Holographic Anyonic Superfluids

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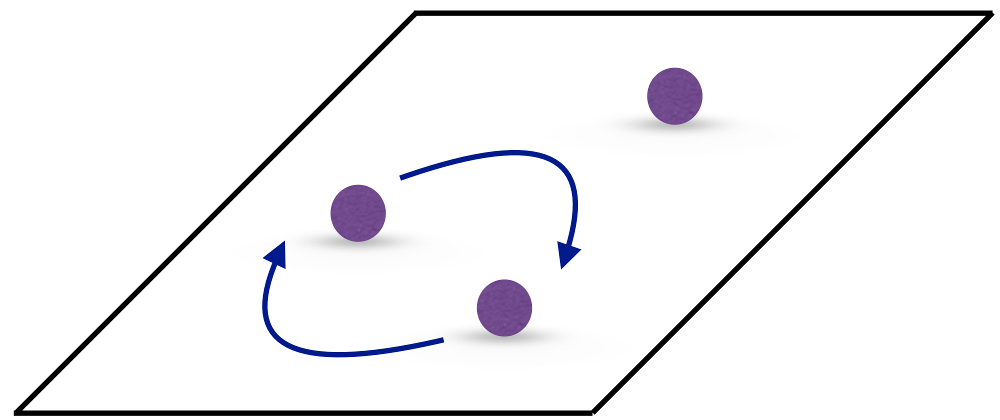
based on 1307.6336 and 1407.3794

Plan

- Anyons and $SL(2,\mathbb{Z})$
- Anyon Superfluids
- A Holographic Anyon Superfluid
- A Flowing Holographic Anyon Superfluid

Anyons

particles in 2+1 dim can have arbitrary statistics



$$|\psi_1\psi_2\rangle = e^{i\theta}|\psi_2\psi_1\rangle$$

- $\theta = 0$ bosons
- $\theta = \pi$ fermions
- $\theta = \pi\, p/q$ anyons

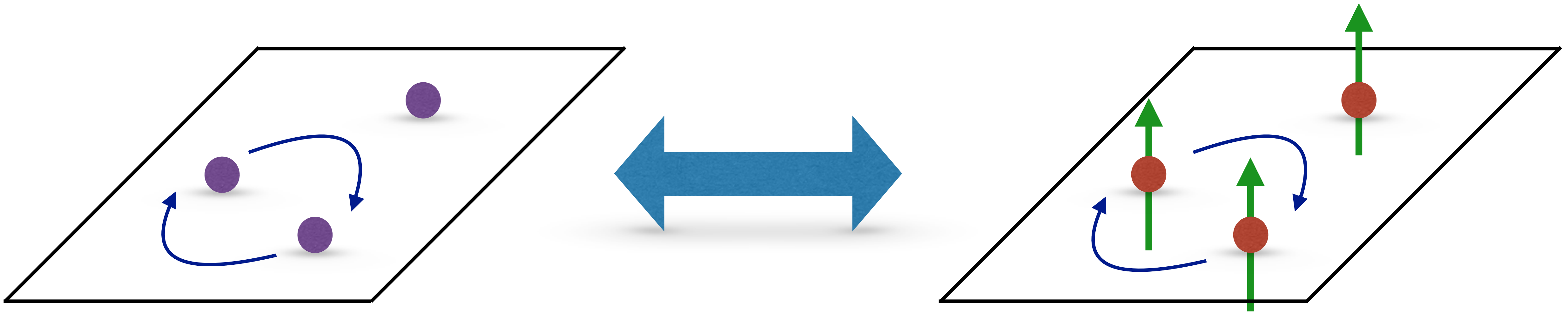
Leinaas, Myrheim

Wilczek

Alternate Description

charged particles with n magnetic fluxes attached

statistical phase $\theta \longleftrightarrow$ Aharonov-Bohm phase πn



anyons

fermions with n fluxes

$$\theta_{stat} = \pi\left(1 - n\right)$$

$$\theta_{stat} + \theta_{AB} = \pi - \pi n$$

Flux attachment and *SL*(2,$\mathbb{Z}$)

2+1 dim CFT

Witten
Burgess, Dolan

- *U*(1) current - J
- external vector - $\mathcal{A}$
- define $\mathcal{B} = \frac{1}{2\pi} * d\mathcal{A}$

mapping to CFT'

- add Chern-Simons term for $\mathcal{A}$: $J' = J + \mathcal{B}$
- make $\mathcal{A}$ dynamical: $J' = \mathcal{B}$
- generate *SL*(2,$\mathbb{Z}$) $\left(\begin{array}{c} J' \\ \mathcal{B}' \end{array}\right) = \left(\begin{array}{cc} a & b \\ c & d \end{array}\right)\left(\begin{array}{c} J \\ \mathcal{B} \end{array}\right)$

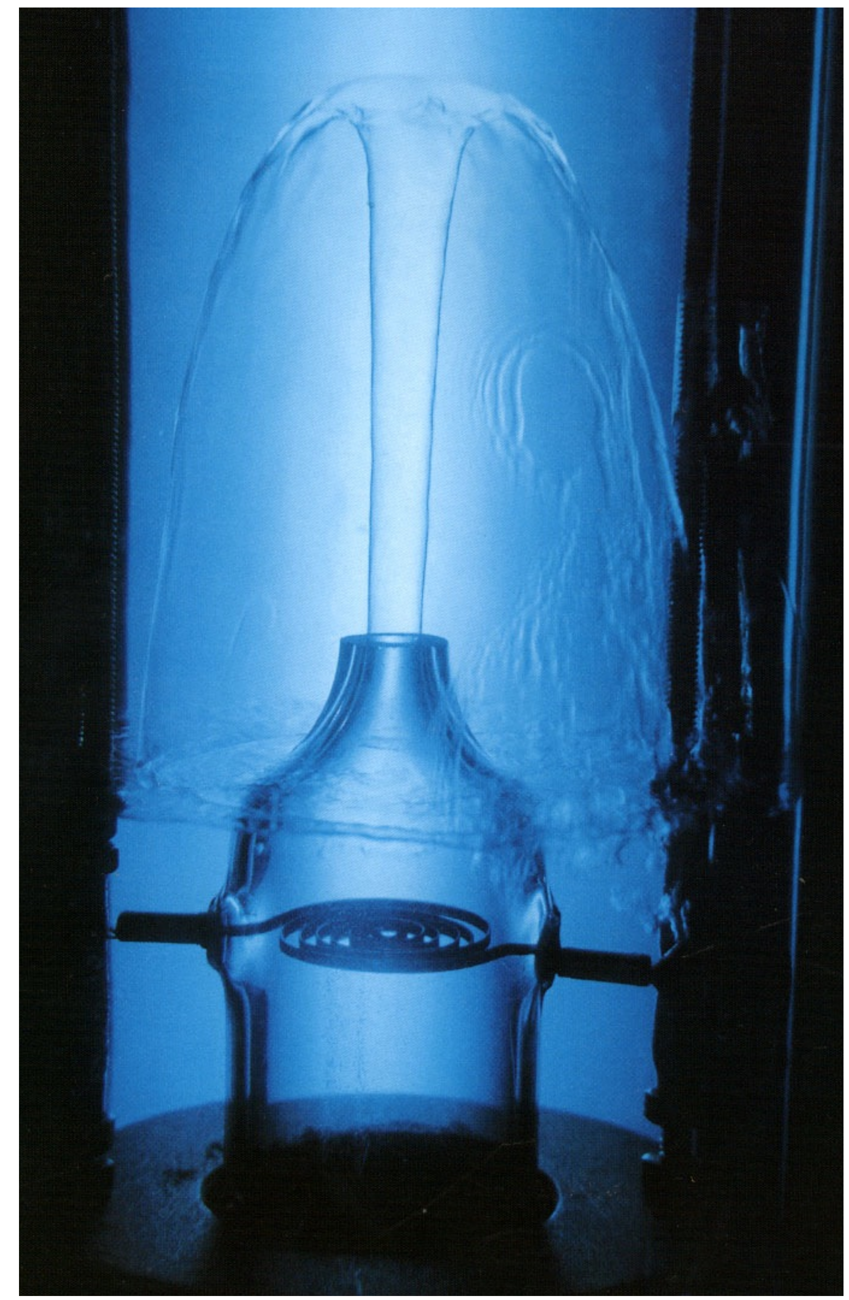
flux attachment: $\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$

Superfluids

- Flow without resistance
- For example:
  - Liquid ^{4}He, $T < 2.17$K
  - Holographic dual of hairy BH
- Spontaneously broken global symmetry

↓

massless mode



^{4}He fountain

Anyon Superfluids

Anyons in $B = 0$ ➡ Superfluid **Laughlin**

Start with:

- QH fluid of fermions, filling fraction ν
- Background E_x ➡ Hall current $J_y = \frac{\nu}{2\pi} E_x$

$SL(2,\mathbb{Z})$ with $\frac{d}{c} = \frac{\nu}{2\pi}$

$J'_y = d\ J_y \neq 0$ current

$B' = E'_x = 0$ no sources

$\theta' = \pi \left(1 - \frac{1}{\nu}\right)$ anyons

Superfluidity without symmetry breaking

Usually:

massless mode ↔ spontaneous symmetry breaking

But, no symmetry breaking in 2+1d **Colman-Mermin-Wagner**

Superfluidity without symmetry breaking

Usually:

massless mode — spontaneous symmetry breaking

But, no symmetry breaking in 2+1d **Mermin-Wagner**

For anyons:

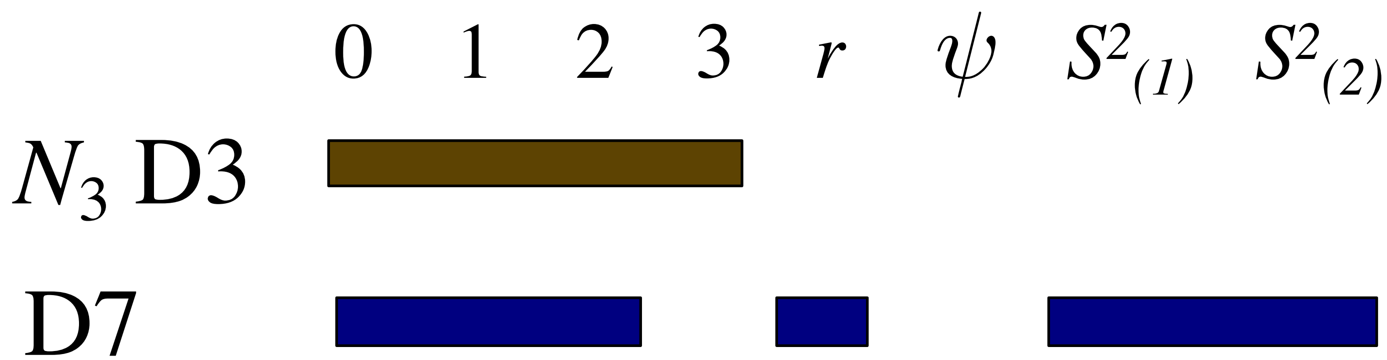
massless mode — spontaneous **fact** violation

$$[T_x, T_y] \neq 0$$

Chen, Wilczek, Witten, Halperin, Giddings

Holographic QH model: D3-D7′

Bergman, NJ, GL, ML



Probe D7:

- fermions on 2+1 dim defect
- wraps $S^2 \times S^2 \subset S^5$
- embedding $\psi(r)$
- $\#ND = 6$ ➔ ~~SUSY~~

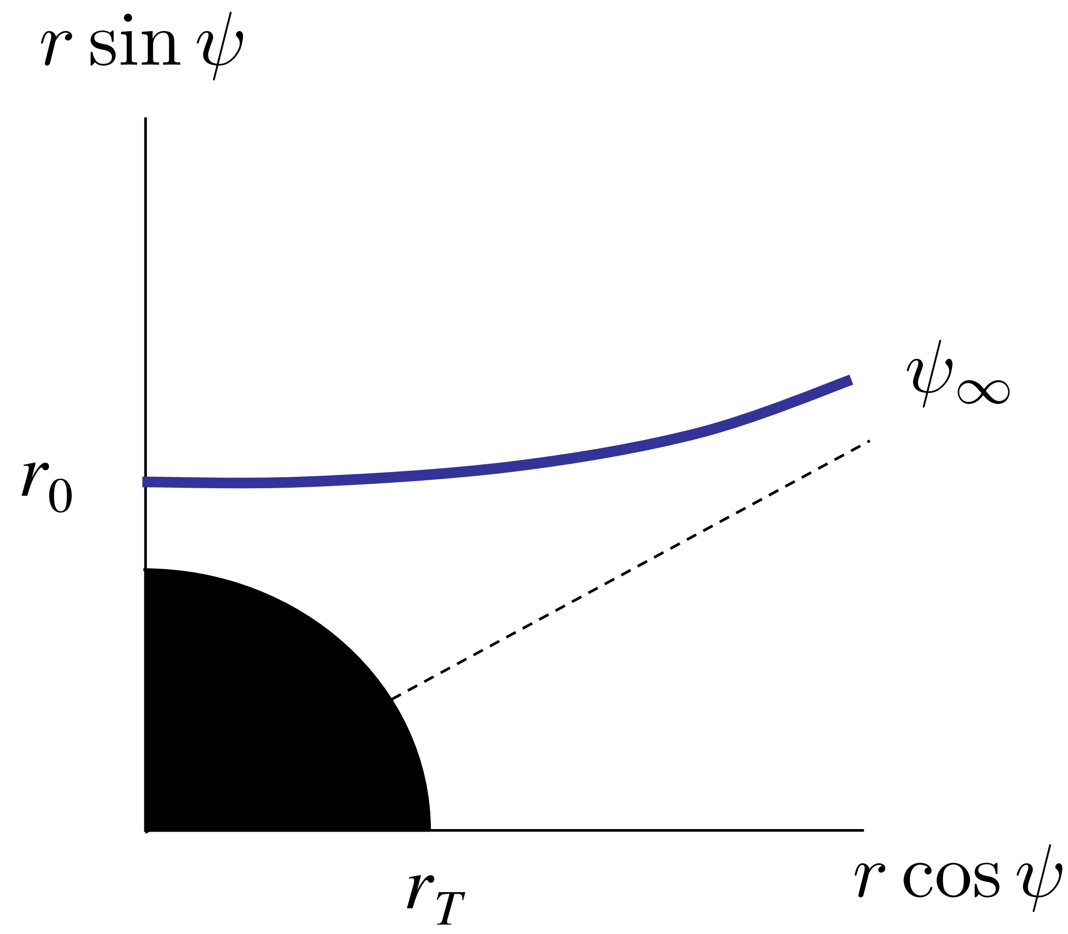
Gauge Field:

$F_{\theta_1\phi_1}$ - stabilization

F_{r0} - charge density $J_0 = D$

F_{12} - magnetic field B

Minkowski Embedding - QH state



- $\nu = 2\pi \frac{D}{B} = 1 - \frac{2\psi_\infty}{\pi} + \frac{1}{4}\sin(4\psi_\infty)$
- gapped
- $\sigma_{xx} = 0$ and $\sigma_{xy} = \frac{\nu}{2\pi}$

Alternative Quantization

Dirichlet conditions:

A fixed at boundary → $\mathcal{B}$ fixed

Neumann conditions:

$\partial_r A$ fixed at boundary → J fixed

$$\mathcal{B}' = J, \quad J' = -\mathcal{B}$$

General mixed conditions:

fix a linear combination of J and $\mathcal{B}$

implements $SL(2,\mathbb{Z})$

Fluctuations

Vary δJ' with fixed $\mathcal{B}'$

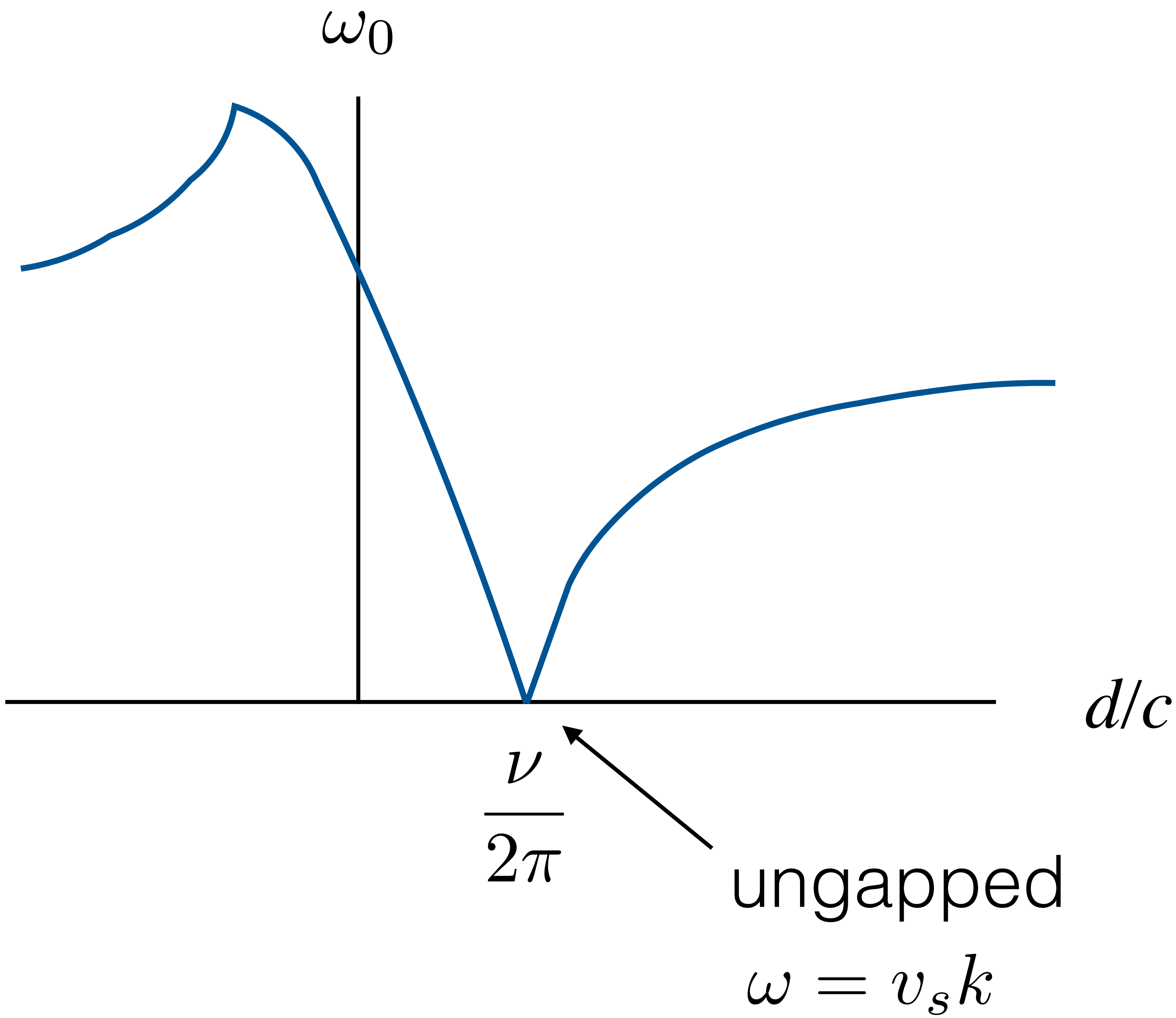
In original $SL(2,\mathbb{Z})$ variables:

$$0 = \delta B' = c\;\delta D + d\;\delta B$$

$$\frac{\delta D}{\delta B} = \frac{d}{c}$$

for $\quad \frac{d}{c} = \frac{\nu}{2\pi} \quad \Rightarrow \quad \delta\nu = 0$

Mass of $\delta J'$ vs. d/c

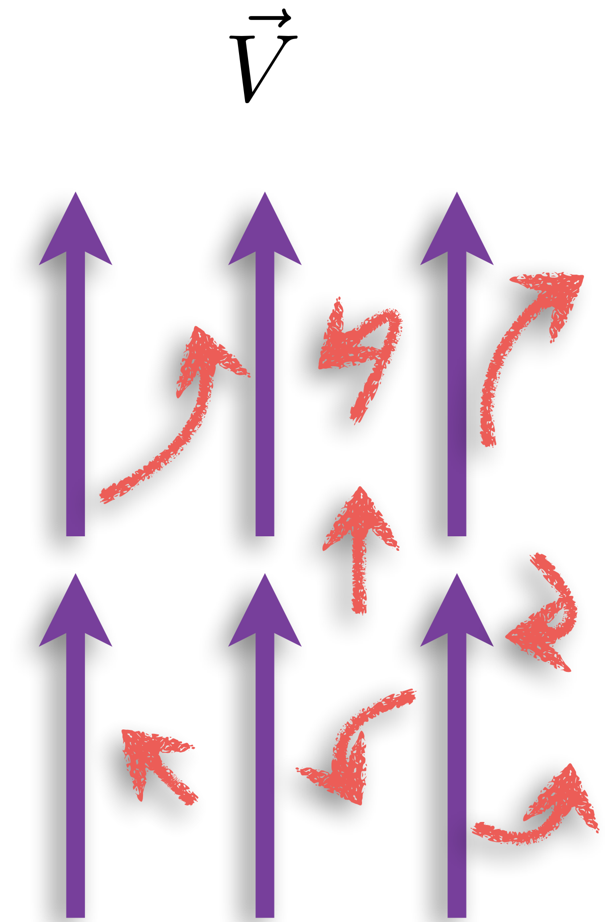


Superfluids can flow

Two component description:

- superfluid with velocity $\vec{V}$
- normal fluid

at low T, gas of phonons



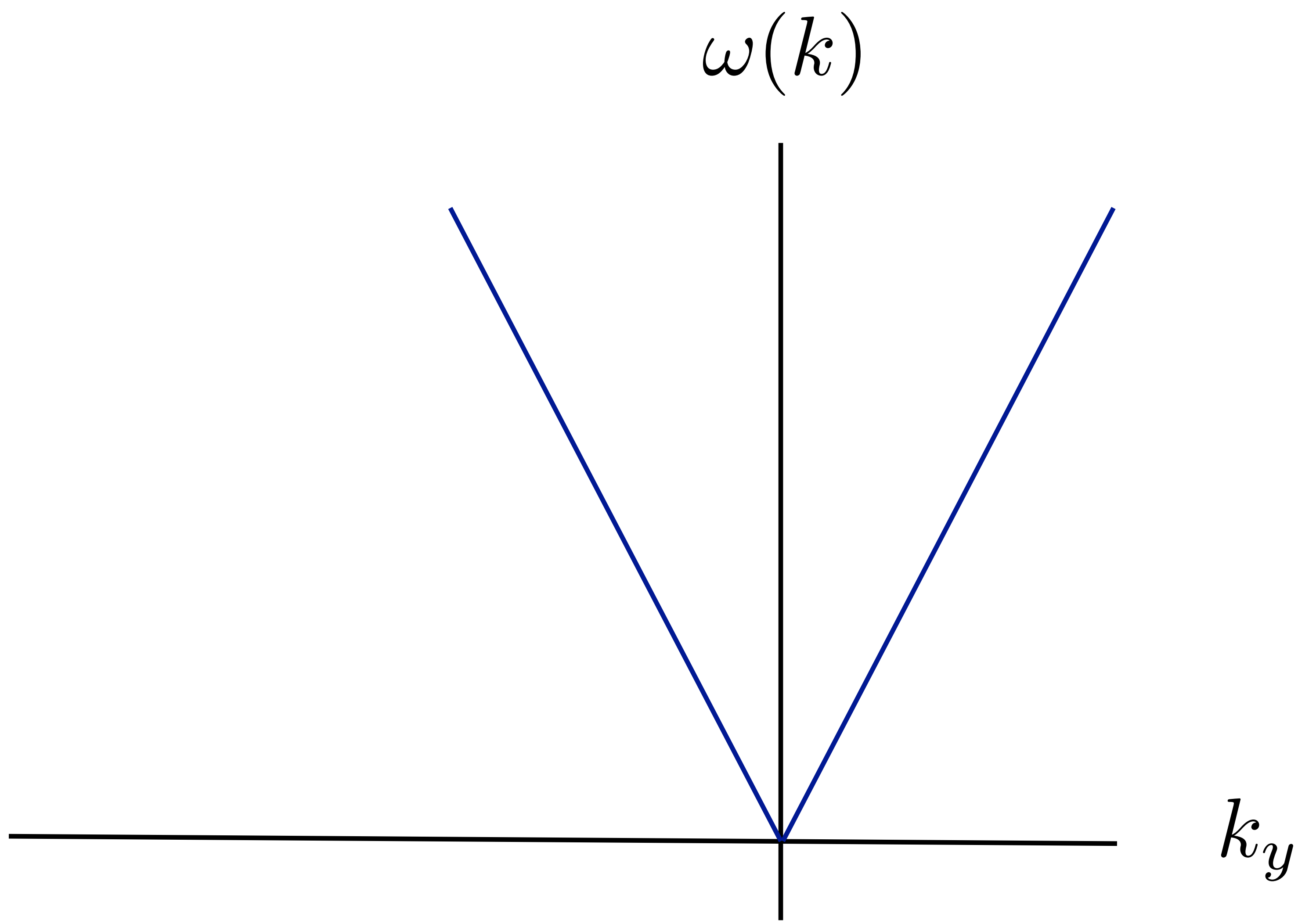
In holographic model:

Electric field E_x → Superfluid velocity V_y

Phonon dispersion

$$V_y = 0$$

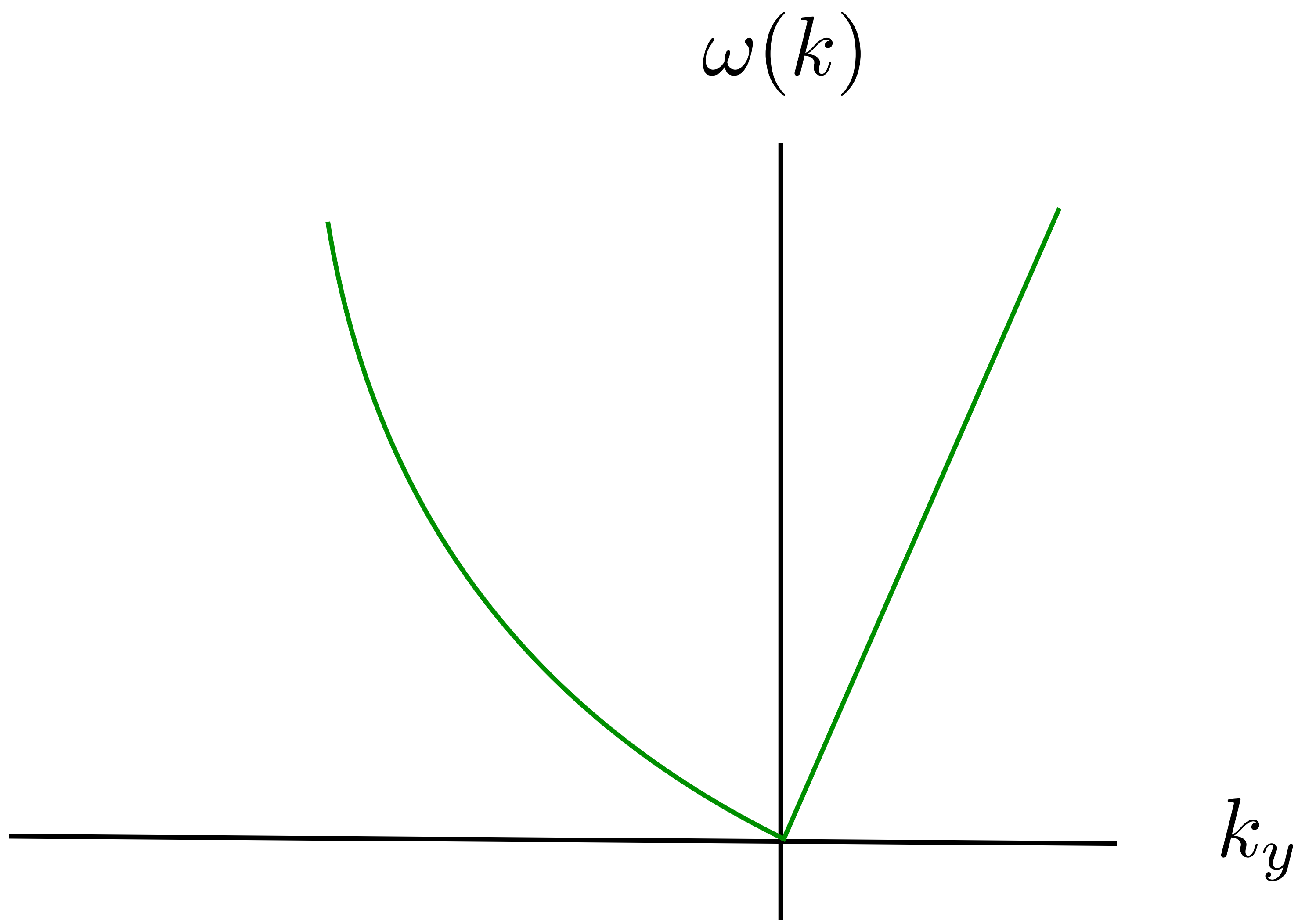
no superflow



Phonon dispersion

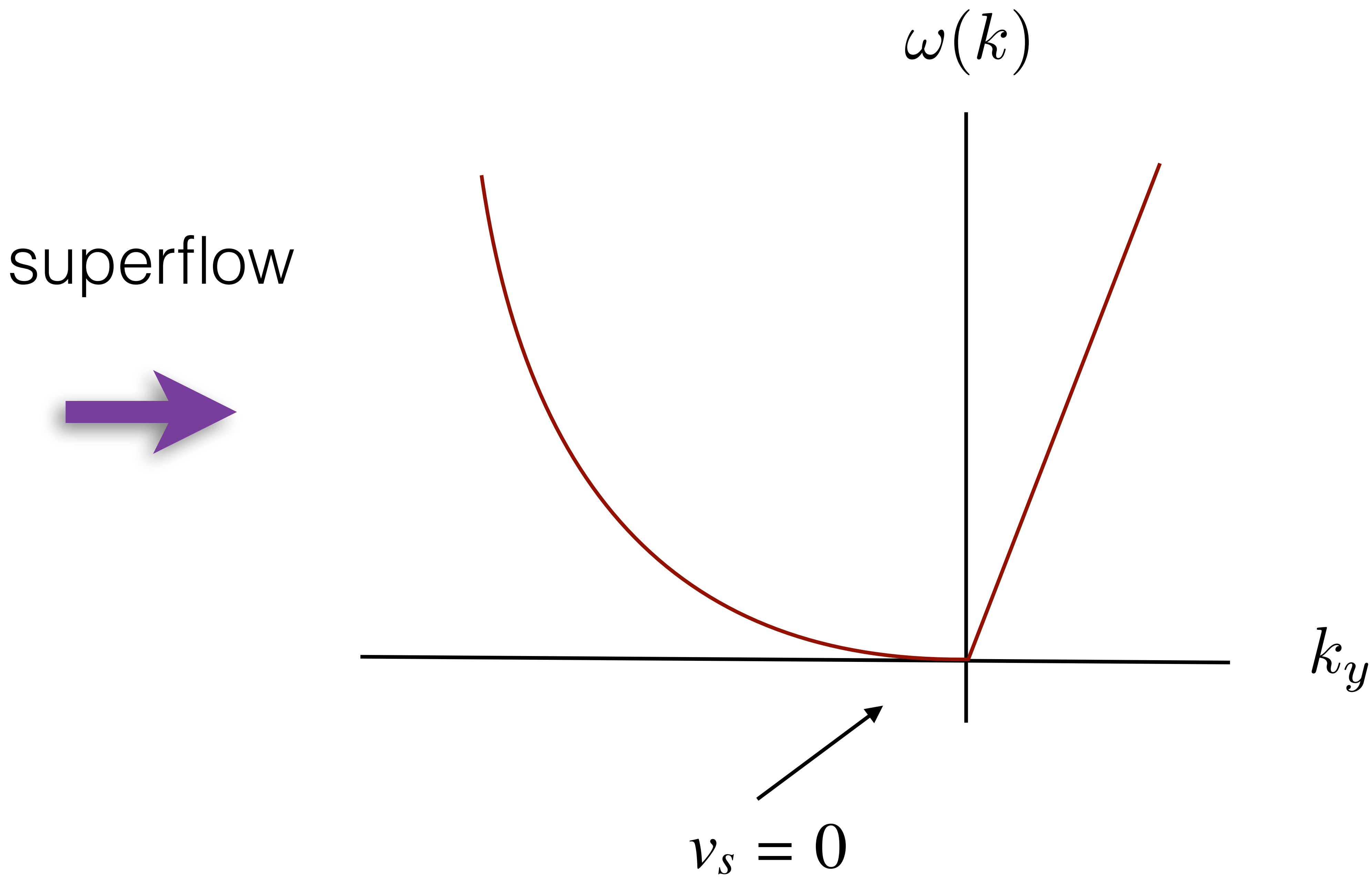
$$V_y > 0$$

superflow



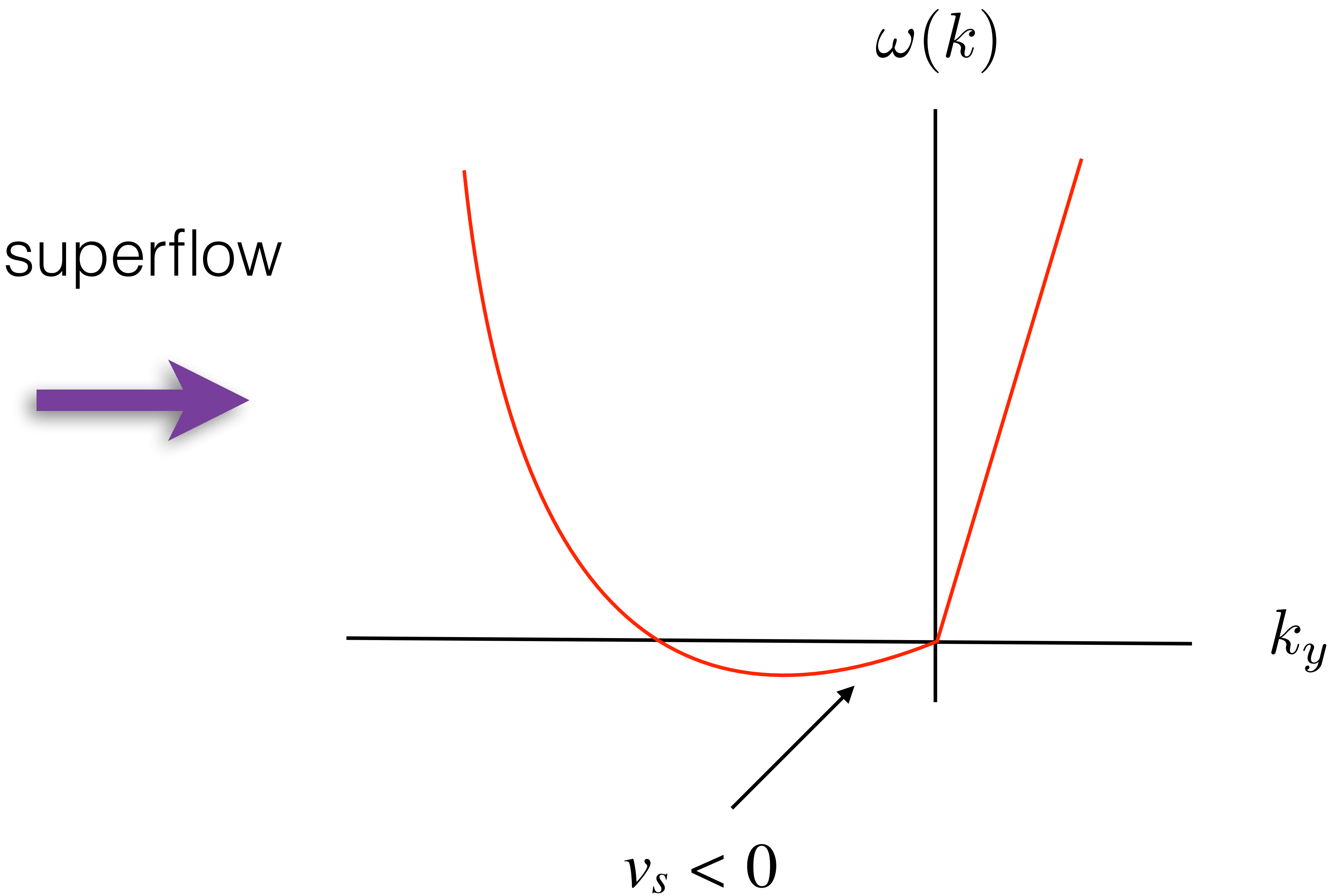
Phonon dispersion

$$V_y = V^{crit}$$



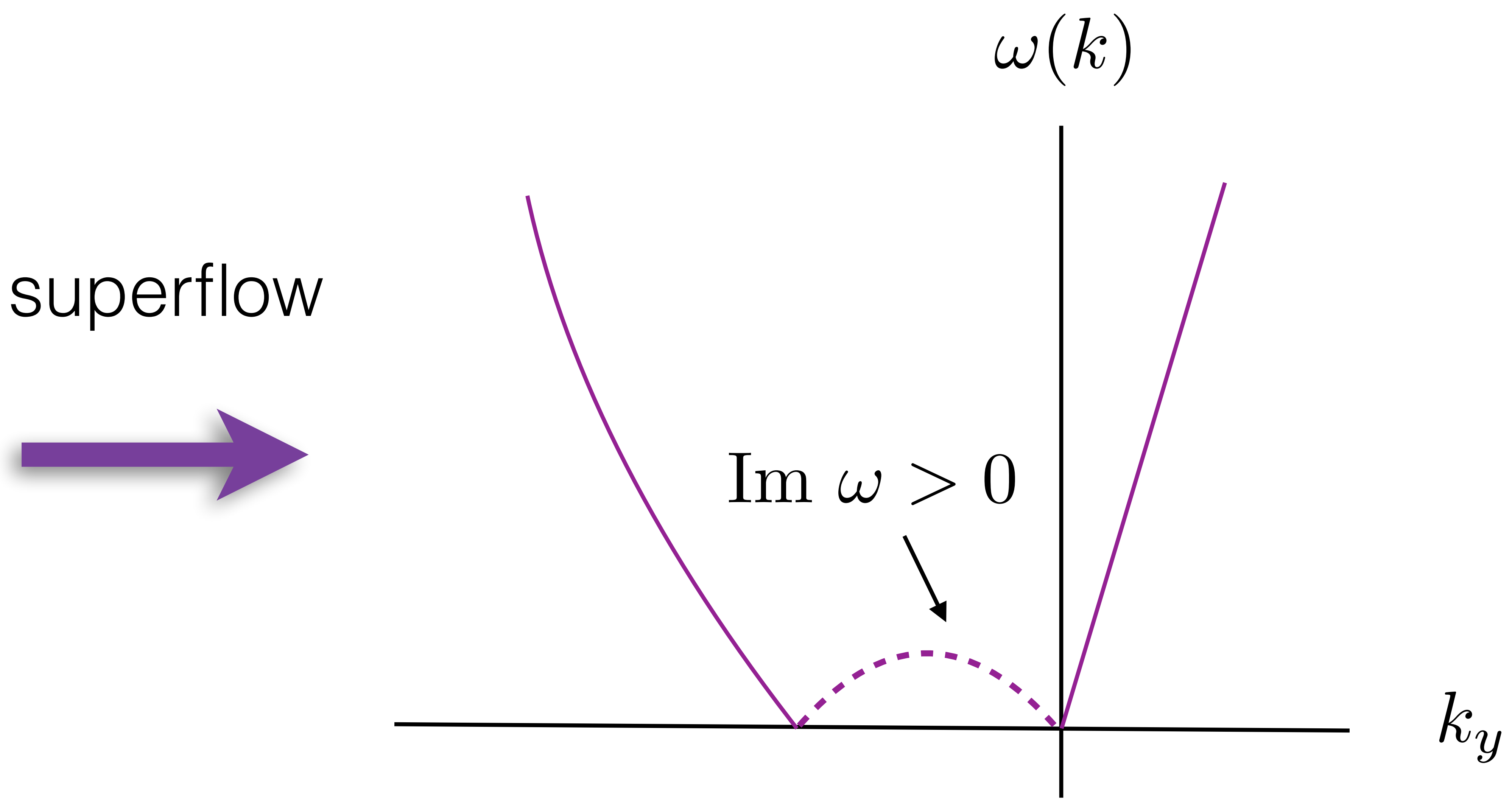
Phonon dispersion

$$V_y > V^{crit}$$

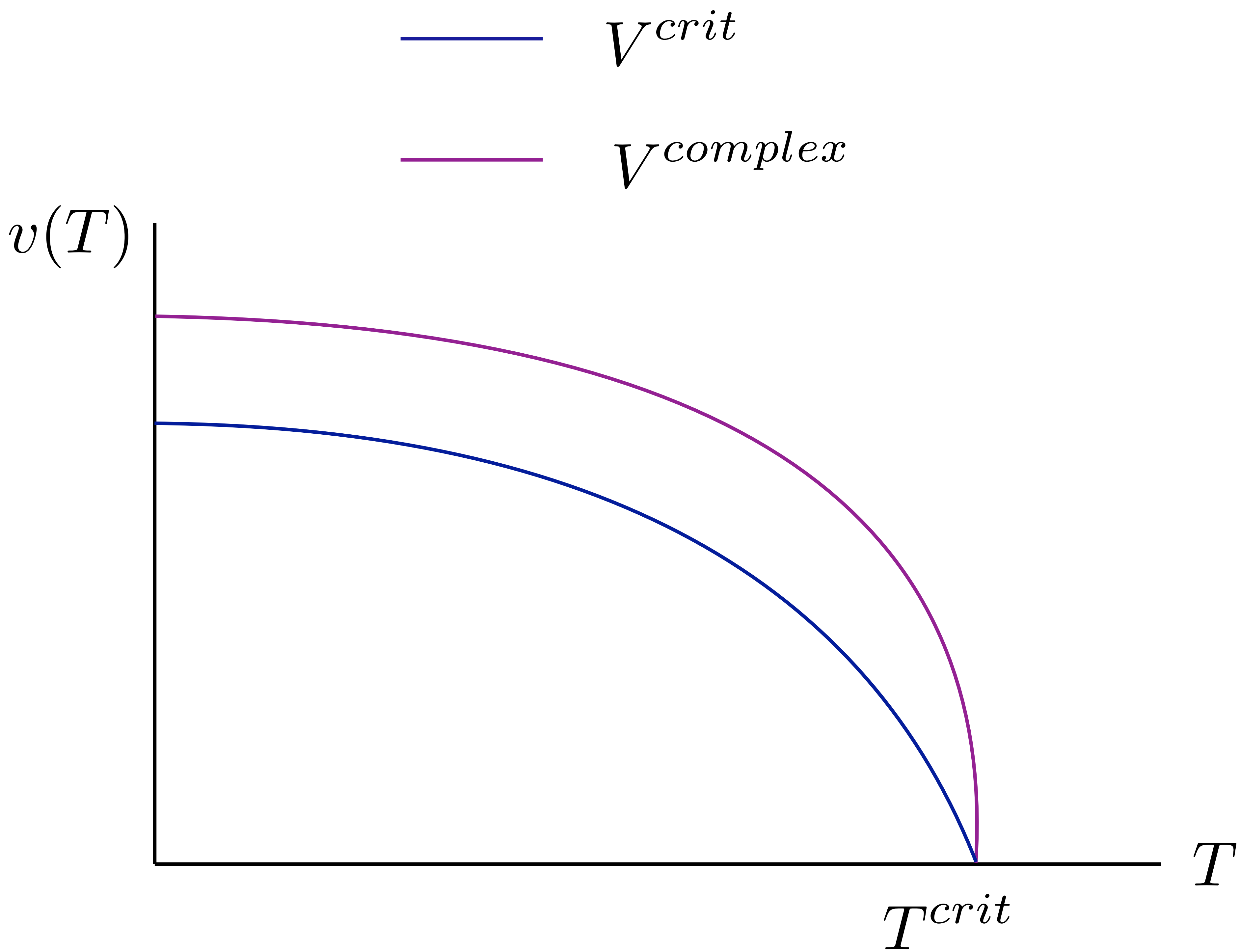


Phonon dispersion

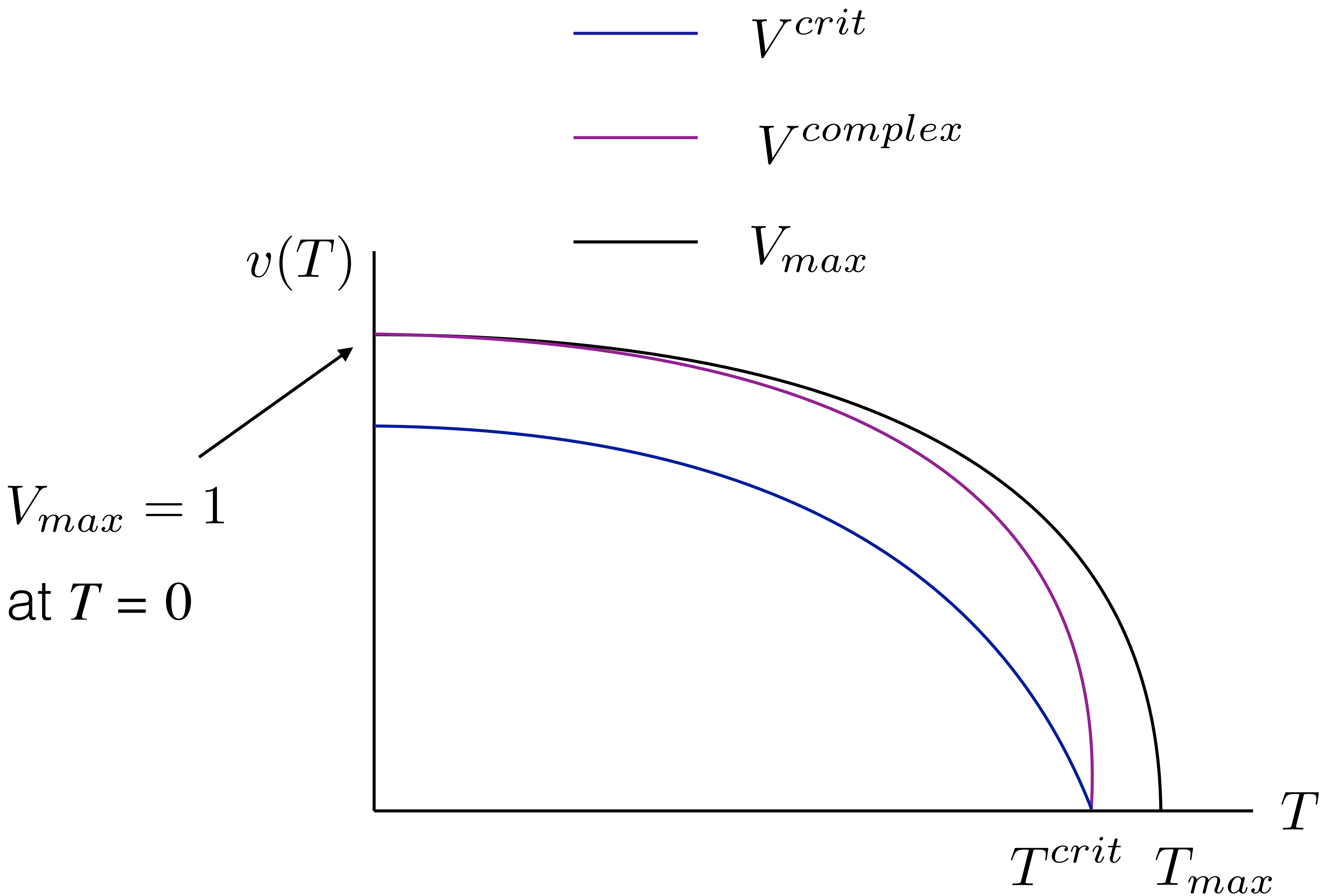
$$V_y > V^{complex} > V^{crit}$$



Velocity vs. Temperature



Velocity vs. Temperature



Summary

- Anyonic superfluid: unconventional superfluid
- Related to QH fluid by $SL(2,\mathbb{Z})$
- Holographic model of strongly-coupled anyon superflulid
  - $T \geq 0$
  - $V^{complex} > V^{crit}$
  - ground state for $V > V^{complex}$?