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Semiholographic non-Fermi liquids

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Holography gives us the possibility of exploring new possible (at least theoretically) phases of matter

It has confirmed ideas proposed before in the CM community but in a more controlled (though less realistic) setting

At finite density, relatively easy to obtain exotic states with attractive features also from a phenomenological point of view Landau's theory of Fermi liquids is remarkably robust, accounting for the properties of most metals in terms of weakly interacting quasiparticles

 $C_v \sim T \qquad \rho \sim T^2$

However many examples are now known of "strange metals" that do not fall into this class





Figure 2. Comparison of the polycrystalline $La_{2-x}Sr_xCuO_4$ resistivity (data points) extracted from Ref. 18 with the "bottleneck" resistivity form of Eq. [10]. Inset slows the low temperature region in detail.

This behavior is often associated with the vicinity to a quantum critical point, and generically points to a short lifetime of excitations





Holography gives many ways of constructing metallic phases with abnormal properties [Sachdev, Hartnoll, Liu, McGreevy, Vegh, Charmousis, Kiritsis, Gouteraux, ...]

The gravitational solution does not show a Fermi surface, so the nature of the charge carriers is obscure

The Fermi surface is exhibited by models that include fermions populating the bulk and backreacting on the geometry but they have FL-like physics [Sachdev, Hartnoll, Tavanfar, Hofman,... Schalm, Zaanen,..., Nitti, Policastro, Vanel...] [Liu, McGreevy, Vegh '09] showed that probe fermions in the background of an extremal AdS-RN black hole exhibits a Fermi surface in the spectral function

They can be understood as "domain-wall" fermions between the asymptotic AdS_4 and $AdS_2 \ge R^2$ regions

The spectral functions has a non-FL character due to the interaction with the critical degrees of freedom of the bulk CFT, similar to Hertz-Millis

The Faulkner-Polchinski model [F-P, 10]

$$S = \int dt \left[\sum_{k} \left(\chi_{\mathbf{k}}^{\dagger} (i\partial_{t} - \epsilon_{\mathbf{k}} + \mu) \chi_{\mathbf{k}} + \sum_{k} (g_{\mathbf{k}} \chi_{\mathbf{k}}^{\dagger} \psi_{\mathbf{k}} + c.c.) \right] + S_{\text{CFT}} \right]$$

 k_{\perp}

Hybridization with fermion of dim
$$\Delta_{\psi} = \frac{\nu + 1}{2}$$

Resummed propagator reproduces Liu et al.

$$G_{\chi\chi} = -\frac{a}{|g|^2 c \omega^{\nu} - v_F k_{\perp}}$$

Almost-solvable model of a non-Fermi liquid Very broad quasiparticles Zero residue at the Fermi surface Lifshitz-like dispersion relation



Similar models: marginal or singular FL (Varma) hidden FL (Anderson) We generalized this model to extract "universal" predictions, i.e. that can be fitted with a finite number of parameters

Most studies focused on single-particle properties, that can be directly measured via ARPES etc.

We want to consider also multiparticle properties and collective excitations

The interacting F-P model

$$S = N^{2}S_{\text{CFT}} + \int dt \left[\sum_{k} \left(\chi_{\mathbf{k}}^{\dagger}(i\partial_{t} - \epsilon_{\mathbf{k}} + \mu) \chi_{\mathbf{k}} + N \sum_{k} (g_{\mathbf{k}}\chi_{\mathbf{k}}^{\dagger}\psi_{\mathbf{k}} + c.c.) + N\eta \sum_{k} \chi^{\dagger}\chi\phi + N\tilde{g} \sum_{k} \chi^{\dagger}\chi\chi^{\dagger}\psi + \sum_{k} (\lambda + V(\mathbf{q})) \chi^{\dagger}\chi\chi^{\dagger}\chi$$

The N counting is chosen to reproduce the FP propagator at O(1) and suppress corrections to the vertex functions

Renormalization group [Polchinski, Sarkar]



$$\Lambda \rightarrow s\Lambda \qquad \mathbf{k} = \mathbf{k}_* + \mathbf{k}_\perp$$
$$[\mathbf{k}_*] = 0 \qquad [\mathbf{k}_\perp] = 1 \qquad [\chi] = -\frac{1}{2}$$

An interaction $\chi^n \Phi$ is relevant or marginal for

$$n \le \frac{3-\nu}{2}$$

The RG argument needs refinement for special configurations (BCS instability)

Self-energy corrections [Lee, 2009: breakdown of large N]



better behavior than in FL as $|\omega| << \omega^{\nu}$

Response function $\Delta \mathcal{L} = \int A_{\mu} (\chi^{\dagger} \gamma^{\mu} \chi + N^2 J^{\mu})$







2PI part: $\Gamma = \Gamma_0 + \Gamma_0 G G \Gamma$

IR leading behavior is determined by the singular part of the two propagator with momenta on the FS

$$G^{AB}(\mathbf{k} - \mathbf{q}/2, \omega)G^{CD}(\mathbf{k} + \mathbf{q}/2, \omega + \Omega) =$$

$$(a^2 Q^{ABCD} + g^{ABCD})(\mathbf{k}, \omega, \mathbf{q}, \Omega) \qquad \Psi^A = \begin{pmatrix} \chi \\ \psi \end{pmatrix}$$

$$Q^{1111} = Q \sim \Theta(-k_{\perp})\delta(\omega)k_{\perp}^{\frac{1}{\nu}-2} \frac{q\cos\theta}{q\cos\theta - \nu\Omega(\frac{|g|^2c}{v_F})^{\frac{1}{\nu}}k_{\perp}^{1-\frac{1}{\nu}}}.$$

compare with
$$Q_{FL} \sim \frac{\delta(k_{\perp})\delta(\omega)}{q\cos\theta - \Omega}$$

The ω integral uses

$$\int \frac{d\omega}{2\pi} \frac{f(\omega)}{\omega^{\nu} - a} = \frac{i}{\nu} a^{\frac{1-\nu}{\nu}} f(a^{1/\nu})$$

The analysis justifies a description in terms of Landau parameters

$$f(\mathbf{k}_*, \mathbf{k}'_*) = a^2 \Gamma^{\chi\chi\chi\chi}(\mathbf{k}_*, \mathbf{k}'_*, \omega = \omega' = 0, \mathbf{q}, \Omega \to 0)$$

that enter in the phenomenological description of a Fermi liquid

$$\tilde{\varepsilon}(p,\sigma) = \varepsilon(p,\sigma) + \sum_{p',\sigma'} f(p,\sigma,p',\sigma') \delta n(p',\sigma')$$

The response is encoded in the Landau-Silin equations

$$\delta n_{k}^{AB}(q) = \int d^{2} k' h^{AB\chi\chi}(\mathbf{k}_{*}, \mathbf{k}_{*}') Q(k', q) E_{\mathbf{k}_{*}'}^{\chi\chi}(q)$$
$$E_{\mathbf{k}_{*}}^{\chi\chi}(q) = \int d^{2} k' \left(h^{\chi\chi AB}(\mathbf{k}_{*}, \mathbf{k}_{*}') e_{\mathbf{k}_{*}'}^{AB}(q) + f(\mathbf{k}_{*}, \mathbf{k}_{*}') Q(k', q) E_{\mathbf{k}_{*}'}^{\chi\chi}(q) \right)$$

non-diagonal structure: $h^{AB\chi\chi} \neq 0$

Zero sound corresponds to a solution of

$$fQ = 1$$

The equations can be solved explicitly for $f = \text{const} = F_0$

Zero-sound linear dispersion $\Omega = v_0 q$ $w = \frac{v_F}{|g|^2 c}$

$$\frac{\nu v_F^2}{F_0 w} = (wk_c)^{\frac{1-\nu}{\nu}} - \frac{\nu v_0}{w} \operatorname{arcsin}\left(\frac{w}{\nu v_0} (wk_c)^{\frac{1-\nu}{\nu}}\right)$$

at large F_0 $v_0 \sim c^{-3/2\nu} \sqrt{F_0}$
in FL $\frac{1}{F_0} = \frac{v_0}{2v_F} \log\left(\frac{v_0 + v_F}{v_0 - v_F}\right) - 1$ $v_0 \sim \sqrt{F_0/3}$

The FL zero sound can be a sharp resonance or can be embedded in the particle-hole continuum and be highly damped



Im(polarization function) integrated over frequency is peaked around

$$\Omega^{1/\nu} = \epsilon_p^{1/\nu} + \epsilon_h^{1/\nu}$$

consistent with contour integral argument, but how sharp is the peak?



 $\nu = 0.7$

Finite temperature

$$G_R(\omega, \mathbf{k}) = \begin{cases} \frac{1}{\omega - v_F k_\perp + icT^{\nu}}, & \text{if } \omega << T \\ \frac{1}{\zeta_R \omega^{\nu} - v_F k_\perp + i\zeta_I \omega^{\nu}}, & \text{if } \omega >> T \end{cases}$$

Optical conductivity $\sigma(\omega) \sim \omega^{-\nu} \qquad \omega >> T$

Entropy density $\frac{S}{V} \sim T^{1-\nu+\nu^2}$

The semiholographic approach offers a viable framework for phenomenological model building

The generalization of Landau theory can be developed and justified from first principles

Testable predictions depending on few parameters Multiple directions for further generalizations, e.g. lattice, different IR sectors Work in progress

Detailed study of the response function Self-energy corrections BCS instability Particle-hole asymmetry Open issues

The coupling g is dimensionful, is it a new scale different from E_F ?

Finite N breakdown [Lee, Mross et al.]

Dynamically determined scaling exponent