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The University of Iceland

Tabletop String Theory

- applications of gauge theory/gravity duality

Lárus Thorlacius

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Motivation

Quantum field theory (QFT) is an extraordinarily successful framework for understanding a wide range of physical phenomena:

- quantum electrodynamics (QED)
- standard model of particle physics
- many body theory for condensed matter systems

Precision tests of QED determine the fine structure constant to a part in 10^8

$$\alpha = \frac{e^2}{4\pi\hbar c} \qquad \alpha^{-1} = 137.03599\ldots$$

Efficient approximation schemes are the key to QFT's success:

- work well when interactions are weak
- strong coupling presents a difficult challenge
- numerical simulations are undermined by fermion sign problem

Gauge theory/gravity duality is a novel *gravitational* approach to strongly coupled QFT

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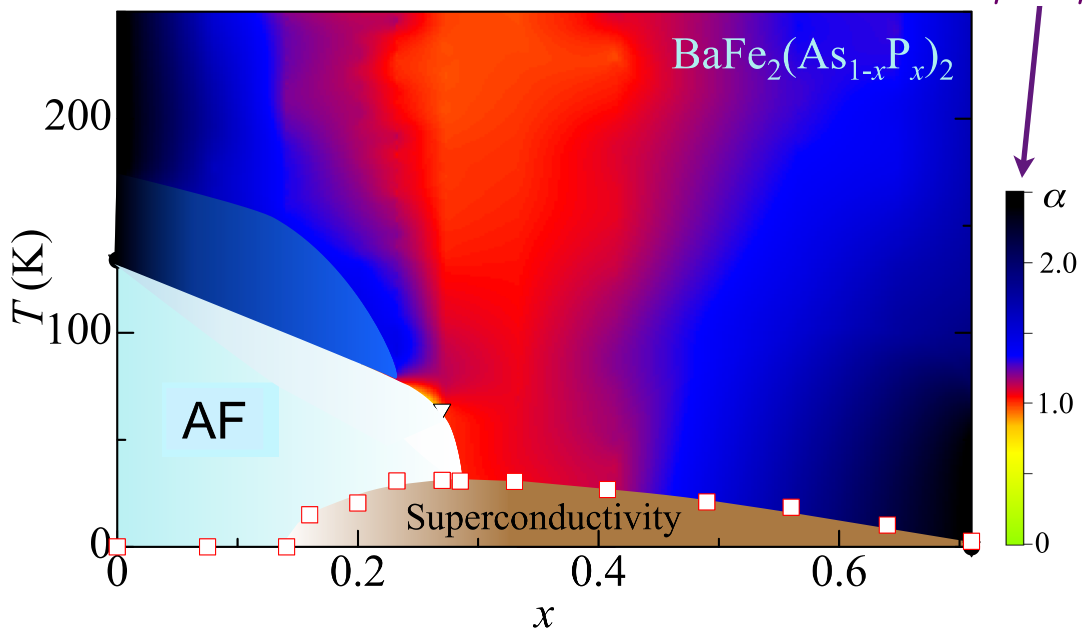
Motivation (continued)

Quantum many particle theory works extremely well!

- explains a wide range of physical phenomena in broad classes of materials
- most known materials are in fact well described by established methods
- **but** there are exceptions...
- physicists want to understand them
- may point the way towards new materials or improved functionality

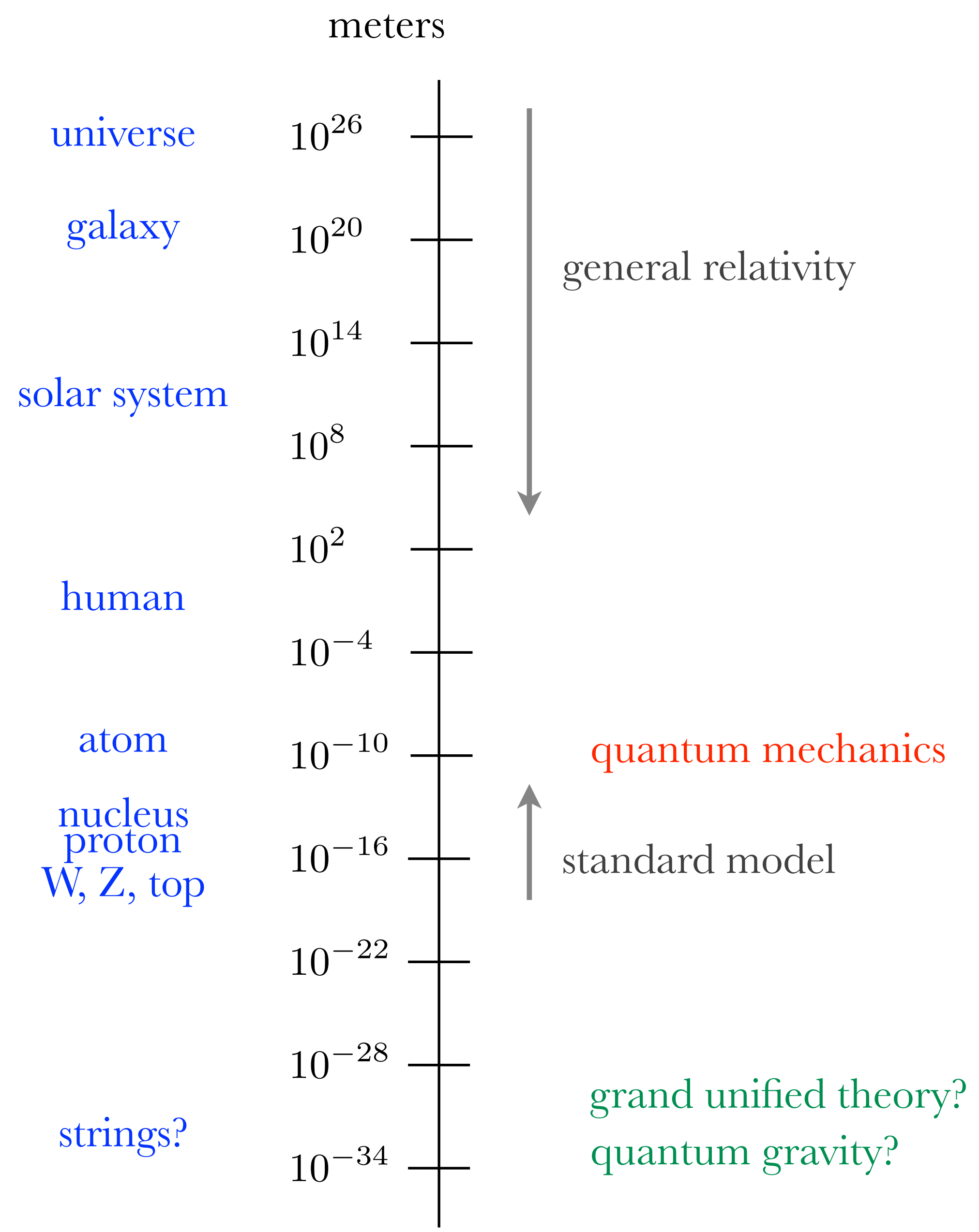
Resistivity in a high T_c superconductor

$\rho \approx \rho_0 + a\,T^{\alpha}$



Can gauge theory/gravity duality provide new insights?

Length scales in Nature



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Outline

Lecture 1

- strong/weak coupling dualities in physics

- open/closed duality in string theory

- Black branes vs. Dirichlet branes

- AdS/CFT (anti-de Sitter/conformal field theory) correspondence

Lecture 2

- scale invariance and quantum critical points

- heavy fermion alloys, high T_c superconductors

- applied AdS/CFT:
    - electrical conductivity
    - holographic superconductors
    - holographic metals
    - ....

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Electromagnetic duality

Maxwell equations in vacuum

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad -\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

Exchanging the electric and magnetic fields

$$\vec{E} \longrightarrow \vec{B}\,, \qquad \vec{B} \longrightarrow -\vec{E}$$

gives back the same set of equations

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Electromagnetic duality (cont.)

Maxwell equations with sources (both electric **and** magnetic)

$$\vec{\nabla}\cdot\vec{E} = \rho_e \qquad \vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t} + \vec{J}_e$$

$$\vec{\nabla}\cdot\vec{B} = \rho_m \qquad -\vec{\nabla}\times\vec{E} = \frac{\partial\vec{B}}{\partial t} + \vec{J}_m$$

Exchanging the electric and magnetic fields

$$\vec{E} \longrightarrow \vec{B}\,, \qquad \vec{B} \longrightarrow -\vec{E}$$

along with the electric and magnetic sources

$$\rho_e \longrightarrow \rho_m\,, \qquad \rho_m \longrightarrow -\rho_e$$

$$\vec{J}_e \longrightarrow \vec{J}_m\,, \qquad \vec{J}_m \longrightarrow -\vec{J}_e$$

again gives back the same set of equations

Dirac quantization condition

Quantum theory: The wave function describing a particle with electric charge e in the presence of a magnetic charge g is well-defined only if

$$e\,g = 2\pi\hbar\,n\,, \qquad (n = 0, \pm 1, \pm 2, \ldots)$$

- Quantization of e follows from the existence of magnetic monopoles

- Dirac condition implies that monopoles would be strongly coupled in ordinary electromagnetism

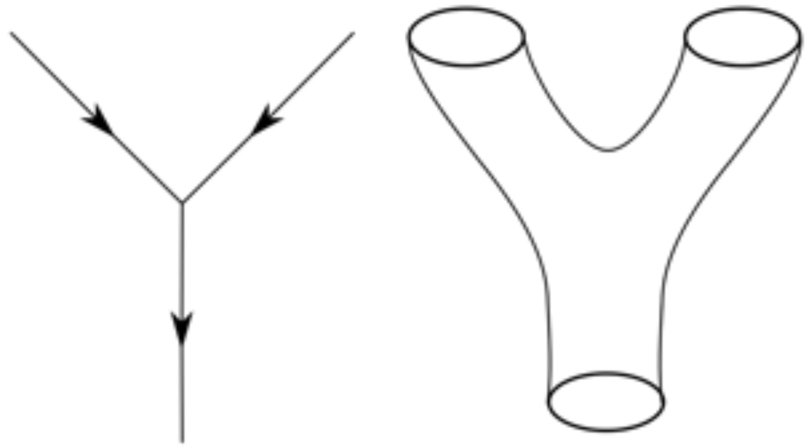
$$g \;\propto\; 1/e$$

- Dual theory has weakly coupled monopoles and strongly coupled electrons

- In particle physics we study generalizations of electromagnetic theory where monopoles occur as soliton solutions of the field equations

Five-minute primer on string theory

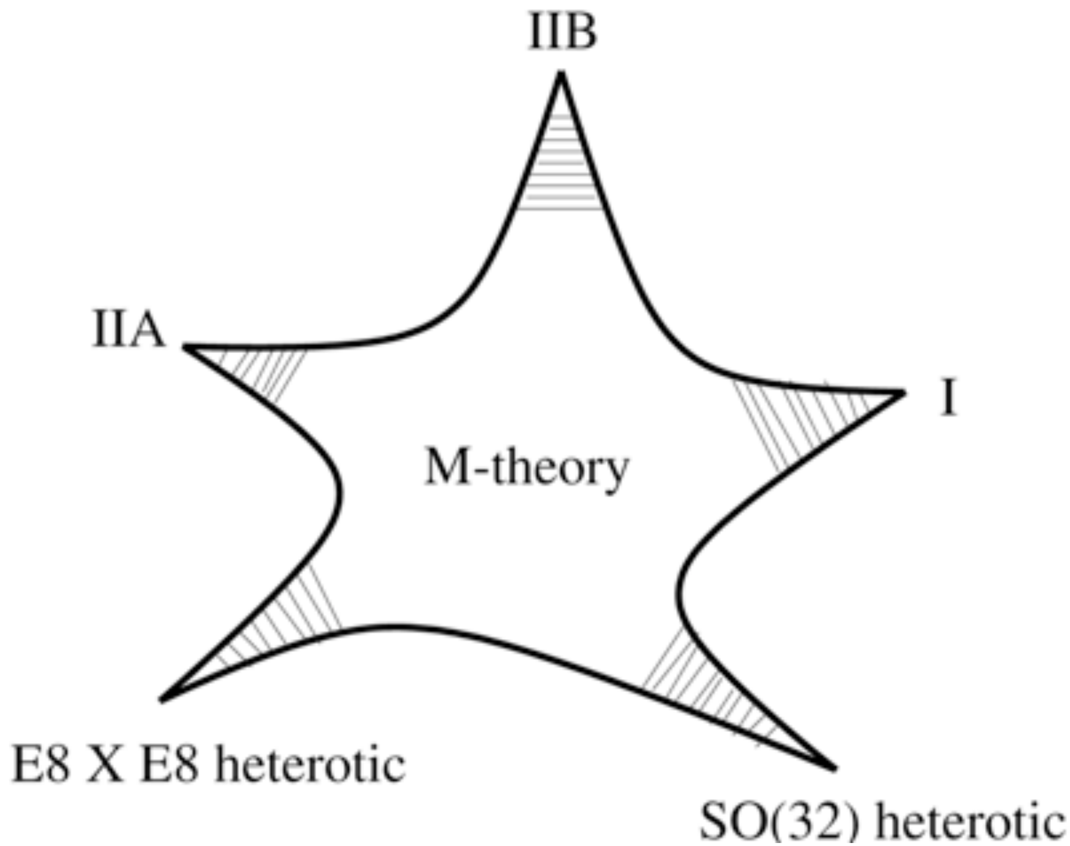
Replace point particles by one-dimensional strings and attempt to work out a quantum theory in flat spacetime.



Does not work unless the space-time has 26 dimensions and even then there are instabilities.

Adding fermions (and supersymmetry) leads to a stable theory in 10 dimensional spacetime.

Consistent string theories in 10 spacetime dimensions

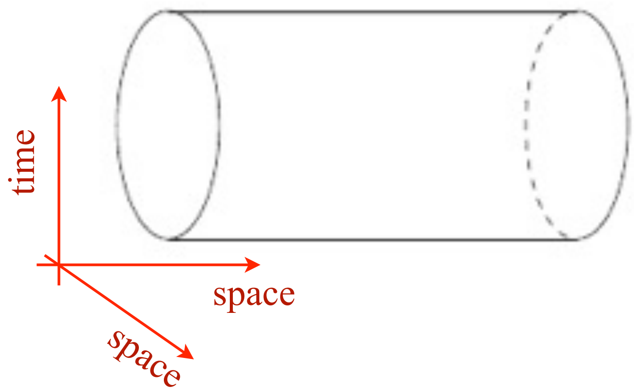


(from A. Sen, 1999)

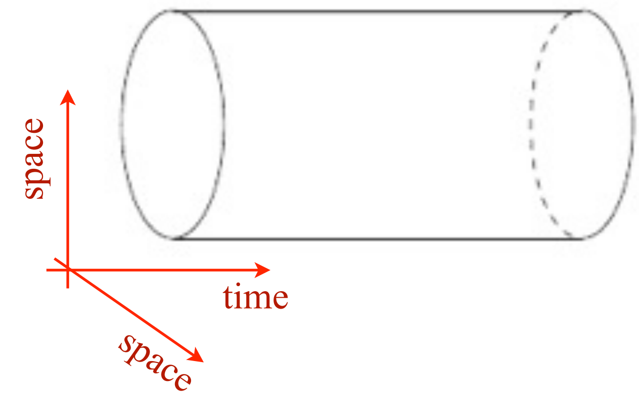
The five string theories are interrelated by a web of strong/weak coupling string dualities

Open/closed string duality

The same string world-sheet can be interpreted in different ways



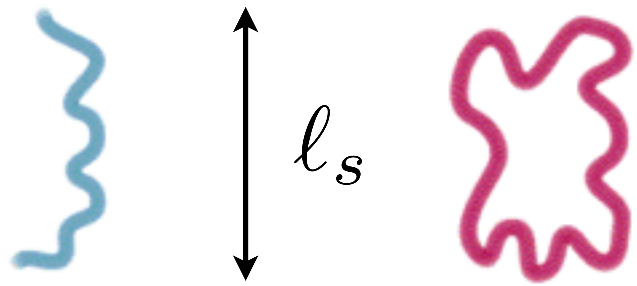
Pair of open strings



Single closed string

A given system can have very different descriptions from the point of view of open vs. closed strings

Low-energy limit of string theory



Strings appear point like in low-energy processes

closed (super-)string theory $\longrightarrow$ (super-)gravity theory

Type II super-gravity -- bosonic fields

metric:	$g_{\mu\nu}$	
NS-NS tensor:	$H_{\mu\nu\lambda}$	
dilaton:	ϕ	
R-R tensors:	$F^{(q)}_{\mu_1\ldots\mu_q}$	$q \in \{1,3,5\}$

correspond to massless modes of closed strings

String theory generalization of monopoles

String theory contains a variety of higher dimensional objects called p-branes.

A black p-brane solution of the field equations of 10-d supergravity describes a space-time where charged matter is confined to a p+1 dimensional hyperplane.

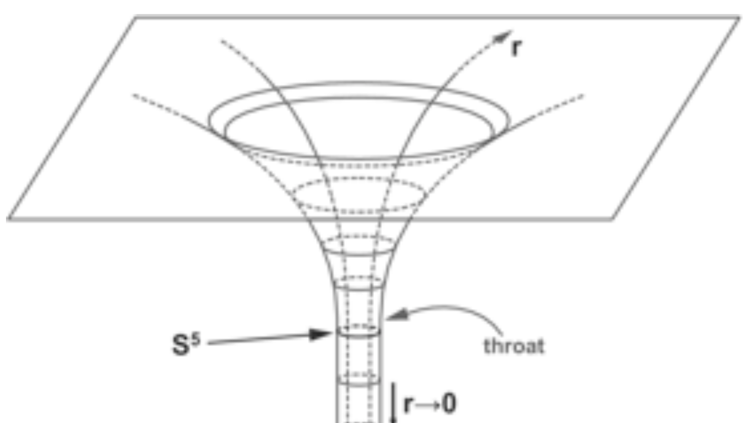
Higher-dimensional generalization of a charged black hole in general relativity.

The allowed charge-to-mass ratio of a black p-brane has an upper bound.

Space-time geometry outside a maximally charged (a.k.a. extremal) 3-brane:

far field: M_{10} ten-dimensional Minkowski spacetime

near horizon: $AdS_5 \times S_5$ product of 5d anti-de Sitter spacetime and a 5-sphere



Dirichlet-branes

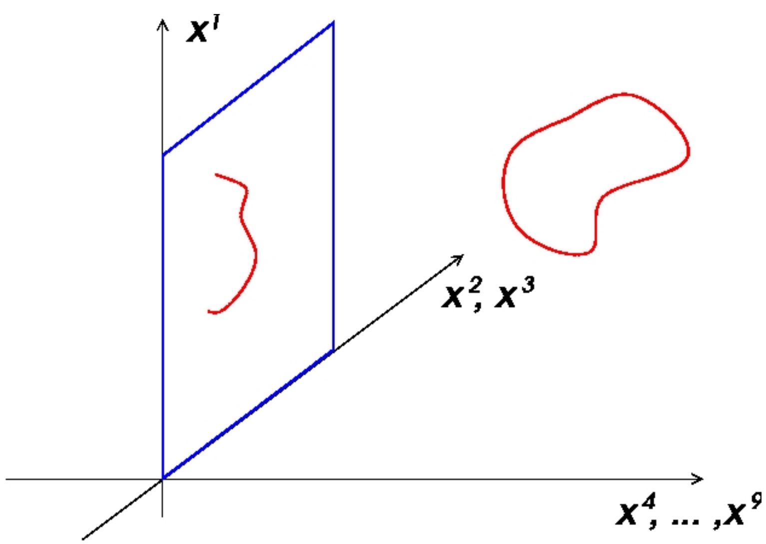
Open strings provide a very different view of p-branes.

Dp-brane: A p+1 dimensional hyperplane where open strings end.

Dp-brane dynamics $\longleftrightarrow$ physics of open strings

Low-energy limit $\longleftrightarrow$ Yang-Mills gauge theory

D3-brane in IIB string theory



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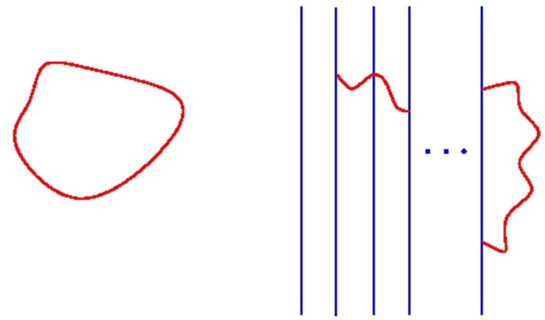
Multiple coincident D3-branes

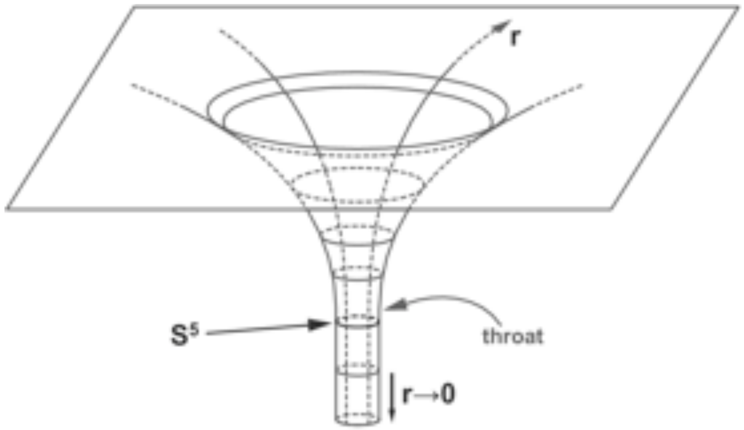


Low-energy dynamics ⟷ massless open string modes

d = 4 , $\mathcal{N} = 4$ supersymmetric U(N) Yang-Mills theory

Closed string description

d = 10, Type IIB supergravity in AdS_5 x S_5 background



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AdS/CFT correspondence

supergravity
in $AdS_5 \times S_5$
background

$N = 4$ supersymmetric
$U(N)$ Yang-Mills theory
in 3+1 dimensions

- strong/weak coupling duality so it is difficult to prove
- the original AdS/CFT conjecture has passed numerous tests and has been generalized in many directions

AdS/CFT prescription

Relates QFT correlation function to string amplitude in AdS_5 background

$$\mathcal{Z}_{\text{string}}\left[\{\phi_i\}\right] = \left\langle \exp\{\int d^4x \sum_i \tilde{\phi}_i(\vec{x})\mathcal{O}_i(\vec{x})\} \right\rangle_{\text{QFT}}$$

String theory partition function

QFT generating functional

$\phi_i(\vec{x}, r)$ is a field (string mode) in AdS_5 background

$\tilde{\phi}(\vec{x}) = \lim_{r\to\infty} \phi_i(\vec{x}, r)$ boundary value

$\mathcal{O}_i(\vec{x})$ is a local operator in QFT

Holographic dictionary:

metric	$g_{\mu\nu}$	$\longleftrightarrow$	$T_{\mu\nu}$	energy momentum tensor
gauge potential	A_μ	$\longleftrightarrow$	J_μ	conserved current
		$\vdots$		

Both sides of the prescription require regularization & renormalization

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Two sides of AdS-CFT

1. Quantum gravity via gauge theory

   - emergent spacetime
   - Hawking information paradox

2. Gravitational approach to strongly coupled field theories

   - strongly coupled QFT in D spacetime dimensions equivalent to weakly coupled gravity in D+1 dimensions
   - recipe for correlation functions at finite temperature
   - transport coefficients, damping rates
   - involves novel black hole geometries

Applied AdS-CFT

Investigate strongly coupled quantum field theories via classical gravity

- growing list of applications:

  - hydrodynamics of quark gluon plasma
  - holographic QCD
  - quantum critical systems
    - strongly correlated electron systems
    - cold atomic gases
  - out of equilibrium dynamics
  - ....

Bottom-up approach: Look for interesting behavior in simple models

- Assume that classical gravity in (asymptotically) AdS spacetime is dual to some strongly coupled QFT.
- Use AdS/CFT techniques to compute QFT correlation functions.
- Add gauge and matter fields to gravity theory to model interesting physics.
- Back-reaction can modify asymptotic behavior: non AdS - non CFT

Applied AdS-CFT

Investigate strongly coupled quantum field theories via classical gravity

- growing list of applications:

- hydrodynamics of quark gluon plasma
- jet quenching in heavy ion collisions
- quantum critical systems
    - strongly correlated electron systems
    - cold atomic gases
- holographic superconductors
- holographic metals
- out of equilibrium dynamics
- ....

Bottom-up approach: Look for interesting behavior in simple models

Lecture 2

M.C. Escher's
Circle Limit IV (1960)

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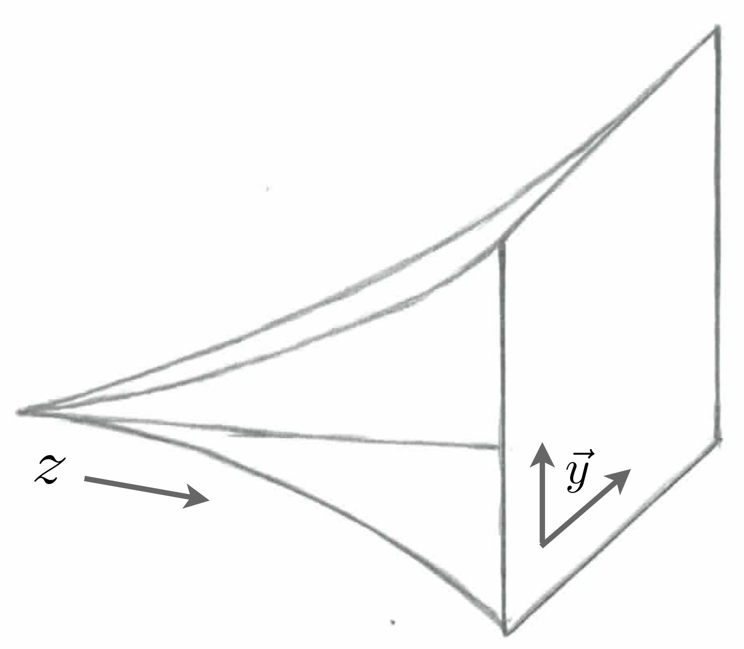
Anti-de Sitter geometry

$$ds^2 = b^2 \left(z^2(-dt^2 + d\vec{y}^2) + \frac{dz^2}{z^2} \right)$$



Metric is invariant under Lorentz transformations on $(t, \vec{y})$

and also under the scaling $(t, z, y^i) \to \left(c\,t, \frac{z}{c}, c\,y^i \right)$, $c > 0$

UV - IR connection

The map $z \to 1/\xi$ gives $$ds^2 = \frac{b^2}{\xi^2} \left(-dt^2 + d\vec{y}^2 + d\xi^2 \right)$$

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Applied AdS-CFT

- Assume that classical gravity in (asymptotically) AdS spacetime is dual to some strongly coupled QFT
- Use AdS-CFT techniques to calculate QFT correlation functions at strong coupling
- Add gauge fields and matter fields to the gravity theory to model interesting physics
- Quantum critical systems have scale invariance + strong correlations

  $\longrightarrow$ natural starting point for applied AdS-CFT

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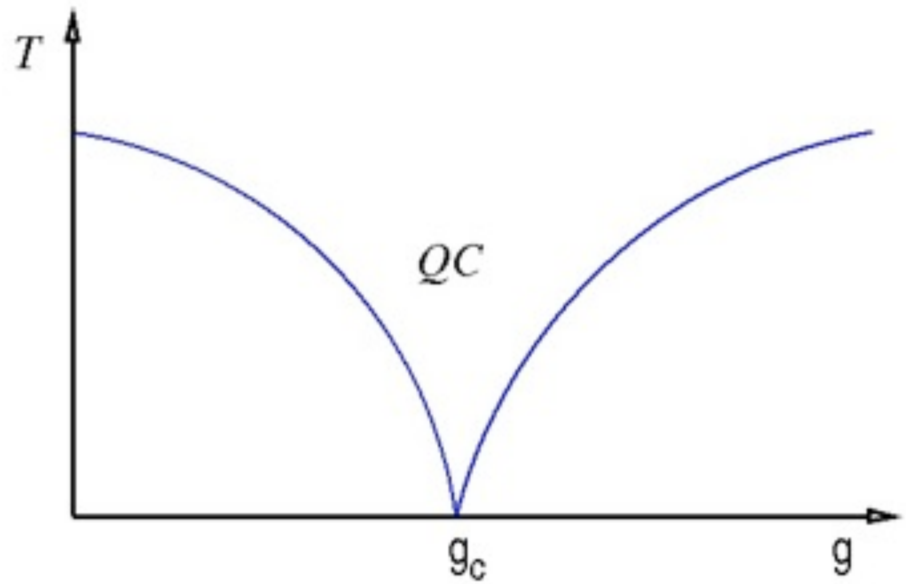
Quantum critical points



Typical behavior at $T = 0$

characteristic energy $\delta \sim (g - g_c)^{z\nu}$

coherence length $\xi \sim (g - g_c)^{-\nu}$

$\delta \sim \xi^{-z}$ z = dynamical scaling exponent

Scale invariant theory at finite T : $\xi = c\,T^{-1/z}$

Deformation away from fixed pt.: $\lambda_i \sim (\text{length})^{-1}$ QCP has $\lambda_i = 0$

Quantum critical region : $\xi = T^{-1/z}\,\eta(T^{-1/z}\lambda_i)$ $\eta(0) = c$

Physical systems with z = 1, 2, and 3 are known -- non-integer values of z are also possible

$z = 1$ scaling symmetry is part of SO(d+1,1) conformal group = isometries of adS_{d+1}

$z > 1$ scale invariance without conformal invariance - asymptotically Lifshitz spacetime

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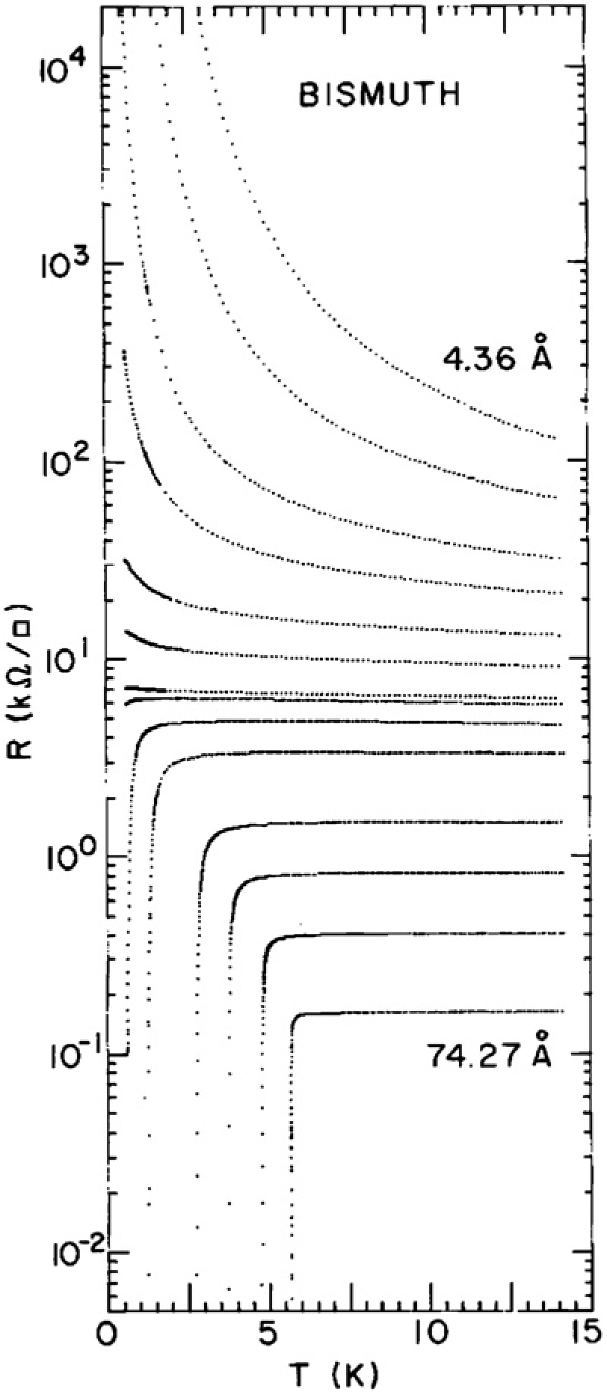


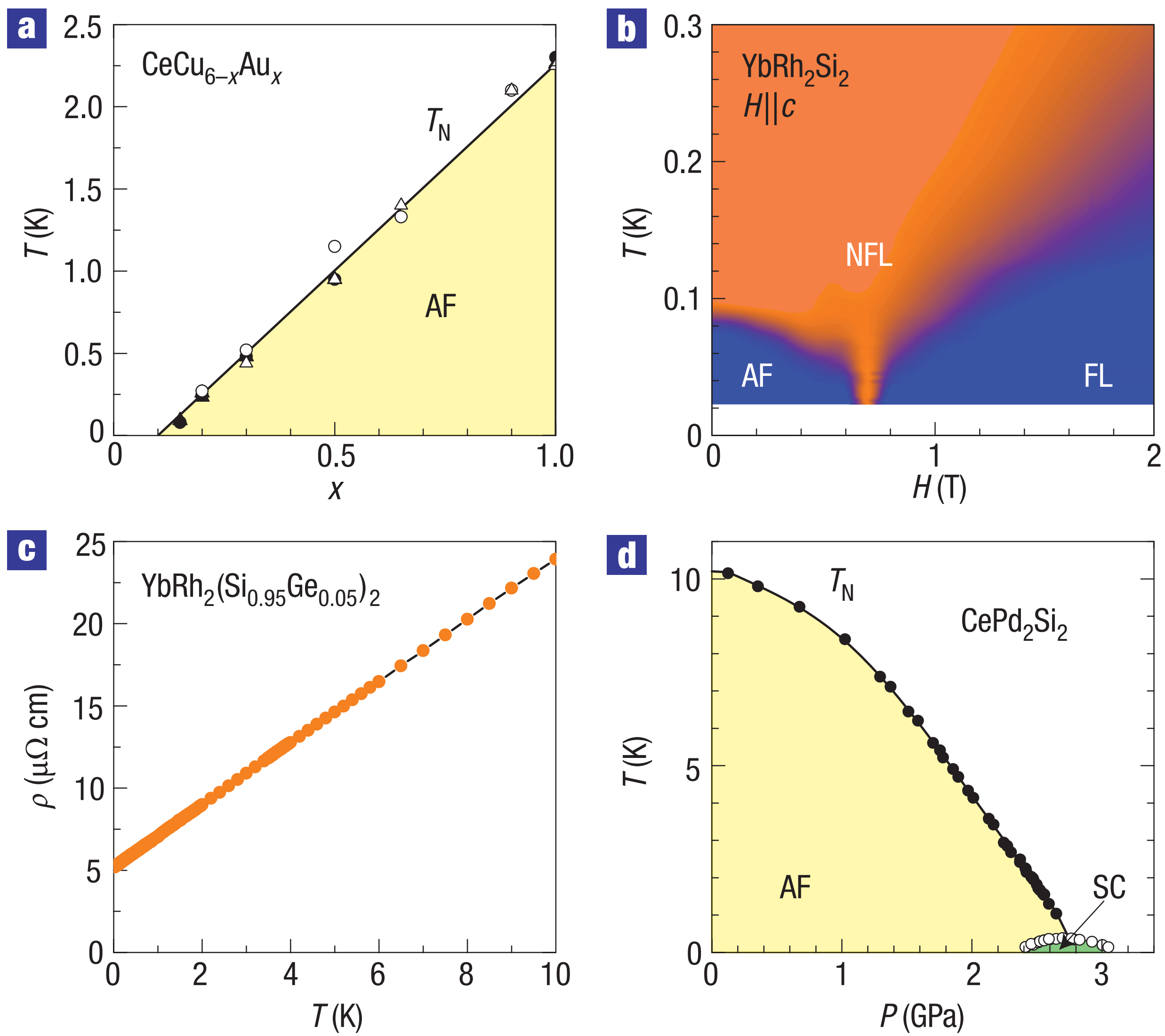
Classic example of a QCP

Resistivity vs. temperature in thin films of bismuth

T = 0 state changes from insulating to superconducting at a critical thickness

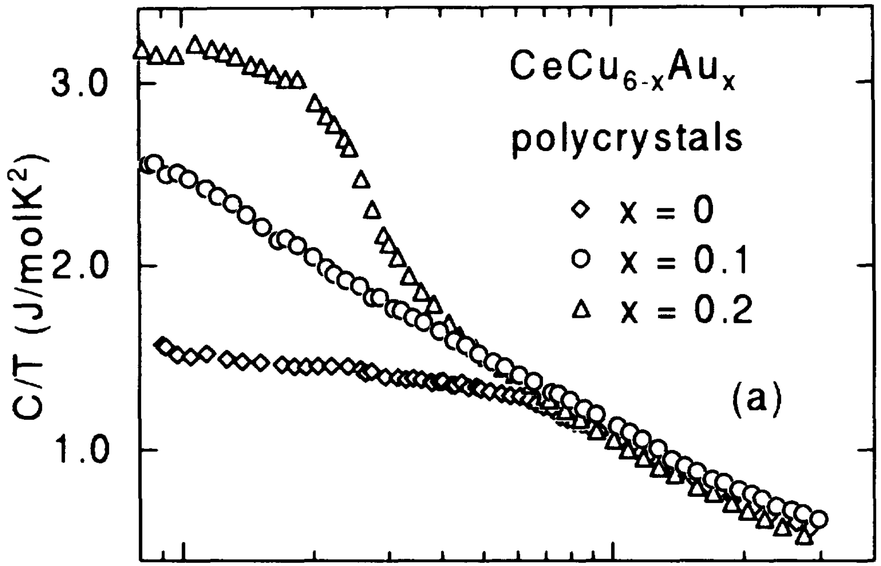
From D.B. Haviland, Y. Liu and A.M. Goldman, Phys. Rev. Lett. **62** (1989) 2180.

Quantum criticality in heavy fermion materials

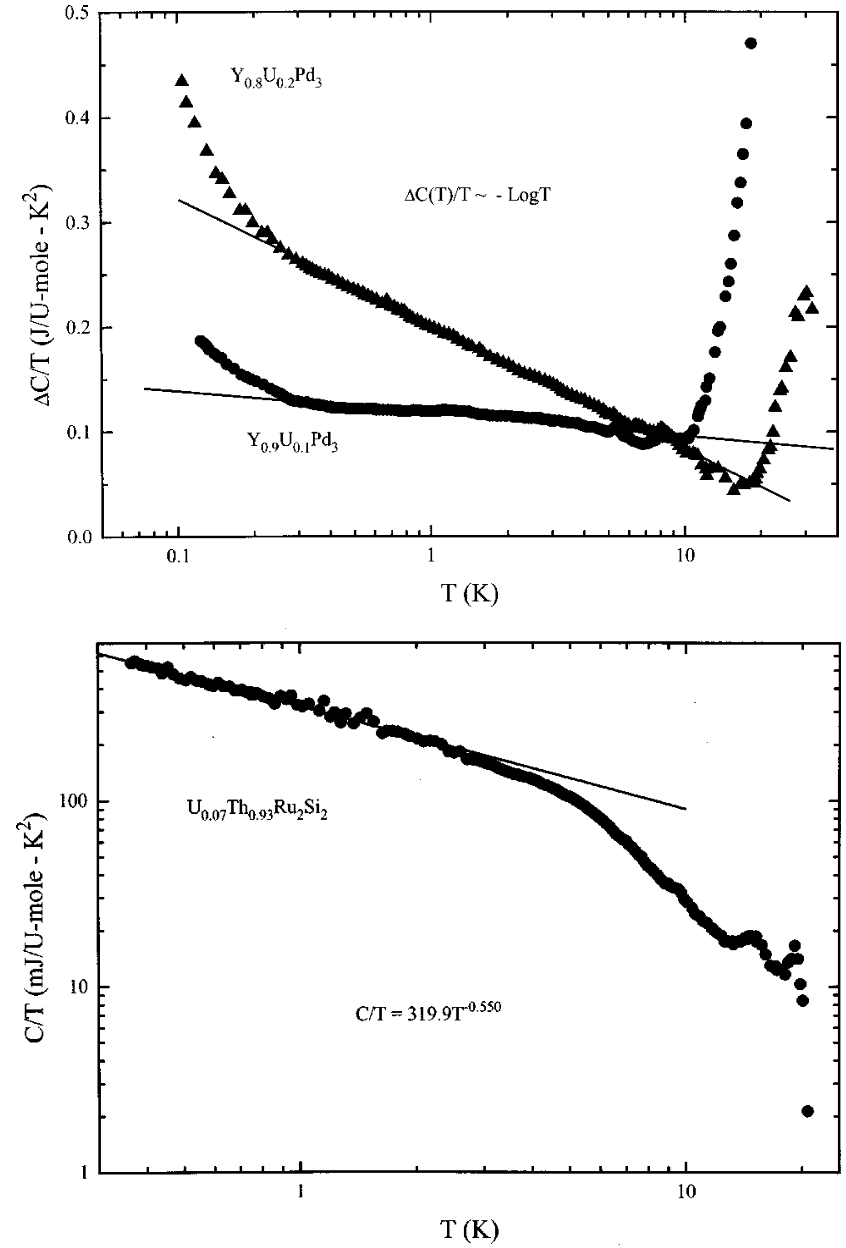


From P. Gegenwart, Q. Si and F. Steglich, Nature Phys. **4** (2008) 186.

Some measured c/T values in heavy fermion metals

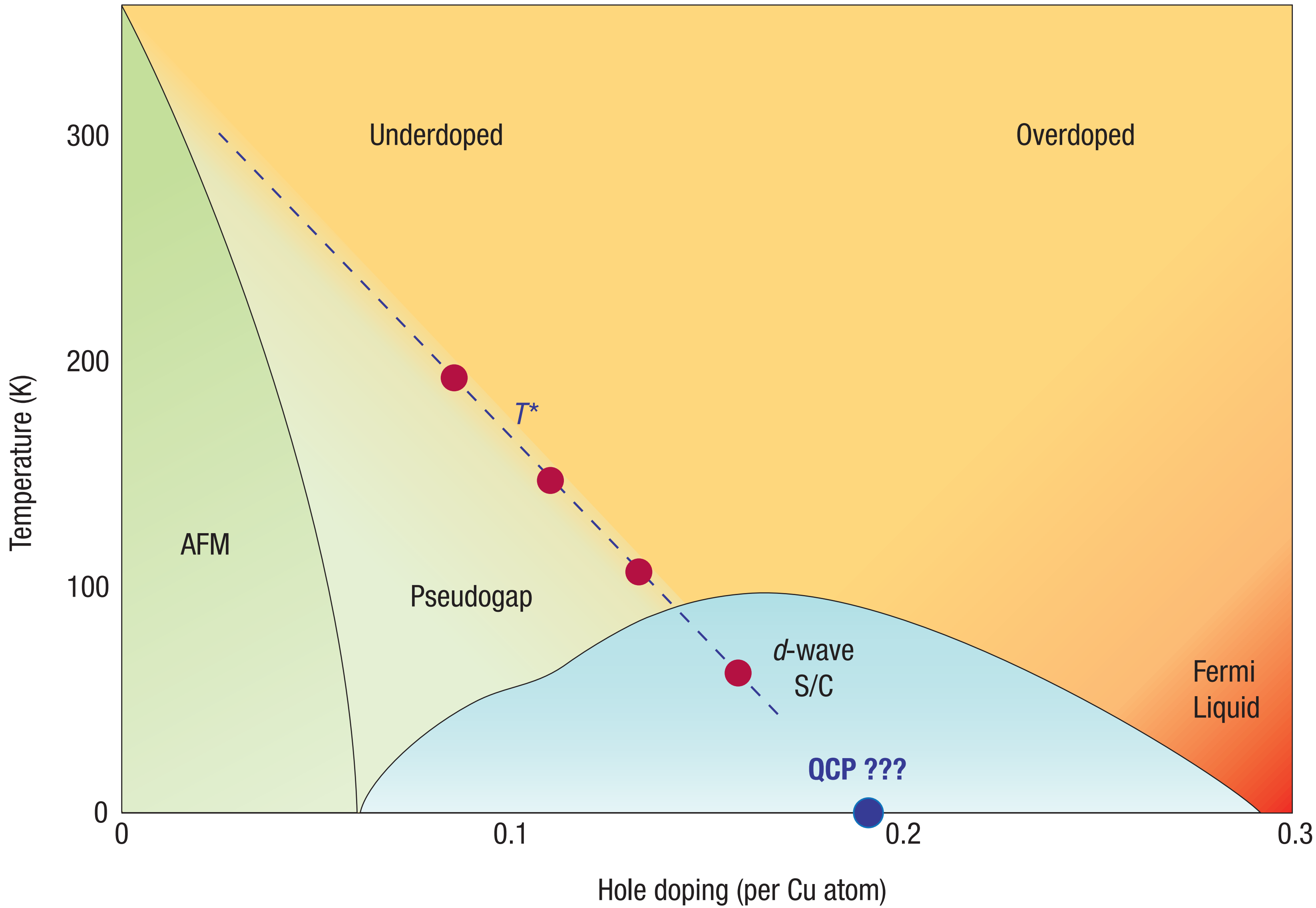


H. Lohneysen et al. *PRL* **72** (1994) 3262.



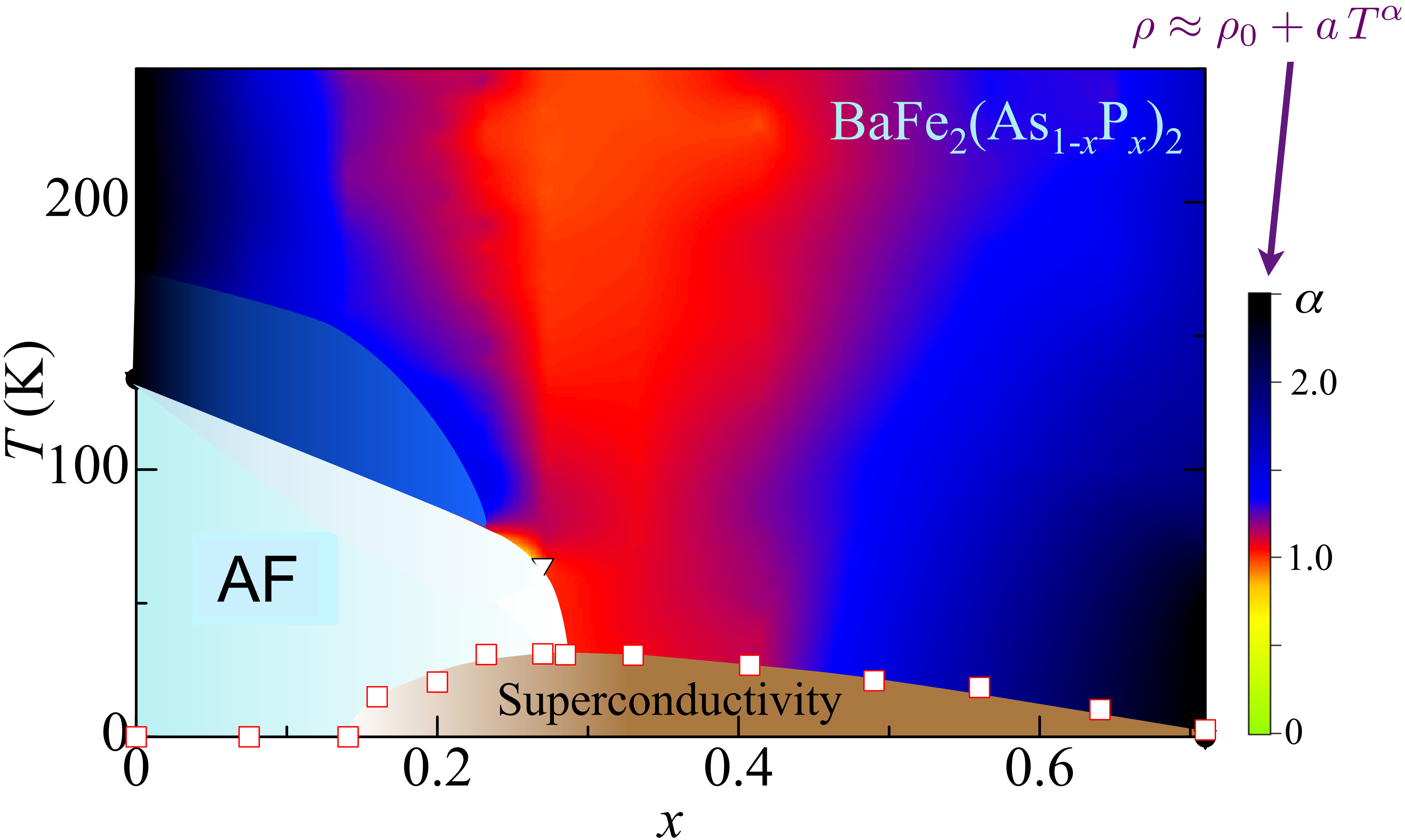
From G.Stewart, *Rev.Mod.Phys*. **73** (2001) 797.

Quantum criticality in high T_c superconductors



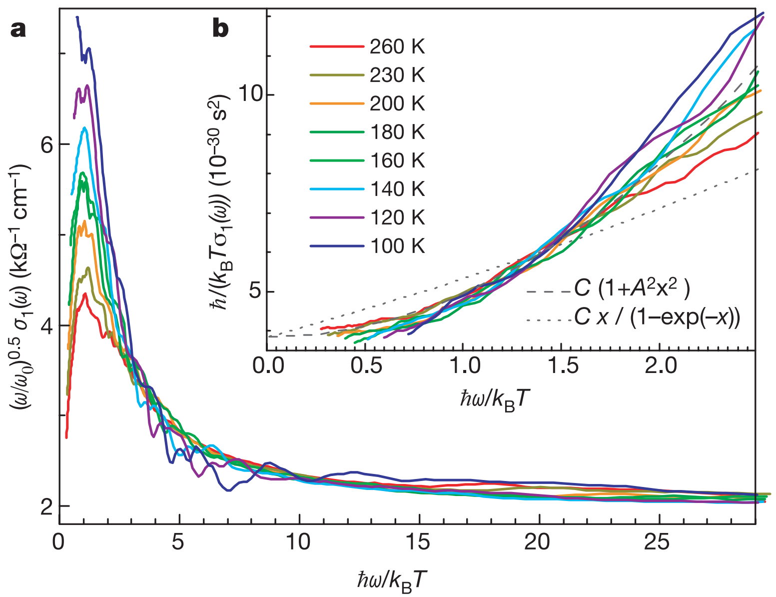
From D.M. Broun, Nature Phys. **4** (2008) 170.

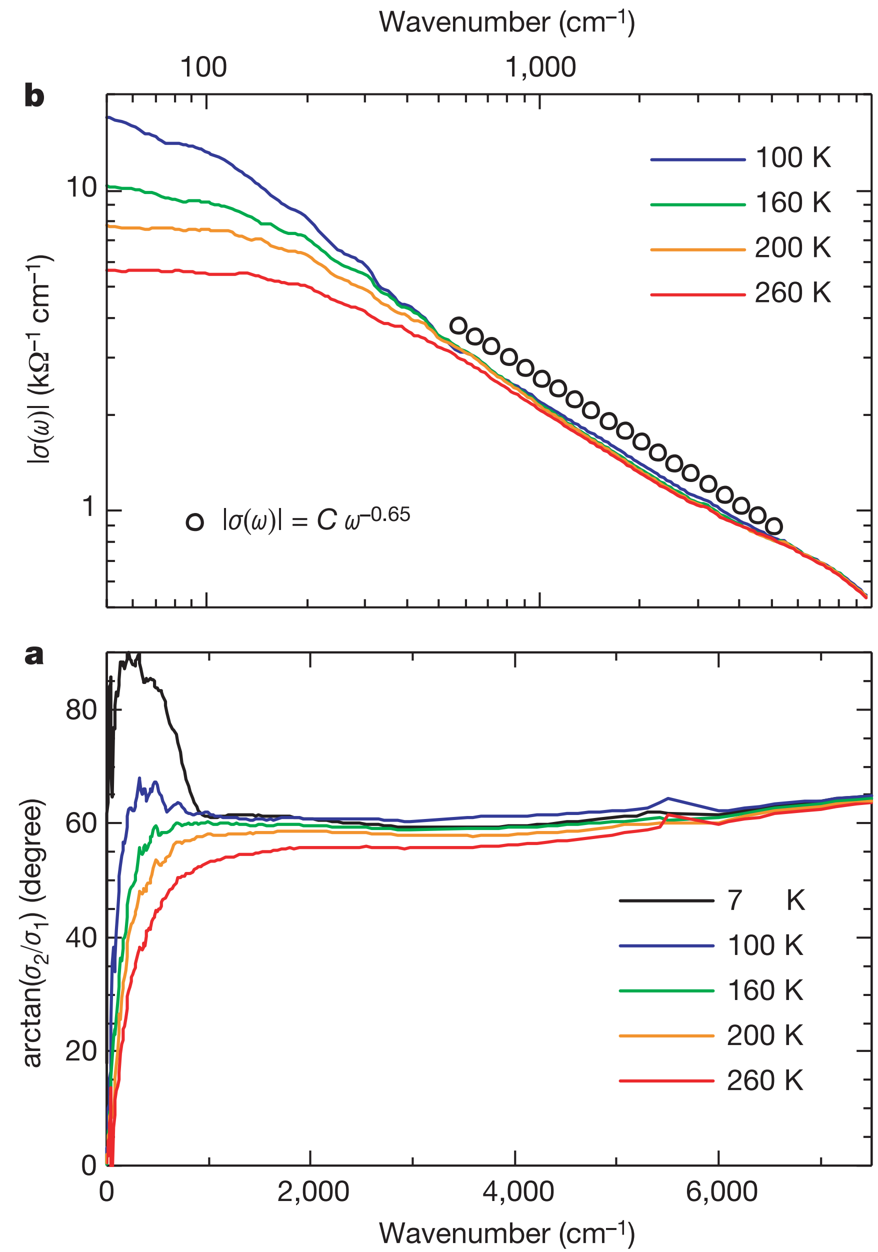
Linear resistivity in strange metal region



From S. Kashara et al., *Phys. Rev. B*. **81** (2010) 184519.

Optical conductivity in $Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$





Drude form at low frequency:

$$\mathrm{Re}\,\sigma(\omega,T) \sim T^{-1}\left(1 + A^2\left(\frac{\omega}{T}\right)^2\right)^{-1}$$

Universal power law at intermediate frequency:

$$\sigma(\omega,T) \approx B\,(-i\omega)^{-2/3}$$

From D. van der Marel et al., *Nature* **425** (2003) 271.

Gravity duals at finite temperature

periodic Euclidean time: $\tau \simeq \tau + \beta, \quad \beta = \frac{1}{T}$

β introduces an energy scale: scale symmetry is broken

thermal state in field theory: black hole with $T_{\text{Hawking}} = T_{\text{qft}}$

finite charge density in dual field theory: electric charge on BH

magnetic effects in dual field theory: dyonic BH

$z = 1$: planar AdS-Reissner-Nordström black hole

$z > 1$: planar charged Lifshitz black hole

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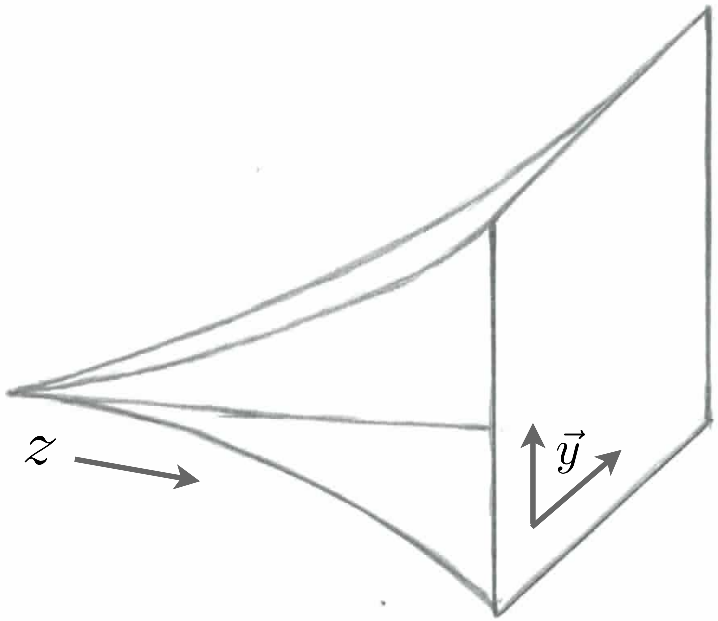
Anti-de Sitter geometry

$$ds^2 = b^2 \left(z^2(-dt^2 + d\vec{y}^{\,2}) + \frac{dz^2}{z^2} \right)$$



Metric is invariant under Lorentz transformations on $(t, \vec{y})$

and also under the scaling $\quad (t, z, y^i) \to \left(c\, t, \frac{z}{c}, c\, y^i \right), \quad c > 0$

UV - IR connection

The map $z \to 1/\xi$ gives $\quad ds^2 = \frac{b^2}{\xi^2} \left(-dt^2 + d\vec{y}^{\,2} + d\xi^2 \right)$

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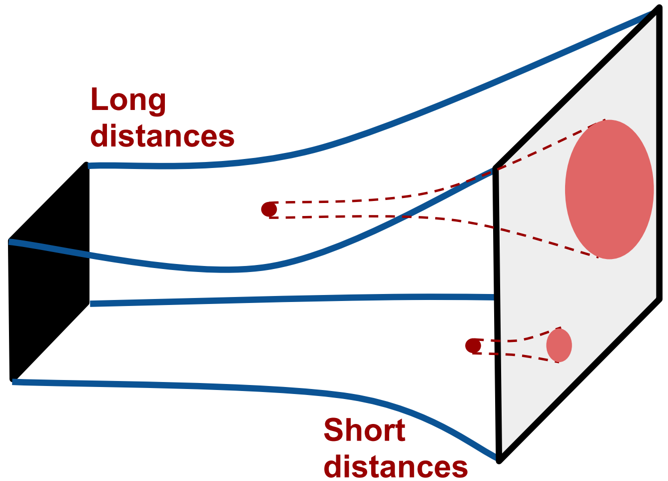
Planar AdS black holes

AdS spacetime: $$ds^2 = \frac{1}{\xi^2}(-dt^2 + d\vec{y}^2 + d\xi^2) \quad \longleftarrow \quad b = 1$$

Planar AdS black hole: $$ds^2 = \frac{1}{\xi^2}(-f(\xi)dt^2 + d\vec{y}^2 + \frac{d\xi^2}{f(\xi)})$$

AdS-Schwarzschild $$f(\xi) = 1 - \left(\frac{\xi}{\xi_0}\right)^3 \quad \longleftarrow \quad n = 3$$

AdS-Reissner-Nordström $$f(\xi) = 1 - \left(1 + \frac{\rho^2}{2}\right)\left(\frac{\xi}{\xi_0}\right)^3 + \frac{\rho^2}{2}\left(\frac{\xi}{\xi_0}\right)^4$$



(from S. Hartnoll, arXiv:1106.4342)

Electrical conductivity from AdS/CFT

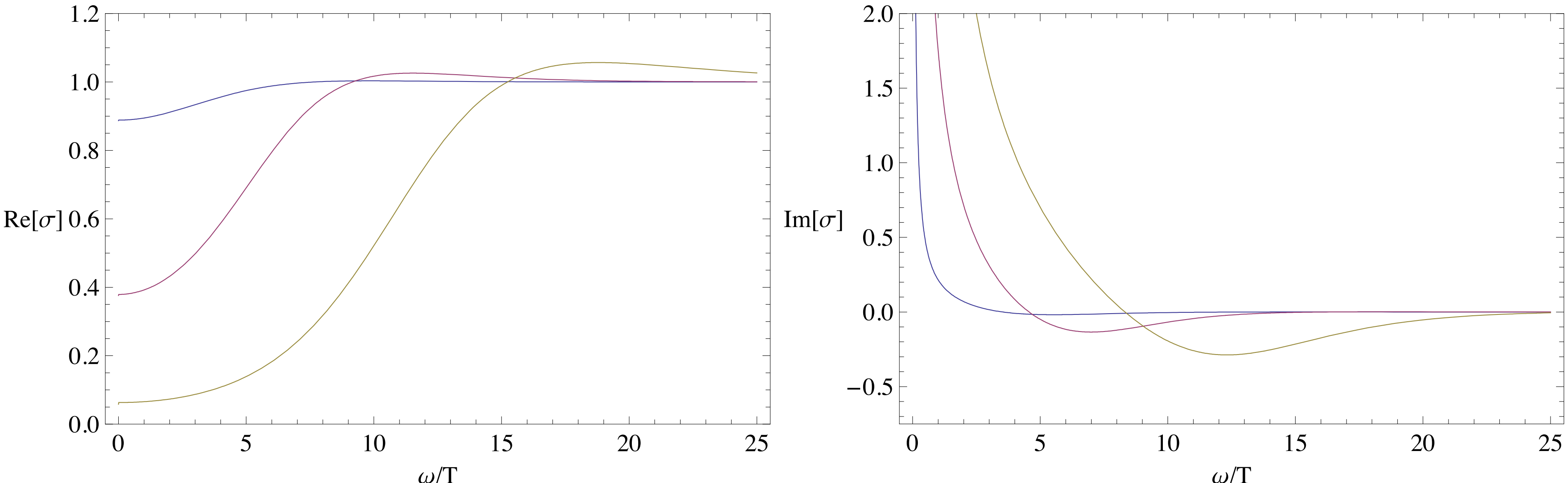
Holographic dictionary: $A_\mu \longleftrightarrow J_\mu$ U(1) current

Solve Maxwell's equations in black hole background

-- with "in-going" boundary conditions at black hole horizon

Asymptotic behavior: $A_x(\omega,\vec{k},\xi) \approx a_x^{(0)}(\omega,\vec{k}) + a_x^{(1)}(\omega,\vec{k})\xi + \dots$

Calculation simplifies at $\vec{k}=0$: $\sigma_{xx}(\omega) = -\frac{i}{\omega}\frac{a_x^{(1)}}{a_x^{(0)}}$



Figures from S. Hartnoll, Class. Quant. Grav. 26 (2009) 224002

Delta function peak in $\mathrm{Re}\ \sigma$ at $\omega = 0$ due to translation invariance

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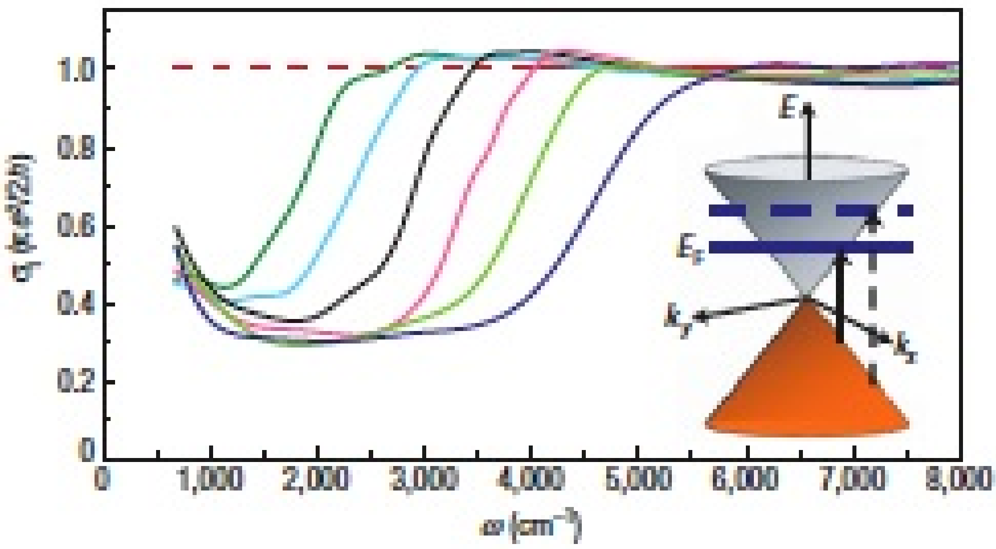
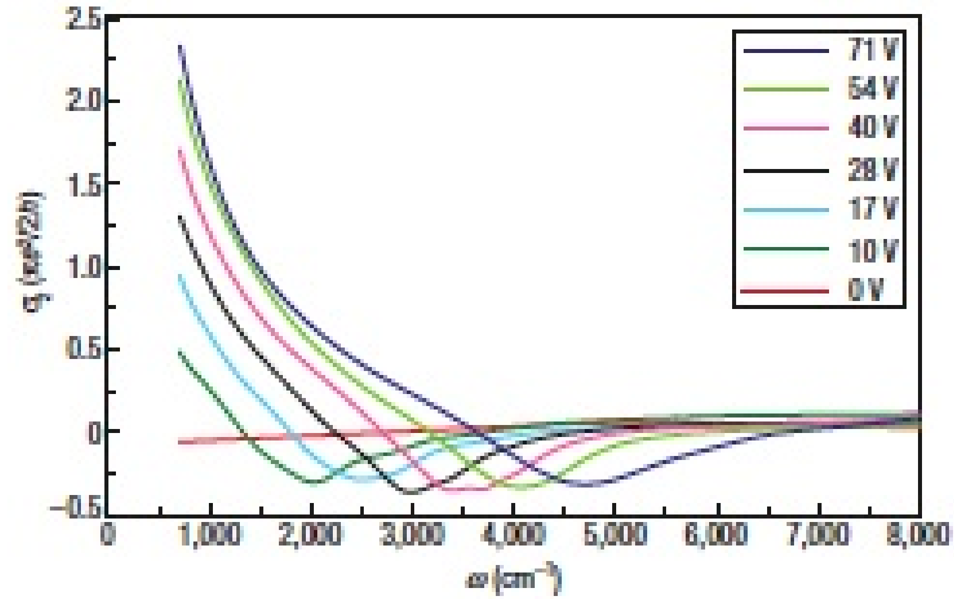
Experimental results in graphene





(figures from S. Sachdev, arXiv:0711.3015)

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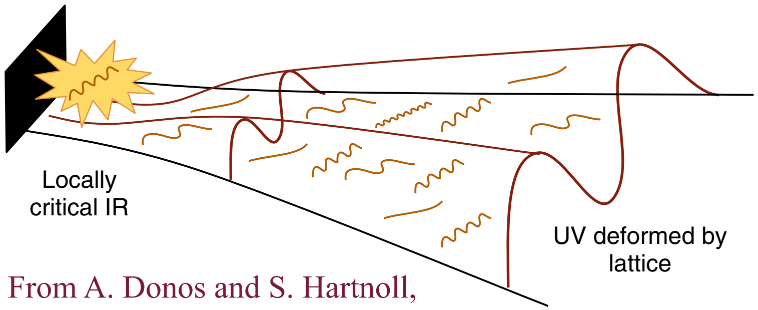
Holographic lattices

Break translation symmetry via UV boundary conditions

- Scalar field lattice: $\psi \to \psi_1(x,y)\,\frac{1}{r} + \psi_2(x,y)\,\frac{1}{r^2} + \ldots$

$$\psi_1 = A_0 \cos(k_0 x)$$

- Ionic lattice: $A_t \to \mu\ (1 + A_0 \cos(k_0 x)) + \ldots$



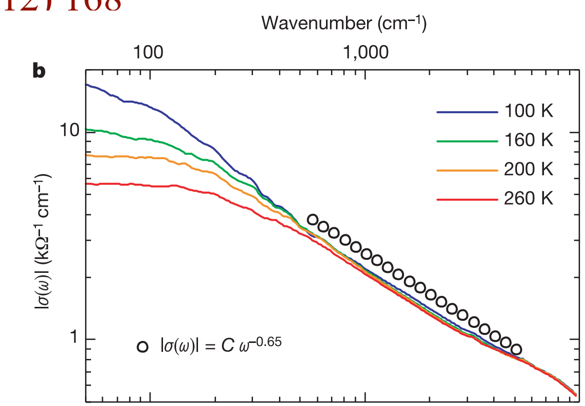
From A. Donos and S. Hartnoll, *Phys. Rev. D* **86** (2012) 124046

Optical conductivity G. Horowitz, J. Santos and D. Tong, *JHEP* **1207** (2012) 168

- low frequency: $\sigma(\omega) = \dfrac{K\tau}{1 - i\omega\tau}$

- intermediate frequency: $|\sigma(\omega)| = \dfrac{B}{\omega^{2/3}} + C$

- high frequency: $\sigma(\omega) \to \text{constant}$



Linear resistivity can be obtained from some holographic lattice models

A. Donos and S. Hartnoll, *PRD* **86** (2012) 124046

Holographic superconductors

Couple a charged scalar field to gravitational system

instability at low T : black brane with scalar "hair"

AdS/CFT prescription: hair corresponds to sc condensate

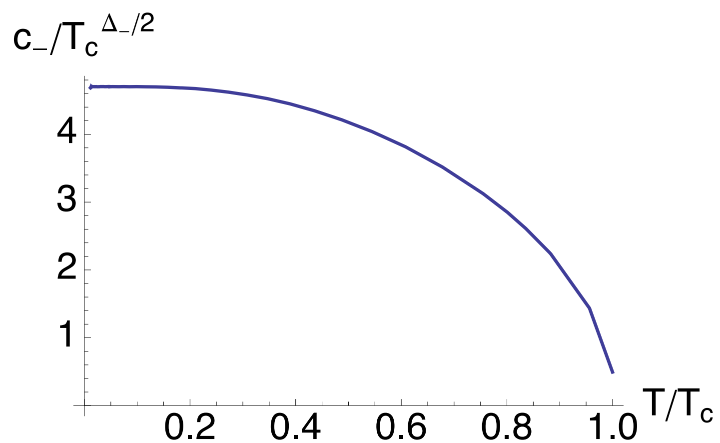
transport properties: solve classical wave equation in bh background

add magnetic field: dyonic black hole -- holographic sc is type II

conformal system: start from AdS-RN exact solution

z > 1 systems: work with Lifshitz black branes

Numerical results for superconducting condensate:



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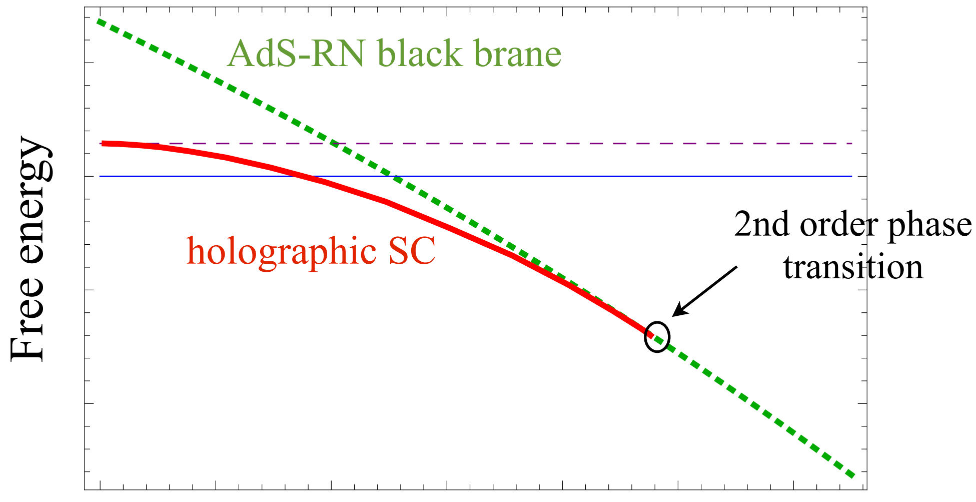
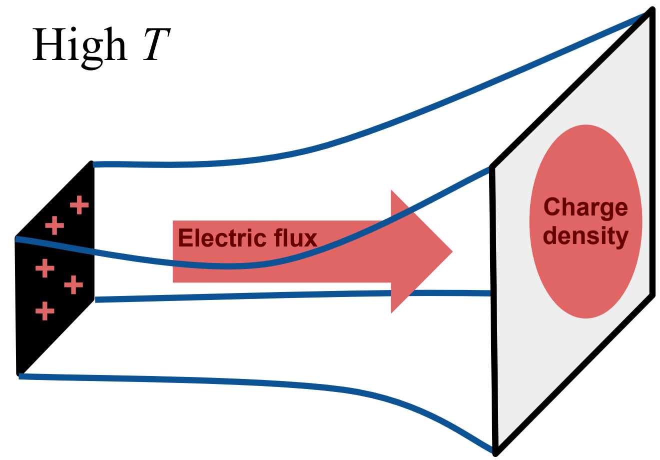
Thermodynamic stability

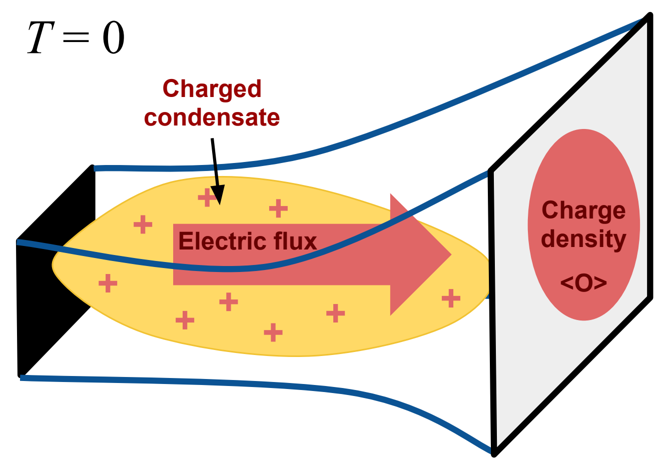


(from G. Horowitz and B. Way, arXiv:1007.3714)

High T

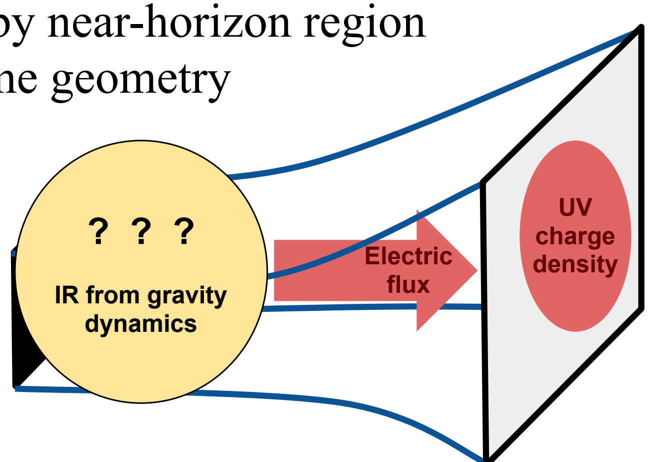


$T = 0$



(from S. Hartnoll, arXiv:1106.4342)

Low T dynamics at finite density is governed by near-horizon region in spacetime geometry



(from S. Hartnoll, arXiv:1106.4342)

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Zero temperature entropy

Low temperature limit is described by a near extremal black brane

$z = 1:$ Extremal RN black brane has non-vanishing entropy

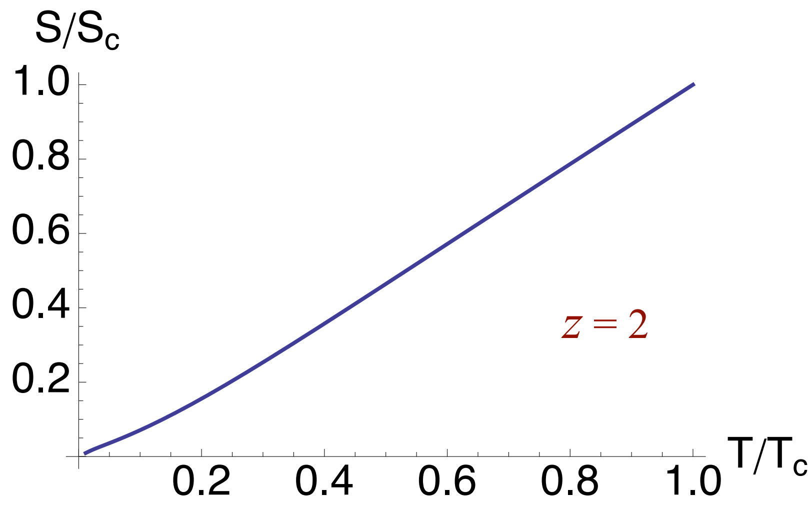
BUT

black brane with charged scalar hair has vanishing entropy density in extremal limit

G.Horowitz and M.Roberts (2009)

$z > 1:$ Lifshitz black brane with hair also has vanishing entropy density in extremal limit

E. Brynjólfsson, U. Danielsson, L.T., T. Zingg, (2010)



Holographic metals

Include charged fermions in the bulk: $S_{\text{matter}} = -\int \mathrm{d}^4x\sqrt{-g}\ \{\bar{\Psi}\not{D}\Psi + m\bar{\Psi}\Psi\}$

Fermion probe calculations: S.-S. Lee (2008); H.Liu, J.McGreevy, D.Vegh (2009); T. Faulkner, H. Liu, J. McGreevy, D.Vegh (2009); M.Cubrovic, J.Zaanen, K.Schalm (2009)

Dirac equation: $(\not{D} + m)\Psi = 0 \qquad D_M = \partial_M + \frac{1}{4}\omega_{abM}\Gamma^{ab} - iqA_M$

Boundary fermions: $\psi_\pm(t,\vec{x}) = \lim_{r\to\infty}\Psi_\pm(t,\vec{x},r) \qquad \Gamma^3\Psi_\pm = \pm\Psi_\pm$

$$\Psi_\pm(t,\vec{x},r) = \frac{1}{(2\pi)^3}\int d\omega\, d^2k\tilde{\Psi}_\pm(\omega,\vec{k},r)e^{-i\omega t+i\vec{k}\cdot\vec{x}}$$

Adapt AdS/CFT prescription to compute $G_R(\omega,k)$

Single fermion spectral function $\quad A(\omega,k) = \frac{1}{\pi}\mathrm{Im}\left(\mathrm{Tr}\left[i\sigma^3 G_R(\omega,k)\right]\right)$

can be directly compared to ARPES data.

Holographic Fermi surface

$$G_R(\omega,k)^{-1} \sim \omega - v_F(k-k_F) - i\Gamma + \ldots$$

$$v_F,\, k_F \sim \mu \qquad \Gamma \sim \omega^{2\nu} \qquad \nu = \sqrt{m^2 - q^2 + \frac{k_F^2}{\mu^2}}$$

Depending on the probe parameters we can have:

$\nu > \frac{1}{2}$ long-lived quasiparticles — Landau Fermi liquid: $\Gamma \sim \omega^2$

$\nu < \frac{1}{2}$ no stable quasiparticles

$\nu = \frac{1}{2}$ log suppressed quasiparticle residue — marginal Fermi liquid

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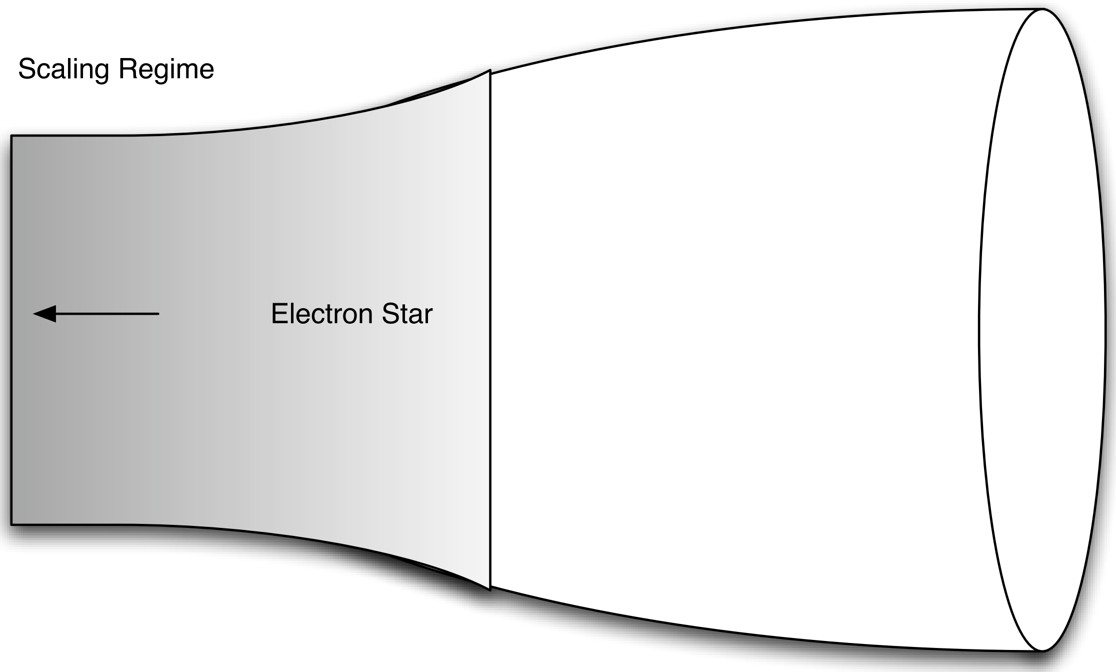
Going beyond fermion probe approximation

Fermion many-body problem in AdS_4 No easier than original problem!

Thomas-Fermi approximation: Treat fermions as a continuous charged fluid

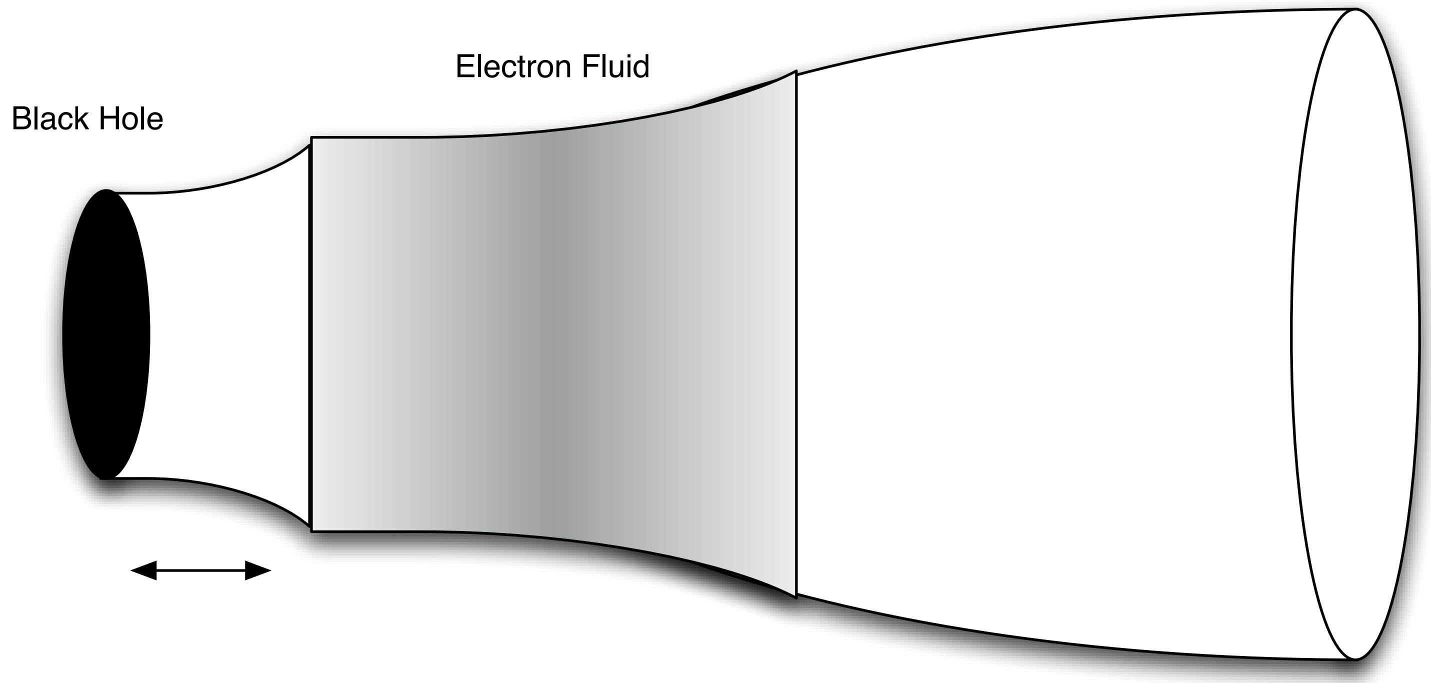
S. Hartnoll, J. Polchinski, E. Silverstein, D. Tong (2009)
S. Hartnoll, A. Tavanfar (2010)

T = 0 configuration is an *electron star*



Electron stars at finite temperature

S. Hartnoll, P. Petrov (2010);
V. Giangreco Puletti, S. Nowling, L.T., T. Zingg (2010)



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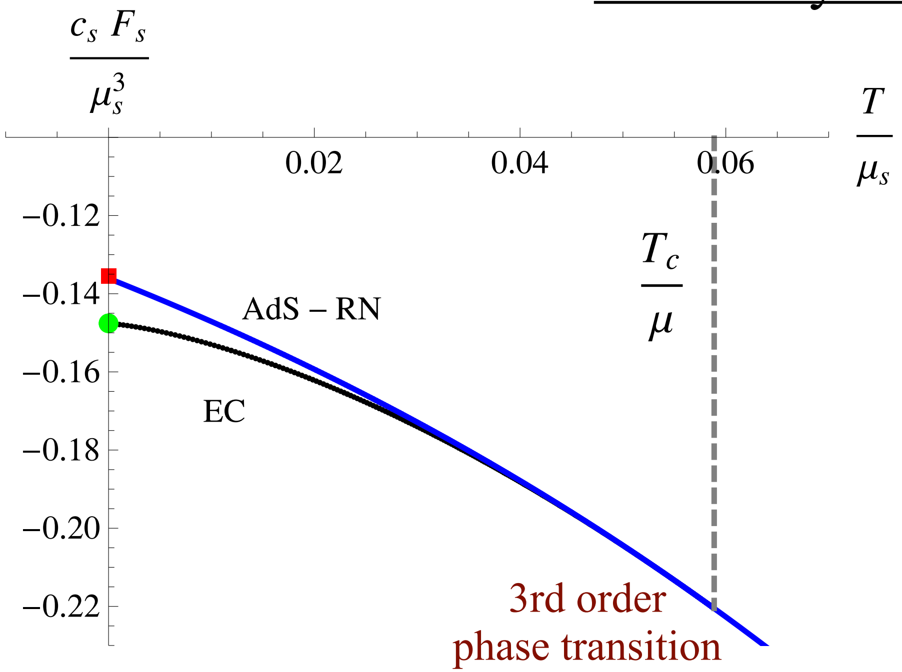
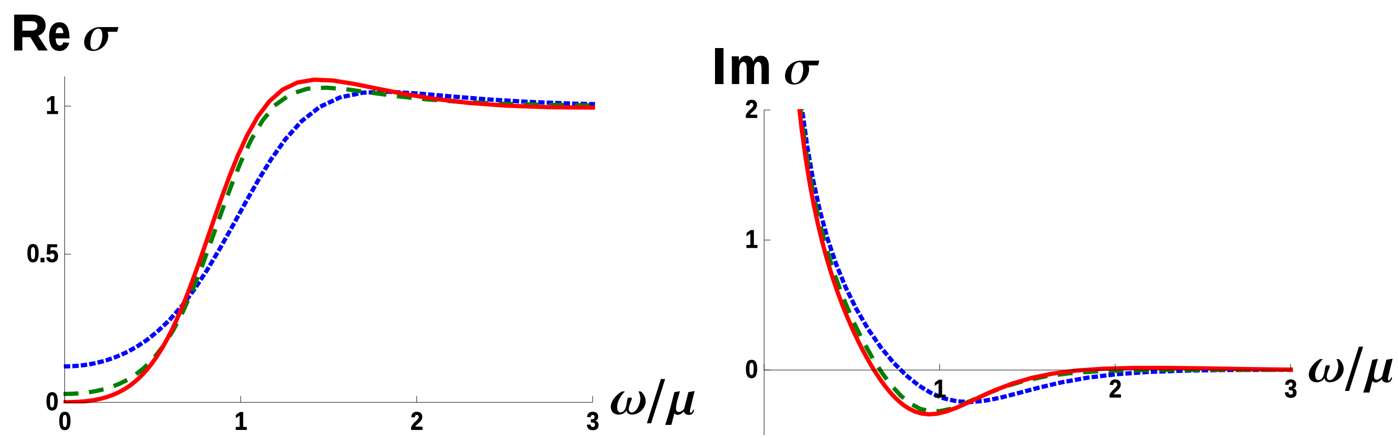
Thermodynamic stability

Hartnoll and Petrov 2010
Giangreco, Nowling, L.T., Zingg 2010



AdS-Reissner Nordström background is unstable to forming an electron cloud at low T

Electrical conductivity



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Summary and open questions

- Gauge theory/gravity correspondence provides a handle on some strongly coupled field theories.

- It is motivated by the study of supersymmetric solitons in string theory but bottom-up models only involve classical gravity & simple matter fields.

- Can also be used to study thermalization in out-of-equilibrium systems (holographic quenching, etc.)

- Moving towards more realistic models:

  - consider dyonic black holes to include magnetic effects

  - introduce modulated sources at AdS boundary to model lattice effects

  - quantum electron stars (bulk fermions beyond probe or TF approximation)

  - holographic p- and d-wave superconductors

- Many open questions:

  - why is classical gravity valid?

  - what plays role of N (# of colors)?

  - disordered systems?

  - designer gravity?

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