(Super)Yang-Mills theories at finite density of heavy quarks and strong coupling

Javier Tarrío

with A. Faedo, A. Kundu and D. Mateos

Holographic Methods and Applications

Reykjavík, August 2014

Motivation

- We are interested in studying the effects of fundamental charge density at strong coupling.
- Stacks of Dp-branes in flat space give the holographic dual of (super)YM theories.
- Restricting to SUGRA fields means considering mostly adjoint matter, we need stringy sources for fundamental matter.
- A finite charge density is dual to strings dissolved in the flavor branes with one end at the Dp-branes.
- One approximation: Non-dynamic quarks $\rightarrow$ only strings in the holographic description.

Quick reminder of holographic duals to SYM theories

[Itzhaki et al. '98]

- Stacks of N_c Dp-branes: solution to type II supergravity with a compact internal manifold

$$\mathrm{d}s^2 = e^{-\frac{2(8-p)}{p}\eta} \, g_{\mu\nu} \, \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{2\eta} \, L^2 \, \mathrm{d}\Omega_{8-p}^2 \ .$$

- Maldacena limit: large N_c and fixed (dimensionful) YM coupling.

- 10d curvature in string frame and string coupling must be small, thus

$$g_{YM}^{\frac{2}{3-p}} \, N_c^{\frac{1}{7-p}} \ll \frac{u}{\ell_s^2} \ll g_{YM}^{\frac{2}{3-p}} \, N_c^{\frac{1}{3-p}} \qquad \text{for } p < 3 \ ,$$

$$g_{YM}^{\frac{2}{3-p}} \, N_c^{\frac{1}{3-p}} \ll \frac{u}{\ell_s^2} \ll g_{YM}^{\frac{2}{3-p}} \, N_c^{\frac{1}{7-p}} \qquad \text{for } p > 3 \ .$$

Setup

- The action is given by SUGRA with a stringy source term

$$S_{total} = S_{IIA/IIB} - \frac{n_q}{2\pi\ell_s^2} \int \left(\sqrt{-G_{tt}\, G_{rr}}\, \mathrm{d}t \wedge \mathrm{d}r - B_2 \right) \wedge \Xi_8 \ .$$

- In the adjoint sector we include the stack of N_c adjoint degrees of freedom by an RR form

$$F_{8-p} = (7-p)\, L^{7-p}\, \omega_{8-p} \ ,$$

related to N_c by a quantization condition.

Setup

- The presence of strings necessarily sources additional supergravity fields

$$\mathrm{d}\left(e^{-2\phi} * H_3\right) + F_2 \wedge F_6 - \frac{1}{2} F_4 \wedge F_4 = -\frac{2\kappa_{10}^2}{2\pi\ell_s^2}\, n_q\, \Xi_8 \ ,$$

$$\mathrm{d}\left(e^{-2\phi} * H_3\right) + F_1 \wedge F_7 - F_3 \wedge F_5 = -\frac{2\kappa_{10}^2}{2\pi\ell_s^2}\, n_q\, \Xi_8 \ ,$$

  and the flux can be absorbed by

$$F_p = (-1)^{\left[\frac{p+1}{2}\right]} \frac{q}{L}\, \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^p \ .$$

- An interpretation in terms of dissolved baryonic branes where N_c strings end arises naturally.

UV geometry

- For $q = n_q = 0$ the solution is that of a stack of Dp-branes, dual to (S)YM theories [Itzhaki et al. '98]
- For finite charge density the UV is governed by that geometry, but subleading corrections exist

$$e^{\phi} = \left(\frac{u}{L}\right)^{\frac{(p-3)(7-p)}{4}} \left[1 - \alpha_\phi\, q \left(\frac{L}{u}\right)^{6-p} + v_\phi \left(\frac{L}{u}\right)^{7-p} + \cdots\right] .$$

- The RG flow of the charged system flows to a new IR. What is the end point?

IR geometry

- There is an exact solution with a HV-Lif metric

$$\mathrm{d}s^2 = \left(\frac{r}{L}\right)^{-\frac{2\,\theta}{p}} \left(-\frac{r^{2z}}{L^{2z}}\,\mathrm{d}t^2 + \frac{r^2}{L^2}\,\mathrm{d}x_p^2 + \#^2\, q^{\frac{2\,(3-p)}{p}}\, \frac{L^2}{r^2}\,\mathrm{d}r^2\right) \ ,$$

  where

$$z = \frac{16-3p}{4-p} \ , \qquad \theta = \frac{p\,(3-p)}{4-p} \ ,$$

  and running scalars

$$e^{\phi} \sim q^{\frac{p-7}{2}} \left(\frac{r}{L}\right)^{\frac{p\,(p-7)}{2\,(p-4)}} \qquad e^{\eta} \sim q^{\frac{3-p}{8}} \left(\frac{r}{L}\right)^{\frac{p\,(3-p)}{8\,(p-4)}} \ .$$

- Black hole solution also exists with $f = 1 - (r_h/r)^{p+z-\theta}$.

A closer look into 4+1 (S)YM

- Can be understood as the $z \to \infty$ limit of the generic solution with $z/\theta = -1$. see [Hartnoll & Shaghoulian '12]

$$\mathrm{d}s^2 = \left(\frac{r}{L}\right)^{\frac{1}{2}} \left(-\frac{r^2}{L^2} f(r)\mathrm{d}t^2 + \mathrm{d}x_4^2 + \#^2\, q^{-\frac{1}{2}}\, \frac{L^2}{r^2} \frac{\mathrm{d}r^2}{f(r)}\right) \, .$$

- Conformal to $AdS_2 \times \mathbb{R}^4$. Running scalars. $f(r) = 1 - r_h^2/r^2$.

A closer look into 4+1 (S)YM

- Can be understood as the $z \to \infty$ limit of the generic solution with $z/\theta = -1$. see [Hartnoll & Shaghoulian '12]

$$\mathrm{d}s^2 = \left(\frac{r}{L}\right)^{\frac{1}{2}} \left(-\frac{r^2}{L^2} f(r)\mathrm{d}t^2 + \mathrm{d}x_4^2 + \#^2\, q^{-\frac{1}{2}} \frac{L^2}{r^2} \frac{\mathrm{d}r^2}{f(r)}\right) .$$

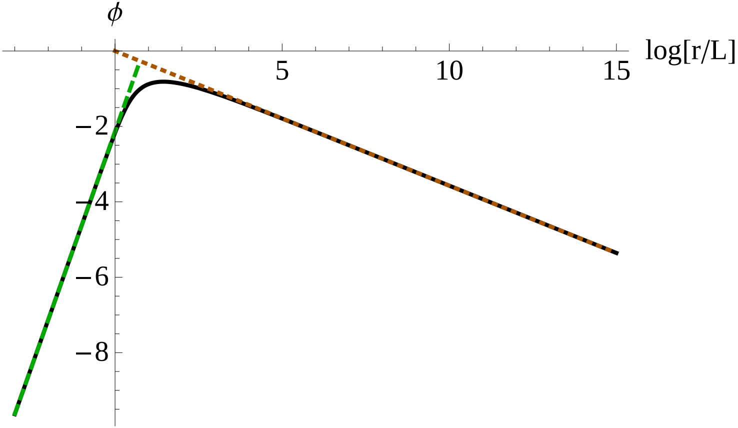
- Conformal to $\mathrm{AdS}_2 \times \mathbb{R}^4$. Running scalars. $f(r) = 1 - r_h^2/r^2$.

- Uplift to M-theory gives an $\mathrm{AdS}_7 \to \mathrm{AdS}_3 \times \mathbb{R}^4$ DW solution.

$$\mathrm{d}s^2 = -\frac{r^2}{\mathcal{L}^2} f(r)\mathrm{d}t^2 + \frac{r^2}{\mathcal{L}^2} d\psi^2 + \frac{\mathcal{L}^2}{r^2}\frac{\mathrm{d}r^2}{f(r)} + \mathrm{d}x_4^2 + \frac{3}{2}\mathcal{L}^2 \mathrm{d}\Omega_4^2 ,$$

$$F_4 = \frac{\sqrt{2}}{3^{1/4}} \frac{1}{\mathcal{L}} \left[\mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}x^4 + \frac{3}{2}\mathcal{L}^4 \omega_4\right] .$$

RG flow in d=2+1 (S)YM



RG flow in d=2+1 (S)YM

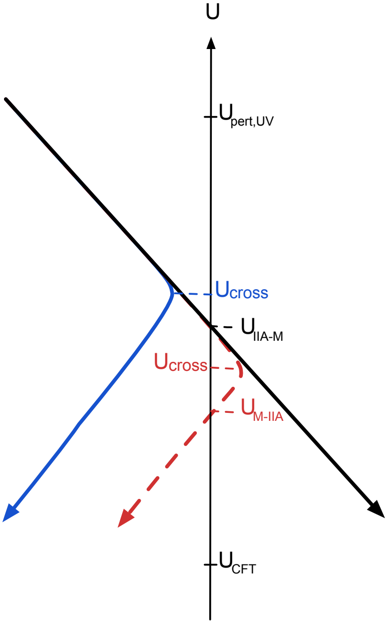
- For fixed λ, different scales arise depending on n_q/N_c^2.
- Crossover scale at

$$U_{cross} \sim \lambda^{\frac{1}{2}} \left(\frac{n_q}{N_c^2}\right)^{\frac{1}{4}} \ .$$

- Critical density for the need of an M-theory description

$$\left(\frac{n_q}{N_c^2}\right)_{crit} \sim \lambda^2 N_c^{-\frac{16}{5}} \ .$$

- UV and IR described by IIA supergravity + sources.



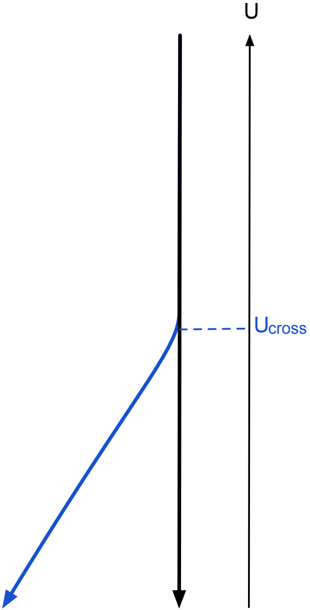
RG flow in d=3+1 (S)YM [Kumar '12]

- The d=3+1 case introduces a scale on a conformal theory, so all cases are equivalent.

- Crossover scale at

$$U_{cross} \sim \lambda^{\frac{2}{3}} \left(\frac{n_q}{N_c^2} \right)^{\frac{1}{3}} .$$

- Description given in terms of IIB supergravity + sources.



RG flow in d=4+1 (S)YM

- Crossover scale at

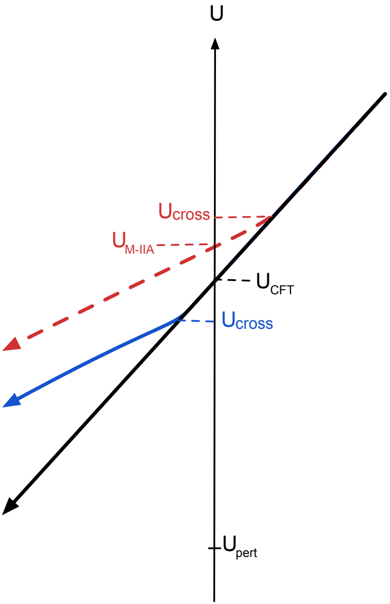
$$U_{cross} \sim \lambda \left(\frac{n_q}{N_c^2} \right)^{\frac{1}{2}} \; .$$

- Critical density for the need of an M-theory description

$$\left(\frac{n_q}{N_c^2} \right)_{crit} \sim \lambda^{-4} \, N_c^{\frac{8}{3}} \; .$$

- UV described by 11d M-theory.

- IR given by 10d IIA SUGRA.



Low temperature thermodynamics

- At low temperatures

$$T \ll \lambda^{\frac{4-p}{2(6-p)}} \left(\frac{n_q}{N_c^2}\right)^{\frac{5-p}{2(6-p)}}$$

only the HV-Lifshitz geometry matters (but UV time!) and

$$s \sim N_c^2 \left(\frac{n_q}{N_c^2}\right)^{\frac{7p-4\theta}{z}} (\lambda\, T)^{\frac{p-\theta}{z}} = N_c^2 \left(\frac{n_q}{N_c^2}\right)^{\frac{2(6-p)}{16-3p}} (\lambda\, T)^{\frac{p}{16-3p}} \ .$$

- At large temperatures Dp-brane thermodynamics

$$s \sim N_c^2\, \lambda^{\frac{p-3}{5-p}}\ T^{\frac{9-p}{5-p}} \ .$$

Conclusions and outlook

- Supergravity with strings gives an accurate description of $d \leq 6$ (S)YM with an external charge density, $n_q \sim N_c^2$, at strong coupling and large N_c.

- The RG flow drives the theory to an IR characterized by dynamical and hyperscaling-violating exponents (+ scalars)

p	1	2	3	4	5
z	$13/3$	5	7	∞	-1
θ	$2/3$	1	0	$-z$	10

- Would be interesting to consider solutions where the sources are taken into account by the NS form, $H_3 \neq 0$.

- Next step: understand how this picture changes when the charge is dynamic. [Work in progress]