A holographic Dual for logarithmic Field Theories with $z = 2$ Lifshitz Scaling

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[tobɪɑs t͡sɪŋk]

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based on 1310.4778

logarithmic?

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logarithmic?

[Gurarie '93]

- irreducible representations which are not diagonalizable

logarithmic?

[Gurarie '93]

- irreducible representations which are not diagonalizable

$$\begin{aligned}
L_0 \cdot \phi^{(0)} &= h\phi^{(0)} \\
L_0 \cdot \phi^{(1)} &= h\phi^{(1)} + \phi^{(0)} \\
L_0 \cdot \phi^{(2)} &= h\phi^{(2)} + \phi^{(1)} \\
&\vdots \\
L_0 \cdot \phi^{(r)} &= h\phi^{(r)} + \phi^{(r-1)}
\end{aligned}$$

logarithmic?

[Gurarie '93]

- irreducible representations which are not diagonalizable

$$L_0 \cdot \vec{\Phi} = \underbrace{[h \cdot I + N]}_{J_h} \vec{\Phi}$$

logarithmic?

[Gurarie '93]

- irreducible representations which are not diagonalizable

$$L_0 \cdot \vec{\Phi} \quad = \quad \underbrace{[h \cdot I + N]}_{J_h} \vec{\Phi}$$

- applications include :
  gravitationally dressed CFTs, (multi)critical polymers, percolation, two-dimensional (magnetohydrodynamic) turbulence, the (fractional) quantum Hall effect, the sandpile model, disordered systems ...

logarithmic 2-point functions in CFT

$$\begin{aligned}
\langle\phi^{(0)}(z)\phi^{(0)}(0)\rangle &= 0\\
\langle\phi^{(0)}(z)\phi^{(1)}(0)\rangle &= 0\\
\langle\phi^{(0)}(z)\phi^{(2)}(0)\rangle &= \frac{c_0}{z^{2h}}\\
\langle\phi^{(1)}(z)\phi^{(1)}(0)\rangle &= \frac{c_0}{z^{2h}}\\
\langle\phi^{(1)}(z)\phi^{(2)}(0)\rangle &= \frac{c_0 + c_1 \ln z}{z^{2h}}\\
\langle\phi^{(1)}(z)\phi^{(2)}(0)\rangle &= \frac{c_0 + c_1 \ln z + c_2 (\ln z)^2}{z^{2h}}
\end{aligned}$$

Lifshitz/anisotropic scaling symmetry

$$
\begin{aligned}
t &\rightarrow \lambda^z\, t \\
x_i &\rightarrow \lambda\, x_i
\end{aligned}
$$

Lifshitz/anisotropic scaling symmetry

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- characteristic for (quantum) critical phenomena

Lifshitz/anisotropic scaling symmetry

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$$

- characteristic for (quantum) critical phenomena
- a metric that incorporates Lifshitz scaling :

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{dx^2 + dy^2 + dr^2}{r^2}$$

$\longrightarrow$ natural place to start constructing gravity duals of theories that are invariant under Lifshitz scaling (in the UV)

[Kachru, Liu, Mulligan '08]

Motivation

Condensed Matter :

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  BUT : *not all systems of interest are conformally invariant*

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String Theory :

- gravitational Lifshitz models can be uplifted to string theory

[Donos, Gauntlett '10]
[Chemissany, Hartong '12]

$\longrightarrow$ *better understanding of a new class of string vacua*

Action

$$\begin{aligned}
\mathcal{S} &= \mathcal{S}_{EH} + \mathcal{S}_{Proca} \\
\mathcal{S}_{EH} &= \frac{1}{2\kappa}\int_M (R - 2\Lambda) v_M \\
\mathcal{S}_{Proca} &= -\frac{1}{4\kappa}\int_M \left(dP \wedge *dP + c\, P \wedge *P\right) \\
v_M &= e_3 \wedge e_1 \wedge e_2 \wedge e_0
\end{aligned}$$

Lifshitz solutions :

$$\begin{aligned}
c &= \frac{2z}{L^2} \\
\Lambda &= -\frac{z^2 + z + 4}{2L^2}
\end{aligned}$$

Lifshitz Background

asymptotic Lifshitz fixed point :

$$
\begin{aligned}
e_0 &= \frac{1}{r^z}\, dt \\
e_1 &= \frac{1}{r}\, dx \\
e_2 &= \frac{1}{r}\, dy \\
e_3 &= -\frac{1}{r}\, dr \\
P &= \sqrt{\frac{2(z-1)}{z}}\,\frac{1}{r^z}\, dt
\end{aligned}
$$

Holographic n-point Functions

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$$\langle \mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)\rangle = (-i)^{n+1}\frac{\delta^n \mathcal{S}^{on-shell}}{\delta J_1(x_1)\cdots\delta J_n(x_n)}\bigg|_{J=0}$$

Holographic n-point Functions

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$$\mathcal{S}^{on-shell,ren} = \mathcal{S}^{bulk}+\mathcal{S}^{GH}+\mathcal{S}^{ct}+\mathcal{S}^{ct,anom}$$

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for 2-point functions it suffices to consider the linearized e.o.m. ($\longrightarrow$ only need $\mathcal{S}^{bulk}$ to quadratic order)

[Son, Starinets '02]

Linearized Einstein-Proca equations

$$\delta g \sim \left[\begin{array}{c:c:c} \overbrace{\Omega}^{t} & \overbrace{\Sigma}^{x,y} & \overbrace{0}^{r} \\ \hdashline \Sigma & h & 0 \\ \hdashline 0 & 0 & 0 \end{array}\right]\begin{array}{l} \}t \\ \}x,y \\ \}r \end{array} \qquad \delta P \sim \left(\begin{array}{c} a \\ \hdashline b \\ \hdashline c \end{array}\right)\begin{array}{l} \}t \\ \}x,y \\ \}r \end{array}$$

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Linearized Einstein-Proca equations

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- 10(9) coupled linear PDEs
- 4 (linear) constraints from Bianchi identities

Linearized Einstein-Proca equations

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- 10(9) coupled linear PDEs
- 4 (linear) constraints from Bianchi identities

- transversal/shear modes ($\perp k$) :

$$\check{h}, \quad \check{\Sigma}, \quad \check{b}$$

- longitudinal/sound modes ($\parallel k$) :

$$\Omega, \quad a, \quad c, \quad \mathrm{tr}\, h, \quad \hat{h}, \quad \hat{\Sigma}, \quad \hat{b}$$

Perturbations

$$\delta\Xi \sim r^{\Delta} \sum_{k,l,m,n\geq 0} \Xi^{(k,l,m,n)}\left[\xi^{(0)}\right] r^{2k+2lz+m\lambda_1+n\lambda_2}$$

$$\lambda_{1,2} = \frac{1}{2}\left(z+2\mp\sqrt{9z^2-20z+20}\right)$$

Perturbations

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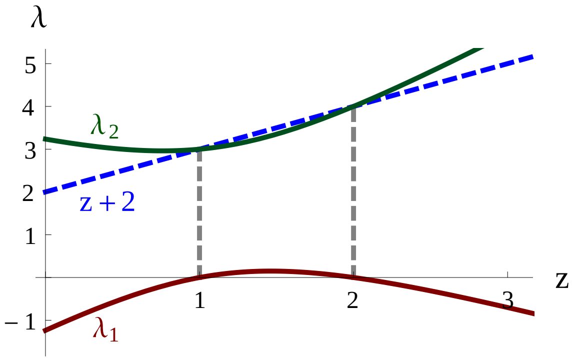
$$\lambda_{1,2} = \frac{1}{2}\left(z+2\mp\sqrt{9z^2-20z+20}\right)$$

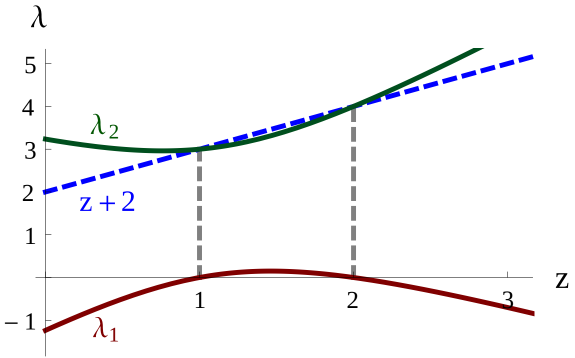
- $\Delta = 0$: sources for $\mathcal{E}$, ϵ^j and Π^j_a
- $\Delta = 2(z-1)$: source for p_a
- $\Delta = z+2$: expectation values for $\mathcal{E}$, ϵ^j and Π^j_a
- $\Delta = 3z$: expectation value for p_a
- $\Delta = \lambda_1(z)$: source of a scalar operator O_P
- $\Delta = \lambda_2(z)$: expectation value for O_P

[Ross, Saremi '09]

$z = 2$



$z = 2$



$$\delta\Xi \sim \sum_{n\geq 0} \Xi^{(n)}[\xi^{(0)}, \tilde{\xi}^{(0)}]\, r^{2n} + \ln r \sum_{n\geq 0} \tilde{\Xi}^{(n)}[\tilde{\xi}^{(0)}]\, r^{2n}$$

Quadratic on-shell Action

$$\mathcal{S}^{on-shell} = \frac{1}{2\kappa}\frac{1}{(2\pi)^3}\int \langle X, \mathcal{M}X\rangle \, d\omega \, d^2k + \mathcal{O}\left[X^3, (\omega+k^2)^2\right]$$

$$X = \left(\Omega^{(0)}, \widetilde{\Omega}^{(0)}, \check{\Sigma}^{(2)}, \hat{\Sigma}^{(2)}, \check{\Sigma}^{(0)}, \hat{\Sigma}^{(0)}, \operatorname{tr} h^{(0)}, \hat{h}^{(0)}, \check{h}^{(0)}, \check{u}, \hat{v}, \check{v}, T, \widetilde{T}\right)$$

$$\mathcal{M} = \frac{1}{2}\begin{bmatrix}
 & & & & & & & & & & & & & 1 \\
 & & & & & & & & & & & & 1 & \\
 & & & & & & & & & & & 1 & & \\
 & & & & & & & & & & 1 & & & \\
 & & & & & & & & & 1 & & & & \\
 & & & & & & & & & & & & & \frac{\omega}{k^2} \\
 & & & & & & & & & & \omega & & & -2 \\
 & & & & & & & & & & -2\omega & & & 2 \\
 & & & & & & & & & & & 2\omega & & \\
 & & & & 1 & & & & & & & & & \\
 & & & 1 & & & \omega & -2\omega & & & & & & \\
 & & 1 & & & & & & 2\omega & & & & & \\
 & 1 & & & & & & & & & & & & \\
1 & & & & & \frac{\omega}{k^2} & -2 & 2 & & & & & &
\end{bmatrix}$$

Shear Channel

$$\check{\Pi}, \quad \check{\epsilon}, \quad \check{p}$$

Shear Channel

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$$\check{\Pi}, \quad \check{\epsilon}, \quad \check{p}$$

2-point functions are related :

$$\begin{aligned} \langle \check{p}\,\check{p} \rangle &= \lambda \langle \check{\Pi}\,\check{p} \rangle = \lambda^2 \langle \check{\Pi}\,\check{\Pi} \rangle \\ \langle \check{p}\,\check{\epsilon} \rangle &= \lambda \langle \check{\Pi}\,\check{\epsilon} \rangle \end{aligned} \qquad \Bigg| \; \lambda = \frac{k^2}{\omega}$$

Shear Channel

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equations for $\check{h}, \check{\Sigma}$ and $\check{b}$ can be reduced to 2 decoupled master equations :

$$\begin{aligned} v^2 \mathcal{X}'' - v \mathcal{X}' + v\,(2v - \lambda)\, \mathcal{X} &= 0 \\ v^2 \mathcal{Y}'' + v\left(v - \tfrac{\lambda}{2}\right) \mathcal{Y} &= 0 \end{aligned} \quad \Bigg| \; v = \frac{r^2}{2\omega}$$

Shear Channel: Master Equations

$$v^2\mathcal{X}'' - v\mathcal{X}' + v\,(2v-\lambda)\,\mathcal{X} = 0$$

$$v^2\mathcal{Y}'' + v\left(v - \frac{\lambda}{2}\right)\mathcal{Y} = 0$$

Shear Channel: Master Equations

$$v^2\mathcal{X}'' - v\mathcal{X}' + v(2v-\lambda)\mathcal{X} = 0$$
$$v^2\mathcal{Y}'' + v\left(v - \frac{\lambda}{2}\right)\mathcal{Y} = 0$$

solutions with ingoing boundary conditions :

$$\mathcal{X} = B_{\mathcal{X}} e^{-iv}\,\Gamma\left(1 - \frac{i\lambda}{4}\right) U\left(-\frac{i\lambda}{4}, 0, 2iv\right)$$
$$\mathcal{Y} = B_{\mathcal{Y}} e^{-iv}\,v^2\,\Gamma\left(\frac{6-i\lambda}{4}\right) U\left(\frac{6-i\lambda}{4}, 1, 2iv\right)$$

Shear Channel: Master Equations

$$v^2 \mathcal{X}'' - v\mathcal{X}' + v\,(2v - \lambda)\,\mathcal{X} = 0$$

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$$\mathcal{Y} = B_{\mathcal{Y}} e^{-iv}\,v^2\,\Gamma\left(\frac{6 - i\lambda}{4}\right) U\left(\frac{6 - i\lambda}{4}, 1, 2iv\right)$$

$$\check{\Xi}(v) = p^0(v) + p^i(v)\mathcal{X} + p^{ii}(v)\mathcal{Y} + p^{iii}(v)\mathcal{Y}' + p^{iv}(v)\int \mathcal{Y}$$

Shear Channel: 2-point functions & QNM

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$$
\begin{aligned}
\langle \check{p}\, \check{p} \rangle &= \frac{i\omega^2\lambda^2}{32(2\pi)^3\kappa}\left[\psi_1 + 4\ln(-i\omega)\right] \\
\langle \check{p}\, \check{\epsilon} \rangle &= -\frac{i\omega^3\lambda^2\psi_2}{32(2\pi)^3\kappa} \\
\langle \check{\epsilon}\, \check{\epsilon} \rangle &= -\frac{i\omega^4\lambda}{512(2\pi)^3\kappa}\left[\lambda(\lambda^2+4)\psi_1 - 8(\lambda^2+2)\psi_2 + 4\lambda(\lambda^2+4)\ln(-i\omega)\right]
\end{aligned}
$$

$$
\begin{aligned}
\psi_1 &= 2i\lambda - 6 + 8\gamma - \lambda^2\psi\left(\frac{i\lambda}{4}\right) + (\lambda^2+4)\,\psi\left(\frac{i\lambda+2}{4}\right) \\
\psi_2 &= \frac{1}{4}\left[2i\lambda^2 - 4\lambda^2 + 8i - \lambda\,(\lambda^2+4)\,\psi\left(\frac{i\lambda}{4}\right) + \lambda\,(\lambda^2+4)\,\psi\left(\frac{i\lambda+2}{4}\right)\right]
\end{aligned}
$$

Shear Channel: 2-point functions & QNM

$$\langle \check{p}\,\check{p} \rangle = \frac{i\omega^2\lambda^2}{32(2\pi)^3\kappa}\left[\psi_1 + 4\ln(-i\omega)\right]$$

$$\langle \check{p}\,\check{\epsilon} \rangle = -\frac{i\omega^3\lambda^2\psi_2}{32(2\pi)^3\kappa}$$

$$\langle \check{\epsilon}\,\check{\epsilon} \rangle = -\frac{i\omega^4\lambda}{512(2\pi)^3\kappa}\left[\lambda(\lambda^2+4)\psi_1 - 8(\lambda^2+2)\psi_2 + 4\lambda(\lambda^2+4)\ln(-i\omega)\right]$$

$$\psi_1 = 2i\lambda - 6 + 8\gamma - \lambda^2\psi\left(\frac{i\lambda}{4}\right) + (\lambda^2+4)\,\psi\left(\frac{i\lambda+2}{4}\right)$$

$$\psi_2 = \frac{1}{4}\left[2i\lambda^2 - 4\lambda^2 + 8i - \lambda\,(\lambda^2+4)\,\psi\left(\frac{i\lambda}{4}\right) + \lambda\,(\lambda^2+4)\,\psi\left(\frac{i\lambda+2}{4}\right)\right]$$

quasinormal frequencies :

$$\omega = -\frac{i}{2n}k^2, \quad n > 2$$

Sound Channel

$$\mathcal{E}, \quad \widetilde{\mathcal{E}}, \quad \mathrm{tr}\Pi, \quad \hat{\Pi}, \quad \hat{\epsilon}, \quad \hat{p}$$

Sound Channel

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$$\mathcal{E}, \quad \widetilde{\mathcal{E}}, \quad \mathrm{tr}\Pi, \quad \hat{\Pi}, \quad \hat{\epsilon}, \quad \hat{p}$$

relations :

$$\begin{aligned}
\langle \mathcal{E}\,\mathcal{E}\rangle &= -\frac{1}{2}\langle \mathcal{E}\,\mathrm{tr}\Pi\rangle = \frac{1}{\omega}\langle \hat{\epsilon}\,\mathcal{E}\rangle = \frac{1}{4}\langle \mathrm{tr}\Pi\,\mathrm{tr}\Pi\rangle = -\frac{1}{2\omega}\langle \hat{\epsilon}\,\mathrm{tr}\Pi\rangle = \frac{1}{\omega^2}\langle \hat{\epsilon}\,\hat{\epsilon}\rangle \\
\langle \widetilde{\mathcal{E}}\,\mathcal{E}\rangle &= -\frac{1}{2}\langle \widetilde{\mathcal{E}}\,\mathrm{tr}\Pi\rangle = \frac{1}{\omega}\langle \widetilde{\mathcal{E}}\,\hat{\epsilon}\rangle \\
\langle \hat{p}\,\mathcal{E}\rangle &= -\frac{1}{2}\langle \hat{p}\,\mathrm{tr}\Pi\rangle = \frac{1}{\omega}\langle \hat{p}\,\hat{\epsilon}\rangle = \lambda\langle \hat{\Pi}\,\mathcal{E}\rangle = -\frac{\lambda}{2}\langle \hat{\Pi}\,\mathrm{tr}\Pi\rangle = \frac{\lambda}{\omega}\langle \hat{\Pi}\,\hat{\epsilon}\rangle \\
\langle \hat{p}\,\widetilde{\mathcal{E}}\rangle &= \lambda\langle \hat{\Pi}\,\widetilde{\mathcal{E}}\rangle \\
\langle \hat{p}\,\hat{p}\rangle &= \lambda\langle \hat{\Pi}\,\hat{p}\rangle = \lambda^2\langle \hat{\Pi}\,\hat{\Pi}\rangle
\end{aligned}$$

Sound Channel: Master Equation

$$\mathfrak{D}^3 \frac{1}{v} \mathfrak{D}^2 \frac{1}{v^2} \mathfrak{D}^1 \mathcal{Z} \quad = \quad 0$$

$$\mathfrak{D}^1 \quad = \quad v^2 \partial_v^2 + v \left(v - \frac{\lambda}{2} \right)$$

$$\mathfrak{D}^2 \quad = \quad v^2 \partial_v^2 + v \partial_v + v \left(v + 2i - \frac{\lambda}{2} \right)$$

$$\mathfrak{D}^3 \quad = \quad v^2 \partial_v^2 + v \partial_v + v \left(v - 2i - \frac{\lambda}{2} \right)$$

Sound Channel: Master Equation

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$$\mathfrak{D}^3 \frac{1}{v} \mathfrak{D}^2 \frac{1}{v^2} \mathfrak{D}^1 Z \quad = \quad 0$$

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\mathfrak{D}^1 &= v^2 \partial_v^2 + v \left(v - \frac{\lambda}{2} \right) \\
\mathfrak{D}^2 &= v^2 \partial_v^2 + v\partial_v + v \left(v + 2i - \frac{\lambda}{2} \right) \\
\mathfrak{D}^3 &= v^2 \partial_v^2 + v\partial_v + v \left(v - 2i - \frac{\lambda}{2} \right)
\end{aligned}$$

$$\begin{aligned}
\hat{\Xi}(v) &= \sum_{0 \le j \le 3} q^i_j(v) \Upsilon^j(v) + q^{ii}_j(v) \Upsilon^{j\prime}(v) + q^{iii}_j(v) \int \Upsilon^j(v) \\
\mathfrak{D}^j \Upsilon^j &= 0
\end{aligned}$$

Sound Channel: 2-point functions

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$$\langle \mathcal{E}\,\mathcal{E}\rangle = -\frac{i\omega^2\lambda(\lambda^2+4)}{8\kappa(2\pi)^3\Phi}$$

$$\langle \widetilde{\mathcal{E}}\mathcal{E}\rangle = \frac{i\omega^2\lambda(\lambda^2+4)}{8\kappa(2\pi)^3\Phi}\ln(-i\omega) + \frac{i\omega^2\left[(\lambda^2+4)\psi_3+\lambda\psi_4\right]}{16\kappa(2\pi)^3\Phi}$$

$$\langle \widetilde{\mathcal{E}}\widetilde{\mathcal{E}}\rangle = -\frac{i\omega^2\lambda(\lambda^2+4)}{8\kappa(2\pi)^3\Phi}\ln(-i\omega)^2 - \frac{i\omega^2\left[(\lambda^2+4)\psi_3+\lambda\psi_4\right]}{8\kappa(2\pi)^3\Phi}\ln(-i\omega) - \frac{i\omega^2\psi_3\psi_4}{8\kappa(2\pi)^3\Phi}$$

$$\langle \hat{p}\,\mathcal{E}\rangle = \frac{i\omega^2\lambda^2}{\kappa(2\pi)^3\Phi}$$

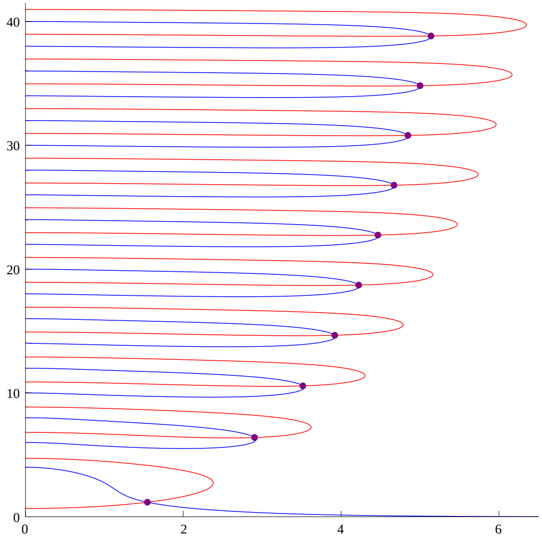
$$\langle \hat{p}\,\widetilde{\mathcal{E}}\rangle = -\frac{2i\omega^2\lambda\left[\psi_3+\lambda\ln(-i\omega)\right]}{\kappa(2\pi)^3\Phi}$$

$$\langle \hat{p}\,\hat{p}\rangle = -\frac{i\omega^2\lambda^2}{8\kappa(2\pi)^3(\lambda^2+4)}\left[\frac{64\lambda}{\Phi} - \psi_4 - (\lambda^2+4)\ln(-i\omega)\right]$$

$$\Phi = 2i\lambda^2 + 8\lambda + 8i - \left(\lambda^2+4\right)\lambda\psi\left(\frac{i\lambda}{4}\right) + \left(\lambda^2+4\right)\lambda\psi\left(\frac{i\lambda+2}{4}\right)$$

$$\text{QNM} \longleftrightarrow \Phi = 0$$

Sound Channel: quasinormal Frequencies



logarithmic Correlators

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momentum space

$$
\begin{aligned}
\langle \mathcal{E}\,\mathcal{E} \rangle &= -\frac{i\omega^2 \lambda(\lambda^2+4)}{8\kappa(2\pi)^3\Phi} \\
\langle \widetilde{\mathcal{E}}\mathcal{E} \rangle &= \frac{i\omega^2 \lambda(\lambda^2+4)}{8\kappa(2\pi)^3\Phi} \ln(-i\omega) + \frac{i\omega^2 \left[(\lambda^2+4)\psi_3 + \lambda\psi_4\right]}{16\kappa(2\pi)^3\Phi} \\
\langle \widetilde{\mathcal{E}}\widetilde{\mathcal{E}} \rangle &= -\frac{i\omega^2 \lambda(\lambda^2+4)}{8\kappa(2\pi)^3\Phi} \ln(-i\omega)^2 - \frac{i\omega^2 \left[(\lambda^2+4)\psi_3 + \lambda\psi_4\right]}{8\kappa(2\pi)^3\Phi} \ln(-i\omega) - \frac{i\omega^2 \psi_3\psi_4}{8\kappa(2\pi)^3\Phi}
\end{aligned}
$$

logarithmic Correlators

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$$\langle \widetilde{\mathcal{E}}\widetilde{\mathcal{E}}\rangle = -\frac{i\omega^2\lambda(\lambda^2+4)}{8\kappa(2\pi)^3\Phi}\ln(-i\omega)^2 - \frac{i\omega^2\left[(\lambda^2+4)\psi_3+\lambda\psi_4\right]}{8\kappa(2\pi)^3\Phi}\ln(-i\omega) - \frac{i\omega^2\psi_3\psi_4}{8\kappa(2\pi)^3\Phi}$$

position space $\chi = x^2/t$

$$\langle \mathcal{E}(t,x)\,\mathcal{E}(0,0)\rangle = \frac{\theta(t)}{t^4}\,g^{(0)}(\chi)$$

$$\langle \widetilde{\mathcal{E}}(t,x)\,\mathcal{E}(0,0)\rangle = \frac{\theta(t)\ln t}{t^4}\,g^{(0)}(\chi) + \frac{\theta(t)}{t^4}\,g^{(1)}(\chi)$$

$$\langle \widetilde{\mathcal{E}}(t,x)\,\widetilde{\mathcal{E}}(0,0)\rangle = \frac{\theta(t)\ln^2 t}{t^4}\,g^{(0)}(\chi) + \frac{\theta(t)\ln t}{t^4}\,g^{(2)}(\chi) + \frac{\theta(t)}{t^4}\,g^{(3)}(\chi)$$

logarithmic Correlators

momentum space

$$\langle \mathcal{E}\,\mathcal{E}\rangle = -\frac{i\omega^2\lambda(\lambda^2+4)}{8\kappa(2\pi)^3\Phi}$$

$$\langle \widetilde{\mathcal{E}}\mathcal{E}\rangle = \frac{i\omega^2\lambda(\lambda^2+4)}{8\kappa(2\pi)^3\Phi}\ln(-i\omega) + \frac{i\omega^2\left[(\lambda^2+4)\psi_3+\lambda\psi_4\right]}{16\kappa(2\pi)^3\Phi}$$

$$\langle \widetilde{\mathcal{E}}\widetilde{\mathcal{E}}\rangle = -\frac{i\omega^2\lambda(\lambda^2+4)}{8\kappa(2\pi)^3\Phi}\ln(-i\omega)^2 - \frac{i\omega^2\left[(\lambda^2+4)\psi_3+\lambda\psi_4\right]}{8\kappa(2\pi)^3\Phi}\ln(-i\omega) - \frac{i\omega^2\psi_3\psi_4}{8\kappa(2\pi)^3\Phi}$$

position space $\chi = x^2/t$

$$\langle \mathcal{E}(t,x)\,\mathcal{E}(0,0)\rangle = \frac{\theta(t)}{t^4}\,g^{(0)}(\chi)$$

$$\langle \widetilde{\mathcal{E}}(t,x)\,\mathcal{E}(0,0)\rangle = \frac{\theta(t)\ln t}{t^4}\,g^{(0)}(\chi) + \frac{\theta(t)}{t^4}\,g^{(1)}(\chi)$$

$$\langle \widetilde{\mathcal{E}}(t,x)\,\widetilde{\mathcal{E}}(0,0)\rangle = \frac{\theta(t)\ln^2 t}{t^4}\,g^{(0)}(\chi) + \frac{\theta(t)\ln t}{t^4}\,g^{(2)}(\chi) + \frac{\theta(t)}{t^4}\,g^{(3)}(\chi)$$

$\longrightarrow$ cf. LCFT & e.g. [Bergshoeff, Haan, Merbis, Rosseel, '11]

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