Holography with small N=4

Entropy of asymptotically flat black holes in gauged supergravity

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Statement

There is an "old and forgotten" two-dimensional superconformal algebra with $(0,4)$. [Schwimmer and Seiberg (1988)]

Who cares?

This algebra was thought to be uninteresting.

Question

What is the holographic dual ?

Black holes

I will show an interesting application, describing the microscopics of asymptotically flat black holes in gauged supergravity.

AdS/CFT

Holography and symmetries in various dimensions

$$AdS_7 \times S^4 \quad \longleftrightarrow \quad (0,2) - M5$$
$$AdS_5 \times S^5 \quad \longleftrightarrow \quad N = 4 - D3$$
$$AdS_4 \times S^7 \quad \longleftrightarrow \quad N = 8 - M2$$

Two-dimensional CFT's arise from

$$AdS_3 \times S^3 \times T^4/K3 \quad \longleftrightarrow \quad small(4,4) - D1/D5$$
$$AdS_3 \times S^3 \times S^3 \times S^1 \quad \longleftrightarrow \quad large(4,4) - \ ?\ (see\ \ Tong)$$
$$AdS_3 \times S^2 \times CY_3 \quad \longleftrightarrow \quad small(0,4) - MSW\ \ (M5)$$

Holography and black holes

AdS/CFT has allowed us to compute (BPS) black hole entropies microscopically.

This is because the near-horizon geometry of a p-brane is AdS_{p+2}.

BPS Black Holes and MSW

The M5 brane can be wrapped over a four-cycle to give a BPS string in $D = 5$, with near horizon geometry

$$AdS_3 \times S^2 \times CY_3$$

Upon reducing over S^1 to $D = 4$ it gives a BPS black hole with near horizon geometry

$$AdS_2 \times_f S^1 \times S^2 \times CY_3$$

The entropy of this black hole was computed by Maldacena, Strominger Witten ('97) using AdS_3/CFT_2 and the Cardy formula for the $(0,4)$ dual CFT. Near-extremal (Strominger '97):

$$AdS_3 \leftrightarrow BTZ$$

.

R-symmetries in sugra

I will focus on MSW setups

$$AdS_3 \times S^2 \times CY_3 \quad \overset{black\ hole}{\longleftrightarrow} \quad \mathbb{R}^{1,3} \times S^1 \times CY_3$$

$N = 2, D = 5$ R-symmetry group $SU(2)_\rho$.
$N = 4, D = 3$ gives another $SU(2)_\eta$. This is a gauge symmetry.
Total R-symmetry group:

$$SO(4)_R = SU(2)_\rho \times SU(2)_\eta$$

$D = 3$ Gravitini:

$$\psi_\mu^{\alpha\dot{\alpha}} \qquad \alpha, \dot{\alpha} = 1, 2$$

R-symmetries in Dual (0,4) CFT

The holographic dual has only $N=4$ in the right moving sector. Currents:

$$T(z), \quad J^i(z);\ i=1,2,3, \quad G^a(z), G^{*a}(z); a=1,2\ .$$

R-symmetry $SO(4)_R = SU(2)_\rho \times SU(2)_\eta$, with $J^i : (1,3)$ and $c=12k$.

Possible boundary conditions (Schwimmer & Seiberg 1988):

$$\begin{aligned} J^\pm(z) &= e^{\pm 2i\pi\eta} J^\pm(e^{2\pi i}z) \\ G^1(z) &= e^{i\pi(\rho+\eta)} G^1(e^{2\pi i}z) \\ G^2(z) &= e^{i\pi(\rho-\eta)} G^2(e^{2\pi i}z) \end{aligned}$$

Spectral flow

The η-parameter can be gauged away (spectral flow), but the ρ-parameter lead to inequivalent algebras:

$$L'_n = L_n - \eta J^3_n + \frac{1}{2}\eta^2 k\delta_{n,0}$$

$$J'^3_n = J^3_n - \eta k\delta_{n,0} \qquad J'^{\pm}_{n+\eta} = J^{\pm}_{n+\eta}$$

$$G'^1_{n+(\rho+\eta)/2} = G^1_{n+\rho/2} \,, \qquad G'^2_{n+(\rho-\eta)/2} = G^2_{n+\rho/2}$$

NS-sector: $\rho = 0, \eta = 1$. Vacuum dual to AdS_3
R-sector: $\rho = \eta = 0$. Vacuum dual to massless BTZ.
Nobody ever really cared about $\rho \neq 0$, but today we do. Which geometry/black hole are we computing the entropy of ???

Back to $N = 2, D = 5$ sugra

Remember that the $SU(2)_\rho$ corresponded to the R-symmetry in $D = 5$. Giving twisted boundary conditions means giving twisted boundary conditions on the circle S^1 in $AdS_3 = AdS_2 \times_f S^1$:

$$AdS_3 \times S^2 \times CY_3 \quad \overset{black\ hole}{\longleftrightarrow} \quad \mathbb{R}^{1,3} \times S^1 \times CY_3$$

This means we are doing a Scherk-Schwarz twist along S^1 upon compactifying from $D = 5$ to $D = 4$.

Scherk-Schwarz Baby version

Consider a massless complex scalar field with $U(1)$ symmetry on $\mathbb{R}^{1,3} \times S^1$ coupled to gravity

$$\begin{aligned} L &= -\partial_{\hat{\mu}}\phi\partial^{\hat{\mu}}\bar{\phi} \\ &= -\Big(\partial_\mu\phi\partial^\mu\bar{\phi} + g^{\mu z}\partial_\mu\phi\partial_z\bar{\phi} + g^{z\mu}\partial_z\phi\partial_\mu\bar{\phi} + g^{zz}\partial_z\phi\partial_z\bar{\phi}\Big) \end{aligned}$$

Give twisted boundary condition (Scherk-Schwarz)

$$\phi(x, z + 2\pi R) = e^{2\pi i\rho}\phi(x, z) \Leftarrow \partial_z\phi = i\frac{\rho}{R}\phi \ .$$

Resulting Lagrangian has KK-charged field and positive definite potential

$$L = -|D_\mu\phi|^2 - V(\phi, \bar{\phi}) \ , \qquad V = \frac{\rho^2}{R^2}|\phi|^2 \ .$$

Mass and charge $m = q = \frac{\rho}{R}$.

Scherk-Schwarz in sugra I

The Scherk-Schwarz twist yields no-scale supergravity in $D = 4$ with $V \geq 0$ [..., Hull, ..., Ferrara et al.; Looyestijn, Plauschinn, SV].
We twist the $U(1) \subset SU(2)_R = SU(2)_\rho$ symmetry.
All particles with R-charge become massive: gravitini and hypers.
The potential is given by a sum of positive terms,

$$
\begin{aligned}
\frac{V}{\rho^2} \quad = \quad & \frac{1}{R^3}\mathcal{N}^r\bar{\mathcal{N}}^s \; G_{r\bar{s}} + \frac{1}{4R^3\mathcal{V}^2}\left[\xi^T N^T \xi\right]^2 \\
& \frac{1}{2R^3\mathcal{V}}\xi^T \; N^T \; \mathcal{M} \; (\mathrm{Im}\,\mathcal{M})^{-1}\,\mathcal{M}^T \; N\xi \\
& \frac{1}{4R^6}\; \left(M^A{}_C\phi^C\right)\; \left(M^B{}_D\phi^D\right)\; \mathcal{K}_{\mathcal{AB}} \; .
\end{aligned}
$$

We found Minkowski vacua with radius R and Calabi-Yau volume $\mathcal{V}$ flat directions.

Scherk-Schwarz in sugra II

Vector multiplet scalars and vectors remain massless. The gaugino gets eaten by the gravitino. Expanding around the vacuum, we get massive and charged hypermultiplets and gravitini, with $m = q$ and

$$q = n\, q_{1/2}\ , \quad n \in \mathbb{Z}\ , \qquad q_{1/2} = \frac{\rho}{R}\ , \qquad \rho << 1\ .$$

Actually, $n \in \{1, 3, 4, 8, 12\}$ for the model at hand with $h_{1,2} = 1$ and hypers span $G_2/SO(4)$.
In the vacuum, the resulting bosonic Lagrangian looks like ungauged supergravity ($\rho = 0$) for which black hole solutions known. They are the RN black holes, but now embedded in a theory with light charged matter.

Black holes and Scherk-Schwarz - Micro

- Wrapping the M5 brane over a four-cycle still gives a BPS black string in $D = 5$, where susy is unbroken. Hence there is a still a dual CFT, the $(0, 4)$ MSW.
- Wrap the string around the circle with R-twisted boundary conditions still gives a black hole, with the same near horizon geometry $(BTZ \times S^2)$, but susy is broken by the boundary conditions.
- The black hole will Hawking radiate, have temperature, but Hawking particles also consist of charged light matter. Possibly also Schwinger radiation takes place.

MSW and Scherk-Schwarz

We can use the representation theory and Cardy formula to compute the entropy for near-extremal black holes:

$$\begin{aligned} S_{CFT} &= S_L + S_R \\ &= 2\pi \left[\sqrt{\frac{c_L n_L}{6}} + \sqrt{\frac{c_R}{6}\left(n_R + h_0\right)} \right] \ , \end{aligned}$$

with $h_0 = \frac{c_R}{12}(\rho - \frac{\rho^2}{2})$.
For $\rho = 0$, standard result for near-extremal. Additional correction in ρ. Remember: twist parameter is equal to coupling constant in sugra. Prediction for a macroscopic correction to the entropy ?!

Conclusions

- Found holographic dual of ρ-twisted $(0,4)$ algebra, realized as MSW with twisted boundary conditions.
- Bulk interpretation: Scherk-Schwarz twist on 5D circle.
- Description of black hole entropy in models with light charged particles and spontaneously broken supersymmetry.

Future directions

- Study stability of the vacuum. Susy breaking parameter is $m_{1/2} = \frac{\rho}{R}$ and ρ is continuous and taken very small.
- Instabilities and Schwinger radiation. Threshold: BF-bound in AdS_2:

$$(m^2 - q^2)Q^2 + \frac{1}{4} < 0$$

- Scherk-Schwarz twist that only partially breaks susy. This is possible $N = 4 \rightarrow N = 2$: Strominger-Vafa setup (D1-D5).
- Generalizations to AdS vacua/BH with no susy??