What is topological matter & why do we care?

Part 1: what are topological insulators?

Eddy Ardonne
thanks to:
Hans Hansson
Topological insulators in the media!

Fourth season of Big Bang theory: ‘The thespian catalyst’.
Topological insulators in the media!

Fourth season of Big Bang theory: ‘The thespian catalyst’.

So, who knows what a topological insulator is?
Condensed matter physics

What phases of matter do exist?

How does matter go from one phase to another?

Daily life example: water and ice
Condensed matter physics

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Other examples: magnets
Condensed matter physics

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Other examples: superconductors

Kamerling Onnes et al., 1911
Condensed matter physics

Some materials (such as copper) are metallic, they conduct current very well:
Condensed matter physics

Some materials (such as copper) are metallic, they conduct current very well:

Some materials (such as wood) are insulating, they do not conduct current:
Condensed matter physics

Some materials (such as copper) are metallic, they conduct current very well:

![Copper conductor](image1)

Some materials (such as wood) are insulating, they do not conduct current:

![Wood insulator](image2)

Both conductors and insulators are important, but the behaviour in between is really interesting: *semi-conductors*!
Between metals and insulators

Semiconductors can be used to make very interesting devices, transistors!
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Bardeen et al., 1947
Between metals and insulators

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Impact on society is hard to quantify.

Bardeen et al., 1947
Topological insulators in words

A topological insulator is also in between metals and insulators.
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They are *insulating inside* (in ‘the bulk’).
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They are *conducting on the surface*
Topological insulators in words

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They are *insulating inside* (in ‘the bulk’)

They are *conducting on the surface*

The conductance on the surface is insensitive to dirt, disturbances, etc.

We say that the conductance is **protected** for **topological** reasons!
Crash course on topology
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A football player would say: 

![Soccer ball](soccer_ball.png) ≠ ![American football](american_football.png)
Crash course on topology

A football player would say: ≠

A topologist would say: =
Crash course on topology

A football player would say: ≠

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Knot theory, some history

Inspired by Peter Tait’s experiments on smoke rings, Sir William Thomson developed the idea that the different atoms are related to different knots (1867)!

The knots were thought to be different vortex rings in the aether. Tait started to classify all the different knots. However, the aether doesn’t exist, so this idea failed.
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Knots play an important role in topological quantum computation!
Knots, some examples

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But are these the same or not?
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But are these the same or not?

Yes, but for a long time they were listed as different in the literature!
Topological invariant: winding number

An example of a topological invariant is the winding number. How many times winds a curve around the origin?
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An example of a topological invariant is the winding number. How many times winds a curve around the origin?

winding number: +2
An example of a topological invariant is the winding number. How many times winds a curve around the origin?

winding number: not defined

- corresponds to a phase transition

start

finish
An example of a topological invariant is the winding number. How many times winds a curve around the origin?

winding number: -1
The quantum Hall effect

The first topological state that was observed is the ‘quantum Hall effect’, dating back to 1980.

The electrons are confined to a two-dimensional plane, between to semi-conductors. The quantum Hall effect occurs at very low temperatures, 1 Kelvin or lower, and in a strong magnetic field, 10 Tesla.
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The electrons are confined to a two-dimensional plane, between two semiconductors. The quantum Hall effect occurs at very low temperatures, 1 Kelvin or lower, and in a strong magnetic field, 10 Tesla.

Charged particles that move in a magnetic field experience a ‘Lorentz force’, perpendicular to the field and direction they move in.

Example: Earth magnetic field, giving rise to the northern light:
The classical Hall effect

When there is a current through a thin strip, there is voltage drop along the current. In a magnetic field, there is also a voltage perpendicular to the current, the Hall voltage.

The resistance is the ratio of the voltage to the current:

\[ R_L = \frac{V_L}{I} \quad R_H = \frac{V_H}{I} \]

The (classical) Hall resistance is proportional to the magnetic field:
The quantum Hall effect

In very clean samples, at low temperatures, and high fields, the Hall conductance becomes quantized:

\[ \sigma_H = 1/R_H = \nu \frac{e^2}{h} \]
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Red curve: Hall resistance
Green curve: longitudinal resistance
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Hall resistance is a topological invariant
The quantum Hall effect

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The quantum Hall liquid is an insulator, with a chiral edge mode!

The electrons on the edge can only move in one direction, which can not be changed, not even by dirt, disorder, etc. They can simply not turn back. This is why the quantization is so precise!
Are there other topological states?

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It was long believed that the magnetic field is necessary. Without it, there are two edge modes, moving in opposite directions. These modes can interact with each other, destroying the topological properties and the quantized conductance.

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To explain this, we have to make a step back, and look at the difference between metals and insulators.
Energy levels

The energy levels of atoms, obtained by solving the Schrödinger equation, are discrete. For instance, for the hydrogen atom.

\[ \mathcal{H}\Psi = E\Psi \]
Band theory

What happens if the electrons move in a periodic potential, such as in solids?
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Bloch theorem says that the energy levels form continuous bands of states, that are periodic:

$$\Psi(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} u(\vec{r})$$

‘plane’ wave
periodic function

The energy of the states depends smoothly on the ‘wave vector’ $k$, forming a ‘band’, as opposed to discrete levels.
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‘plane’ wave periodic function

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To what level the bands are filled, determines the properties of solids!
Often, there is a gap between different bands. To which energy one has to fill each band (called the Fermi level), depends on the number of electrons that are not bound to an atom.
Metals v.s. insulators

In a metal, applying an electric fields give electrons slightly more energy, allowing it to conduct.
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In an insulator, the only way to give an electron more energy, is to excite it over the gap, which costs a lot of energy. Therefore, insulators do not conduct for low electric fields!
Quantum Hall effect revisited

Using band theory, we can now also explain the situation in the quantum Hall effect.

There is a single energy level that connects the valence and conduction band. As long as the Fermi level is in the gap, there is a state that conducts!

The slope of the level gives the direction in which the electron moves, so we also find that this level is chiral.
Kane & Mele’s model

Kane and Mele realized that topological phases can exist, even without a strong magnetic field.

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$$|\text{electron} >= \alpha |\uparrow> + \beta |\downarrow>$$

In the absence of magnetic fields, energy levels come in pairs, with exactly the same energy.

Magnetic fields can be external, or due to the motion of the electrons themselves, called spin-orbit coupling.
Kane & Mele’s model

For certain values of the momentum, there are no magnetic fields, so the energy levels must come in pairs! Example: point A.
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There are two, topologically distinct ways of drawing the diagram!

So there are two types of insulators, one with a pair of edge modes! These modes are chiral, and move in opposite direction.
Topological insulators: CdTe v.s. HgTe

Kane and Mele worked on graphene, which is always trivial.

Zhang & coworkers designed an experiment:
CdTe is trivial, while HgTe is topological

Zhang et al., 2006 (Picture: Physics Today)
Topological insulators: CdTe v.s. HgTe

Zhang & coworkers designed an experiment:
Take a sandwich of CdTe-HgTe-CdTe, and vary the thickness!

Zhang et al., 2006 (Picture: Physics Today)
Topological insulators: CdTe v.s. HgTe

Trivial case

Topological case

Molenkamp et al., 2007 (Picture: Physics Today)
The measurements show that the conductance is quantized in the topological case. Other experiments show that the system indeed has chiral edge modes.
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Conclusion: this really is a topological insulator! By now, many types are observed experimentally, in several groups!
Conclusions part I

In condensed matter, often experiments come (way) before the theory (superconductivity, quantum Hall effect for instance).

Topological insulators were predicted on theoretical grounds, and observed afterwards.

The topology in the problem allows for very precise predictions, that can be verified.

Topological insulators are extremely interesting from a fundamental perspective.

Can topological insulators be used for devices?