Tabletop String Theory

- applications of gauge theory/gravity duality

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Motivation

Quantum field theory (QFT) is an extraordinarily successful framework for understanding a wide range of physical phenomena:

- quantum electrodynamics (QED)
- standard model of particle physics
- many body theory for condensed matter systems

Precision tests of QED determine the fine structure constant to a part in $10^8$

$$\alpha = \frac{e^2}{4\pi\hbar c} \quad \Rightarrow \quad \alpha^{-1} = 137.03599 \ldots$$

Efficient approximation schemes are the key to QFT’s success:
- work well when interactions are weak
- strong coupling presents a difficult challenge
- numerical simulations are undermined by fermion sign problem

Gauge theory/gravity duality is a novel *gravitational* approach to strongly coupled QFT
Motivation (continued)

Quantum many particle theory works extremely well!
- explains a wide range of physical phenomena in broad classes of materials
- most known materials are in fact well described by established methods
- but there are exceptions...
- physicists want to understand them
- may point the way towards new materials or improved functionality

Resistivity in a high $T_c$ superconductor

Can gauge theory/gravity duality provide new insights?

\[ \rho \approx \rho_0 + \alpha T^\alpha \]
Length scales in Nature

- universe: $10^{26}$ meters
- galaxy: $10^{20}$ meters
- solar system: $10^{14}$ meters
- human: $10^{2}$ meters
- atom: $10^{-10}$ meters
- nucleus, proton, W, Z, top: $10^{-16}$ meters
- strings?: $10^{-34}$ meters

General relativity
- quantum mechanics
- standard model
- grand unified theory?
- quantum gravity?
Outline

Lecture 1
- strong/weak coupling dualities in physics
- open/closed duality in string theory
- Black branes vs. Dirichlet branes
- AdS/CFT (anti-de Sitter/conformal field theory) correspondence

Lecture 2
- scale invariance and quantum critical points
- heavy fermion alloys, high $T_c$ superconductors
- applied AdS/CFT:
  - electrical conductivity
  - holographic superconductors
  - holographic metals
Electromagnetic duality

Maxwell equations in vacuum

\[ \nabla \cdot \vec{E} = 0 \quad \nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} \]

\[ \nabla \cdot \vec{B} = 0 \quad -\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \]

Exchanging the electric and magnetic fields

\[ \vec{E} \rightarrow \vec{B}, \quad \vec{B} \rightarrow -\vec{E} \]

gives back the same set of equations
Electromagnetic duality (cont.)

Maxwell equations with sources (both electric and magnetic)

\[ \nabla \cdot \vec{E} = \rho_e \quad \nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{J}_e \]

\[ \nabla \cdot \vec{B} = \rho_m \quad -\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} + \vec{J}_m \]

Exchanging the electric and magnetic fields

\[ \vec{E} \rightarrow \vec{B} , \quad \vec{B} \rightarrow -\vec{E} \]

along with the electric and magnetic sources

\[ \rho_e \rightarrow \rho_m , \quad \rho_m \rightarrow -\rho_e \]

\[ \vec{J}_e \rightarrow \vec{J}_m , \quad \vec{J}_m \rightarrow -\vec{J}_e \]

again gives back the same set of equations
Dirac quantization condition

Quantum theory: The wave function describing a particle with electric charge $e$ in the presence of a magnetic charge $g$ is well-defined only if

$$e g = 2\pi \hbar n, \quad (n = 0, \pm 1, \pm 2, \ldots)$$

- Quantization of $e$ follows from the existence of magnetic monopoles

- Dirac condition implies that monopoles would be strongly coupled in ordinary electromagnetism

$$g \propto 1/e$$

- Dual theory has weakly coupled monopoles and strongly coupled electrons

- In particle physics we study generalizations of electromagnetic theory where monopoles occur as soliton solutions of the field equations
Five-minute primer on string theory

Replace point particles by one-dimensional strings and attempt to work out a quantum theory in flat spacetime.

Does not work unless the space-time has 26 dimensions and even then there are instabilities.

Adding fermions (and supersymmetry) leads to a stable theory in 10 dimensional spacetime.
Consistent string theories in 10 spacetime dimensions

The five string theories are interrelated by a web of strong/weak coupling string dualities

(from A. Sen, 1999)
Open/closed string duality

The same string world-sheet can be interpreted in different ways

Pair of open strings

Single closed string

A given system can have very different descriptions from the point of view of open vs. closed strings
Low-energy limit of string theory

Strings appear point-like in low-energy processes

closed (super-)string theory $\rightarrow$ (super-)gravity theory

Type II super-gravity -- bosonic fields

metric: $g_{\mu\nu}$

NS-NS tensor: $H_{\mu\nu\lambda}$

dilaton: $\phi$

R-R tensors: $F_{\mu_1\ldots\mu_q}^{(q)}$ $q \in \{1, 3, 5\}$

correspond to massless modes of closed strings
String theory generalization of monopoles

String theory contains a variety of higher dimensional objects called p-branes.

A black p-brane solution of the field equations of 10-d supergravity describes a space-time where charged matter is confined to a p+1 dimensional hyperplane.

Higher-dimensional generalization of a charged black hole in general relativity.

The allowed charge-to-mass ratio of a black p-brane has an upper bound.

Space-time geometry outside a maximally charged (a.k.a. extremal) 3-brane:

far field: \( M_{10} \) ten-dimensional Minkowski spacetime

near horizon: \( \text{AdS}_5 \times S_5 \) product of 5d anti-de Sitter spacetime and a 5-sphere
Dirichlet-branes

Open strings provide a very different view of p-branes.

Dp-brane: A \( p+1 \) dimensional hyperplane where open strings end.

Dp-brane dynamics \( \iff \) physics of open strings

Low-energy limit \( \iff \) Yang-Mills gauge theory

D3-brane in IIB string theory

\[ x^1 \]
\[ x^2, x^3 \]
\[ x^4, \ldots, x^9 \]
Multiple coincident D3-branes

Low-energy dynamics $\leftrightarrow$ massless open string modes

$d = 4$, $N = 4$ supersymmetric U($N$) Yang-Mills theory

Closed string description

$d = 10$, Type IIB supergravity in AdS$_5 \times S_5$ background
AdS/CFT correspondence

- strong/weak coupling duality so it is difficult to prove
- the original AdS/CFT conjecture has passed numerous tests and has been generalized in many directions
AdS/CFT prescription

Relates QFT correlation function to string amplitude in AdS$_5$ background

\[ \mathcal{Z}_{\text{string}}[\{\phi_i\}] = \left\langle \exp \left\{ \int d^4x \sum_i \tilde{\phi}_i(\vec{x}) \mathcal{O}_i(\vec{x}) \right\} \right\rangle_{\text{QFT}} \]

String theory partition function
QFT generating functional

\[ \phi_i(\vec{x}, r) \quad \text{is a field (string mode) in AdS}_5 \text{ background} \]

\[ \tilde{\phi}(\vec{x}) = \lim_{r \to \infty} \phi_i(\vec{x}, r) \quad \text{boundary value} \]

\[ \mathcal{O}_i(\vec{x}) \quad \text{is a local operator in QFT} \]

Holographic dictionary:

- metric \( g_{\mu\nu} \) \( \leftrightarrow \) energy momentum tensor \( T_{\mu\nu} \)
- gauge potential \( A_\mu \) \( \leftrightarrow \) conserved current \( J_\mu \)

Both sides of the prescription require regularization & renormalization.
Two sides of AdS-CFT

1. Quantum gravity via gauge theory
   - emergent spacetime
   - Hawking information paradox

2. Gravitational approach to strongly coupled field theories
   - strongly coupled QFT in D spacetime dimensions equivalent to weakly coupled gravity in D+1 dimensions
   - recipe for correlation functions at finite temperature
   - transport coefficients, damping rates
   - involves novel black hole geometries
Applied AdS-CFT

Investigate strongly coupled quantum field theories via classical gravity

- growing list of applications:
  - hydrodynamics of quark gluon plasma
  - holographic QCD
  - quantum critical systems
    - strongly correlated electron systems
    - cold atomic gases
  - out of equilibrium dynamics
  - ....

Bottom-up approach: Look for interesting behavior in simple models

- Assume that classical gravity in (asymptotically) AdS spacetime is dual to some strongly coupled QFT.

- Use AdS/CFT techniques to compute QFT correlation functions.

- Add gauge and matter fields to gravity theory to model interesting physics.

- Back-reaction can modify asymptotic behavior: non AdS - non CFT
Applied AdS-CFT

Investigate strongly coupled quantum field theories via classical gravity
- growing list of applications:
  - hydrodynamics of quark gluon plasma
  - jet quenching in heavy ion collisions
  - quantum critical systems
    - strongly correlated electron systems
    - cold atomic gases
  - holographic superconductors
  - holographic metals
  - out of equilibrium dynamics
  - ....

Bottom-up approach: Look for interesting behavior in simple models
M.C. Escher’s Circle Limit IV (1960)
Anti-de Sitter geometry

\[ ds^2 = b^2 \left( z^2 (-dt^2 + d\bar{y}^2) + \frac{dz^2}{z^2} \right) \]

Metric is invariant under Lorentz transformations on \((t, \bar{y})\)

and also under the scaling \((t, z, y^i) \rightarrow \left( ct, \frac{z}{c}, cy^i \right)\), \(c > 0\)

UV - IR connection

The map \( z \rightarrow 1/\xi \) gives

\[ ds^2 = \frac{b^2}{\xi^2} \left(-dt^2 + d\bar{y}^2 + d\xi^2\right) \]
Applied AdS-CFT

- Assume that classical gravity in (asymptotically) AdS spacetime is dual to some strongly coupled QFT.
- Use AdS-CFT techniques to calculate QFT correlation functions at strong coupling.
- Add gauge fields and matter fields to the gravity theory to model interesting physics.
- Quantum critical systems have scale invariance + strong correlations → natural starting point for applied AdS-CFT.
Quantum critical points

Scale invariant theory at finite $T$:

$$\xi = c T^{-1/z}$$

Deformation away from fixed pt.:

$$\lambda_i \sim (\text{length})^{-1}$$

QCP has $\lambda_i = 0$

Quantum critical region:

$$\xi = T^{-1/z} \eta(T^{-1/z} \lambda_i)$$

$$\eta(0) = c$$

Physical systems with $z = 1$, 2, and 3 are known -- non-integer values of $z$ are also possible

$z = 1$ scaling symmetry is part of $SO(d+1,1)$ conformal group = isometries of $adS_{d+1}$

$z > 1$ scale invariance without conformal invariance - asymptotically Lifshitz spacetime
1.3. Quantum critical points in the real world

Quantum phase transitions are believed to be important in describing superconducting–insulator transitions in thin metallic films, as is demonstrated pictorially by rotating figure 2 $90^\circ$ counter-clockwise. The rotated diagram is meant to resemble closely figure 1 where phase one is an insulator, phase two is a superconductor, and $g$ corresponds to the thickness of the film. The insulating transition is a cross-over, while the superconducting transition might be of Kosterlitz–Thouless type. There exists a critical thickness for which the system reaches the quantum critical point at $T = 0$.

One of the most exciting (and also controversial) prospects for the experimental relevance of quantum phase transitions is high-temperature superconductivity. Consider the parent compound $\text{La}_2\text{CuO}_4$ of one of the classic high-$T_c$ superconductors, $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$. $\text{La}_2\text{CuO}_4$ is actually not a superconductor at all but an anti-ferromagnetic insulator at low temperatures. The physics of this layered compound is essentially two dimensional. The copper atoms

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Classic example of a QCP

Resistivity vs. temperature in thin films of bismuth

$T = 0$ state changes from insulating to superconducting at a critical thickness

Quantum criticality in heavy fermion materials

From P. Gegenwart, Q. Si and F. Steglich,
Some measured c/T values in heavy fermion metals


Quantum criticality in high T_c superconductors

Linear resistivity in strange metal region

\[ \rho \approx \rho_0 + \alpha T^\alpha \]

Drude form at low frequency:

$$\text{Re } \sigma(\omega, T) \sim T^{-1} \left( 1 + A^2 \left( \frac{\omega}{T} \right)^2 \right)^{-1}$$

Universal power law at intermediate frequency:

$$\sigma(\omega, T) \approx B (-i\omega)^{-2/3}$$

Gravity duals at finite temperature

periodic Euclidean time: $\tau \simeq \tau + \beta$, $\beta = \frac{1}{T}$

$\beta$ introduces an energy scale: scale symmetry is broken

thermal state in field theory: black hole with $T_{\text{Hawking}} = T_{\text{qft}}$

finite charge density in dual field theory: electric charge on BH

magnetic effects in dual field theory: dyonic BH

$z = 1$: planar AdS-Reissner-Nordström black hole

$z > 1$: planar charged Lifshitz black hole
Anti-de Sitter geometry

\[ ds^2 = b^2 \left( z^2 (-dt^2 + d\vec{y}^2) + \frac{dz^2}{z^2} \right) \]

Metric is invariant under Lorentz transformations on \((t, \vec{y})\)

and also under the scaling \((t, z, y^i) \to \left( c t, \frac{z}{c}, c y^i \right)\), \(c > 0\)

UV - IR connection

The map \(z \to 1/\xi\) gives

\[ ds^2 = \frac{b^2}{\xi^2} (-dt^2 + d\vec{y}^2 + d\xi^2) \]
Planar AdS black holes

AdS spacetime: \[ ds^2 = \frac{1}{\xi^2} (-dt^2 + d\bar{y}^2 + d\xi^2) \quad \longleftrightarrow \quad b = 1 \]

Planar AdS black hole: \[ ds^2 = \frac{1}{\xi^2} (-f(\xi) dt^2 + d\bar{y}^2 + \frac{d\xi^2}{f(\xi)}) \]

AdS-Schwarzschild \[ f(\xi) = 1 - \left( \frac{\xi}{\xi_0} \right)^3 \quad \longleftrightarrow \quad n = 3 \]

AdS-Reissner-Nordström \[ f(\xi) = 1 - \left( 1 + \frac{\rho^2}{2} \right) \left( \frac{\xi}{\xi_0} \right)^3 + \frac{\rho^2}{2} \left( \frac{\xi}{\xi_0} \right)^4 \]

(from S. Hartnoll, arXiv:1106.4342)
Electrical conductivity from AdS/CFT

Holographic dictionary: \( A_\mu \leftrightarrow J_\mu \) U(1) current

Solve Maxwell’s equations in black hole background

-- with “in-going” boundary conditions at black hole horizon

Asymptotic behavior:

\[
A_x(\omega, \vec{k}, \xi) \approx a_x^{(0)}(\omega, \vec{k}) + a_x^{(1)}(\omega, \vec{k})\xi + \ldots
\]

Calculation simplifies at \( \vec{k} = 0 \):

\[
\sigma_{xx}(\omega) = -\frac{i}{\omega} \frac{a_x^{(1)}}{a_x^{(0)}}
\]

Delta function peak in \( \text{Re} \sigma \) at \( \omega = 0 \) due to translation invariance

Figures from S. Hartnoll, Class. Quant. Grav. 26 (2009) 224002
Experimental results in graphene

(figures from S. Sachdev, arXiv:0711.3015)
Holographic lattices

Break translation symmetry via UV boundary conditions

- Scalar field lattice: \( \psi \rightarrow \psi_1(x, y) \frac{1}{r} + \psi_2(x, y) \frac{1}{r^2} + \ldots \)
  \[ \psi_1 = A_0 \cos(k_0 x) \]

- Ionic lattice: \( A_t \rightarrow \mu (1 + A_0 \cos(k_0 x)) + \ldots \)


- low frequency: \( \sigma(\omega) = \frac{K \tau}{1 - i\omega \tau} \)
- intermediate frequency: \( |\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C \)
- high frequency: \( \sigma(\omega) \rightarrow \text{constant} \)

Linear resistivity can be obtained from some holographic lattice models

Holographic superconductors

Couple a charged scalar field to gravitational system

instability at low $T$: black brane with scalar “hair”

AdS/CFT prescription: hair corresponds to sc condensate

transport properties: solve classical wave equation in bh background

add magnetic field: dyonic black hole -- holographic sc is type II

conformal system: start from AdS-RN exact solution

$z > 1$ systems: work with Lifshitz black branes

Numerical results for superconducting condensate:
Thermodynamic stability

\begin{align*}
\text{Free energy} \quad \text{Temperature} \\
\end{align*}

Low $T$ dynamics at finite density is governed by near-horizon region in spacetime geometry

\begin{align*}
\text{AdS-RN black brane} \\
\text{2nd order phase transition}
\end{align*}

High $T$

\begin{align*}
\text{Charge density} \\
\text{Electric flux}
\end{align*}

$T = 0$

\begin{align*}
\text{Charged condensate} \\
\text{Electric flux} \\
\text{Charge density} \langle \mathbf{O} \rangle
\end{align*}

\begin{align*}
\text{UV charge density}
\end{align*}

\begin{align*}
\text{Electric flux}
\end{align*}

\begin{align*}
\text{IR from gravity dynamics}
\end{align*}

\begin{align*}
\text{Electric flux}
\end{align*}

\begin{align*}
\text{UV charge density}
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\begin{align*}
\text{(from S. Hartnoll, arXiv:1106.4342)}
\end{align*}
Zero temperature entropy

Low temperature limit is described by a near extremal black brane

\[ z = 1 : \] Extremal RN black brane has non-vanishing entropy

**BUT**

black brane with charged scalar hair has vanishing entropy density in extremal limit

G. Horowitz and M. Roberts (2009)

\[ z > 1 : \] Lifshitz black brane with hair also has vanishing entropy density in extremal limit

Holographic metals

Include charged fermions in the bulk: \[ S_{\text{matter}} = -\int d^4x \sqrt{-g} \{ \bar{\Psi} \slashed{D} \Psi + m \bar{\Psi} \Psi \} \]


Dirac equation: \[ (\slashed{D} + m) \Psi = 0 \]
\[ D_M = \partial_M + \frac{1}{4} \omega_{abM} \Gamma^{ab} - iqA_M \]

Boundary fermions: \[ \psi_\pm(t, \vec{x}) = \lim_{r \to \infty} \Psi_\pm(t, \vec{x}, r) \quad \Gamma^3 \Psi_\pm = \pm \Psi_\pm \]
\[ \Psi_\pm(t, \vec{x}, r) = \frac{1}{(2\pi)^3} \int d\omega \, d^2k \bar{\Psi}_\pm(\omega, \vec{k}, r) e^{-i\omega t + i\vec{k} \cdot \vec{x}} \]

Adapt AdS/CFT prescription to compute \( G_R(\omega, k) \)

Single fermion spectral function \( A(\omega, k) = \frac{1}{\pi} \text{Im} (\text{Tr} [i \sigma^3 G_R(\omega, k)]) \)
can be directly compared to ARPES data.
Holographic Fermi surface

\[ G_R(\omega, k)^{-1} \sim \omega - v_F(k - k_F) - i\Gamma + \ldots \]

\[ v_F, k_F \sim \mu \quad \Gamma \sim \omega^{2\nu} \quad \nu = \sqrt{m^2 - q^2 + \frac{k_F^2}{\mu^2}} \]

Depending on the probe parameters we can have:

\[ \nu > \frac{1}{2} \quad \text{long-lived quasiparticles} \]

\[ \nu < \frac{1}{2} \quad \text{no stable quasiparticles} \]

\[ \nu = \frac{1}{2} \quad \text{log suppressed quasiparticle residue} \]

Landau Fermi liquid: \( \Gamma \sim \omega^2 \)

marginal Fermi liquid
Going beyond fermion probe approximation

Fermion many-body problem in AdS_4

No easier than original problem!

Thomas-Fermi approximation: Treat fermions as a continuous charged fluid

S. Hartnoll, A. Tavanfar (2010)

T = 0 configuration is an electron star
Electron stars at finite temperature

S. Hartnoll, P. Petrov (2010);
Different sections of the text:

1. **Thermodynamic stability**
   - AdS-Reissner Nordström background is unstable to forming an electron cloud at low T.
   - 3rd order phase transition.

2. **Electrical conductivity**
   - Graphs showing real and imaginary parts of conductivity as functions of frequency.
Summary and open questions

- Gauge theory/gravity correspondence provides a handle on some strongly coupled field theories.

- It is motivated by the study of supersymmetric solitons in string theory but bottom-up models only involve classical gravity & simple matter fields.

- Can also be used to study thermalization in out-of-equilibrium systems (holographic quenching, etc.)

- Moving towards more realistic models:
  - consider dyonic black holes to include magnetic effects
  - introduce modulated sources at AdS boundary to model lattice effects
  - quantum electron stars (bulk fermions beyond probe or TF approximation)
  - holographic p- and d-wave superconductors

- Many open questions:
  - why is classical gravity valid?
  - what plays role of N (# of colors)?
  - disordered systems?
  - designer gravity?