Dark Energy Interactions
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Theory and phenomenology of a non-minimally coupled fluid

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Refs:
DB, Liberati, Sindoni, 2011,
DB, Pettorino, Liberati, Baccigalupi, 2012,
DB, Colombo, Liberati, 2014
DB, Liberati, in preparation

Motivations

Why fluids?

- Good description of dynamics in cosmology
- Parametrization of microscopic degrees of freedom

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<u>Flat space-time</u>

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$\eta_{\mu\nu}$	No scale

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Example: Bose-Einstein Condensate [DB, Colombo, Liberati, 2014]

Non-minimally coupled fluid

[DB, Liberati, Sindoni, 2011]

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g}R[1+\alpha_s\zeta(\rho/\rho_*)]+$$
$$+\frac{\alpha_R c^3}{16\pi G}\int d^4x\sqrt{-g}\xi(\rho/\rho_*)R_{\mu\nu}u^\mu u^\nu + S_{fluid}[g,m]$$

[Brown, 1993]

$$S_{fluid} = \int d^4x\left[-\sqrt{-g}\rho(n,s)+J^\alpha L_\alpha\right]$$

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What should we expect

- Not a scalar-tensor theory: $\rho(|J|/\sqrt{-g}, s)$
- Higher derivatives of fluid variables
- Modified TD properties: $f \neq \mu - sT$ $\qquad \dfrac{\partial\rho}{\partial s} \neq nT$

Cosmological background evolution

[DB, Liberati, Pettorino, work in progress]

Conformally coupled fluid

$$\dot{\rho} + 3H(\rho + p) = 0$$

$$3H^2 \left[M_{Pl}^2 + \alpha_S \left(\zeta - 3(\rho + p)\zeta' \right) \right] = \rho$$

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Disformally coupled fluid

$$\dot{\rho} + 3H(\rho + p) = 0$$

$$\langle R \rangle \equiv R_{\mu\nu} u^\mu u^\nu$$

$$3H^2 \left[M_{Pl}^2 + \frac{3}{2}\alpha_R \xi' (\rho + p)\right] = \rho + \frac{\alpha_R}{2}\xi \langle R \rangle$$

$$\langle R \rangle \left[M_{Pl}^2 + \frac{\alpha_R}{4}\left(4\xi + 3\xi'(\rho + p)\right)\right] = \frac{\rho}{2} - \frac{3}{4}\alpha_R \left[\ddot{\xi} - H\dot{\xi}\right]$$

Newtonian limit

[DB, Liberati, Sindoni, 2011]

- Consider small perturbation around flat space-time $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$

Modified Poisson equation

$$\nabla^2 \phi_N = 4\pi G_N \left(\rho - \frac{\alpha_R}{2} \nabla^2 \xi - \frac{\alpha_S}{2} \nabla^2 \zeta \right)$$

- For flat fluid distributions or other fluids dominance effects are negligible

More degrees of freedom

$$-\frac{1}{2} \nabla^2 \gamma_{ij} = \frac{8\pi G_N}{c^2} \left[\alpha_S \left(\frac{1}{2} \eta_{ij} \nabla^2 \zeta + \partial_i \partial_j \zeta \right) - \frac{\alpha_R}{2} \eta_{ij} \nabla^2 \xi \right]$$

- Deviation from GR. Constraints from $1 - \psi/\phi \ll 1$

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More degrees of freedom

- Spherical symmetry

$$\nabla_r^2 \psi = \frac{8\pi G_N}{c^2} \left[\frac{\alpha_S}{2} \nabla_r^2 \zeta - \frac{\alpha_R}{2} \nabla_r^2 \xi + \alpha_S \partial_r^2 \zeta \right]$$

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Fluid evolution

[DB, Pettorino, Liberati, Baccigalupi, 2012]
[DB, Liberati, in prep., 2014]

Conformally coupled fluid

$$\dot{\rho} + \theta(\rho + p) = 0$$

$$\dot{u}^\sigma = -\frac{1}{\rho + p} H^{\sigma\nu}\nabla_\nu p + \underline{H^{\sigma\nu}\nabla_\nu \ln(1 - \alpha_C \zeta' R)}$$

Effective pressure —

Mixing terms —

Higher derivatives —

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$$\dot{u}^\sigma \left[\rho - \alpha_R \left(\frac{1}{2}\xi'\rho - \xi\right)\langle R \rangle\right] = -\alpha_R h^{\sigma\beta} \underline{\left\{R_{\beta\alpha} u^\alpha \left(\dot{\xi} + \theta\xi\right) + R_{\beta\alpha}\dot{u}^\alpha\right\}}$$

$$+\alpha_R \{\xi - \rho\xi'\} u^\alpha R_{\alpha\gamma} h^{\sigma\beta} \nabla_\beta u^\gamma$$

$$+\alpha_R u^\alpha u^\gamma h^{\sigma\beta} \underline{\left\{\xi \nabla_\alpha R_{\beta\gamma} - \frac{1}{2}\rho\xi' \nabla_\beta R_{\alpha\gamma}\right\}}$$

$$\underline{+\frac{1}{2}\alpha_R \{\rho\xi' \langle R \rangle\} h^{\sigma\beta} \nabla_\beta \xi'}$$

Effective pressure —

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Conclusions

NMC fluids provides interesting alternative description of dark sector

What we know

- Generalized EOMs from a variational principle
- Extra force acting on NMC fluid
- Equations are higher order in fluid variables
- Modified Planck mass
- Newtonian limit: gradient correction to Poisson and $\phi \neq \psi$

Open questions/issues

- Generically can have stability issues: add counter terms a-la Horndeski or EFT approach. (To appear soon!)
- What about thermodynamic properties of the NMC fluid?
- Screening?
- Investigate viable cosmological solutions

Thanks!

BACKUP SLIDES

Einstein equations

Tensor equations

$$\left(M_{Pl}^2 + \alpha_S \zeta\right) G_{\mu\nu} = T_{\mu\nu} + \alpha_S \left[-g_{\mu\nu} \Box \zeta + \nabla_\mu \nabla_\nu \zeta - \frac{R}{2} (\rho + p) \zeta' H_{\mu\nu} \right]$$

$$+ \frac{\alpha_R}{2} \left[\langle R \rangle \left(\xi - (\rho + p) \xi' \right) H_{\mu\nu} + \langle R \rangle \xi u_\mu u_\nu \right.$$

$$\left. - \Box t_{\mu\nu} + 2 \nabla_\sigma \nabla_{(\mu} t_{\nu)}{}^\sigma - g_{\mu\nu} \nabla_\rho \nabla_\sigma t^{\sigma\rho} \right]$$

Trace equation

$$-R (M_{Pl}^2 + \alpha_S \zeta) = -\rho + 3p - 3\alpha_S \left[\Box \zeta + \frac{1}{2} (\rho + p) \zeta' R \right]$$

$$- \frac{\alpha_R}{2} \left[2 \nabla_\alpha \nabla_\beta t^{\alpha\beta} + \Box t + \langle R \rangle \left(3 \rho \xi' - 2 \xi \right) \right]$$

Fluid action and equations

Action

$$S[g_{\mu\nu}, J^\mu, \varphi, \theta, s, \alpha^A, \beta_A] =$$

$$= \int d^4x \left\{ -\sqrt{-g}\rho(|J|/\sqrt{-g}) + J^\mu \left(\varphi_{,\mu} + s\theta_{,\mu} + \beta_A \alpha^A_{,\mu} \right) \right\}$$

Equations of motion

$$T^{\alpha\beta} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\alpha\beta}}$$

$$\frac{\delta S}{\delta J^\mu} : \quad \mu = f + sT \qquad \frac{\delta S}{\delta \theta} : \quad \nabla_\mu (snu^\mu) = 0$$

$$\frac{\delta S}{\delta \varphi} : \quad \nabla_\mu (nu^\mu) = 0 \qquad \frac{\delta S}{\delta s} : \quad \frac{\partial \rho}{\partial s} = nT$$