What is topological matter & why do we care?

Part 2: fractionalization & applications?

Eddy Ardonne
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Famous example: polyacetylene (Su, Schrieffer, Heeger, 1979)
Polyacetylene consists of a chain of carbon atoms, with alternating single and double bonds, long and short.

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actual positions
Polyacetylene

We can simplify the picture by just drawing the bonds:

A \begin{array}{c}
\end{array} \quad \begin{array}{c}
\end{array} \quad \begin{array}{c}
\end{array} \quad \begin{array}{c}
\end{array} \quad \begin{array}{c}
\end{array} \quad \begin{array}{c}
\end{array} \quad \begin{array}{c}
\end{array} \quad \begin{array}{c}
\end{array} \quad A
Polyacetylene

We can simplify the picture by just drawing the bonds:

A

B

We see there are two ways of staggering!
Polyacetylene

We can simplify the picture by just drawing the bonds:

A

[Drawing of a line of double bonds]

B

[Drawing of a line of double bonds]

We see there are two ways of staggering!

We can create a kink, a region with two neighbouring double bonds:
Polyacetylene

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A  —— —— —— —— —— —— —— —— —— —— —— —— —— —— A

B  —— —— —— —— —— —— —— —— —— —— —— —— —— —— B

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We can create a kink, a region with two neighbouring double bonds:

A  —— —— —— —— —— —— —— —— —— —— —— —— —— —— B
Polyacetylene

These kinks can move around freely:

A \[ \rightarrow \] B
Polyacetylene

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A  ────►  ────  ────  ────  B
A  ────  ────  ────  ────  B
A  ────  ────  ────  ────  B
A  ────  ────  ────  ────  B
Polyacetylene

To find the charge of the kink, we introduce two of them!

A ———— ———— ———— ———— ———— ———— ———— ———— A

no kink
Polyacetylene

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If the excitations have ‘smaller’ quantum numbers that the constituent particles, one speaks of ‘quantum number fractionalization’
In 1982, the ‘fractional’ quantum Hall effect was discovered, namely, a plateau in the Hall conductance, with fractional value:

\[ \sigma_H = \frac{1}{3} \frac{e^2}{h} \]

This effect was explained in 1983 by Laughlin. This state has particles with fractional charge \( \frac{e}{3} \) and so-called ‘fractional statistics’, they are neither fermions nor bosons!
Fractional quantum Hall effect

Many such fractional phases have been observed!

Eisenstein et. al., 1990
Fractional quantum Hall effect

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Note: odd denominator filling fractions (electrons are fermions)
Measuring the fractional charge

How can one determine the size of hail?
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The bigger the noise, the larger the size of the hail!
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In the quantum Hall contacts, one lets two edges come close, so that the particles can ‘tunnel’ from one edge to the other.

The noise in the tunneling current is proportional to the charge of the particles that tunnel!
Measuring the fractional charge

Schematic setup for a shot-noise experiment: Send in current via contact A, and measure the tunneling current at B
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$$S = e^* \langle I_t \rangle$$
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The power in the noise is proportional to the charge of the tunneling particles $e^*$

$$S = e^* <I_t>$$

Several experiments, in various groups, have confirmed the existence of fractionally charged particles!

Glattli, 1997
Statistics of particles

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Exchange two identical particles: multiply the wave function by a phase!

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But a double exchange is equivalent to doing nothing:

So the phase for a single exchange:
+ sign (bosons)
- sign (fermions)
Statistics of particles in two dimensions

In the argument, we used the third dimension.

In two dimensions, there a double exchange is not the same as doing nothing, and arbitrary phases are allowed! Such particles are called anyons.
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The theory of the fractional quantum Hall effect predict that the particles with fractional charge, also have fractional statistics.

One can calculate the phase if one braids two such particles (Berry phase):

$$\Psi(w_1, w_2) \rightarrow \Psi(w_2, w_1) = e^{i\pi/3} \Psi(w_1, w_2)$$
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Detecting fractional statistics has proven to be much harder than fractional charge. Attempts using interferometry have been made by several groups.
Interferometry experiments
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Mach Zehnder interferometer
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X can influence the interference!
Interference in the quantum Hall effect

In the quantum Hall effect, the particles move along the edge. In the constrictions, the particles can tunnel from one edge to the other.
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To probe statistics, a double point contact setup is an ideal setup, because the two path can interfere with each other, leading to so-called Aharonov-Bohm oscillations.
More exotic statistics?

In 1987, a quantum Hall state with an even denominator Hall conductance was observed!

![Graph showing quantum Hall conductance](image)

**Graph Details**
- **Temperature** $T = 9$ mK
- **Density** $n = 3 \times 10^{11}$ cm$^{-2}$
- **Mobility** $\mu = 31 \times 10^6$ cm$^2$/Vs

**Equation**

$$R_{xx} (\text{k}\Omega) = \frac{2+1/2}{2+1/3} \frac{2+2/3}{2+2/5} \frac{2+2/7}{2+1/5} \frac{2+4/5}{2+3/5} \frac{2+3/5}{2+3/8} \frac{2+3/8}{2+3/9} \frac{2+3/9}{2+3/10} \frac{2+3/10}{2+3/11}$$

Xia et al., 2004
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One has to perform topologically non-trivial operations, such as braids!
Braiding non-abelian anyons

The ‘wave function’ of four anyons can be described as follows

\[ \Psi(w_1, w_2, w_3, w_4) = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \]

The exchange of two anyons is described by a matrix, not just a phase!
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Exchanging 2 and 3: \[ \Psi(w_2 \leftrightarrow w_3) = N \Psi(w_1, w_2, w_3, w_4) \]
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Changing the order of the exchanges, gives a different result!
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Braiding the anyons in the Moore-Read state: implements a NOT gate!
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If one uses this in a computation, one has to perform a measurement in the end. Interferometry can be used for this!
Interferometry in the Moore-Read state

The interference pattern depends on how many anyons are in the interferometer region.

For an odd number of anyons, the interference vanishes completely. This can (in principle!) be used to measure the outcome of the computation.

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Topological quantum computation?

Topological states with non-abelian anyons are rare, and hard to control.

But, recently, similar anyons (Majorana bound states) might have been observed in one-dimensional nano-wires. This could give better control on the system.

The anyons of the Moore-Read state are not universal, the phase gate can not be constructed in a topological protected way.

States that are rich enough might exist (with conductance 12/5), but no experiments have been done to verify this (really hard!)

Topological states are a very interesting alternative to quantum computation, because of there inherent robustness, but the field is much behind other approaches.
Conclusions part II

In topological phases, particles with fractional quantum numbers can sometimes appear!

Fractionalized particles have been predicted to exist in several systems, quantum Hall effect, on surfaces of topological insulators, in nano-wire systems (actually, they seem hard to avoid).

Experiments have seen signatures, that point that such particles really exist. More conclusive experiments are necessary!

If (once?) they are found, there are many ways to exploit them in very interesting devices (theory is way ahead of experiments here)!