Conformal gravity holography in four dimensions

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Outline

Why conformal gravity?

Aspects

- is 2-loop renormalizable [*Stelle*,1977]
- has negative kinetic energy states..

- useful for constructing supergravity theories [*Bergshoeff*, *deRoo*, *deWit*,1981]
- arises as a counterterm in the AdS-CFT correspodence [*Liu and Tseytlin*,1998]
- emerges from twistor string theory [*Berkovits and Witten*,2004]
- studied in the quantum gravity context [*'t Hooft*,2011]
- explanation of galactic rotation curves without need for dark matter [*Mannheim*,2012]
- elimination of ghosts with a particular choice of boundary conditions [*Maldacena*,2011]

What is conformal gravity?

•classical theory of gravity that has conformal invariance
Locally rescale the metric

$$g_{\mu\nu} \quad : \tilde{g}_{\mu\nu} = e^{\omega(x)} g_{\mu\nu} \tag{1}$$

then that the action

$$I_{\mathrm{CG}} = \alpha_{\mathrm{CG}} \int_{\mathcal{M}} d^4x \sqrt{|g|}\, C^{\lambda}{}_{\mu\sigma\nu}\, C_{\lambda}{}^{\mu\sigma\nu} \tag{2}$$

is conformally invariant.

$$C_{\mu\nu\kappa\lambda} = R_{\mu\nu\kappa\lambda} - \frac{1}{2}(g_{\rho[\mu} R_{\nu]\sigma} - g_{\sigma[\mu} R_{\nu]\rho}) + \frac{1}{3} g_{\rho[\mu} g_{\nu]\sigma} R \tag{3}$$

Equations of motion

$$\left(\nabla^{\delta} \nabla_{\gamma} + \frac{1}{2} R^{\delta}{}_{\gamma} \right) C^{\gamma}{}_{\mu\delta\nu} = 0 \qquad (4)$$

Solutions

The most general spherically symmetric solution is the Mannheim-Kazanas-Riegert (MKR) solution:

$$ds^2 = -k(r)dt^2 + \frac{dr^2}{k(r)} + r^2 d\Omega^2_{S^2} \qquad (5)$$

- $k(r) = \sqrt{1 - 12\alpha M} - \frac{2M}{r} - \Lambda r^2 + 2\alpha r$
- α is the Rindler acceleration

Simple classe of solutions:

- conformally flat metrics
- Einstein metrics ($R_{\mu\nu} \sim g_{\mu\nu}$)

$\rightarrow$ Einstein solutions are a subset of solutions of conformal gravity

Boundary conditions

- crucial in physical theories
- in the context of the AdS-CFT correspodence

Starobinsky boundary conditions

Written in the Fefferman-Graham expansion as:

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \frac{\rho^2}{\ell^2}\gamma_{ij}^{(2)} + \frac{\rho^3}{\ell^3}\gamma_{ij}^{(3)} + \dots \quad (6)$$

· γ_{ij} is the 3-dimensional boundary metric
· $\gamma_{ij}^{(n)}$ denotes the order of perturbation
· ρ is the holographic coordinate
· ℓ is the (A)dS radious

- Einstein gravity in dS background [*Starobinsky*,1983]
- conformal gravity in dS and Euclidean AdS background[*Maldacena*,2011]

$\rightarrow$ from all the solutions of conformal gravity the Starobinsky boundary conditions "extract" the solutions of Einstein gravity

For conformal gravity are there more general boundary conditions than the Starobinsky ones such that

- they give a well-defined variational principle?
- they give finite response functions in the dual field theory?

4-dimensional line element

$$ds^2 = \frac{\ell^2}{\rho^2}\left(-\sigma\, d\rho^2 + \gamma_{ij} dx^i dx^j\right) \tag{7}$$

- ℓ is the length scale ($\Lambda = \frac{3\sigma}{\ell^2}$) with $\sigma = -1$ for AdS, $\sigma = 1$ for dS
- $0 < \rho << \ell$ is the holographic coordinate
- γ_{ij} is the 3-dimensional boundary metric

Boundary conditions

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \frac{\rho}{\ell}\gamma_{ij}^{(1)} + \frac{\rho^2}{\ell^2}\gamma_{ij}^{(2)} + \frac{\rho^3}{\ell^3}\gamma_{ij}^{(3)} + \dots \tag{8}$$

- $\gamma_{ij}^{(n)}$ depend on boundary coordinates x^i and not on the holographic coordinate ρ
- fix the $n = 0, 1$ terms on the boundary metric $\gamma_{ij}^{(n)}$ up to a local Weyl rescaling:

$$\delta\gamma_{ij}^{(0)}|_{\partial M} = 2\lambda\gamma_{ij}^{(0)} \ , \ \delta\gamma_{ij}^{(1)}|_{\partial M} = \lambda\gamma_{ij}^{(1)} \tag{9}$$

- leave terms with $n \geq 2$ vary freely

Consistency of boundary conditions

$$I_{\mathrm{CG}} = \alpha_{\mathrm{CG}} \int_M d^4x \sqrt{|g|}\, C^{\lambda}{}_{\mu\nu\sigma} C_{\lambda}{}^{\mu\nu\sigma} \tag{10}$$

- boundary value problem well-posed → add a ”Gibbons-Hawking-York” boundary term e. g. for a Dirichlet boundary value problem
- stationary action for all variations that preserve our boundary conditions → add holographic counterterms

$\rightarrow$ no such terms needed for conformal gravity!
The full action is the bulk action ($\alpha_{\text{CG}} = 1$):

$$\Gamma_{\text{CG}} = I_{\text{CG}} = \int_M d^4x \sqrt{|g|}\, C^{\lambda}{}_{\mu\nu\sigma} C_{\lambda}{}^{\mu\nu\sigma} \tag{11}$$

Rewrite CG action:

$$\Gamma_{\text{CG}}=\int_M d^4x\,\left(2R_{\mu\nu}R^{\mu\nu}-\frac{2}{3}R^2+R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda}-4R_{\mu\nu}R^{\mu\nu}+R^2\right)$$
$$=\int_M d^4x\,\left(2R_{\mu\nu}R^{\mu\nu}-\frac{2}{3}R^2\right)$$
$$+32\pi^2\chi(M)+\int_{\partial M}d^3x\sqrt{|\gamma|}\Big(-8\sigma\mathcal{G}^{ij}K_{ij}+\frac{4}{3}K^3-4KK^{ij}K_{ij}+\frac{8}{3}K^{ij}K_j^kK_{ki}\Big)\quad(12)$$

- $\chi(M)$ is the Euler characteristic for spacetimes with a (conformal) boundary[*Myers*,1987]
- calligraphic quantities are intrinsic to the 3-dimensional surface ∂M
- $K_{ij}=-\frac{\sigma}{2}\mathcal{L}_{n^\mu}\gamma_{ij}$ is the extrinsic curvature and n^μ the outward-pointing normal vector to ∂M

First variation of the action

$$\delta\Gamma_{\mathrm{CG}} = EOM + \int_{\partial M} \sqrt{|\gamma|}\left(\pi^{ij}\delta\gamma_{ij} + \Pi^{ij}\delta K_{ij}\right) \tag{13}$$

- π^{ij} and Π^{ij} are functions of the intrinsic 3-dimensional tensors and the extrinsic curvature
- boundary metric and extrinsic curvature vary independently
- not a Dirichlet boundary value problem

$\rightarrow$ mixed boundary value problem well-posed choosing a compact region $\rho_c \leq \rho$, applying the EOM and using our asymptotic expansion

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \frac{\rho}{\ell}\,\gamma_{ij}^{(1)} + \frac{\rho^2}{\ell^2}\,\gamma_{ij}^{(2)} + \frac{\rho^3}{\ell^3}\,\gamma_{ij}^{(3)} + \dots \qquad (14)$$

we get:

$$\delta\Gamma_{\mathrm{CG}}|_{EOM} = \int_{\partial M}\sqrt{|\gamma^{(0)}|}\Big(\tau^{ij}\delta\gamma_{ij}^{(0)} + P^{ij}\delta\gamma_{ij}^{(1)}\Big) \qquad (15)$$

Inserting our asymtpotic expansion (14) to the electric and magnetic part of the Weyl tensor ($E_{ij} = n_\alpha\, n^\beta\, C^\alpha{}_{i\beta j}, B_{ijk} = n_\alpha\, C^\alpha{}_{ijk}$)

$$E_{ij} = E_{ij}^{(2)} + \frac{\rho}{\ell}\,E_{ij}^{(3)} + \dots \quad , \quad B_{ijk} = \frac{\ell}{\rho}\,B_{ijk}^{(1)} + B_{ijk}^{(2)} + \dots \qquad (16)$$

and using the splitting $\gamma_{ij}^{(n)} = \frac{1}{3}\gamma^{(n)}\gamma_{ij}^{(0)} + \psi_{ij}^{(n)}$, the functions τ_{ij} and P_{ij} take the form:

$$\begin{aligned}\tau_{ij} =& \sigma\big[\tfrac{2}{\ell}\,(E_{ij}^{(3)} + \tfrac{1}{3}E_{ij}^{(2)}\gamma^{(1)}) - \tfrac{4}{\ell}\,E_{ik}^{(2)}\psi_j^{(1)k} + \tfrac{1}{\ell}\,\gamma_{ij}^{(0)}E_{kl}^{(2)}\psi_{(1)}^{kl} + \tfrac{1}{2\ell^3}\,\psi_{ij}^{(1)}\psi_{kl}^{(1)}\psi_{(1)}^{kl} \\ &- \tfrac{1}{\ell^3}\,\psi_{kl}^{(1)}\,(\psi_i^{(1)k}\psi_j^{(1)l} - \tfrac{1}{3}\,\gamma_{ij}^{(0)}\psi_m^{(1)k}\psi_{(1)}^{lm})\big] - 4\,\mathcal{D}^k B_{ijk}^{(1)} + i \leftrightarrow j \\ P_{ij} =& -\tfrac{4\sigma}{\ell}\,E_{ij}^{(2)}\end{aligned} \qquad (17)$$

Holographic response functions

- τ^{ij} is the holographic response function conjugate to the source $\gamma^{(0)}_{ij}$, the conformal gravity analogue to the Brown-York stress tensor
- P^{ij} is the holographic response function conjugate to the source $\gamma^{(1)}_{ij}$
  "Partially massless response" $\rightarrow \gamma^{(1)}_{ij}$ pluged into the linearized EOM exhibits partial masslessness[*Deser*, *Nepomechie and Waldron*,1984,2001]

- are finite as $\rho_c \rightarrow 0$
- satisfy the "trace" conditions

$$\gamma^{(0)}_{ij}\tau^{ij} + \tfrac{1}{2}\psi^{(1)}_{ij}P^{ij} = 0 \quad , \quad \gamma^{(0)}_{ij}P^{ij} = 0 \tag{18}$$

$$\delta\Gamma_{CG}|_{EOM} = \int_{\partial M}\sqrt{|\gamma^{(0)}|}\left(\tau^{ij}\delta\gamma^{(0)}_{ij} + P^{ij}\delta\gamma^{(1)}_{ij}\right) \tag{19}$$

subject to our boundary conditions

$$\delta\gamma^{(0)}_{ij}|_{\partial M} = 2\lambda\gamma^{(0)}_{ij} \quad , \quad \delta\gamma^{(1)}_{ij}|_{\partial M} = \lambda\gamma^{(1)}_{ij} \tag{20}$$

vanishes on-shell $\rightarrow$ no counterterms needed

- for conformal gravity there are more general boundary conditions than the Starobinsky ones
- they are consistent with a well-defined variational principle
- they give finite holographic response functions

- application to MKR solution
- canonical analysis of the theory
- calculation of the asymptotic charges and the asymtptotic symmetry algebra with our proposed boundary conditions
- calculation of the 2-point functions