

Dark Energy Interactions  
NORDITA, October 2014

# Theory and phenomenology of a non-minimally coupled fluid

Dario Bettoni  
Technion

Refs:

- DB, Liberati, Sindoni, 2011,
- DB, Pettorino, Liberati, Baccigalupi, 2012,
- DB, Colombo, Liberati, 2014
- DB, Liberati, in preparation

## Why fluids?

- Good description of dynamics in cosmology
- Parametrization of microscopic degrees of freedom

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# Motivations

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Example: Bose-Einstein Condensate [DB, Colombo, Liberati, 2014]

# Non-minimally coupled fluid

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$$S = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} R [1 + \alpha_s \zeta(\rho/\rho_*)] + \\ + \frac{\alpha_R c^3}{16\pi G} \int d^4x \sqrt{-g} \xi(\rho/\rho_*) R_{\mu\nu} u^\mu u^\nu + S_{fluid}[g, m]$$

$$S_{fluid} = \int d^4x \left[ -\sqrt{-g} \rho(n, s) + J^\alpha L_\alpha \right]$$

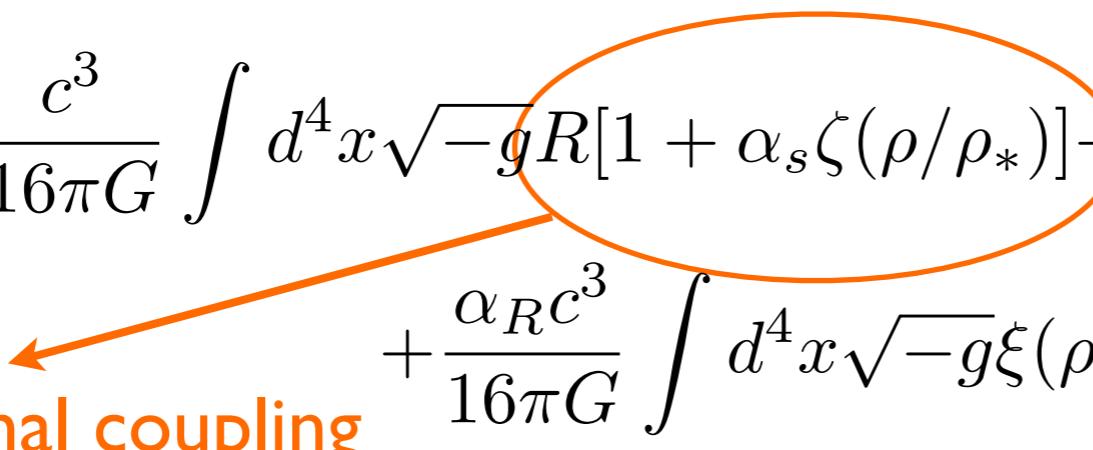
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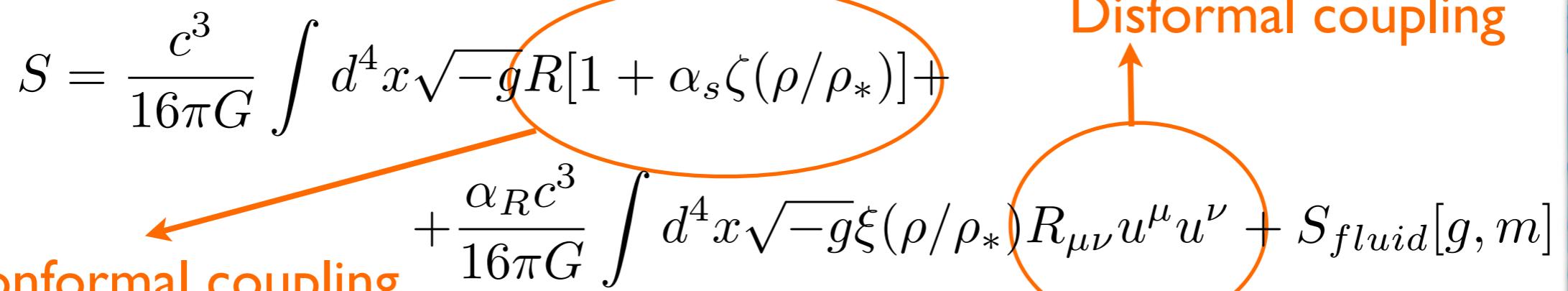
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The diagram shows the action  $S$  as a sum of two terms. The first term, involving  $\alpha_s$ , is highlighted with an orange oval and labeled 'Conformal coupling'. The second term, involving  $\alpha_R$ , is highlighted with another orange oval and labeled 'Disformal coupling' with an upward arrow.

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## What should we expect

- Not a scalar-tensor theory:  $\rho(|J|/\sqrt{-g}, s)$
- Higher derivatives of fluid variables
- Modified TD properties:  $f \neq \mu - sT$

$$\frac{\partial \rho}{\partial s} \neq nT$$

## Conformally coupled fluid

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$$\dot{\rho} + 3H(\rho + p) = 0$$

$$\langle R \rangle \equiv R_{\mu\nu} u^\mu u^\nu$$

$$3H^2 \left[ M_{Pl}^2 + \frac{3}{2}\alpha_R \xi'(\rho + p) \right] = \rho + \frac{\alpha_R}{2}\xi \langle R \rangle$$

$$\langle R \rangle \left[ M_{Pl}^2 + \frac{\alpha_R}{4} (4\xi + 3\xi'(\rho + p)) \right] = \frac{\rho}{2} - \frac{3}{4}\alpha_R [\ddot{\xi} - H\dot{\xi}]$$

- Consider small perturbation around flat space-time  $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$

## Modified Poisson equation

$$\nabla^2 \phi_N = 4\pi G_N \left( \rho - \frac{\alpha_R}{2} \nabla^2 \xi - \frac{\alpha_S}{2} \nabla^2 \zeta \right)$$

- For flat fluid distributions or other fluids dominance effects are negligible

## More degrees of freedom

$$-\frac{1}{2} \nabla^2 \gamma_{ij} = \frac{8\pi G_N}{c^2} \left[ \alpha_S \left( \frac{1}{2} \eta_{ij} \nabla^2 \zeta + \partial_i \partial_j \zeta \right) - \frac{\alpha_R}{2} \eta_{ij} \nabla^2 \xi \right]$$

- Deviation from GR. Constraints from  $1 - \psi/\phi \ll 1$

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## More degrees of freedom

- Spherical symmetry

$$\nabla_r^2 \psi = \frac{8\pi G_N}{c^2} \left[ \frac{\alpha_S}{2} \nabla_r^2 \zeta - \frac{\alpha_R}{2} \nabla_r^2 \xi + \alpha_S \partial_r^2 \zeta \right]$$

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## Conformally coupled fluid

$$\dot{\rho} + \theta(\rho + p) = 0$$

$$\dot{u}^\sigma = -\frac{1}{\rho + p} H^{\sigma\nu} \nabla_\nu p + \underline{H^{\sigma\nu} \nabla_\nu \ln(1 - \alpha_C \zeta' R)}$$

Effective pressure —

Mixing terms —

Higher derivatives —

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## Disformally coupled fluid

$$\dot{\rho} + \theta(\rho + p) = 0$$

$$\dot{u}^\sigma \left[ \rho - \alpha_R \left( \frac{1}{2} \xi' \rho - \xi \right) \langle R \rangle \right] = -\alpha_R h^{\sigma\beta} \underline{\left\{ R_{\beta\alpha} u^\alpha (\dot{\xi} + \theta\xi) + R_{\beta\alpha} \dot{u}^\alpha \right\}}$$

Effective pressure —

$$+ \alpha_R \{ \xi - \rho \xi' \} u^\alpha R_{\alpha\gamma} h^{\sigma\beta} \nabla_\beta u^\gamma$$

Mixing terms —

$$+ \alpha_R u^\alpha u^\gamma h^{\sigma\beta} \underline{\left\{ \xi \nabla_\alpha R_{\beta\gamma} - \frac{1}{2} \rho \xi' \nabla_\beta R_{\alpha\gamma} \right\}}$$

Higher derivatives —

$$+ \frac{1}{2} \alpha_R \{ \rho \xi' \langle R \rangle \} h^{\sigma\beta} \nabla_\beta \xi'$$

# Conclusions

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NMC fluids provides interesting alternative description of dark sector

## What we know

- Generalized EOMs from a variational principle
- Extra force acting on NMC fluid
- Equations are higher order in fluid variables
- Modified Planck mass
- Newtonian limit: gradient correction to Poisson and  $\phi \neq \psi$

## Open questions/issues

- Generically can have stability issues: add counter terms a-la Horndeski or EFT approach. ([To appear soon!](#))
- What about thermodynamic properties of the NMC fluid?
- Screening?
- Investigate viable cosmological solutions

Thanks!

# BACKUP SLIDES

# Einstein equations

## Tensor equations

$$\begin{aligned} (M_{Pl}^2 + \alpha_S \zeta) G_{\mu\nu} = & T_{\mu\nu} + \alpha_S \left[ -g_{\mu\nu} \square \zeta + \nabla_\mu \nabla_\nu \zeta - \frac{R}{2} (\rho + p) \zeta' H_{\mu\nu} \right] \\ & + \frac{\alpha_R}{2} [\langle R \rangle (\xi - (\rho + p) \xi') H_{\mu\nu} + \langle R \rangle \xi u_\mu u_\nu \\ & - \square t_{\mu\nu} + 2 \nabla_\sigma \nabla_{(\mu} t_{\nu)}{}^\sigma - g_{\mu\nu} \nabla_\rho \nabla_\sigma t^{\sigma\rho}] \end{aligned}$$

## Trace equation

$$\begin{aligned} -R(M_{Pl}^2 + \alpha_S \zeta) = & -\rho + 3p - 3\alpha_S \left[ \square \zeta + \frac{1}{2} (\rho + p) \zeta' R \right] \\ & - \frac{\alpha_R}{2} [2 \nabla_\alpha \nabla_\beta t^{\alpha\beta} + \square t + \langle R \rangle (3\rho \xi' - 2\xi)] \end{aligned}$$

# Fluid action and equations

## Action

$$S[g_{\mu\nu}, J^\mu, \varphi, \theta, s, \alpha^A, \beta_A] = \\ = \int d^4x \left\{ -\sqrt{-g} \rho(|J|/\sqrt{-g}) + J^\mu (\varphi_{,\mu} + s\theta_{,\mu} + \beta_A \alpha_{,\mu}^A) \right\}$$

## Equations of motion

$$T^{\alpha\beta} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\alpha\beta}}$$

$$\frac{\delta S}{\delta J^\mu} : \quad \mu = f + sT \quad \quad \quad \frac{\delta S}{\delta \theta} : \quad \nabla_\mu (sn u^\mu) = 0$$

$$\frac{\delta S}{\delta \varphi} : \quad \nabla_\mu (n u^\mu) = 0 \quad \quad \quad \frac{\delta S}{\delta s} : \quad \frac{\partial \rho}{\partial s} = nT$$