



Path-Integral Formulation of the Excursion Set Theory of Dark Matter Halos: Theory vs N-body Simulations



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Outline

- **Dark Matter Halos**
- **Mass Function State of Art**
- **Beyond Press-Schechter Approach**
- **Excursion Set Theory & Path-Integral**
- **Applications to non-Standard Cosmologies**

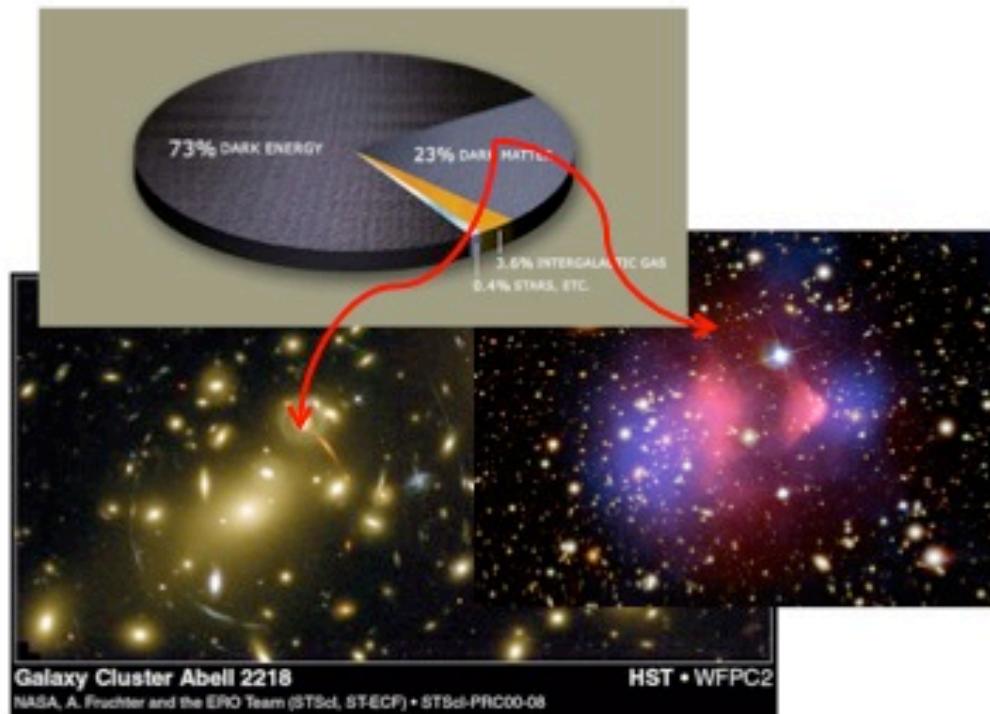
Cosmic Structures

Dark Matter:

- Foster matter clustering
- Resides in virialized clumps

Halos:

- Building blocks of cosmic structure formation
- Shape baryon distribution



Halo properties

- Statistics of the initial density field
- Non-linear gravitational collapse (cosmology and DM properties)

Observationally Testable
Key to understand galaxy formation

Halo Counting Statistics

Mass Function

- #halos with mass $[M, M+dM]$ at given z
- fraction of mass elements $F(M)$

$$\frac{dn}{dM} = \frac{1}{V} \times \frac{dF}{dM}$$

Factorized Form

$$\frac{dn}{dM} = \frac{\rho}{M^2} \frac{d \ln \sigma^{-1}}{d \ln M} f(\sigma)$$

- N-body Simulations
- Analytical Modeling

- Mean matter density ρ
- Variance of Linear Density Field: $S = \sigma^2(M)$
- Multiplicity function $f(\sigma) = 2 S dF/dS$

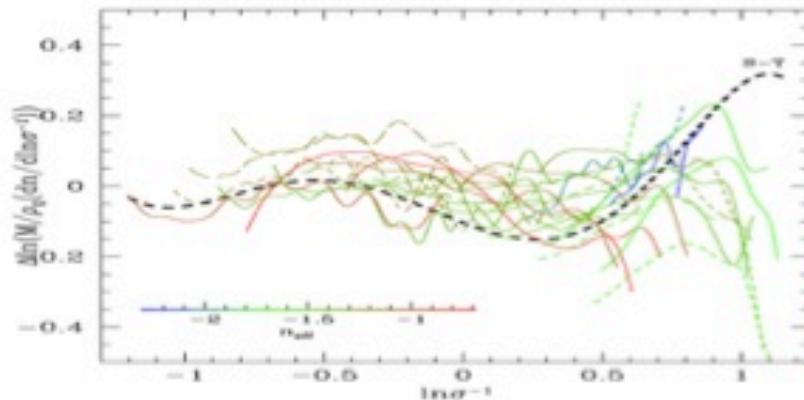
N-body Results

Universal Fitting Formula

$$f_{\text{FoF}(0.2)}(\sigma) = 0.315 \exp\left[-\left(\ln \sigma^{-1} + 0.61\right)^{3.8}\right]$$

- Redshift & Cosmology Independent

Jenkins et al., MNRAS, 321, 372 (2001)



Parametrizations (Λ CDM) & Limitations to Universality

$$f_{\text{FoF}(0.2)}(\sigma) = A(\sigma^{-a} + b) e^{-c/\sigma^2}$$

- Calibrated to z=0, FoF-mass correction, ≈10% scatter Warren et al., ApJ, 646, 881 (2006)
- Calibrated to z=0,.5,1 , FoF-mass correction, ≈2% scatter Crocce et al., MNRAS, 403, 1353 (2008)
- Calibrated to z=0,0.25,1, explicit cosmology (δ_c / σ), ≈2% scatter Courtin et al., MNRAS, 410, 1911 (2011)

$$f_{SO}(\sigma) = A[(\sigma / b)^{-a} + 1] e^{-c/\sigma^2}$$

- Calibrated to z=0,0.5,1.25,2.5, Δ-dependent fit, ≈5% scatter Tinker et al., ApJ, 688, 709 (2008)

What have we learnt?

Properties $f(\sigma)$

- Sheth-Tormen functional form provides excellent fit to N-body results (SOD or FOF)
- Coefficients vary with z & cosmology (exact value depends on halo detection algorithm, i.e. halo enclosed overdensity)
- Dependencies result of characteristics of non-linear collapse

$$f(\sigma) = A \left[\left(\frac{\sigma}{b} \right)^{-a} + 1 \right] e^{-c/\sigma^2},$$

Questions:

What physically mean A , a , b , c ?

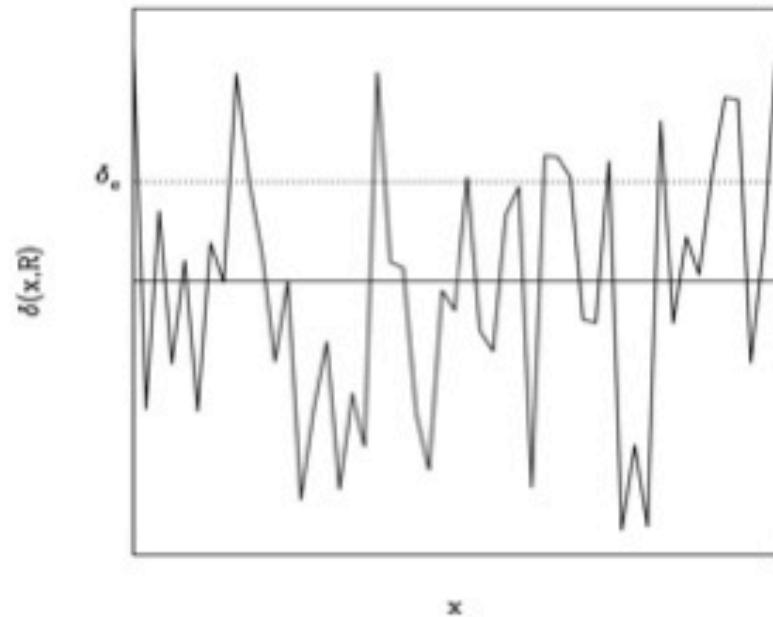
How can we predict $f(\sigma)$?

Can we reproduce simulation results?

Press-Schechter Formalism

Press & Schechter, ApJ, 187, 425 (1975)

Halo Mass Distribution & Linear Density Field:



- Filtering scale and mass

$$M = \bar{\rho} V(R)$$

- Statistics of the smoothed density field

$$\Pi(\delta, R)$$

- Linearly extrapolated collapse threshold

$$\delta_c$$

Fraction of mass in halos $> M$

$$F_{PS}(R[M]) = \int_{\delta_c}^{\infty} d\delta \Pi(\delta, R[M])$$

Press-Schechter Mass Function

Gaussian Field:

- Variance of the smoothed field

$$\sigma^2(R) \equiv S(R) = \frac{1}{2\pi^2} \int k^2 P(k) |\tilde{W}(k, R)|^2 dk$$

- PDF

$$\Pi(\delta, S[R]) = \frac{1}{\sqrt{2\pi S[R]}} e^{-\delta^2/2S[R]}$$

- Spherical collapse threshold δ_c

- Fraction of mass in halos

$$F_{PS}(R) = \int_{\delta_c}^{\infty} d\delta \Pi(\delta, S(R)) = \frac{1}{2} \operatorname{Erfc} \left[\frac{\delta_c}{\sigma(R)\sqrt{2}} \right]$$

- PS multiplicity function

$$f_{PS}(\sigma) = \frac{1}{\sqrt{2\pi}} \frac{\delta_c}{\sigma} e^{-\delta_c^2/(2\sigma^2)}$$

Cloud-in-Cloud Problem

Asymptotic Behavior: • In the limit $R \rightarrow 0$ all mass must be in collapsed structures, $F(0)=1$

- In the PS calculation half of the mass is miscounted

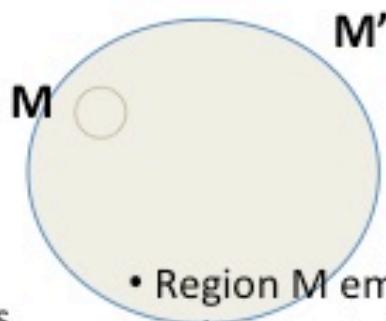
$$F_{PS}(R) = \frac{1}{2} \operatorname{Erfc} \left[\frac{\delta_c}{\sigma(R)\sqrt{2}} \right] \xrightarrow[R \rightarrow 0]{} \frac{1}{2}$$

Problem: in the PS approach there is no mass ordering

- No distinction between different configurations (cloud-in-cloud)



- Halo of mass M embedded in a larger non-collapsed region contributes to mass function at mass M



- Region M embedded in halo $M' > M$. M should not be counted in mass function since already included at M'

Excursion Set Theory

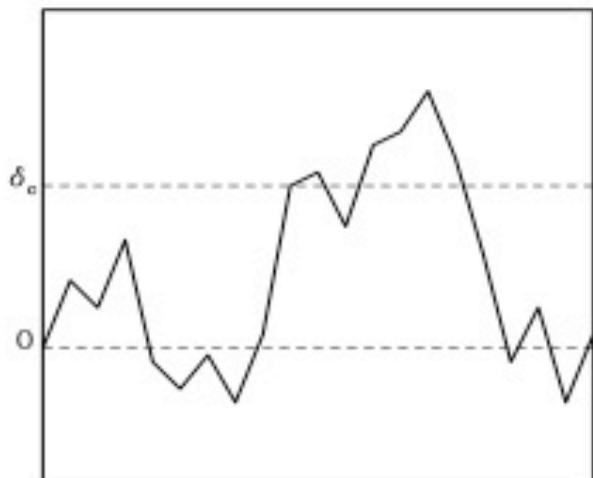
Bond et al., ApJ, 379, 440 (1991)

$$\delta(x, R) = \frac{1}{(2\pi)^3} \int d^3k \ \delta(k) \tilde{W}(k, R) e^{-ikx}$$

- At any point x , δ performs a random walk as function of R

- Langevin Equation:

$$\frac{\partial \delta}{\partial R} = \zeta(R) \text{ and } \zeta(R) = \frac{1}{(2\pi)^3} \int d^3k \ \delta(k) \frac{\partial \tilde{W}}{\partial R} e^{-ikx}$$



- random walks start at $R = \infty$ ($S=0$) with $\delta = 0$ evolving toward smaller R (larger S)
- $\zeta(R)$ depends on $\Pi(\delta)$ and $W(x, R)$
- Halos of mass M corresponds to trajectories crossing the threshold at $S(M)$
- Cloud-in-Cloud solved by requiring first crossing

Excursion Set Mass Function

Stochastic Problem:

- Computation of the probability distribution of random walks with absorbing boundary, $\Pi(\delta, \delta_c, S)$
- Multiplicity function obtained from the first-crossing rate

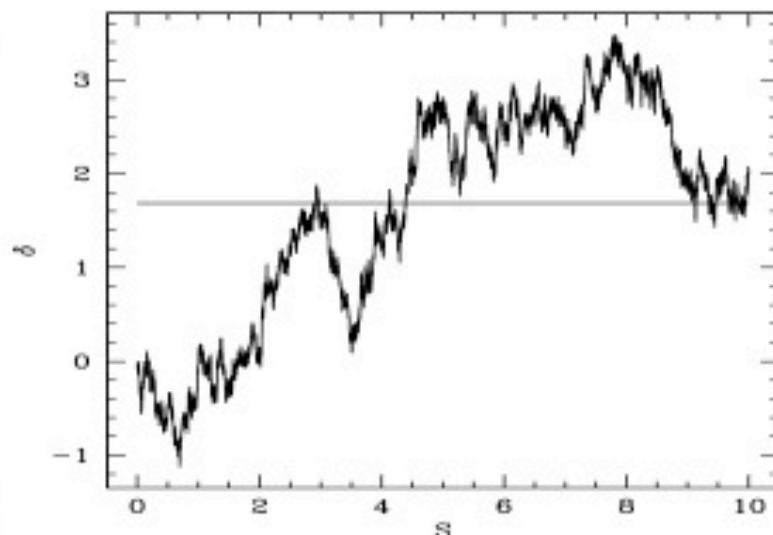
$$F(S) \equiv \int_0^S \frac{dF}{dS'} dS' = 1 - \int_{-\infty}^{\delta_c} \Pi(\delta, \delta_c, S) d\delta \quad \rightarrow \quad \frac{dF}{dS} = - \frac{\partial}{\partial S} \int_{-\infty}^{\delta_c} \Pi(\delta, \delta_c, S) d\delta$$

Cloud-in-cloud: $\lim_{S \rightarrow \infty} \Pi(\delta, \delta_c, S) = 0$

Sharp-k filter: $\tilde{W}(k, R) = \theta(1/R - k)$

- Markovian random walks

$$\frac{\partial \delta}{\partial S} = \eta(S) \quad \text{with} \quad \langle \eta(S) \rangle = 0 \\ \langle \eta(S) \eta(S') \rangle = \delta_D(S - S')$$



Extended Press-Schechter

Fokker-Planck Equation:

$$\frac{\partial \Pi}{\partial S} = \frac{1}{2} \frac{\partial^2 \Pi}{\partial \delta^2} \quad \text{with} \quad \begin{aligned} \Pi(\delta, 0) &= \delta_D(\delta) \\ \Pi(\delta_c, S) &= 0 \end{aligned}$$

Solution:

$$\Pi(\delta, S) = \frac{1}{\sqrt{2\pi S}} \left[e^{-\delta^2/(2S)} - e^{-(2\delta_c - \delta)^2/(2S)} \right] \quad \text{for } \delta < \delta_c$$

Multiplicity Function:

$$\frac{dF}{dS} = -\frac{\partial}{\partial S} \int_{-\infty}^{\delta_c} \Pi(\delta, \delta_c, S) d\delta \quad \longrightarrow$$

$$f_{EPS}(\sigma) = \sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma} e^{-\delta_c^2/(2\sigma^2)}$$

Non-Spherical Halo Collapse

Ellipsoidal Collapse

- Initial Gaussian fluctuations are non-spherical Doroshkevich, *Astroph.*, 3, 175 (1970)
- Ellipsoidal halos collapse and shear

$$\frac{d^2 a_i}{dt^2} = \frac{8}{3} \pi G \bar{\rho}_\Lambda a_i - 4 \pi G \bar{\rho}_m a_i \left[\frac{1}{3} + \frac{\Delta(t)}{3} + \frac{b'_i(t)}{2} \Delta(t) + \lambda'_i(t) \right]$$

$$\lambda_1 = \frac{\delta}{3} (1 - 3e + p)$$

$$\lambda_2 = \frac{\delta}{3} (1 - 2p)$$

$$b'_i = -\frac{2}{3} + a_1 a_2 a_3 \int_0^\infty \frac{d\tau}{(a_i^2 + \tau) \prod_{m=1}^3 (a_m^2 + \tau)^{1/2}}$$

$$\lambda_3 = \frac{\delta}{3} (1 + 3e + p)$$

- Dynamics dependent of initial size of the collapsing region
- Critical Overdensity is mass dependent e.g. Eisenstein & Loeb, *ApJ*, 439, 520 (1995)

Ellipsoidal Collapse Barrier

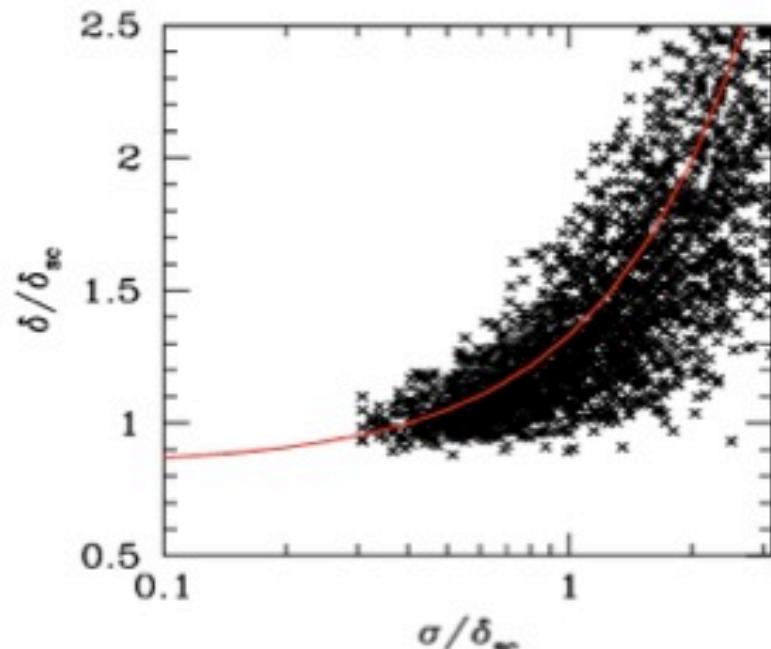
Stochastic Barrier

- Ellipsoidal parameters are random variables with characteristic probability distribution
- Density threshold is a random variable

Sheth & Tormen Mass Function

- Ellipsoidal Collapse Average Threshold:
- Monte Carlo Solution Excursion Set Theory
- MC Fitting Function Solution Calibrated to Gif simulations

Sheth & Tormen, MNRAS, 308, 199 (1999)
Sheth & Tormen, MNRAS, 329, 61 (2002)



$$\langle B(S) \rangle = \delta_c [1 + \beta (S/S_*)^\gamma]$$

Sheth, Mo & Tormen, MNRAS,
323, 1 (2001)

$$f_{ST}(\sigma) = A \left[1 + \left(a \frac{\delta_c^2}{2\sigma^2} \right)^{-p} \right] \sqrt{\frac{a\delta_c^2}{2\pi\sigma^2}} e^{-a\delta_c^2/2\sigma^2}$$

Fuzzy Barrier

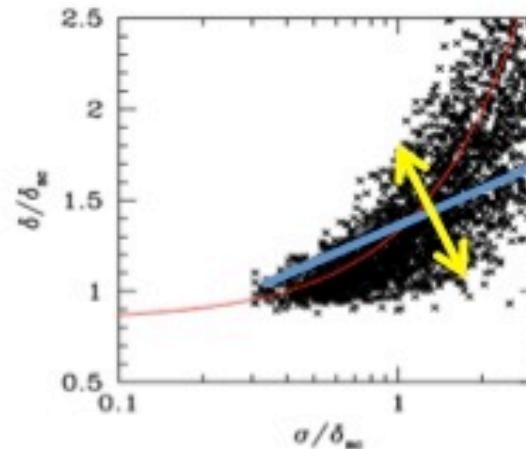
Non-Spherical Collapse:

- Diffusive Drifting Barrier $\langle B(S) \rangle = \delta_c + \beta S$ $\langle [B(S) - \langle B(S) \rangle]^2 \rangle^{1/2} = \sqrt{D_B} \sigma$

β = rate of average deviation from spherical collapse

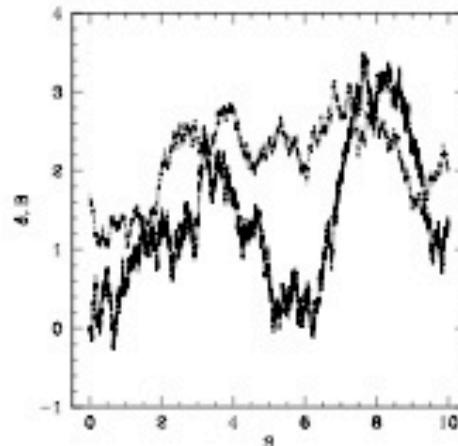
D_B = scatter of the collapse condition around mean

Introduce: $Y=B-\delta$



Sharp-k filter case:

$$\frac{\partial Y}{\partial S} = \beta + \eta(S) \quad \langle \eta(S) \rangle = 0$$
$$\langle \eta(S)\eta(S') \rangle = (1+D_B)\delta_D(S-S')$$



EPS Inaccuracies

Filter & Mass Definition: $M(R) = \bar{\rho} V(R)$ with $V(R) = \int d^3x W(x, R)$

- Unambiguously defined only for sharp-x filter:

$$V(R) = \int d^3x W(x, R) = 4/3 \pi R^3$$

- Consistent with filtering of linear density field entering in dn/dM :

$$\sigma^2(R) \equiv S(R) = \frac{1}{2\pi^2} \int k^2 P(k) |\tilde{W}(k, R)|^2 dk$$

$$\tilde{W}(k, R) = 3 \frac{\sin(kR) - (kR)\cos(kR)}{(kR)^3}$$

EPS Filter: sharp-k filter $\tilde{W}(k, R) = \theta(1/R - k)$

- Inconsistent with $S(R)$ filtering
- It leaves M undefined

$$V(R) = 12 \pi R^3 \int_0^\infty du \left[\frac{\sin u}{u} - \cos u \right]$$

Excursion Set with Sharp-x:

- Correlated random walks
- Numerical MC solutions

MC Solution

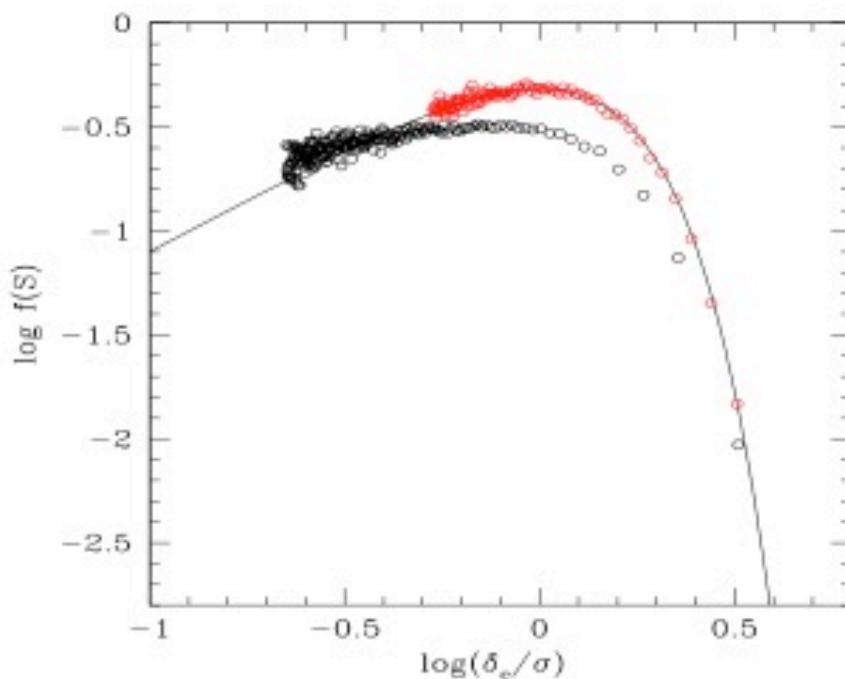
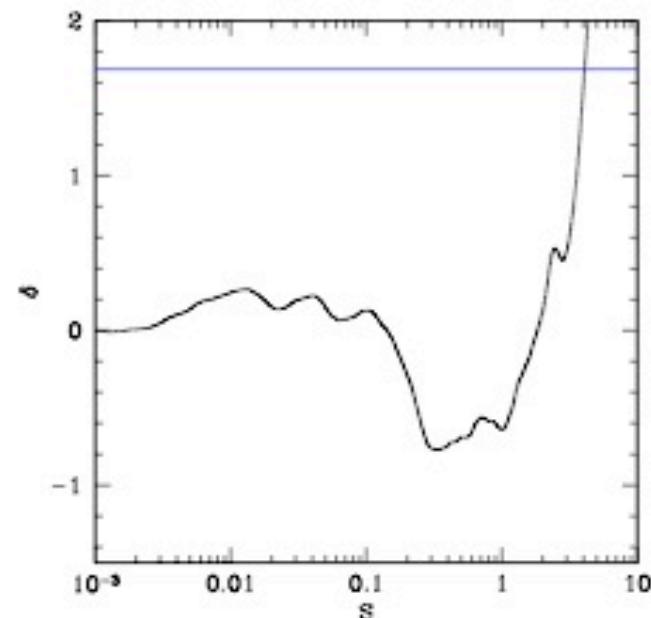
Langevin Equation:

$$\frac{\partial \delta}{\partial \ln k} = Q(\ln k) \tilde{W}(k, R)$$

$$\langle Q(\ln k) \rangle = 0$$

$$\langle Q(\ln k) Q(\ln k') \rangle = \Delta^2(k) \delta_D(\ln k - \ln k')$$

Spherical Collapse Case:



Correlation Function

Generic Filter:

$$\langle \delta[R(S)]\delta[R(S')] \rangle = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P(k) T^2(k) \tilde{W}[k, R(S)] \tilde{W}[k, R(S')]$$

Sharp-k Filter:

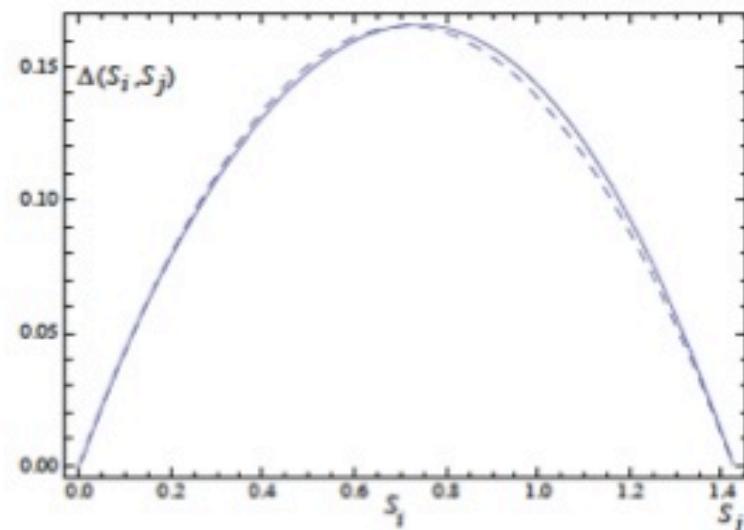
$$\langle \delta(S)\delta(S') \rangle = \int_0^S ds \int_0^{S'} ds' \langle \eta(s)\eta(s') \rangle = \min(S, S')$$

Introduce:

$$\Delta(S, S') = \langle \delta(S)\delta(S') \rangle - \min(S, S')$$

For LCDM power-spectrum:

$$\Delta(S, S') \cong \kappa \frac{S(S' - S)}{S'} \quad \text{where } \kappa \approx 0.47$$



Maggiore & Riotto, ApJ, 711, 907 (2010)

Path-Integral Approach to Excursion Set

Maggiore & Riotto, ApJ, 711, 907 (2010)

Discrete Random Walks

- Trajectory over a discrete “time” interval $\{Y_0, Y_1, \dots, Y_n\}$ with $S_k = k \epsilon$ and $k=1,..,n$

Ensemble Probability Density

$$p(Y_0, \dots, Y_n, S_n) = \left\langle \delta_D[Y(S_1) - Y_1] \cdot \dots \cdot \delta_D[Y(S_n) - Y_n] \right\rangle = \int D\lambda e^{i \sum \lambda_i Y_i} \left\langle e^{-i \sum \lambda_i Y(S_i)} \right\rangle$$

Partition function

$$e^Z = \left\langle e^{-i \sum \lambda_i Y(S_i)} \right\rangle \quad \text{with} \quad Z = \sum_{p=1}^{\infty} \frac{(-i)^p}{p!} \sum_{i_1=1}^n \dots \sum_{i_p=1}^n \lambda_{i_1} \dots \lambda_{i_p} \left\langle Y(S_{i_1}) \dots Y(S_{i_p}) \right\rangle_c$$

Probability Distribution

$$\Pi_e(Y_0, Y_n, S_n) = \int_0^{\infty} dY_1 \dots \int_0^{\infty} dY_{n-1} p(Y_0, \dots, Y_n, S_n)$$

κ -expansion around Markovian solution

Probability Distribution

$$\frac{dF}{dS} = -\frac{\partial}{\partial S} \int_0^\infty \Pi_{RW}(Y, Y_0, S) d\delta$$

$$\Pi_\varepsilon(Y_0, Y_n, S_n) = \int_0^\infty dY_1 \dots \int_0^\infty dY_{n-1} p(Y_0, \dots, Y_n, S_n)$$

Expansion to $O(\kappa)$

$$\Pi_\varepsilon(Y_0, Y_n, S_n) = \int_0^\infty dY_1 \dots \int_0^\infty dY_{n-1} \int D\lambda \left(1 - \frac{1}{2} \sum_{i,j} \lambda_i \lambda_j \Delta_{ij} \right) e^{i \sum_k \lambda_k [Y_k - \bar{B}_k]} e^{-i \sum_{n,m} \lambda_n \lambda_m A_{nm}}$$

Markovian solution and non-Markovian corrections

$$\Pi_\varepsilon(Y_0, Y_n, S_n) = \Pi_\varepsilon^M(Y_0, Y_n, S_n) + \Pi_\varepsilon^{\kappa^{(1)}}(Y_0, Y_n, S_n)$$

Memory and Memory-of-Memory Terms

First Order Corrections:

$$\Pi_{\varepsilon}^{\kappa^{(1)}}(Y_0, Y_n, S_n) = \Pi_{\varepsilon}^m(Y_0, Y_n, S_n) + \Pi_{\varepsilon}^{m-m}(Y_0, Y_n, S_n)$$

Memory:

$$\Pi_{\varepsilon}^m(Y_0, Y_n, S_n) = - \sum_{i=1}^{n-1} \Delta_{in} \partial_n \left[\Pi_{\varepsilon}^{M,f}(Y_0, 0, S_i) \Pi_{\varepsilon}^{M,f}(0, Y_n, S_n - S_i) \right]$$

Memory-of-Memory:

$$\Pi_{\varepsilon}^{m-m}(Y_0, Y_n, S_n) = \sum_{i < j} \Delta_{ij} \left[\Pi_{\varepsilon}^{M,f}(Y_0, 0, S_i) \Pi_{\varepsilon}^{M,f}(0, 0, S_j - S_i) \Pi_{\varepsilon}^{M,f}(0, Y_n, S_n - S_j) \right]$$

- Markovian solution around the barrier $\Pi_{\varepsilon}^{M,f}$

Continuous Limit: $\sum_{i=1}^{n-1} \rightarrow \lim \frac{1}{\varepsilon} \int_0^S dS_i$ & $\sum_{i < j} \rightarrow \lim \frac{1}{\varepsilon^2} \int_0^S dS_i \int_{S_i}^S dS_j$

Excursion Set of Drifting Diffusive Barrier

PSC & Achitouv, PRL, 106, 241302 (2011)

PSC & Achitouv, PRD, 84, 023009 (2011)

Connected $\langle Y(S_i) \rangle_c \equiv \bar{B}(S_i) = \delta_c + \beta S_i$

Correlators $\langle Y(S_i)Y(S_j) \rangle = (1 + D_B) \min(S_i, S_j) + \Delta(S_i, S_j)$

Markovian Density: $p_0(Y_0, \dots, Y_n, S_n) = \int D\lambda e^{i \sum_k \lambda_k [Y_k - \bar{B}_k]} e^{-i \sum_{n,m} \lambda_n \lambda_m A_{nm}}$

$$p_0(Y_0, \dots, Y_n, S_n) = \psi_\varepsilon(\Delta Y) p_0(Y_0, \dots, Y_{n-1}, S_{n-1}) \quad \psi_\varepsilon(\Delta Y) = \frac{1}{\sqrt{2\pi\varepsilon(1+D_B)}} e^{-\frac{(\Delta Y - \beta\varepsilon)^2}{2\varepsilon(1+D_B)}}$$

**Chapman-Kolmogorov
Equation:**

$$\Pi_\varepsilon^M(Y_0, Y_n, S_n) = \int_0^\infty dY_{n-1} \psi_\varepsilon(\Delta Y) \Pi_\varepsilon^M(Y_0, Y_{n-1}, S_{n-1})$$

- In the continuous limit and developing RHS in $S + \varepsilon$ and LHS in $Y - \Delta Y$ we recover:

$$\frac{\partial \Pi_{\varepsilon=0}^M}{\partial S} = -\beta \frac{\partial \Pi_{\varepsilon=0}^M}{\partial Y} + \frac{1+D_B}{2} \frac{\partial^2 \Pi_{\varepsilon=0}^M}{\partial Y^2}$$

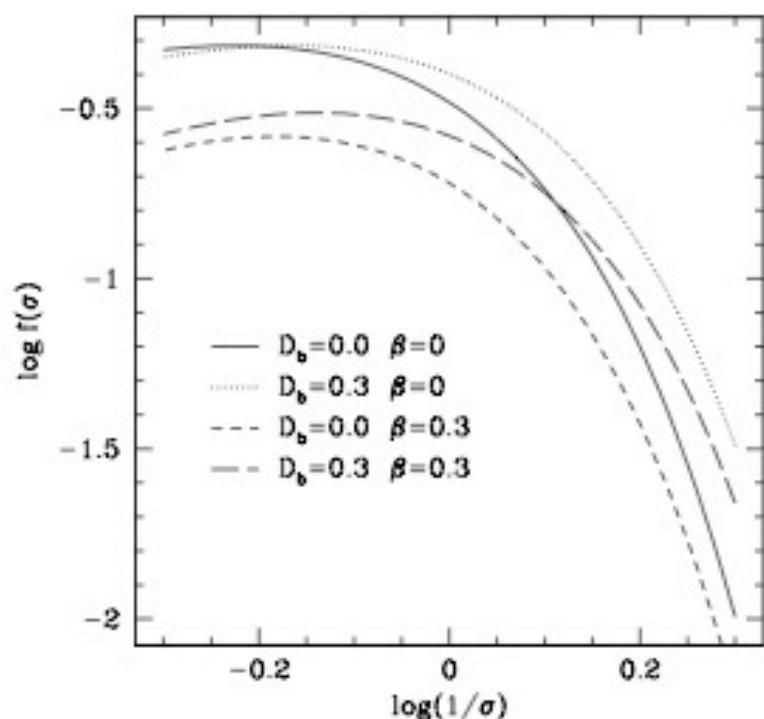
Markovian Contribution

$$\frac{\partial \Pi}{\partial S} = -\beta \frac{\partial \Pi}{\partial Y} + \frac{1+D_B}{2} \frac{\partial^2 \Pi}{\partial Y^2} \quad \begin{aligned} \Pi(Y, 0) &= \delta_D(Y - \delta_c) \\ \Pi(0, S) &= 0 \end{aligned}$$

$$\Pi(Y, S) = \frac{e^{\frac{\beta}{1+D_B}(Y-Y_0-\beta S/2)}}{\sqrt{2\pi S(1+D_B)}} \left[e^{-\frac{(Y-Y_0)^2}{2S(1+D_B)}} - e^{-\frac{(Y+Y_0)^2}{2S(1+D_B)}} \right]$$

Multiplicity Function O($\kappa=0$):

$$f_0(\sigma) = \sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma \sqrt{1+D_B}} e^{-\frac{(\delta_c+\beta \sigma^2)^2}{2\sigma^2(1+D_B)}}$$



non-Markovian Terms

Memory

$$f_1^m(\sigma) = -2\sigma^2 \frac{\kappa Y_0}{(1+D_B)^2} \frac{\partial}{\partial S} \int_0^\infty dY_n \partial_n \left\{ Y_n e^{\frac{-\beta}{1+D_B}(Y-Y_0-\beta S/2)} \operatorname{Erfc}\left[\frac{Y_0+Y_n}{\sqrt{2S(1+D_B)}}\right] \right\} = 0$$

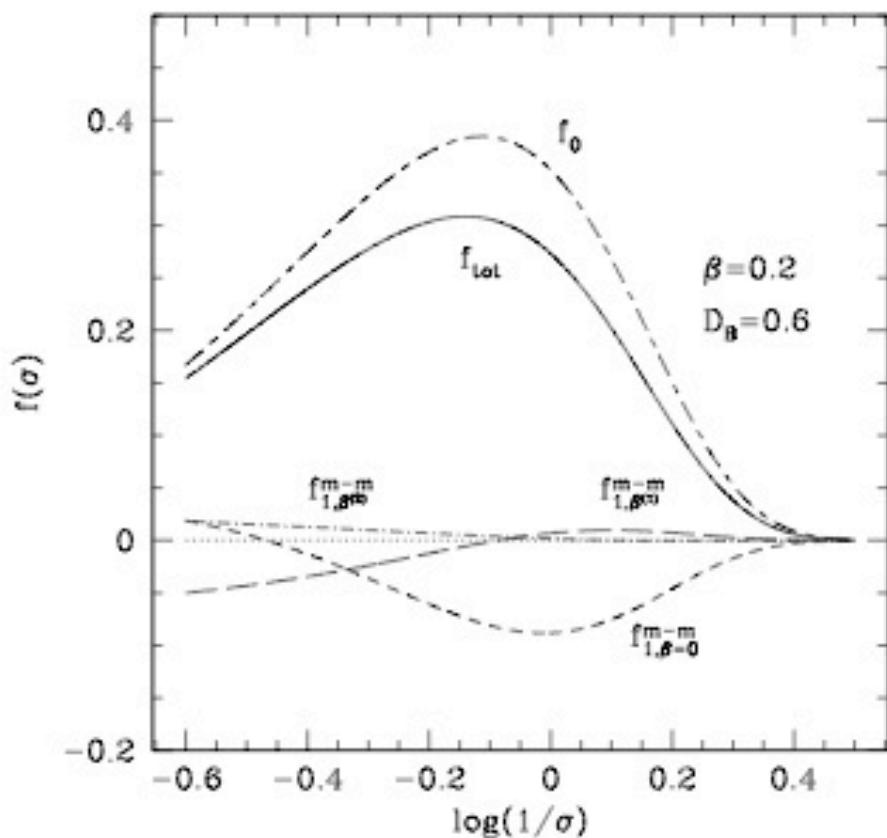
Memory-of-Memory

$$a = \frac{1}{1+D_B}$$

$$f_{1,\beta=0}^{m-m}(\sigma) = -\kappa \frac{a\delta_c}{\sigma} \sqrt{\frac{2a}{\pi}} \left[e^{-\frac{a\delta_c^2}{2\sigma^2}} - \frac{1}{2} \Gamma\left(0, \frac{a\delta_c^2}{2\sigma^2}\right) \right]$$

$$f_{1,\beta^{(1)}}^{m-m}(\sigma) = -a\delta_c \beta \left[a\kappa \operatorname{Erfc}\left(\delta_c \sqrt{\frac{a}{2\sigma^2}}\right) + f_{1,\beta=0}^{m-m}(\sigma) \right]$$

$$f_{1,\beta^{(2)}}^{m-m}(\sigma) = -a\beta \left[\frac{\beta}{2} \sigma^2 f_{1,\beta=0}^{m-m}(\sigma) + \delta_c f_{1,\beta^{(1)}}^{m-m}(\sigma) \right]$$



non-Markovian Terms

Memory

$$f_1^m(\sigma) = -2\sigma^2 \frac{\kappa Y_0}{(1+D_B)^2} \frac{\partial}{\partial S} \int_0^\infty dY_n \partial_n \left\{ Y_n e^{\frac{-\beta}{1+D_B}(Y-Y_0-\beta S/2)} \operatorname{Erfc}\left[\frac{Y_0+Y_n}{\sqrt{2S(1+D_B)}}\right] \right\} = 0$$

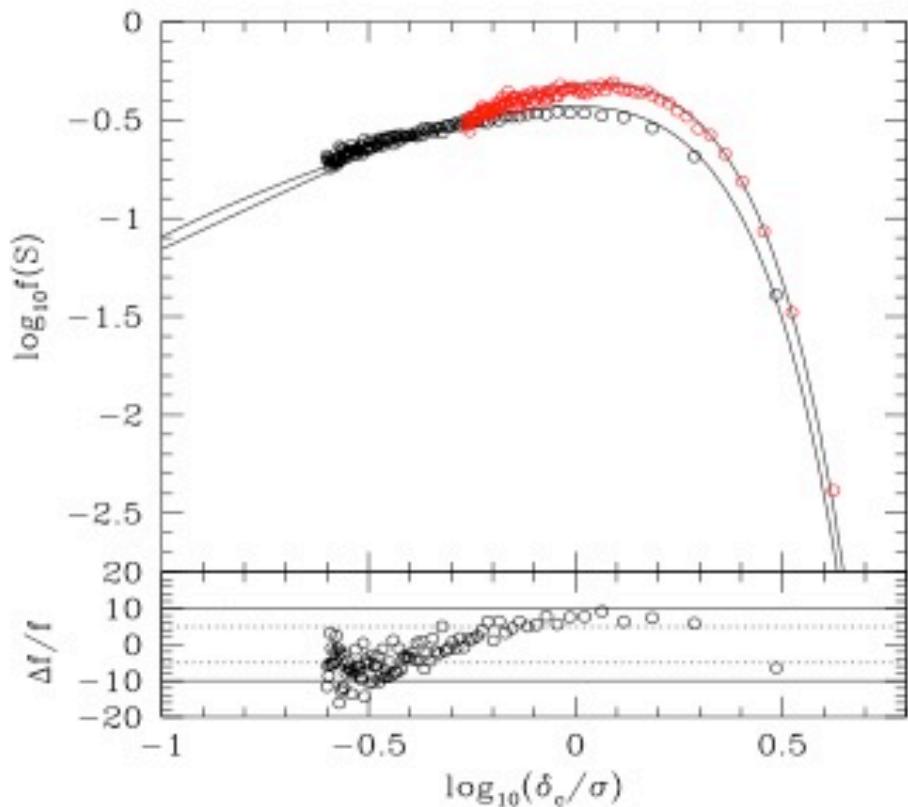
Memory-of-Memory

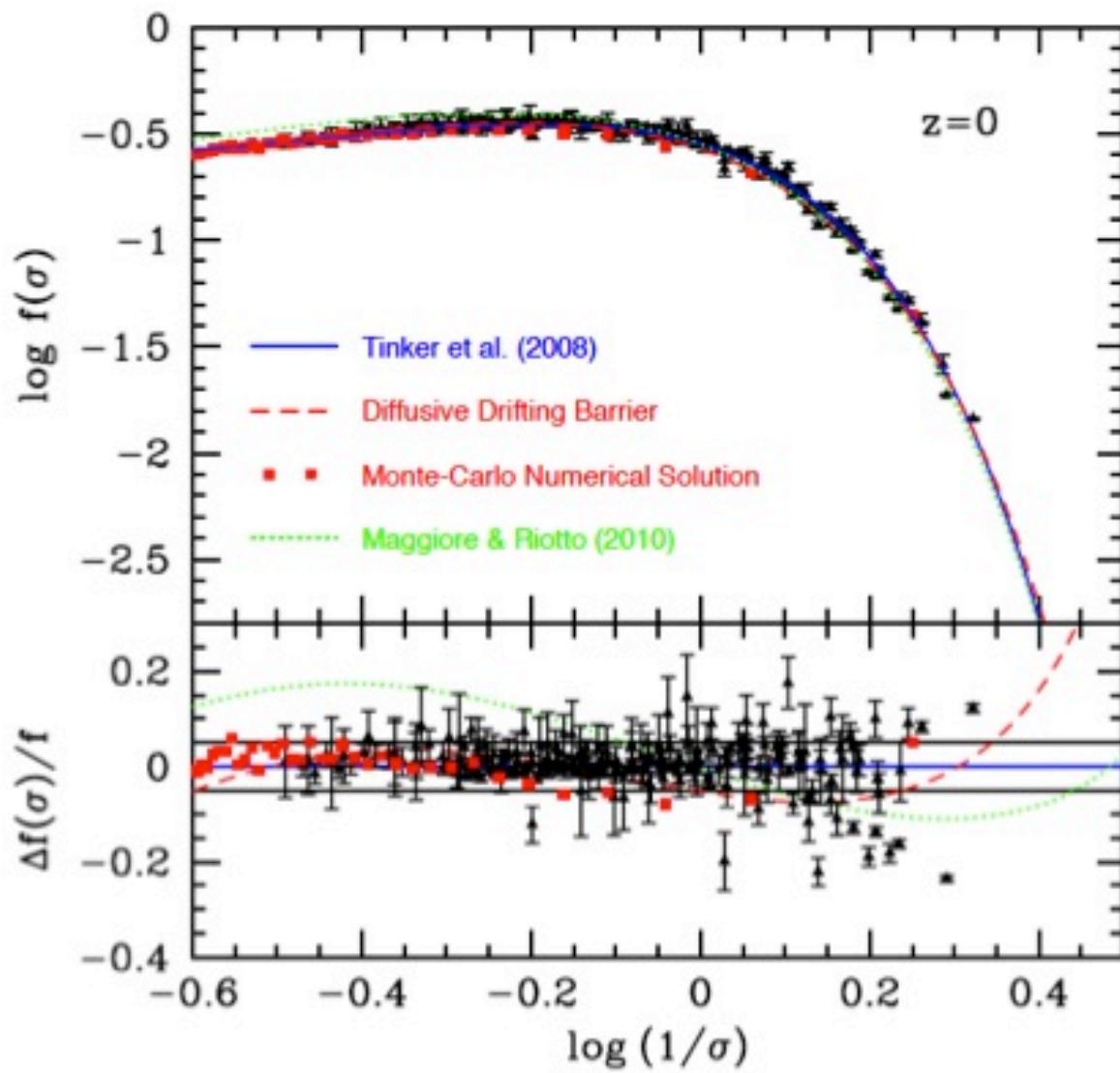
$$a = \frac{1}{1+D_B}$$

$$f_{1,\beta=0}^{m-m}(\sigma) = -\kappa \frac{a\delta_c}{\sigma} \sqrt{\frac{2a}{\pi}} \left[e^{-\frac{a\delta_c^2}{2\sigma^2}} - \frac{1}{2} \Gamma\left(0, \frac{a\delta_c^2}{2\sigma^2}\right) \right]$$

$$f_{1,\beta^{(1)}}^{m-m}(\sigma) = -a\delta_c \beta \left[a\kappa \operatorname{Erfc}\left(\delta_c \sqrt{\frac{a}{2\sigma^2}}\right) + f_{1,\beta=0}^{m-m}(\sigma) \right]$$

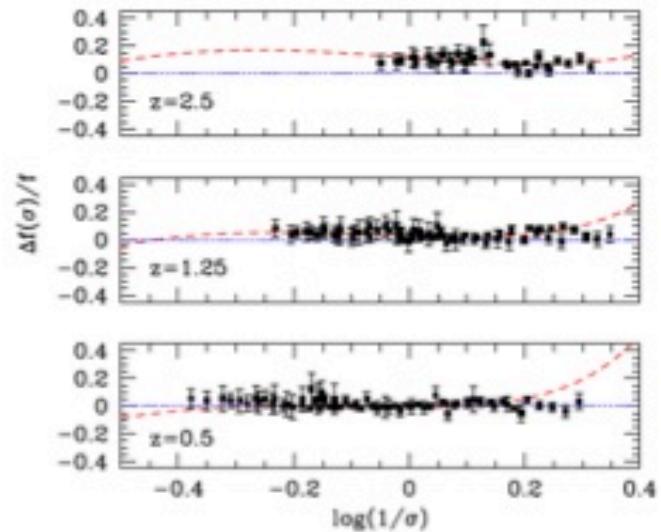
$$f_{1,\beta^{(2)}}^{m-m}(\sigma) = -a\beta \left[\frac{\beta}{2} \sigma^2 f_{1,\beta=0}^{m-m}(\sigma) + \delta_c f_{1,\beta^{(1)}}^{m-m}(\sigma) \right]$$





$$\beta_0 = 0.06$$

$$D_B^0 = 0.3$$



- Good functional fit
(only 2 physical parameters)
- Does the barrier have physical meaning?

Self-Consistency of Excursion Set

Achitouv, Rasera, Sheth, PSC, PRL, 111, 231303 (2013)

Distribution Collapse Threshold

$$g_2 = B / \sqrt{D_B} \quad g_1 = \delta$$

$$g_+ = (g_1 - g_2 \sqrt{D_B}) / \sqrt{1 + D_B}$$

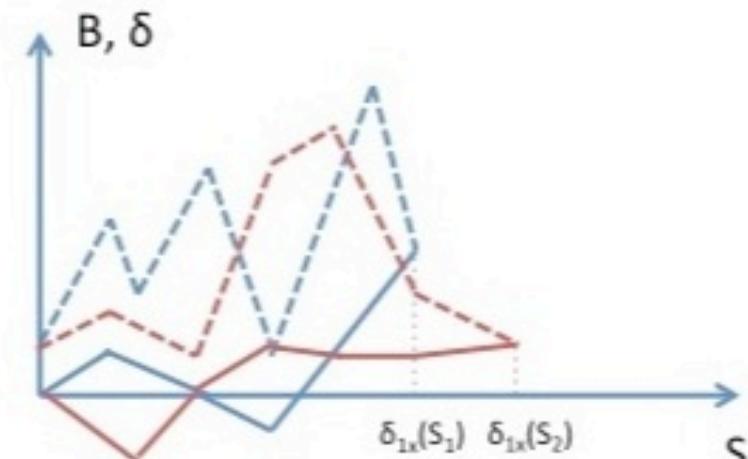
$$g_- = (g_1 \sqrt{D_B} + g_2) / \sqrt{1 + D_B}$$

$$\langle g_+ \rangle = \langle g_- \rangle = 0$$

$$\langle g_+^2 \rangle = \langle g_-^2 \rangle = S \quad \langle g_+ g_- \rangle = 0$$



$$\delta_{1x} = \bar{B}(S) + B = \frac{\bar{B}(S)}{1 + D_B} + g_- \sqrt{\frac{D_B}{1 + D_B}}$$



$$\Pi(\delta_{1x}, S) = \frac{1}{\sqrt{2\pi S} \frac{D_B}{1 + D_B}} \exp \left[-\frac{\left(\delta_{1x} - \frac{\bar{B}(S)}{1 + D_B} \right)^2}{\frac{2SD_B}{1 + D_B}} \right]$$

Self-Consistency of Excursion Set

Achitouv, Rasera, Sheth, PSC, PRL, 111, 231303 (2013)

Distribution Collapse Threshold

$$g_2 = B / \sqrt{D_B} \quad g_1 = \delta$$

$$g_+ = (g_1 - g_2 \sqrt{D_B}) / \sqrt{1 + D_B}$$

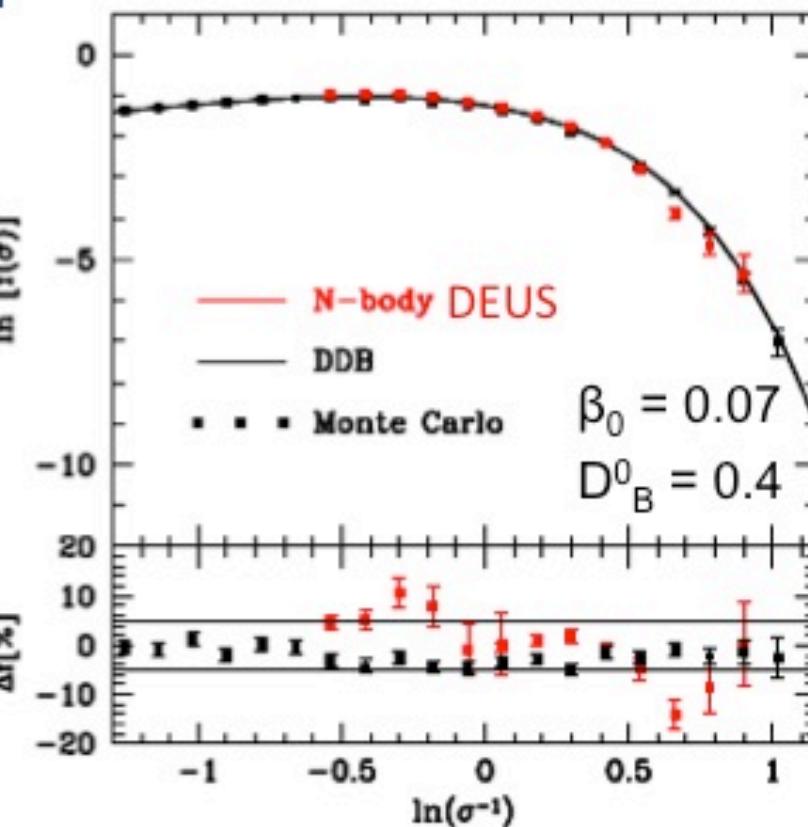
$$g_- = (g_1 \sqrt{D_B} + g_2) / \sqrt{1 + D_B}$$

$$\langle g_+ \rangle = \langle g_- \rangle = 0$$

$$\langle g_+^2 \rangle = \langle g_-^2 \rangle = S \quad \langle g_+ g_- \rangle = 0$$



$$\delta_{1x} = \bar{B}(S) + B = \frac{\bar{B}(S)}{1 + D_B} + g_- \sqrt{\frac{D_B}{1 + D_B}}$$



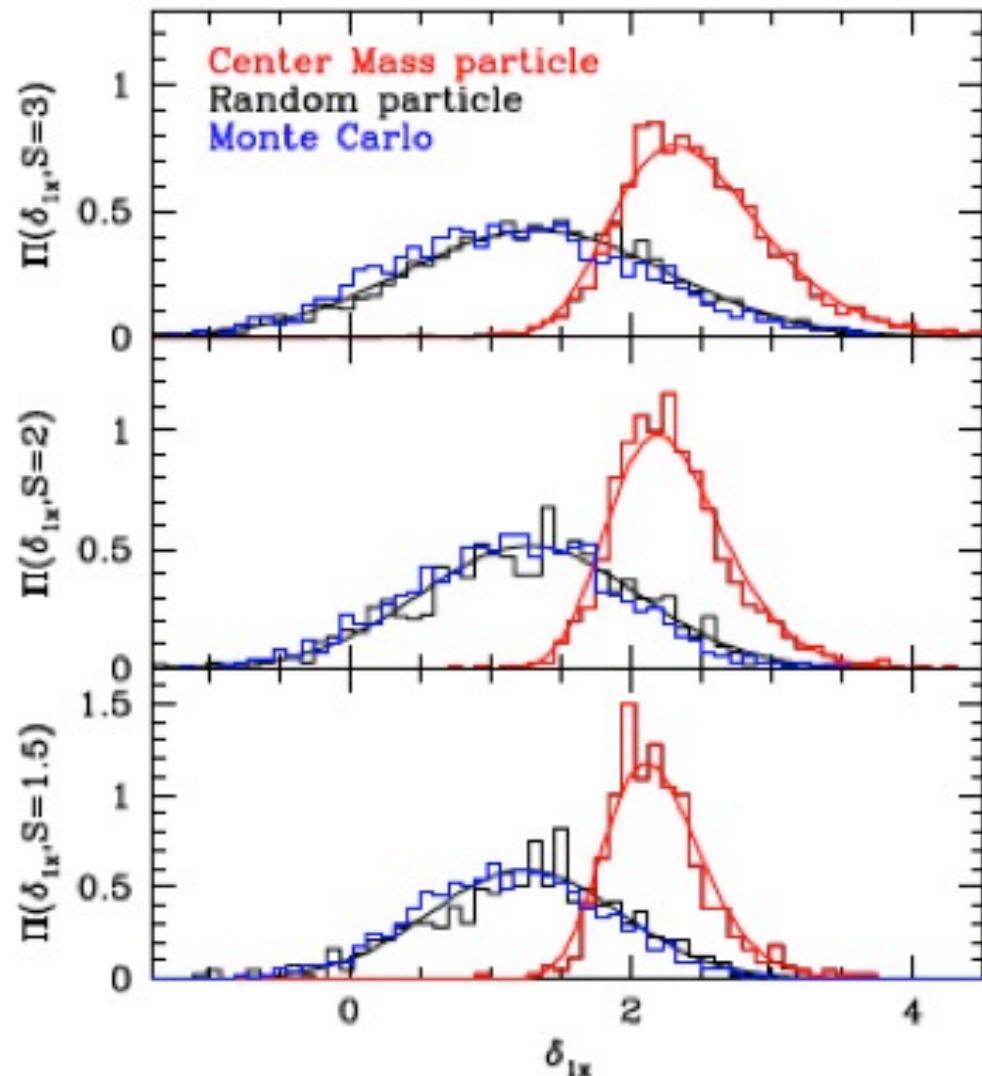
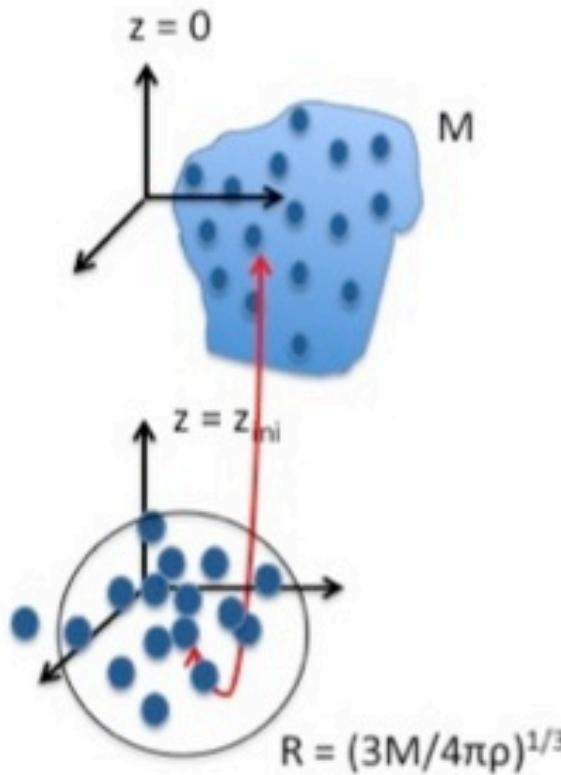
$$\Pi(\delta_{1x}, S) = \frac{1}{\sqrt{2\pi S} \frac{D_B}{1 + D_B}} \exp \left[-\frac{\left(\delta_{1x} - \frac{\bar{B}(S)}{1 + D_B} \right)^2}{\frac{2SD_B}{1 + D_B}} \right]$$

DDB & N-body Halos

Achitouv, Rasera, Sheth, PSC, PRL, 111, 231303 (2013)

Testing the EST barrier

- Random particle in the halo



- Random particle in the initial field (consistent with EST)

Conclusions

- DM Halo mass function is crucial in modern cosmology, physical understanding is needed
- Excursion Set is a self-consistent framework for a theoretical modeling of MF
- Path-Integral Methods allows analytical computation of generic filter corrections and consistent comparison with N-body results
- Focus on physical models of halo collapse and cosmology dependence