Hydrodynamics of Lifshitz fluids and superfluids

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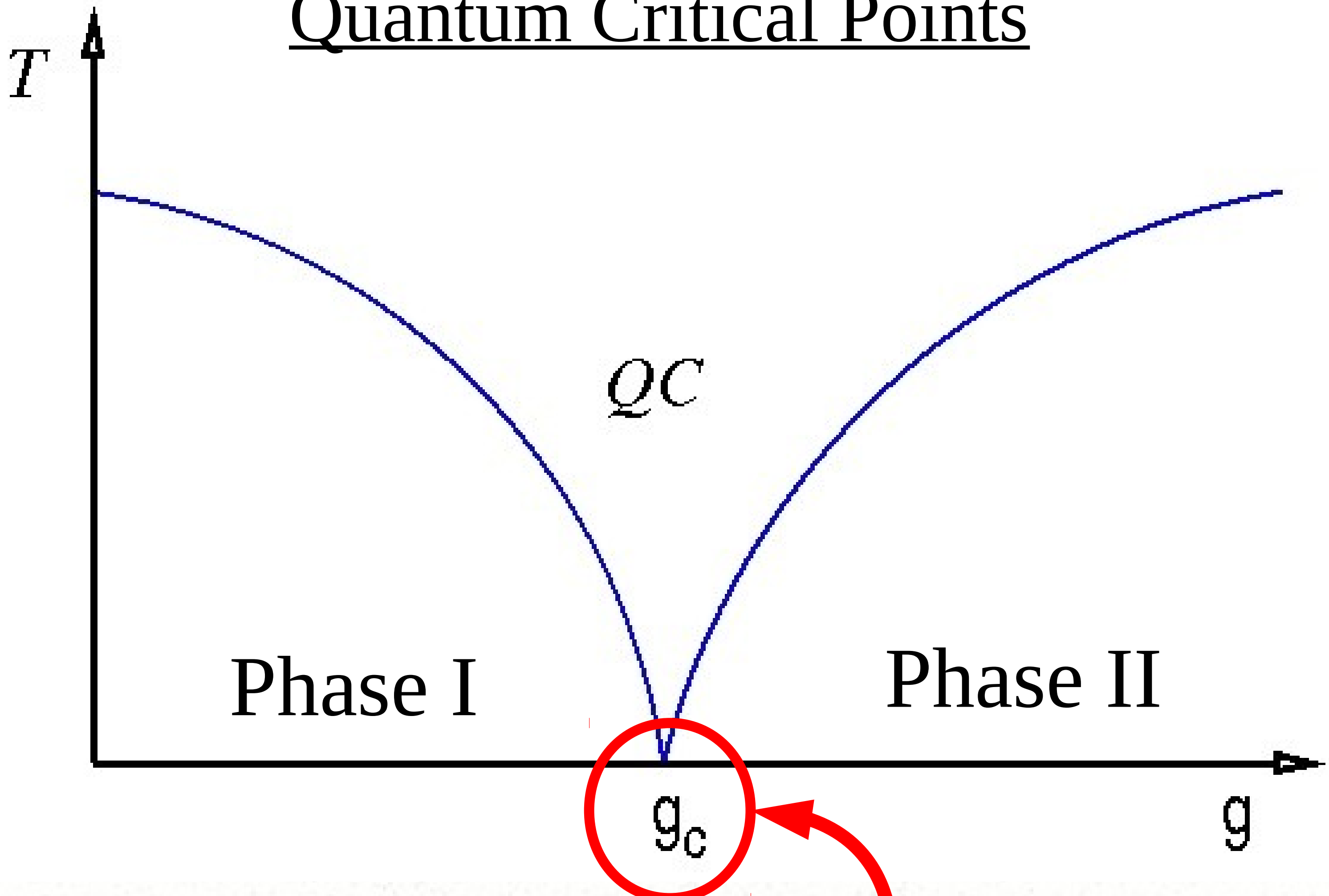
1304.7481,1309.6794, 1312.6380; 1402.2981

Motivation

Quantum Critical Points



Second order phase transition

Examples

- **Spin chain models**
- **O(N) (quantum rotor) models**
- **Hubbard models**
- **Fermi liquids**
- **Bose-Einstein condensates**

Quantum Critical Points

- **Continuum limit: quantum field theory**
- **Scaling symmetry at the critical point:**

$$t \to \lambda^z t, \quad x^i \to \lambda x^i$$

- **Free field example: “Lifshitz theory”**

$$S = \int dt\, d^d x \left[\frac{1}{2} (\partial_t \phi)^2 - \frac{\kappa}{2z} ((\partial_i^2)^{z/2} \phi)^2 \right]$$

Breaking of Lorentz symmetry

- **Couple to external vielbein:**

$$\mathcal{L} = \frac{1}{2}(u^a e_a{}^\mu \partial_\mu \phi)^2 - \frac{\kappa}{4}(P^{ab} e_a{}^\mu e_b{}^\nu \partial_\mu \partial_\nu \phi)^2$$

Diffeomorphism invariance

Broken Lorentz symmetry

$u^a =$ Defines `local rest frame'

- **Canonical theories with scale invariance: relativistic conformal field theories**

- **Many strongly coupled CFTs with gravity duals**

Hydrodynamic description very successful!

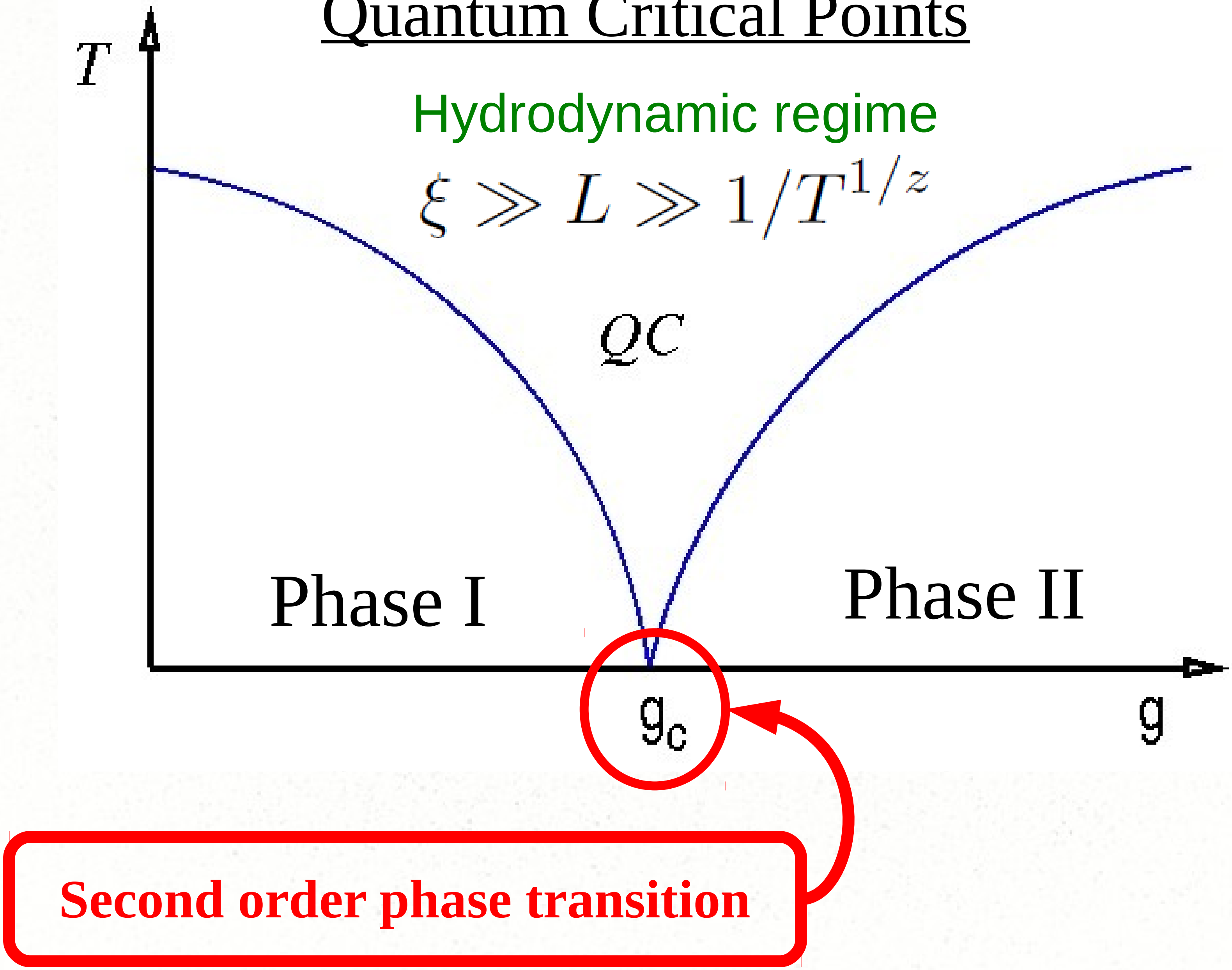
Hydrodynamic modes from AdS black holes

G. Policastro, D.T. Son, A. Starinets

Fluid-gravity correspondence

S. Bhattacharyya, V. Hubeny, S. Minwalla, M. Rangamani

Quantum Critical Points



Can we extend holographic results to Lifshitz?

- **Many gravity duals with Lifshitz symmetry**

- **Einstein gravity plus matter (or higher derivative gravity)**

  S. Kachru, X. Liu, M. Mulligan; M. Taylor + many more

- **Horava gravity**

  S. Janiszewski, A. Karch;
  T. Griffin, P. Horava, C.M. Melby-Thompson

From experience with holographic models, we expect
an universal hydrodynamic description of Lifshitz theories

This was already argued by condensed matter physicists,
but not developed in the same way as for relativistic theories

Sachdev & Ye; Zaanen

We initiate the formulation of hydrodynamics of quantum critical points at finite temperature

Hydrodynamics in QCP

- **Graphene:** Fritz, Schmalian, Muller, Sachdev

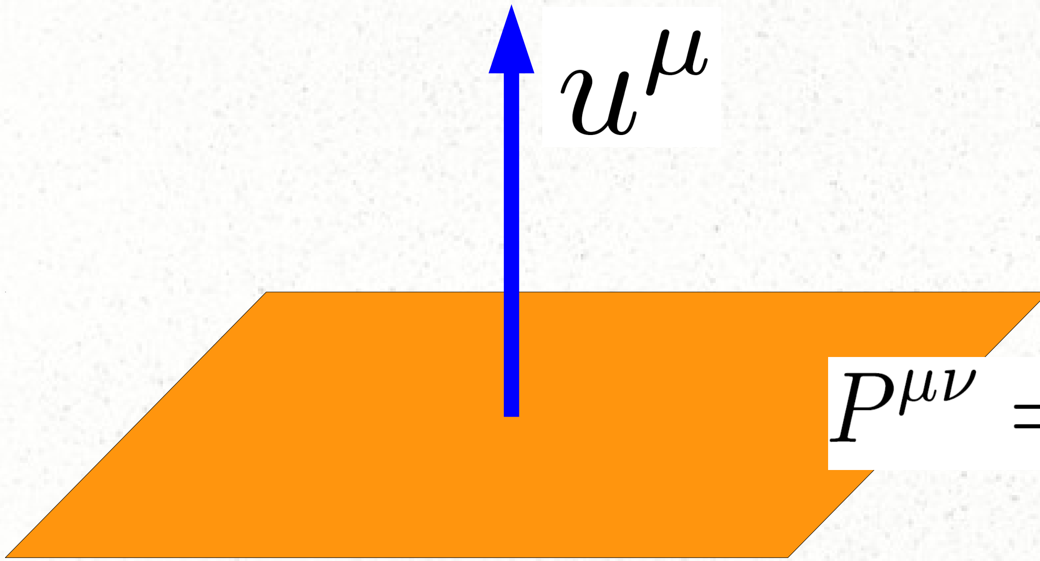
  z=1 (relativistic fermions)

- **Fermions at unitarity:** Cao et al.; Son & Wingate

  z=2 (non-relativistic conformal invariance)

Hydrodynamics

Time-like Killing vector: defines rest frame of the fluid



$$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$$

$$\eta_{\mu\nu} u^\mu u^\nu = -1$$

$$u^\mu = (1, \beta^i)/\sqrt{1-\beta^2} \qquad \beta^i = \frac{v^i}{c}$$

Ward identities

Conservation equations: $$\partial_\mu T^{\mu\nu} = 0$$

"Trace" of energy-momentum tensor:

$$zT_{\mu\nu}u^\mu u^\nu - T_{\mu\nu}P^{\mu\nu} = 0$$

Lorentz symmetry is broken:

$$T^{\mu\nu} \neq T^{\nu\mu}$$

Constitutive relations for ideal fluids

$$T^{\mu\nu} = (\varepsilon + p)u^{\mu}u^{\nu} + p\eta^{\mu\nu}$$

$$J^{\mu} = q_n u^{\mu}$$

$$zT_{\mu\nu}u^{\mu}u^{\nu} - T_{\mu\nu}P^{\mu\nu} = 0$$

Equation of state: $z\varepsilon = dp$

Constitutive relations

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu + p\eta^{\mu\nu} + \pi_S^{(\mu\nu)} + \pi_A^{[\mu\nu]}$$

$$T^{\mu\nu} \neq T^{\nu\mu}$$

$$J^\mu = q_n u^\mu + \nu^\mu$$

First order symmetric terms
(neutral fluid)

$$\pi_S^{(\mu\nu)} = -\eta\sigma^{\mu\nu} - \zeta P^{\mu\nu}\partial_\alpha u^\alpha$$

$$\sigma^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}P_{\alpha\beta}\partial_\sigma u^\sigma\right)$$

Bulk viscosity spoils trace Ward identity:

$$zT_{\mu\nu}u^\mu u^\nu - T_{\mu\nu}P^{\mu\nu} = \zeta\partial_\mu u^\mu$$

In holographic models :

$\zeta \neq 0 \quad \Leftrightarrow$ **Explicit breaking of Lifshitz symmetry**

Constitutive relations in superfluids

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu + p\eta^{\mu\nu} + f\xi^\mu \xi^\nu + \pi_S^{(\mu\nu)} + \pi_A^{[\mu\nu]}$$

$$T^{\mu\nu} \neq T^{\nu\mu}$$

Superfluid velocity

$$J^\mu = q_n u^\mu - f\xi^\mu + \nu^\mu$$

$$u^\mu \xi_\mu = \mu + \mu_{\text{diss}}$$

Broken boost invariance:

$$T^{\alpha\beta} P^{\mu}{}_{\alpha} u_{\beta} \neq T^{\alpha\beta} u_{\alpha} P^{\mu}{}_{\beta}$$

Unbroken rotational invariance:

$$T^{\alpha\beta} P^{\mu}{}_{\alpha} P^{\nu}{}_{\beta} = T^{\alpha\beta} P^{\nu}{}_{\alpha} P^{\mu}{}_{\beta}$$

Asymmetric terms:

$$\pi_A^{[\mu\nu]} = u^{[\mu} V_A^{\nu]}$$

Second law of thermodynamics

$$\partial_t S = \int d^d x \, \partial_t s \geq 0$$

$$\varepsilon + p = Ts + \mu q_n$$

Entropy current: $s^\mu = s u^\mu + \cdots$

Local form: $\boxed{\partial_\mu s^\mu \geq 0}$

Constrains the form of derivative corrections to hydro

Landau,Lifshitz; Son, Surowka; J. Bhattacharya, S. Bhattacharyya, S. Minwalla and A. Yarom, S. Bhattacharyya, S. Jain, S. Minwalla, T. Sharma

Possible terms in first order hydro

$$\zeta^\mu = P^{\mu\nu}\xi_\nu, \quad n^\mu = \frac{\zeta^\mu}{\sqrt{\zeta_\sigma \zeta^\sigma}}, \quad \tilde{P}^{\mu\nu} = P^{\mu\nu} - n^\mu n^\nu$$

- **Tensor:** $\tilde{P}^{\mu\alpha}\tilde{P}^{\nu\beta}\sigma_{\alpha\beta}$
- **Vectors:** $\tilde{P}^{\mu\nu}\frac{\partial_\nu T}{T}, \quad \sigma_{\alpha\beta}\tilde{P}^{\mu\alpha}n^\beta, \quad \boxed{\tilde{P}^{\mu\nu}a_\nu}$
- **Scalars:**

$$\partial_\mu u^\mu, \quad T P^{\mu\nu}\partial_\mu\left(\frac{f\zeta_\nu}{T}\right), \quad \sigma_{\mu\nu}n^\mu n^\nu, \quad \frac{n^\mu \partial_\mu T}{T}, \quad \boxed{n^\mu a_\mu}$$

New transport coefficients: depend on acceleration

$$a^\mu = u^\alpha \partial_\alpha u^\mu$$

Number of dissipative transport coefficients

fluid	non-Lifshitz	3
fluid	Lifshitz	5
superfluid	non-Lifshitz	14
superfluid	Lifshitz	22

(Assuming time reversal invariance)

Future directions

- **Effects due to new transport coefficients**
- **Transport coefficients beyond first order**
- **Additional conserved currents**
- **Fluids in a curved space: Weyl anomaly?**
- **Anomalous currents**