Holographic computations in higher spin theories

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Summary

Perturbative procedure for higher spin interactions quickly becomes tedious

Alternative: Reproduce interactions from AdS/CFT
(ultimately simple CFT)

To start: aiming at quartic vertex for scalars.

Outline

- A problem of higher spin interactions
- AdS/CFT for higher spin theories
- New methods for Witten diagrams
- Towards quartic vertices from AdS/CFT

Higher Spin Interactions

Higher Spin fields

Higher spin fields are fundamental fields of spin greater than two.

Ultimate goal: make them interact.

Fronsdal theory

Massless $m^2 = 0$

$$\partial^\mu \partial_\mu \phi^{a(s)} = 0, \quad \phi^\rho{}_\rho{}^{a(s-2)} = 0, \quad \partial_\mu \phi^{\mu a(s-1)} = 0.$$

Gauge invariance

$$\delta \phi^{a(s)} = \partial^a \xi^{a(s-1)}, \quad \xi^\rho{}_\rho{}^{a(s-3)} = 0, \quad \partial_\mu \xi^{\mu a(s-2)} = 0.$$

Off-shell description: Fronsdal theory

$$\phi^{bcdea(s-4)} \eta_{bc} \eta_{de} = 0, \qquad \xi^{bca(s-3)} \eta_{bc} = 0.$$

Gauge invariant action $\delta \phi^{a(s)} = \partial^a \xi^{a(s-1)}$

$$S[\phi] = \int d^d x \left(\partial^b \phi^{a(s)} \partial_b \phi_{a(s)} + \ldots \right).$$

[Fronsdal'78]

Interactions. Deformation procedure

Free theory

$$\delta_0 S_2 = 0, \quad S_2 \sim \phi \cdot \phi, \quad \delta_0 \phi \sim \xi.$$

Target nonlinear theory. Perturbative construction

$$\delta_{nl} S_{nl} = 0.$$

$$S_{nl} = S_2 + g S_3 + g^2 S_4 + \dots, \quad S_3 \sim \phi \cdot \phi \cdot \phi, \quad S_4 \sim \phi \cdot \phi \cdot \phi \cdot \phi,$$

$$\delta_{nl} \phi = (\delta_0 + g\delta_1 + g^2 \delta_2 + \dots)\phi, \quad \delta_1 \phi \sim \xi \cdot \phi, \quad \delta_2 \phi \sim \xi \cdot \phi \cdot \phi,$$

$$g^1 : \quad \delta_0 S_3 + \delta_1 S_2 = 0,$$

$$g^2 : \quad \delta_0 S_4 + \delta_1 S_3 + \delta_2 S_2 = 0, \quad \dots .$$

Progress in higher spin vertices

A lot of people have been working on that (and keep working):
Bengtsson, Bengtsson, Brink, Berends, Burgers, van Dam, Fradkin, Vasiliev, Metsaev, Boulanger, Bekaert, Cnockaert, Leclercq, Sundell, Alkalaev, Skvortsov, Manvelyan, Mkrtchyan, Ruehl, Sagnotti, Taronna, Joung, Lopez, Fotopoulos, Tsulaia, Zinoviev, Buchbinder, Snegirev, Henneaux, Lucena Gomez, Rahman, Campoleoni, Dempster, D. P. . . .

Highlights of cubic vertices for massless fields

Local cubic vertices in the Minkowski space

Minkowski space, $d > 4$, $s_1 \leqslant s_2 \leqslant s_3$, $N(\partial)$ - number of derivatives

$$N_{min} = s_2 + s_3 - s_1 \leqslant N(\partial) \leqslant s_1 + s_2 + s_3 = N_{max}.$$

[Metsaev'91'93]

AdS space is more suitable for higher spin interactions.

Towards Quartic Vertices

Cubic vertices

$$g^1 : \quad \delta_0 S_3 + \delta_1 S_2 = 0$$

Linear in deformations
The spectrum is not fixed.

Quartic vertices

$$g^2 : \quad \delta_0 S_4 + \delta_1 S_3 + \delta_2 S_2 = 0$$

Quadratic in deformations $\Rightarrow$ fixes the spectrum
Requires infinite set of fields
Number of possible vertices is much larger

Believe: Local quartic vertices do not exist in the Minkowski space

[Metsaev'95], [Taronna'11], [Dempster, Tsulaia'12]

AdS/CFT and Higher Spins

AdS/CFT

AdS_{d+1}	CFT_d
isometry group	conformal group
fields	operators
$g_{\mu\nu}$	$T_{\mu\nu}$
m^2	$\Delta(\Delta - d)$
Witten diagrams	correlation functions

[Maldacena'97], [Gubser, Klebanov, Polyakov'98], [Witten'98].

AdS/CFT for Higher Spins

The minimal Vasiliev theory

[Vasiliev'90, '03]

in AdS_4 is dual to the free $O(N)$ vector model.

$$S = \frac{1}{2}\int d^3x \sum_{a=1}^{N} \partial_\mu \phi^a \partial^\mu \phi^a.$$

Conserved currents

$$J_{\mu_1 \dots \mu_s} = \sum_{a=1}^{N} \phi^a \partial_{(\mu_1} \dots \partial_{\mu_s)} \phi^a + \dots .$$

are dual to gauge fields

$$\delta\phi_{\mu_1 \dots \mu_s} = \nabla_{(\mu_1} \xi_{\mu_2 \dots \mu_s)}.$$

[Sezgin, Sundell'02], [Klebanov, Polyakov'02].

Evidences

Symmetries

Boundary. Linear differential operators $L(x, \frac{\partial}{\partial x})$ that preserve $\Box\phi = 0$

$$L(x, \partial/\partial x): \quad \Box\phi(x) = 0 \quad \Rightarrow \quad \Box L\phi(x) = 0$$

Bulk. Higher spin gauge algebra (Eastwood-Vasiliev algebra)

They coincide!

The algebra is unique

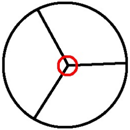
[Vasiliev'90], [Boulanger, D.P., Skvortsov, Taronna'13].

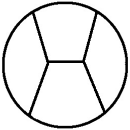
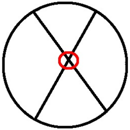
[Maldacena, Zhiboedov'11].

Check of certain 3pt functions

[Giombi, Yin'09,'10].

Quartic Vertex from AdS/CFT

$\langle J_1 J_2 J_3 \rangle =$  $\Rightarrow \quad V_3$

$\langle J_1 J_2 J_3 J_4 \rangle =$  $+$  $\Rightarrow \quad V_4$

New Methods for Witten Diagrams

Ambient Space and the Split Representation

Ambient space

AdS_{d+1} is spanned by a hypersurface $X^2 = -1$ in $\mathbb{R}^{d+2}$
Boundary is spanned by $P^2 = 0$. X — bulk, P — boundary points.

Idea: e.-f. of ∇ don't care $\rightarrow$ Use e.-f. of $\Box$

$$(\Box + (d/2)^2 + \nu^2)\Omega_\nu(X, X') = 0.$$

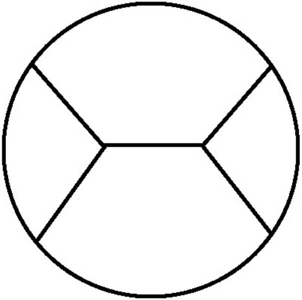
$$\Omega_\nu(X, X') \propto \int dP \frac{1}{(-2P\cdot X)^{d/2+i\nu}} \frac{1}{(-2P\cdot X')^{d/2-i\nu}}$$

$\frac{1}{(-2P\cdot X)^\Delta}$ is a bulk-to-boundary propagator with $m^2 = \Delta(\Delta - d)$.

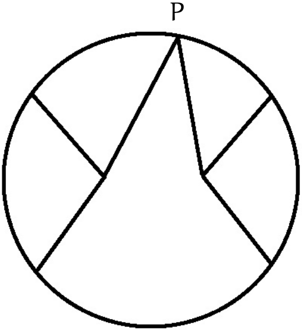
$$\Pi_\Delta(X, X') = \int \frac{d\nu}{\nu^2 + (\Delta - h)^2}\Omega_\nu(X, X').$$

Slit representation. Analogy with CFT

Factorisation of 4pt Witten diagrams



$$= \int d\nu\, a(\nu) \int dP$$



Leonhardt, Ruhl, Manvelyan, Costa, Paulos, Penedones, …

Results for Higher Spins

Bulk-to-boundary spin s

$$\Pi_{\Delta,s}(X,P;W,Z) = C_{\Delta,J}\frac{(-2(P\cdot X)(W\cdot Z)+2(W\cdot P)(Z\cdot X))^s}{(-2P\cdot X)^{\Delta+s}},$$

where W and Z are tangent auxiliary vectors in X and P.

Harmonic traceless basis

$$\Omega_{\nu,s}(X,X';W,W') \propto \int dP\Pi_{h+i\nu,s}(X,P;W,Z)\Pi_{h-i\nu,s}(X',P;W',\left\{\frac{\partial}{\partial Z}\right\})$$

$$\Pi_{\Delta,S} = \sum_{l=0}^{s}\int d\nu a_{s,l}(\nu)((W\cdot\nabla)(W'\cdot\nabla'))^{s-l}\Omega_{\nu,l}.$$

+ certain propagators

[Costa, Goncalves, Penedones'14]

Amplitudes: ⟨0000⟩ with HS exchange

Partial wave decomposition

Building block (partial wave)

$$F_{\nu,s} \propto \int dP_5 \langle \mathcal{O}_{\phi_1}(P_1)\mathcal{O}_{\phi_2}(P_2)\mathcal{O}_{h+i\nu,s}(P_5)\rangle\langle \mathcal{O}_{h-i\nu,s}(P_5)\mathcal{O}_{\phi_3}(P_3)\mathcal{O}_{\phi_4}(P_4)\rangle,$$

$$V_3 = h_{a(s)}\phi\nabla_a\ldots\nabla_a\phi.$$

$$\langle \mathcal{O}_{\phi_1}\mathcal{O}_{\phi_2}\mathcal{O}_{\phi_3}\mathcal{O}_{\phi_4}\rangle = \frac{1}{(P_{12})^{\frac{\Delta_1+\Delta_2}{2}}(P_{34})^{\frac{\Delta_3+\Delta_4}{2}}}\times$$

$$\left(\frac{P_{24}}{P_{14}}\right)^{\frac{\Delta_{12}}{2}}\left(\frac{P_{14}}{P_{13}}\right)^{\frac{\Delta_{34}}{2}}\sum_{s=0}^{\infty}\int d\nu\, b_s(\nu)F_{\nu,s}.$$

$F_{\nu,s}$ can be simply expressed in terms of conformal blocks $G_{h+i\nu,s}$ and $G_{h-i\nu,s}$.

[Costa, Goncalves, Penedones'14]

Quartic vertex for scalar fields in HS theory

[work in progress with X. Bekaert, J. Erdmenger and C. Sleight]

- Fronsdal propagator
- 4pt exchange diagram
- sum them and by comparing with CFT extract a quartic vertex

Fronsdal propagator

$$\mathcal{F}_{A(s)}(\Pi) - \frac{1}{4}s(s-1)g_{AA}\mathrm{Tr}\mathcal{F}_{A(s-2)}(\Pi) = \delta(X,X')(g_{AA'})^s$$

where $\mathcal{F}$ is a Fronsdal tensor. One cannot rich the TT gauge! (only on-free-shell)

De Donder gauge

Propagator is double-traceless
equation involves only $\Box$

Traceless gauge

Propagator is traceless
equation involves $\Box$ and $\nabla\cdot$

Manifest trace structure gauge

Propagator $\sim \sum_k (g_{AA}g_{B'B'})^k \times TT$

Fronsdal propagators

Traceless gauge

$$\Pi_{\Delta,S} = \left\{ \sum_{l=0}^{s} \int d\nu a_{s,l}(\nu)((W\cdot\nabla)(W'\cdot\nabla'))^{s-l}\Omega_{\nu,l} \right\},$$

where

$$a_{s,l}(\nu) = -\frac{2h+2s-3}{(2h+s+l-3)(s-l-1)}\frac{1}{\nu^2+(s-2+h)^2}\times$$
$$\frac{2^{s-l}(l+1)_{s-l}(h+l-1/2)_{s-l}}{(s-l)!(2h+l-1)_{s-l}(h+l-i\nu)_{s-l}(h+l+i\nu)_{s-l}}.$$

Fronsdal propagators

Manifest trace structure gauge

$$\Pi_{\Delta,S} = \sum_{j=0}^{[s/2]} \int d\nu\, b_{s,j}(\nu)(g_{AA})^j (g'_{AA})^j \Omega_{\nu,s-2j},$$

where

$$b_{s,j}(\nu) = \frac{(1/2)_{j-1}}{2^{2j+3}\cdot j!} \frac{(s-2j+1)_{2j}}{(h+s-j)_j(h+s-j-3/2)_j} \times$$
$$\frac{(h/2+s/2-j+i\nu)_{j-1}(h/2+s/2-j-i\nu)_{j-1}}{(h/2+s/2-j+1/2+i\nu)_j(h/2+s/2-j+1/2-i\nu)_j}$$

Current Vertices

Cubic $s-0-0$ vertices

$$V_3 \sim \phi_{A(s)} J^{A(s)}, \qquad \nabla_A J^{A(s)} \approx 0.$$

In flat space

$$J^{A(s)} = \phi(X)(\overleftarrow{\partial}^A - \partial^A)^s \phi(X). \tag{1}$$

In AdS

- Ambient space: projection of (1) to AdS hypersurface
- In intrinsic terms

$$J^{A(s)} = \phi(X)(\overleftarrow{\nabla}^A - \nabla^A)^s \phi(X) + \gamma_1 g^{AA} \phi(X)(\overleftarrow{\nabla}^A - \nabla^A)^{s-2} \phi(X) + \dots$$

γ_i are not known in a compact form

[Bekaert, Meunier'10]

4pt exchange diagram

Goal: reduce

$$\int dXdX'J(X',P_1,P_2)\Pi(X',X)J(X,P_3,P_4)$$

to

$$\int dXdX'd\nu\phi(X',P_1)(\nabla'^A)^s\phi(X',P_2)\Omega_{A(s),A'(s)}(\nu)\phi(X,P_3)(\nabla^A)^s\phi(X,P_4).$$

Directly related to conformal partial waves/conformal blocks.
Approaches:

- Traceless HS propagator, plug bulk-boundary propagators for ϕ
- Manifest trace structure propagators, generalize identities for traces of currents $TrJ \approx -\Box J + 4m^2 J$ to AdS.

Result: bulky expressions both ways.

Conclusion

- The standard perturbative procedure for HS interactions quickly gets tedious
- Instead: extract interactions from holography
- Improved previous results on propagators. Preliminary results for an exchange diagram
- Locality of quartic interactions? What is locality?
- The role of improvements for currents (vertices that are trivial on-shell)?
- Mellin representation

Thank You!