hat{\rho}(0)$ over product states – propagate with MPS saddle point
  =>Allows entanglement to grow to some limit
  ii. D
  =>
- Quen ... propagate
  [Ca

Q. Late time hydrodynamics of eigenstate thermalization?

Potential Applications

Fluctuation corrections to MPS time evolution

- Two ways to include fluctuations about saddle point
- Re-summing the effects of fluctuations (usual field theory)
- Increasing bond order
- Re-summing fluctuations may improve low bond-order saddle point and fidelity of time evolution over low bond order states

Q. Fluctuation Corrections to MPS?

Many body localization

- Product state field theory - saddle points describe low energy
- Many body localization is a dynamical phase transition through spectrum
- MPS field theory may permit description of dynamics higher in the spectrum.

Q. Field Theory of mid-spectrum states?

II Towards a Field Theory over Tensor Network States:

Goal and Key Steps

Formulating the Field Theory

Special Cases

Potential Applications

Extensions to Higher Dimensions and Critical Systems

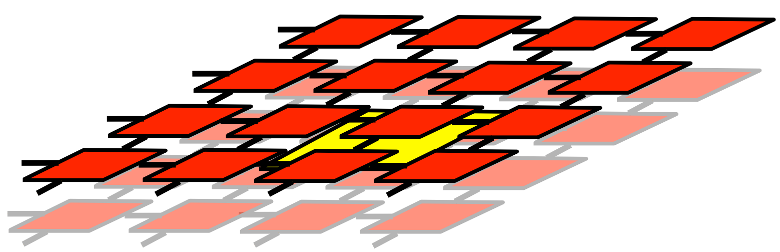
Towards a Field Theory over MPS

Extension to Higher Dimensions

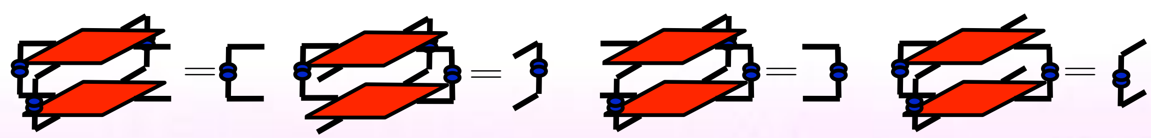
Fundamental difficulty…

- No canonical form in higher dimensions
- Action not local in terms of A
- Decay of entanglement suggests approximately true
- Quasi canonical form
- Connection to multi-band Wannier functions
- [Marzari et al RevModPhys84, 1419 (2012)]?

$$\langle \psi | \hat{\theta} | \psi \rangle =$$

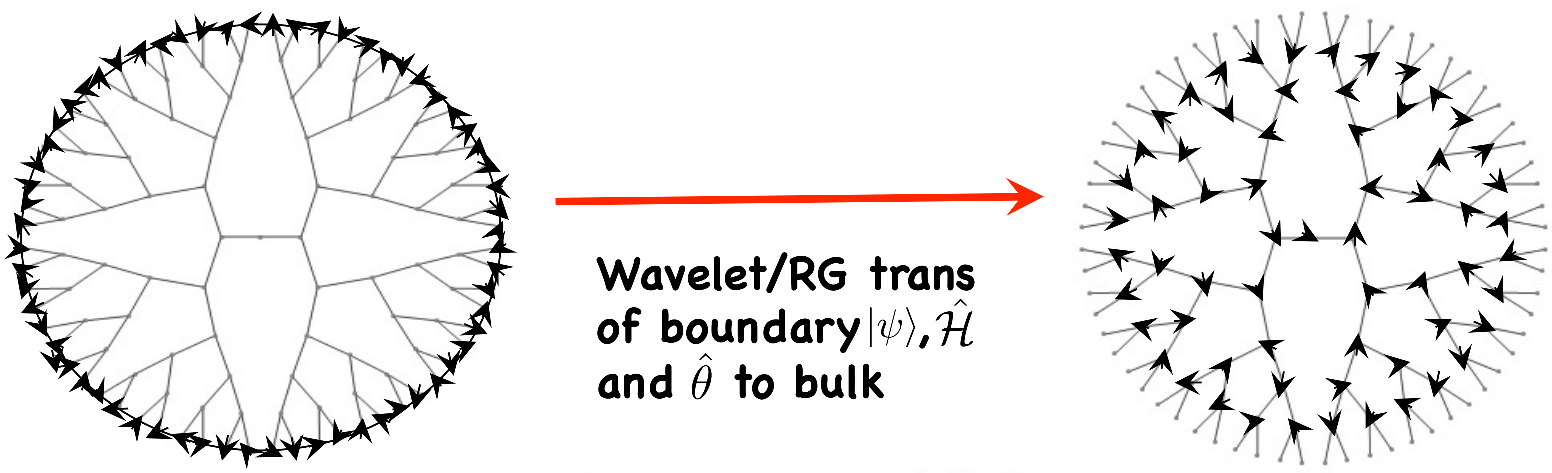


Quasi-Canonical Form:



Extensions to Critical Systems

- Various RG Schemes
- MERA (multiscale entanglement renormalization ansatz) [Vidal, PRL99, 220405(2007)]
- TRG (tensor RG) [Verstrate et al, Adv. Phys 57, 143 (2008)]
- SRG (second RG) [Xie et al PRL103, 16069 (2009)]
- HOTRG (higher order TRG) [Xie et al PRB86, 045139 (2012)]
- Exact Holographic Mapping [Xiao-Liang Qi[ArXiv:1309.6282]



- Relation to AdS/CFT [Swingle, Phys. Rev. D 86, 065007 (2012),ArXiv:1209.3304]
- Applying RG to field theory over MPS ->[S.-S. Lee,NPB 832,56 (2010); 851,143(2011)]

Conclusions

So Far

- Field Theory over Tensor Network States

MPS data

$$\mathcal{Z} = Tr\, e^{-\beta\mathcal{H}} = \int [DA] e^{-\mathcal{S}[A]}$$

- Imports insights about entanglement to field theory
- Managed for 1d – exploring potential applications

Future

- RG-> describing both renormalization of fluctuations and entanglement structure?
- Extension to higher dimensions and critical theories?
- Links to AdS/CFT?