Disformal couplings and cosmological perturbations

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Work in preparation with Jack Morrice, and Tomi Koivisto and Nelson Nunes and Jose Mimoso

1. The setup

We consider extensions to Einstein's theory, in which the action is given, in the Einstein frame, by:

$$\mathcal{S} = \int \sqrt{-g} d^4x \left[\frac{M_{\mathrm{Pl}}^2}{2} \mathcal{R}(g) - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right] + \sum S_i[\tilde{g}^i_{\mu\nu}, \chi_i]$$

with the individual species coupling to the scalar via

$$\tilde{g}^i_{\mu\nu} = C_i(\phi) g_{\mu\nu} + D_i(\phi) \partial_\mu \phi \partial_\nu \phi \ .$$

The gravitational sector can be written in another frame, using a disformal transformation:

$$g_{\mu\nu} = \tilde{C}(\phi) g^{(i)}_{\mu\nu} + \tilde{D}(\phi)\partial_\mu\phi\partial_\nu\phi \ .$$

In this frame, the matter species i is decoupled from the scalar, all other species are (in general) disformally coupled. The gravitational sector looks complicated, but is of Horndeski-form.

Jordan (Horndeski) frame: $X = \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu}$

$$\mathcal{S}_{JF} = \int d^4x \sqrt{-\tilde{g}} \left[\tilde{K} + \tilde{G}_3 \Box\phi + \tilde{G}_4 R + \tilde{G}_{4,X} \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right] \right],$$

with

$$\tilde{K} = -\left(\tilde{X} + V\right) \sqrt{1 + 2\frac{D_m}{C_m} X} + \frac{3M_{\rm Pl}^2}{2} \frac{C_m' + 2C_m' D_m X - 2C_m' C_m - 4C_m' D_m X}{C_m (1 + 2X D_m/C_m)^{3/2}} X ,$$

$$\tilde{G}_3 = \frac{M_{\rm Pl}^2}{2} \frac{4C_m C_m' D_m + C_m D_m D_m' X + C_m D_m^2 X}{C_m^2 (1 + 2D_m X/C_m)^{3/2}} - \mathcal{I}$$

$$\tilde{G}_4 = \frac{M_{\rm Pl}^2}{2} C_m \left(1 + 2\frac{D_m}{C_m} X\right)^{1/2}$$

$$\mathcal{I} = \frac{M_{\rm Pl}^2}{2} \int dX \frac{4C_m C_m' D_m + C_m D_m D_m' X + C_m' D_m^2 X}{C_m^2 \left(1 + 2X D_m/C_m\right)^{3/2}}$$

vdB, Nunes (2014)

2. Coupled Quintessence with a twist

- Consider disformally coupled dark matter only.
- This simplifies a lot but we learn a great deal about the possible role of disformal terms in cosmology.
- Is well motivated from string theory, in which dark matter sits on a moving brane moving on an AdS throat.
- Extends coupled quintessence in an interesting way.

Wetterich 1995, Amendola 1999

Reminder: coupled quintessence

$$\mathcal{S} = \int \sqrt{-g} d^4x \left[\frac{M_{\rm Pl}^2}{2}\mathcal{R}(g) - \frac{1}{2} g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right] + \mathcal{S}_{\rm DM}[\tilde{g},\chi] + \mathcal{S}_{\rm b}[g,...] + ...,$$

$$\tilde{g}_{\mu\nu} = C(\phi) g_{\mu\nu}$$

plus some quintessential potential (exponential, inverse power law…). Here, we include disformal coupling:

$$\tilde{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi$$

Mota, Koivisto ,Zumalacarregui (2012)

Background equations of motion (conformal time):

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{dV}{d\phi} + Q = 0$$

$$\nabla_\mu T^\mu{}_\nu = Q\phi_{,\nu}$$

$$Q = \frac{C'}{2C}T^\mu{}_\mu - \nabla_\nu\left(\frac{D}{C}\phi_{,\mu}T^{\mu\nu}\right) + \frac{D'}{2C}\phi_{,\mu}\phi_{,\nu}T^{\mu\nu}$$

Background equations of motion (conformal time):

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2 \frac{dV}{d\phi} = a^2 Q$$

$$\dot{\rho}_{\rm dm} + 3\mathcal{H}\rho_{\rm dm} = -a^2 Q \dot{\phi}$$

$$a^2 Q = -\frac{\rho_{\rm dm}}{2(C + D(\rho_{\rm dm} - \dot{\phi}^2/a^2))}\left[a^2\frac{dC}{d\phi} - 2D\left(3\mathcal{H}\dot{\phi} + a^2\frac{dV}{d\phi} + \frac{d\log C}{d\phi}\dot{\phi}^2\right) + \frac{dD}{d\phi}\dot{\phi}^2\right]$$

Equations are very similar to standard coupled quintessence! Where are the differences in the physics?

Background evolution:

$$\mathcal{H}^2 = \frac{8\pi G}{3}\left(\rho_b + \rho_{\rm dm} + \rho_{\rm de}\right) a^2$$

Note that $\rho_{\rm dm} \neq a^{-3}$ due to coupling to scalar.

We can define effective (apparent/perceived) equation of state:

$$\mathcal{H}^2 = \frac{8\pi G}{3}\left(\rho_b + \rho_{\rm dm,0} a^{-3} + \rho_{\rm de,eff}\right) a^2$$

$$\rho_{\rm de,eff} = \rho_{\rm de} - \rho_{\rm dm,0} a^{-3} + \rho_{\rm dm}$$

Das, Khoury, Corasaniti (2006)

Background evolution:

$$\mathcal{H}^2 = \frac{8\pi G}{3}\left(\rho_b + \rho_{\rm dm,0}a^{-3} + \rho_{\rm de,eff}\right)a^2$$

$$\rho_{\rm de,eff} = \rho_{\rm de} - \rho_{\rm dm,0}a^{-3} + \rho_{\rm dm}$$

Define: $\dot{\rho}_{\rm de,eff} = -3\mathcal{H}\rho_{\rm de,eff}(1 + w_{\rm eff})$

Using
$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2\frac{dV}{d\phi} = a^2 Q$$
$$\dot{\rho}_{\rm dm} + 3\mathcal{H}\rho_{\rm dm} = -a^2 Q\dot{\phi}$$

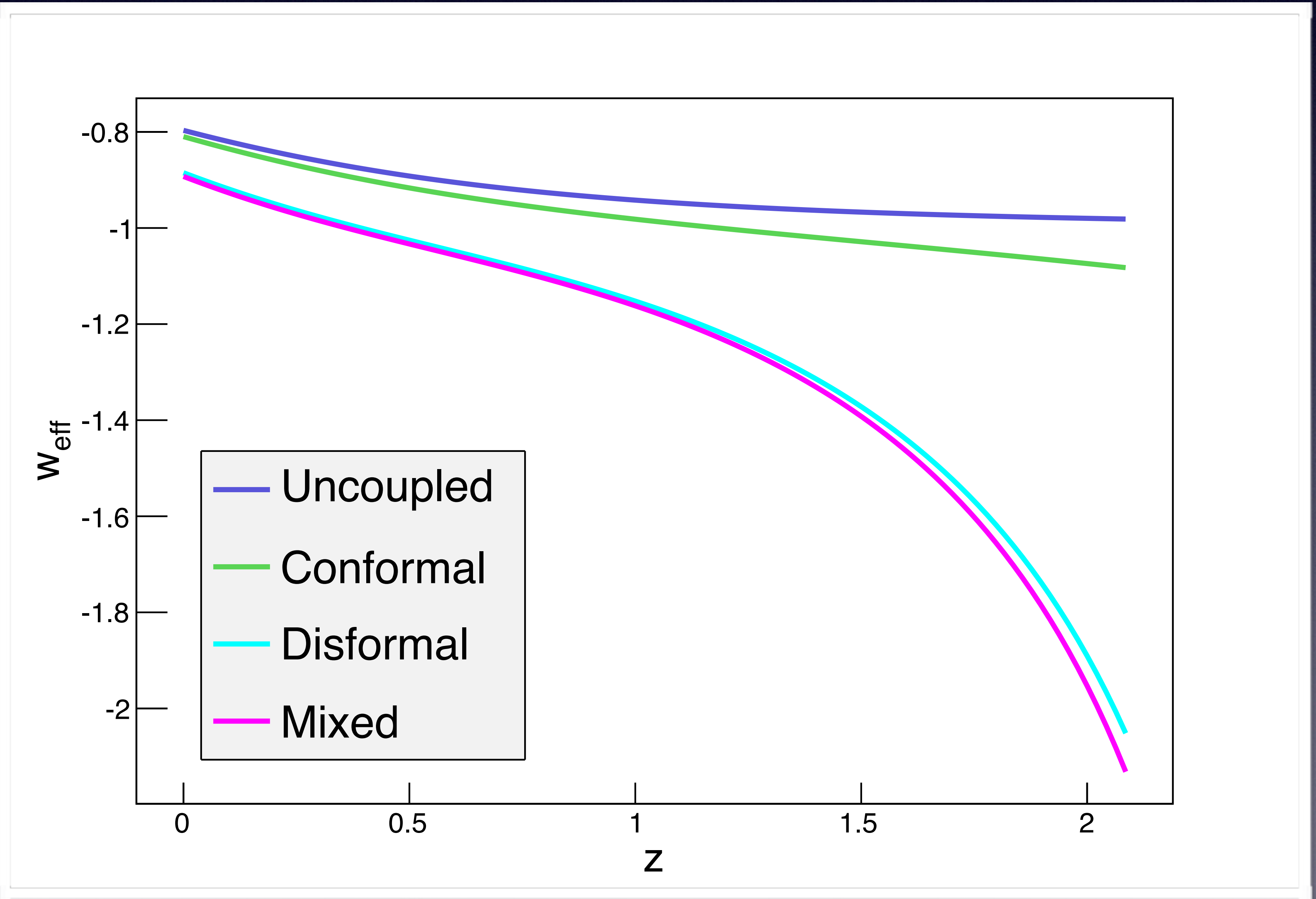
one finds $w_{\rm eff} = \dfrac{p_\phi}{\rho_{\rm de,eff}}$ $\quad p_\phi = \dfrac{1}{2}\dot{\phi}^2 - V(\phi)$

vdB, Morrice (2014)

Background evolution:

Exponential potential with $V = V_0 e^{-2\phi/M_{\rm Pl}}$

$C = e^{2\beta_c \phi}$ $\quad D = \dfrac{1}{M^4}$



Here:

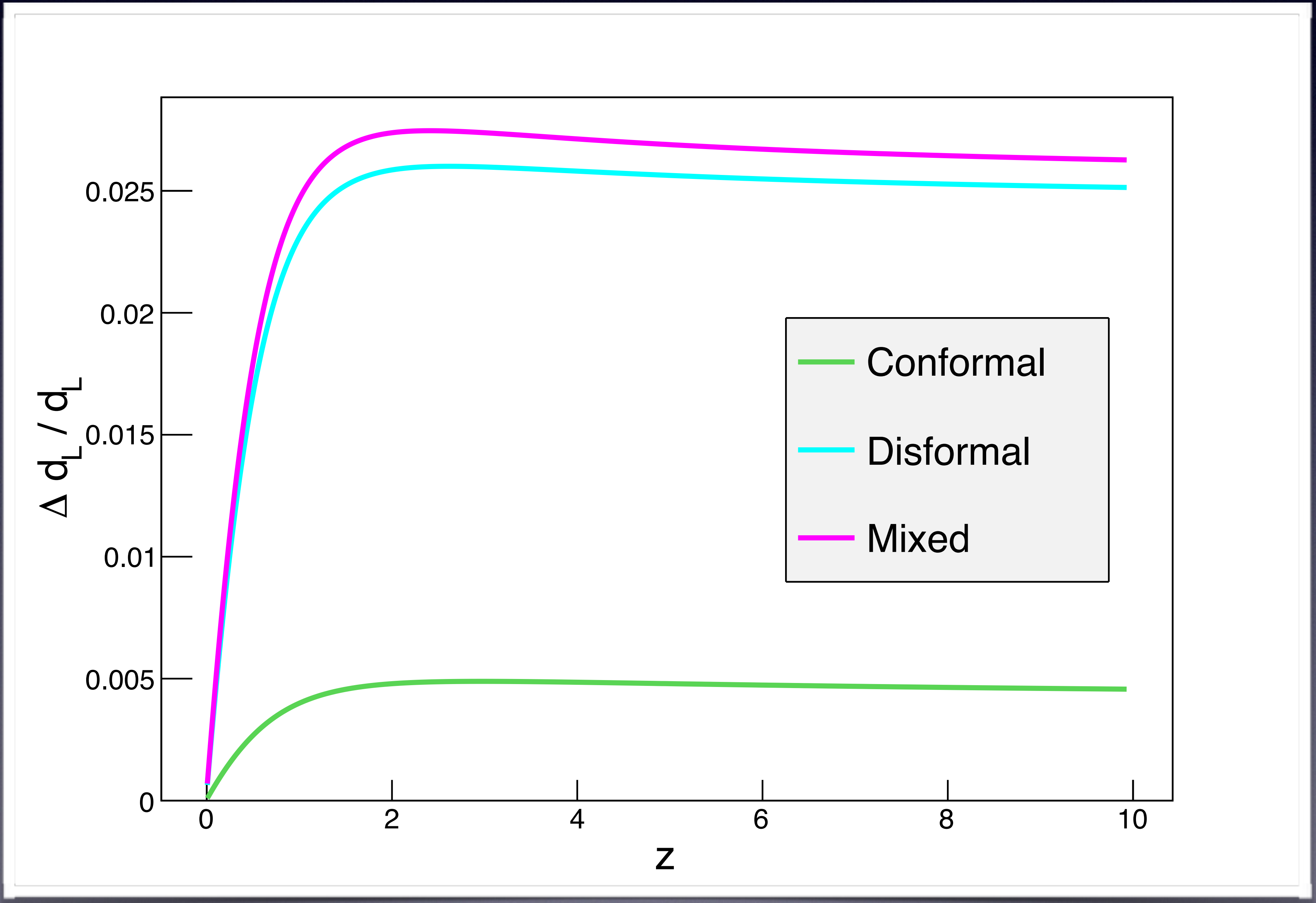
$$\beta_c = 0.05 \frac{1}{M_{\rm Pl}}$$

$$M = 1.8 \text{ meV}$$

vdB, Morrice (2014)

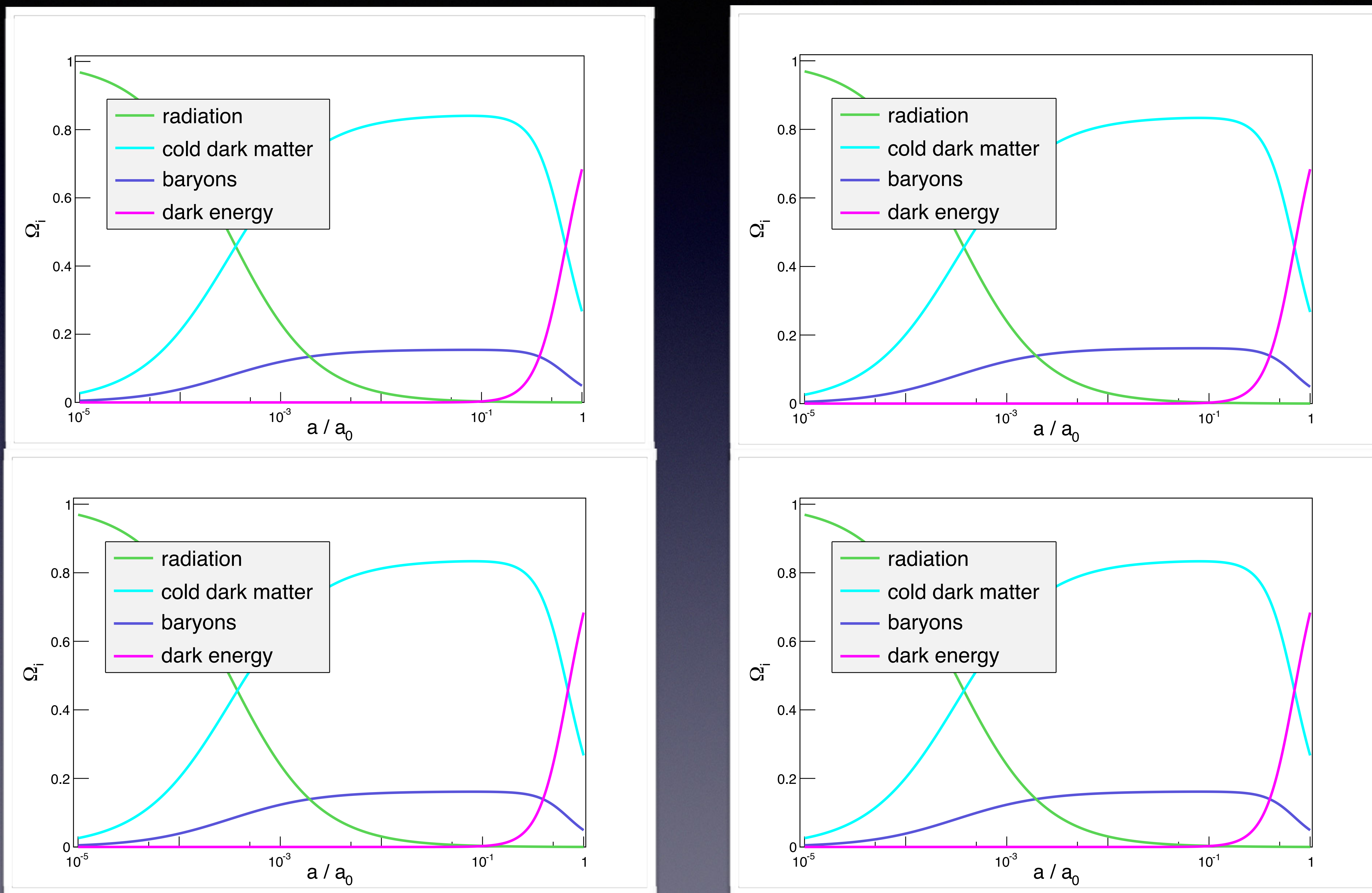
Background evolution:

Luminosity distance: relative change to uncoupled case:



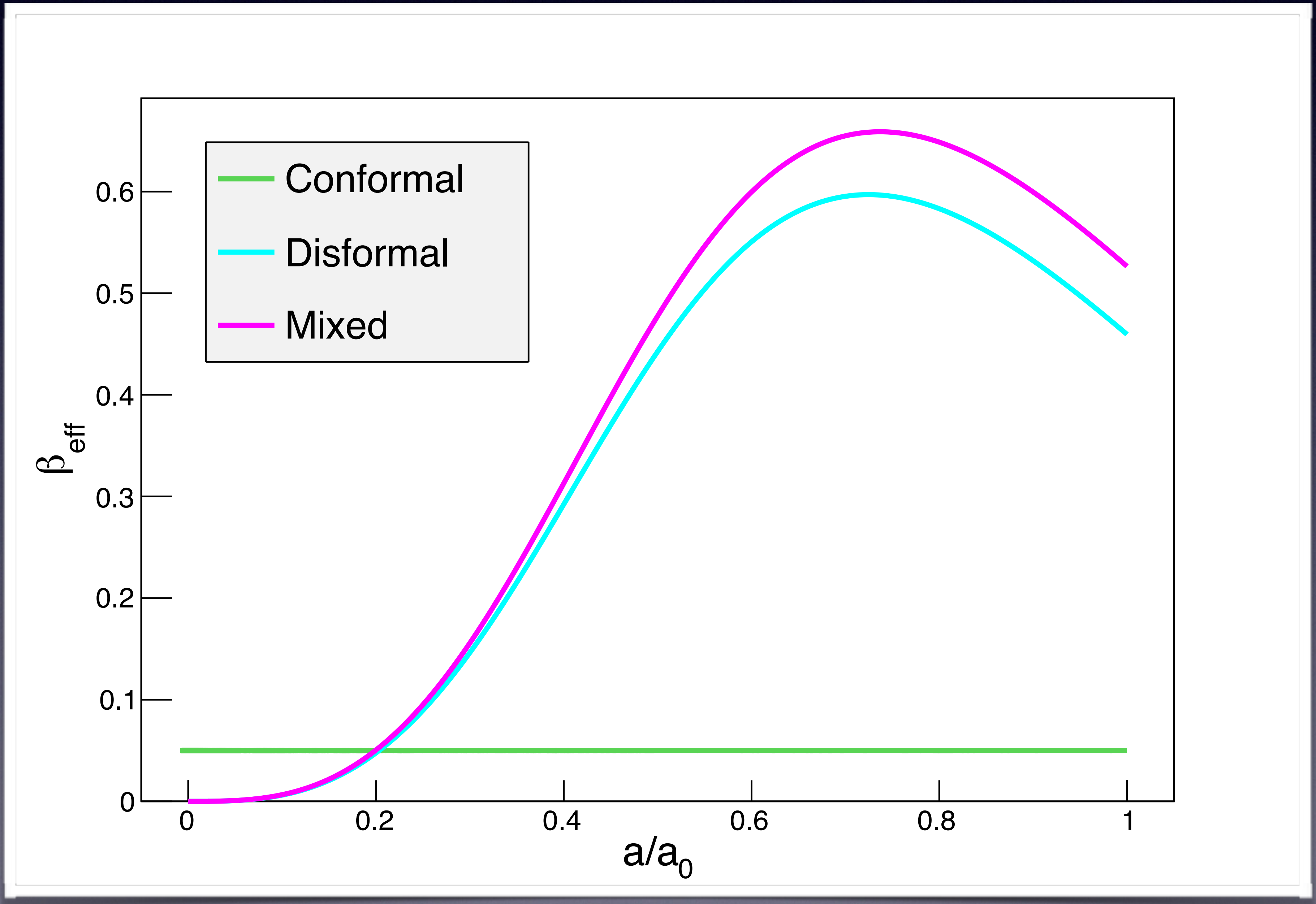
vdB, Morrice (2014)

Background evolution:



Background evolution:

Effective coupling: $$\beta_{\rm eff} = -\frac{Q_0}{\rho_{\rm DM}}$$



Cosmological Perturbations:

Newtonian gauge: $ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j]$

$$\dot{\delta}_i = -(\theta_i - 3\dot{\Phi}) + \frac{Q_0}{\rho_i}\dot{\phi}\,\delta_i - \frac{Q_0}{\rho_i}\delta\dot{\phi} - \frac{\dot{\phi}}{\rho_i}\delta Q$$

$$\dot{\theta}_i = -\mathcal{H}\theta_i + k^2\Psi + \frac{Q_0}{\rho_i}\dot{\phi}\theta_i - \frac{Q_0}{(1+w_i)\rho_i}k^2\delta\phi$$

$$\delta Q = -\frac{\rho_i}{a^2 C + D(a^2\rho_i - \dot{\phi}^2)}[\mathcal{B}_1\delta_i + \mathcal{B}_2\dot{\Phi} + \mathcal{B}_3\Psi + \mathcal{B}_4\delta\dot{\phi} + \mathcal{B}_5\delta\phi]$$

$$\delta\ddot{\phi} + 2\mathcal{H}\delta\dot{\phi} + (k^2 + a^2 V'')\delta\phi = \dot{\phi}(\dot{\Psi} + 3\dot{\Phi}) - 2a^2(V' - Q_0)\Psi + a^2\delta Q$$

vdB, Sculthorpe (2012)   Mota, Koivisto ,Zumalacarregui (2012)

Cosmological Perturbations:

$$\delta Q = -\frac{\rho_i}{a^2 C + D(a^2\rho_i - \dot{\phi}^2)}[\mathcal{B}_1\delta_i + \mathcal{B}_2\dot{\Phi} + \mathcal{B}_3\Psi + \mathcal{B}_4\delta\dot{\phi} + \mathcal{B}_5\delta\phi]$$

$$\mathcal{B}_1 = \frac{a^2 C'}{2} - 3D\mathcal{H}\dot{\phi} - Da^2(V' - Q_0) - D\dot{\phi}^2\left(\frac{C'}{C} - \frac{D'}{2D}\right)$$

$$\mathcal{B}_2 = 3D\dot{\phi} \qquad \mathcal{B}_3 = 6D\mathcal{H}\dot{\phi} + 2D\dot{\phi}^2\left(\frac{C'}{C} - \frac{D'}{2D} + \frac{Q_0}{\rho_i}\right)$$

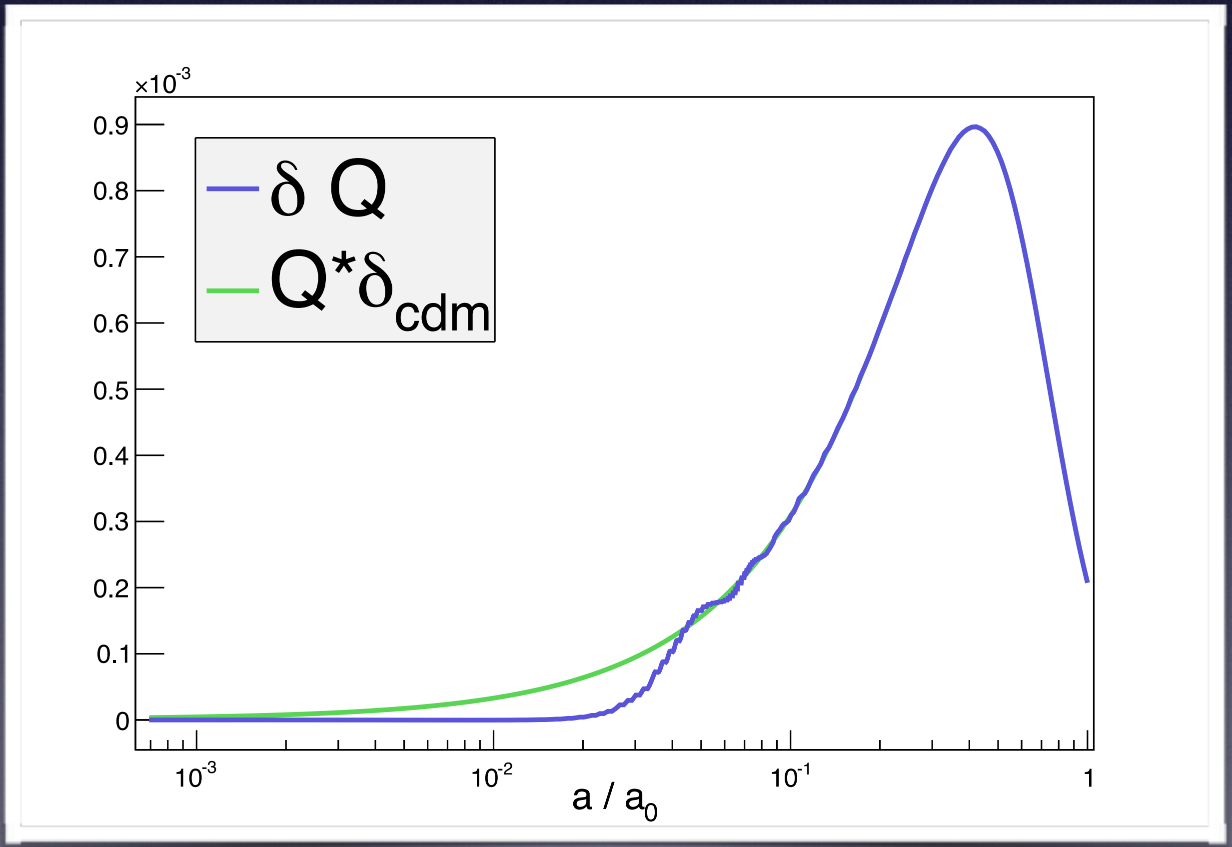
$$\mathcal{B}_4 = -3D\mathcal{H} - 2D\dot{\phi}\left(\frac{C'}{C} - \frac{D'}{2D} + \frac{Q_0}{\rho_i}\right)$$

$$\mathcal{B}_5 = \frac{a^2 C''}{2} - Dk^2 - Da^2 V'' - D'a^2 V' - 3D'\mathcal{H}\dot{\phi}$$
$$- D\dot{\phi}^2\left(\frac{C''}{C} - \left(\frac{C'}{C}\right)^2 + \frac{C'D'}{CD} - \frac{D''}{2D}\right) + (a^2 C' + D'a^2\rho_i - D'\dot{\phi}^2)\frac{Q_0}{\rho_i}$$

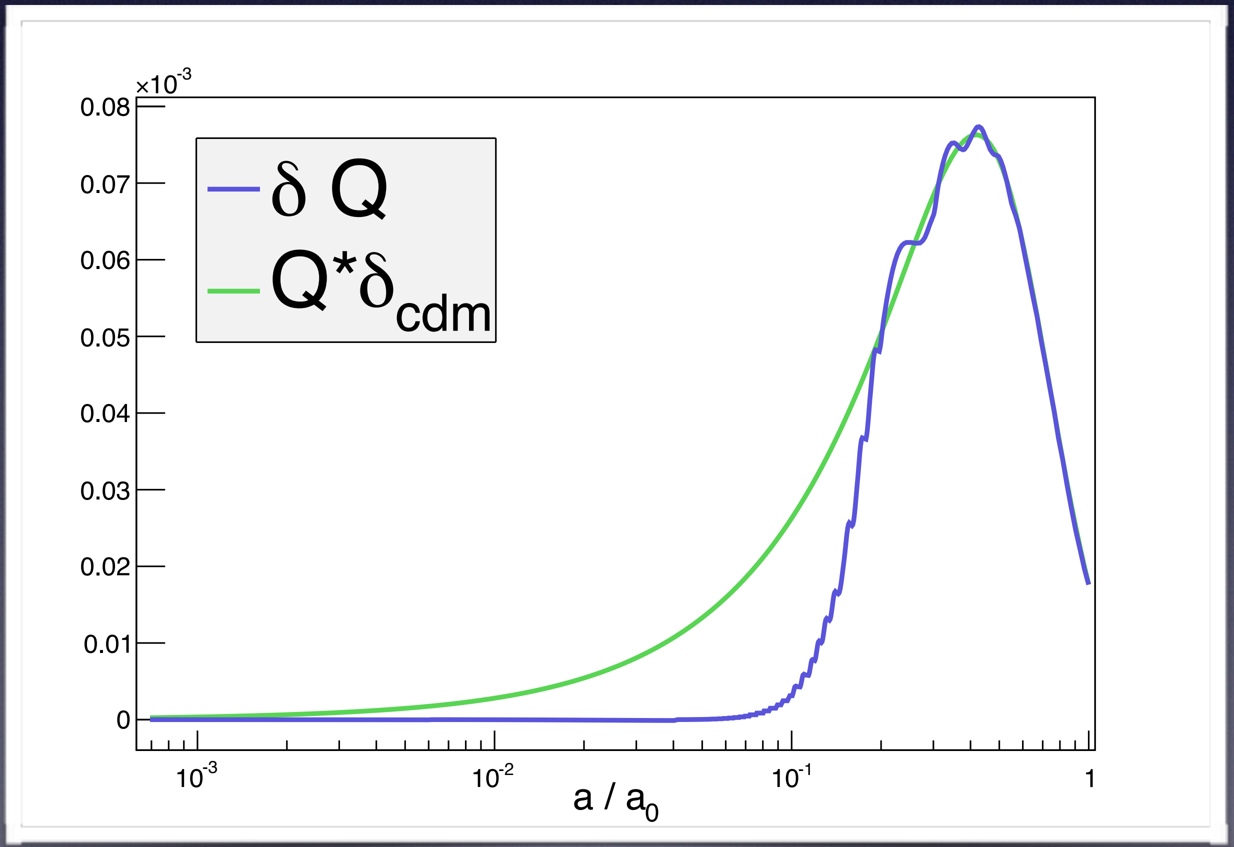
vdB, Sculthorpe (2012) Mota, Koivisto ,Zumalacarregui (2012)

Cosmological Perturbations:

Unlike in the case of pure conformal couplings, the perturbation of the coupling is k-dependent! In the limit of large k, things simplify: $\delta Q \to Q\ \delta_{\rm cdm}$

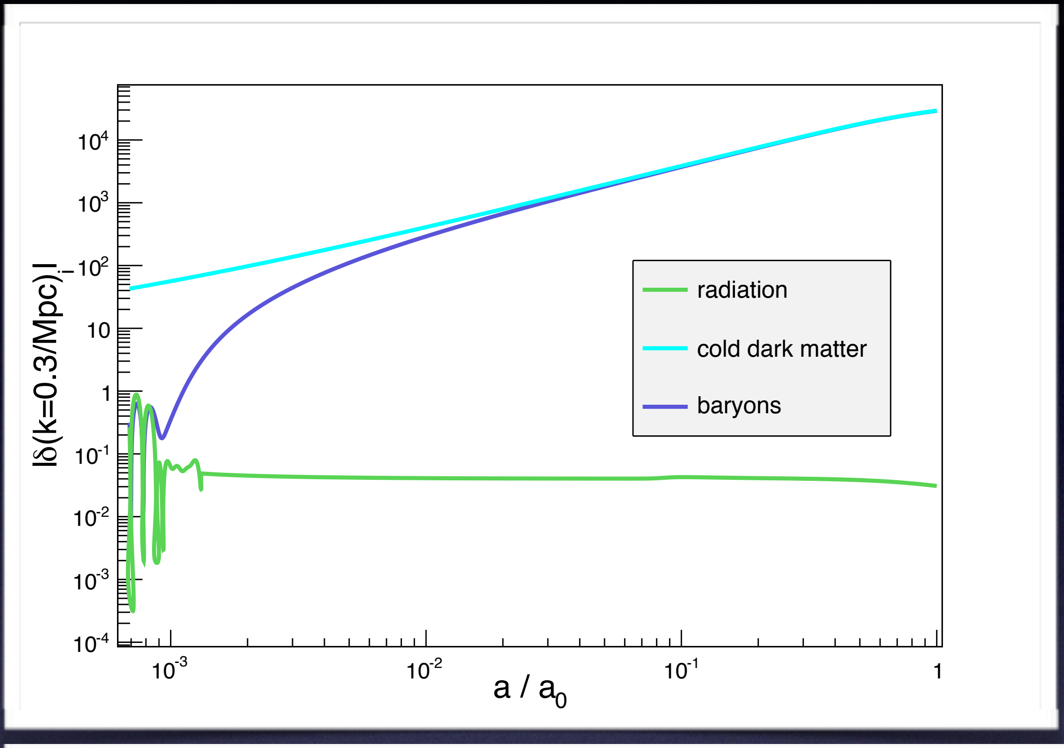


$k = 0.3\ \mathrm{Mpc}^{-1}$

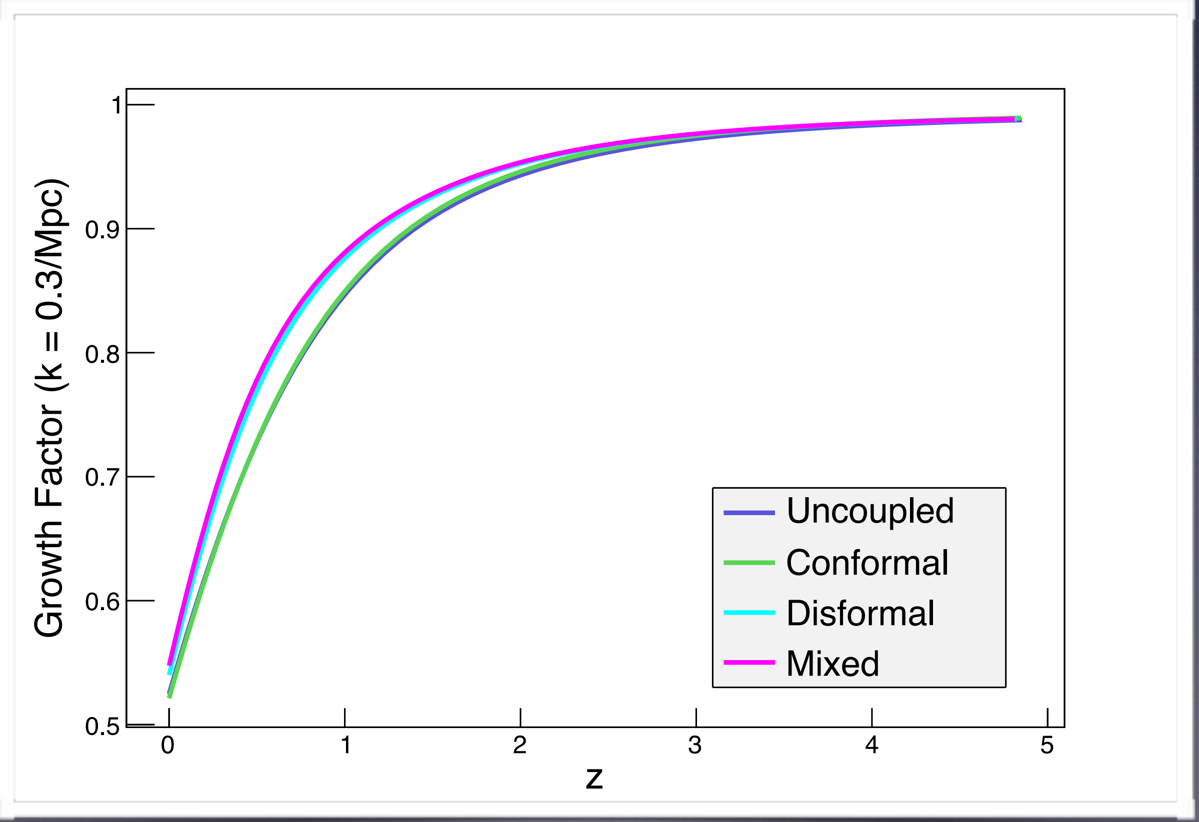


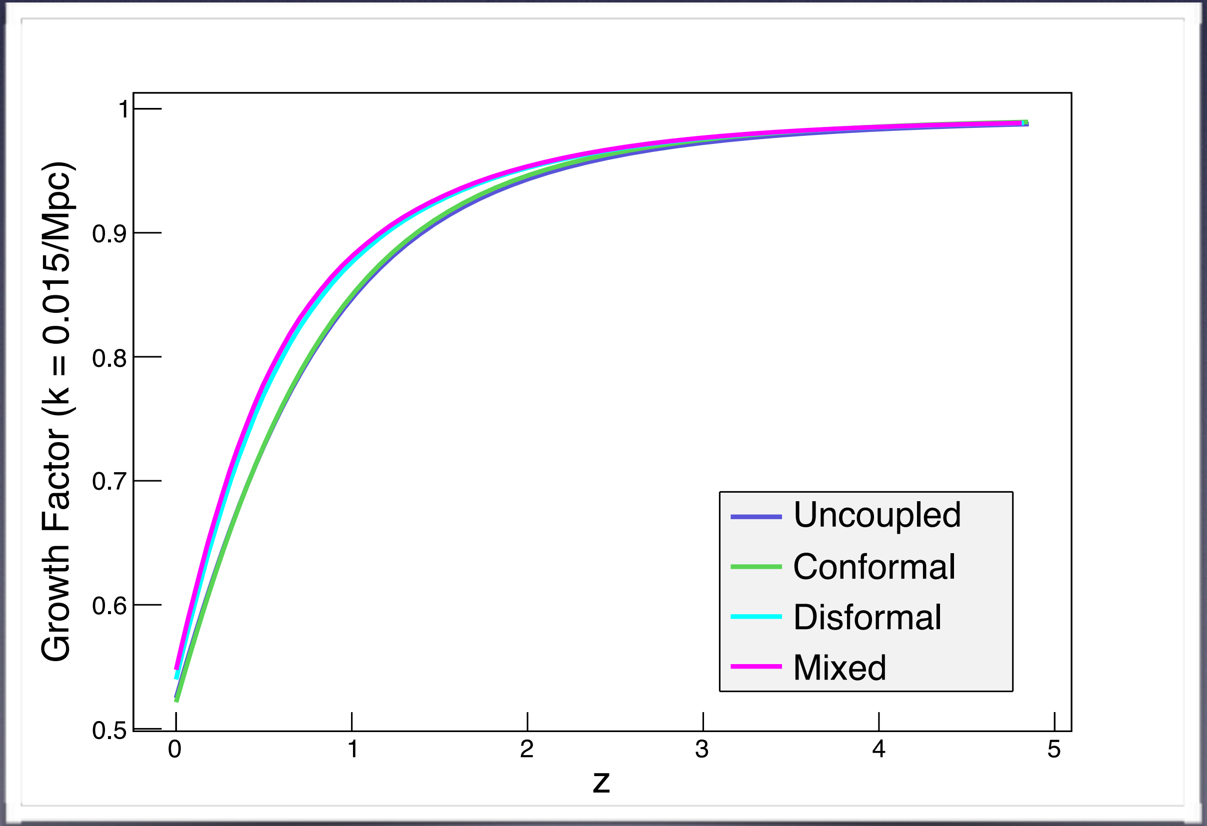
$k = 0.015\ \mathrm{Mpc}^{-1}$

Cosmological Perturbations:



Growth factor: $$f = \frac{d \ln \delta}{d \ln a}$$





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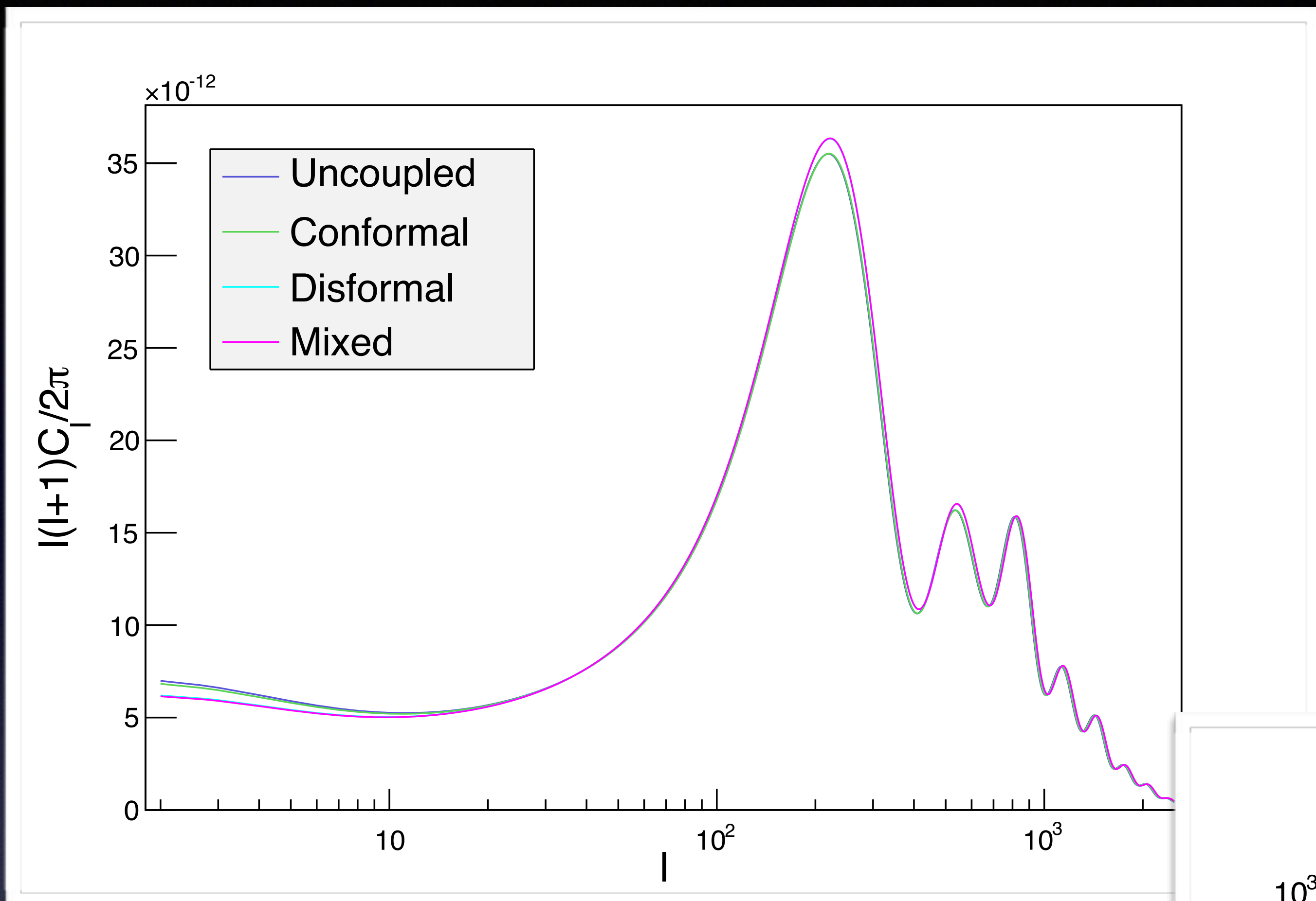

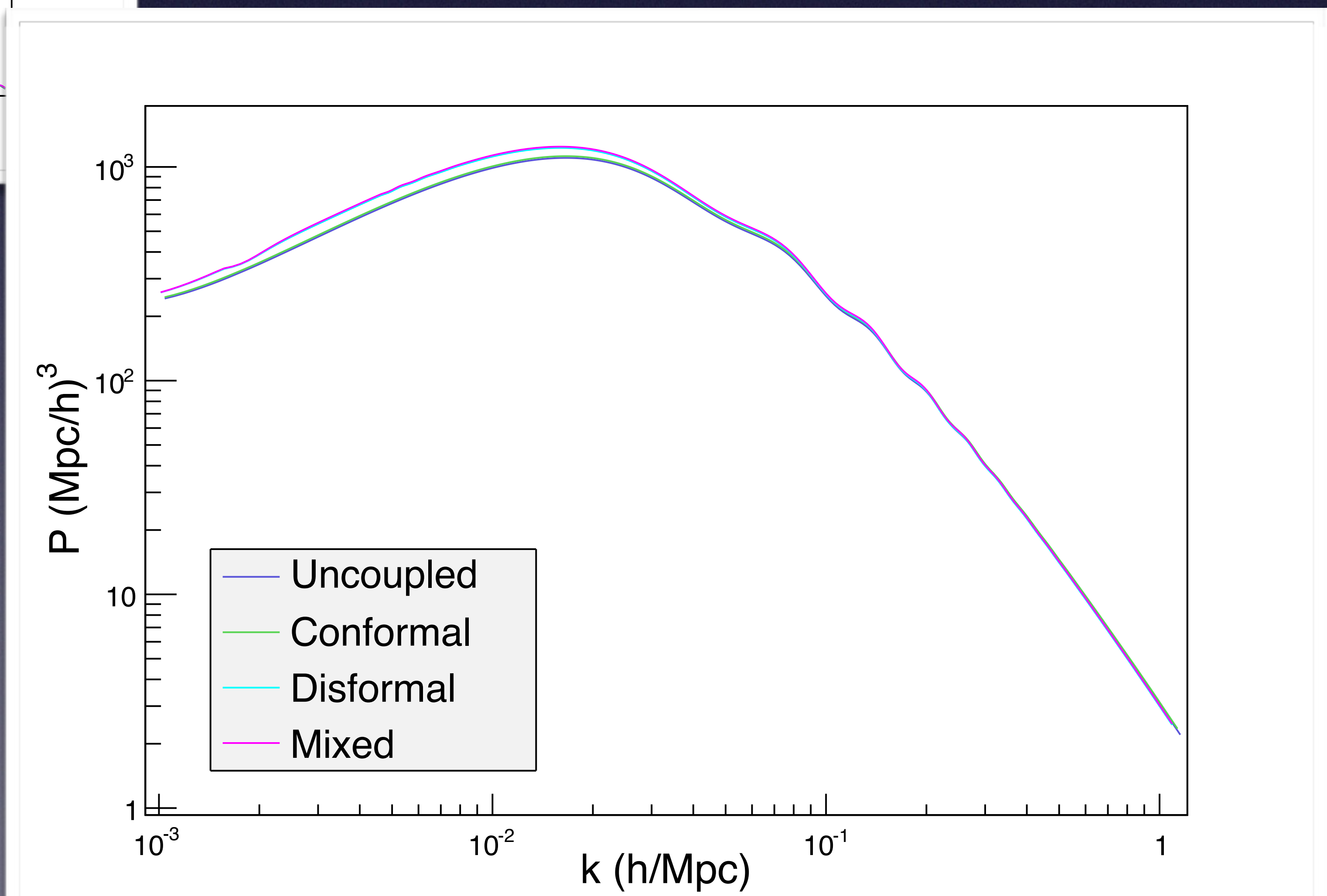

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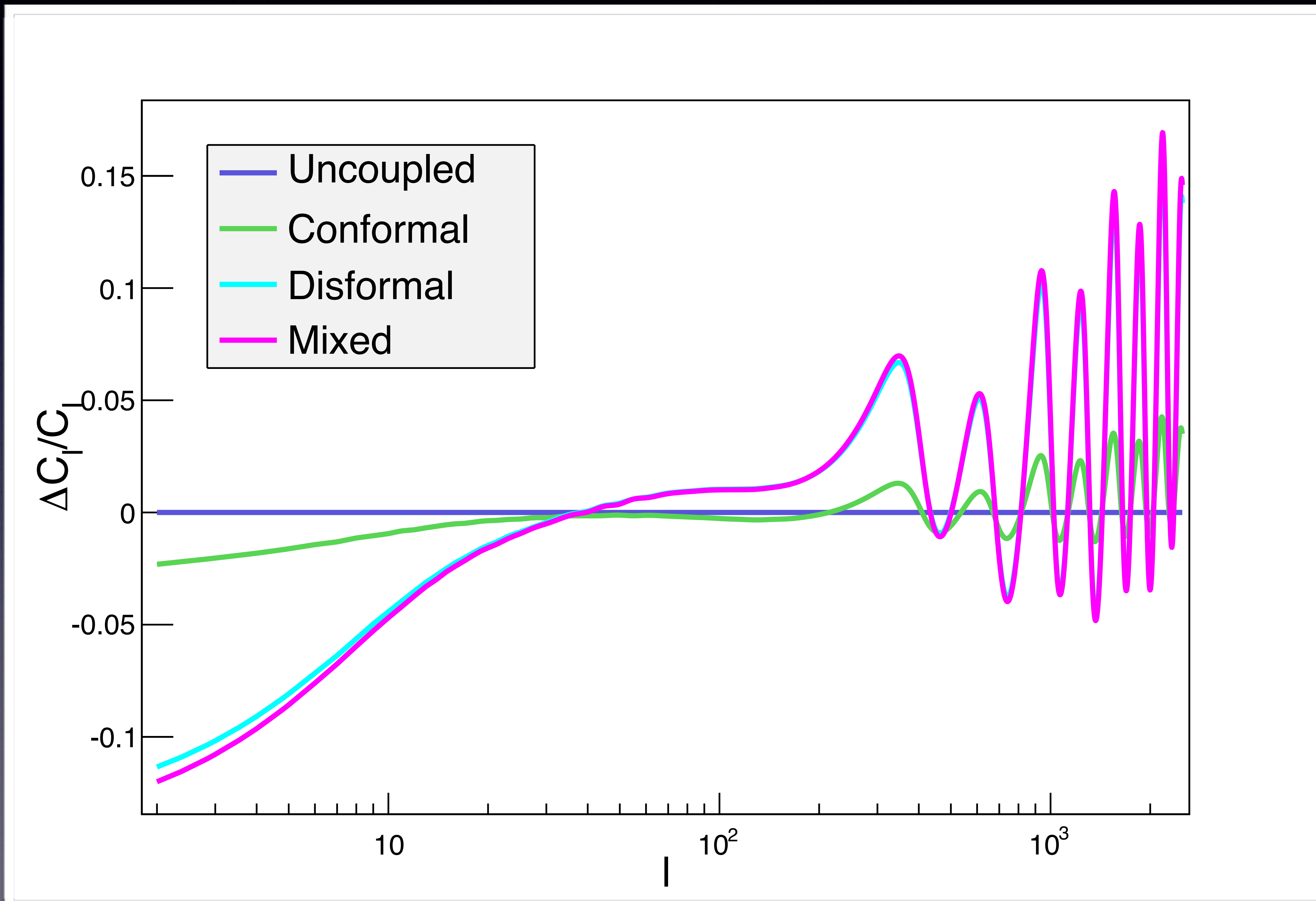
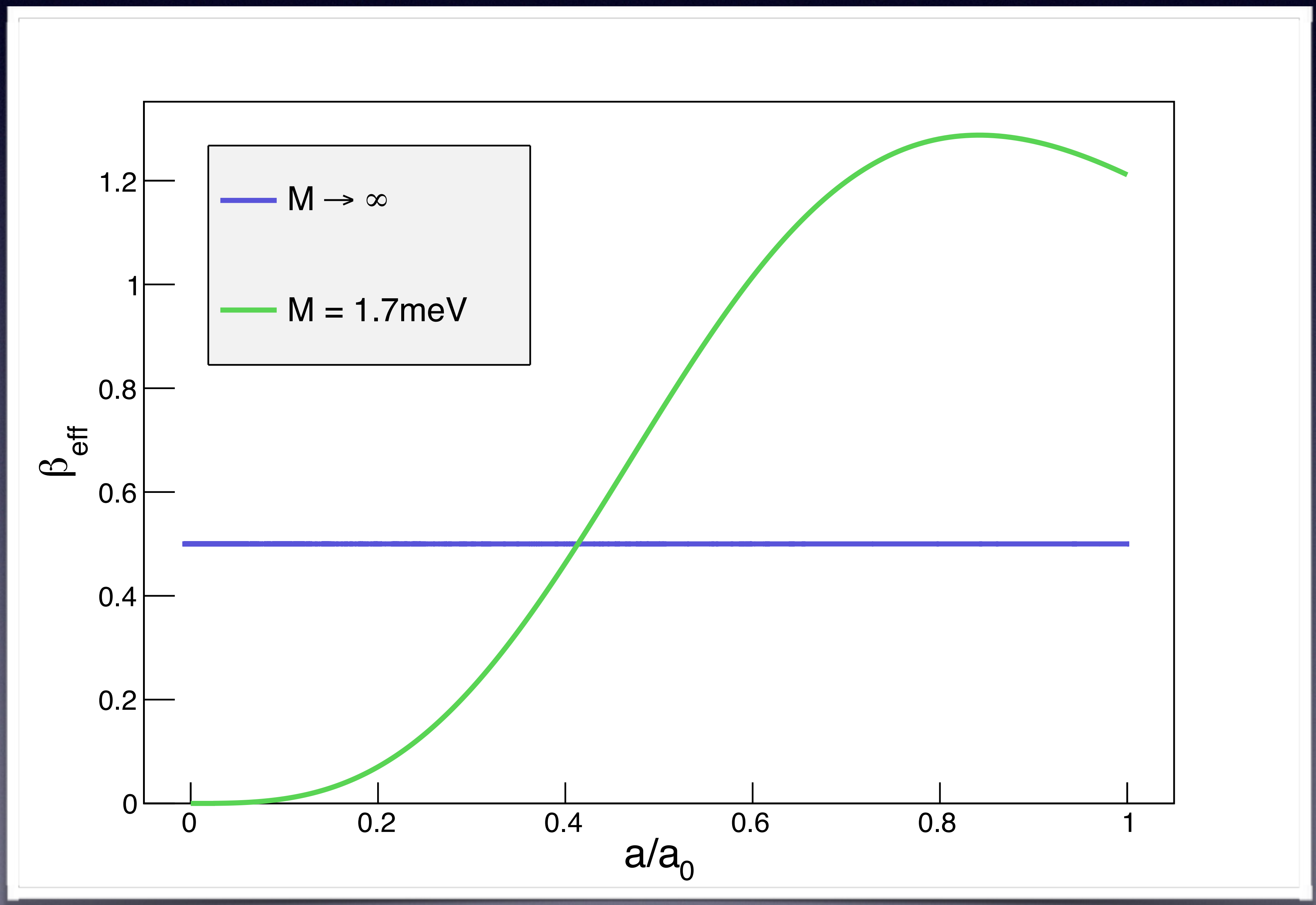


Cosmological Perturbations: A model with large conformal couplings

$$\beta_c = 0.5 \frac{1}{M_{\rm Pl}} \qquad D = \frac{1}{M^4}$$



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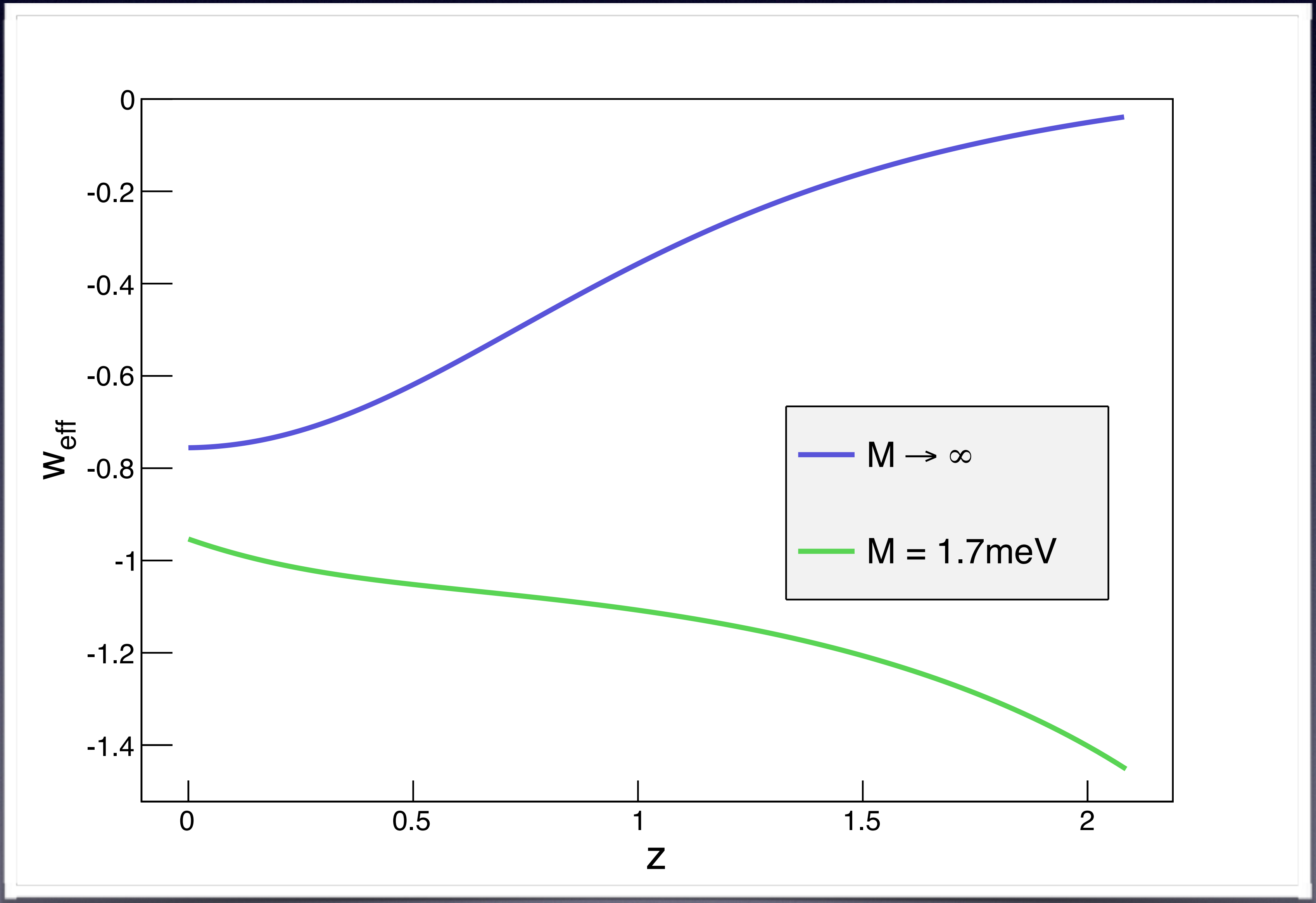

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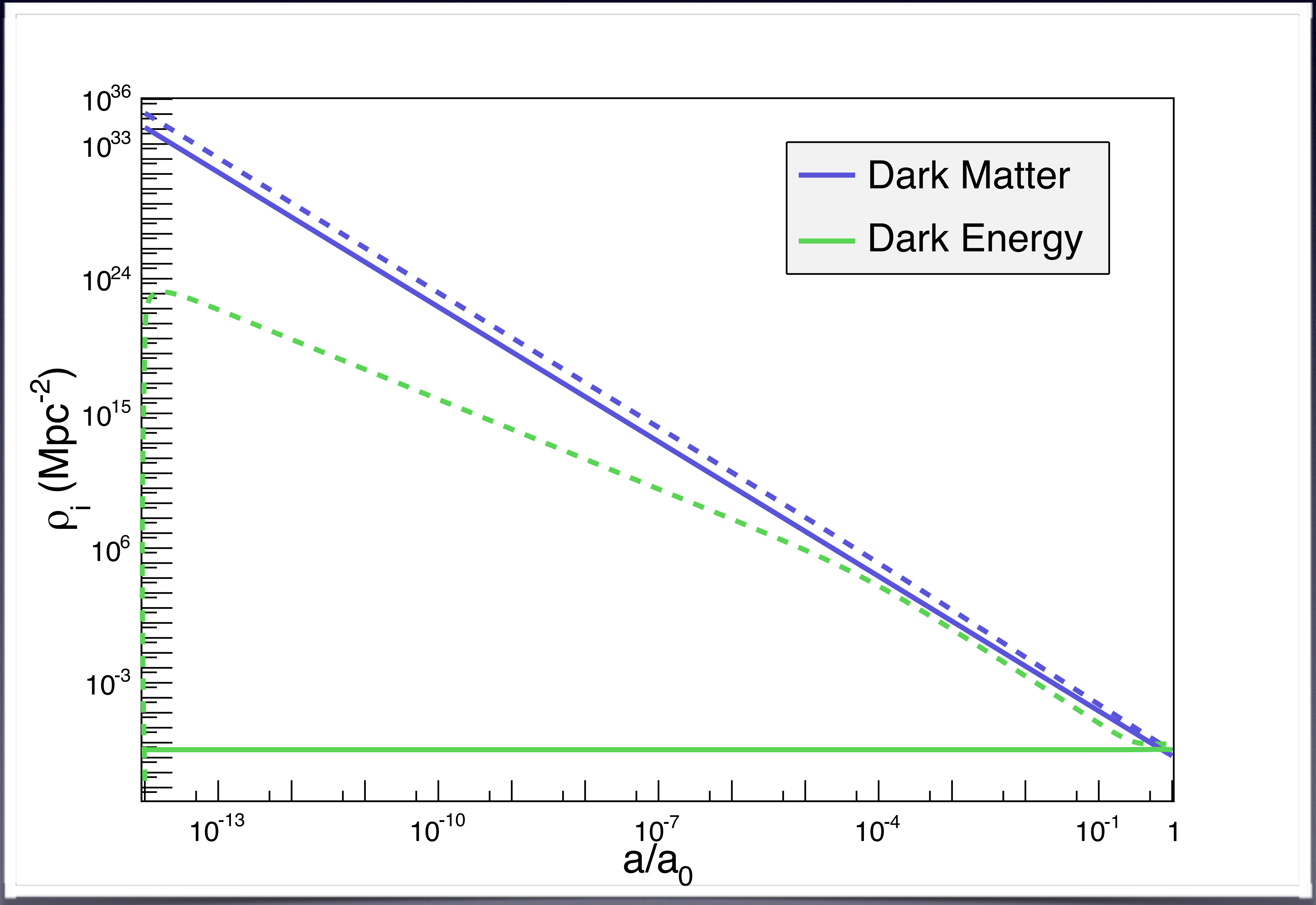

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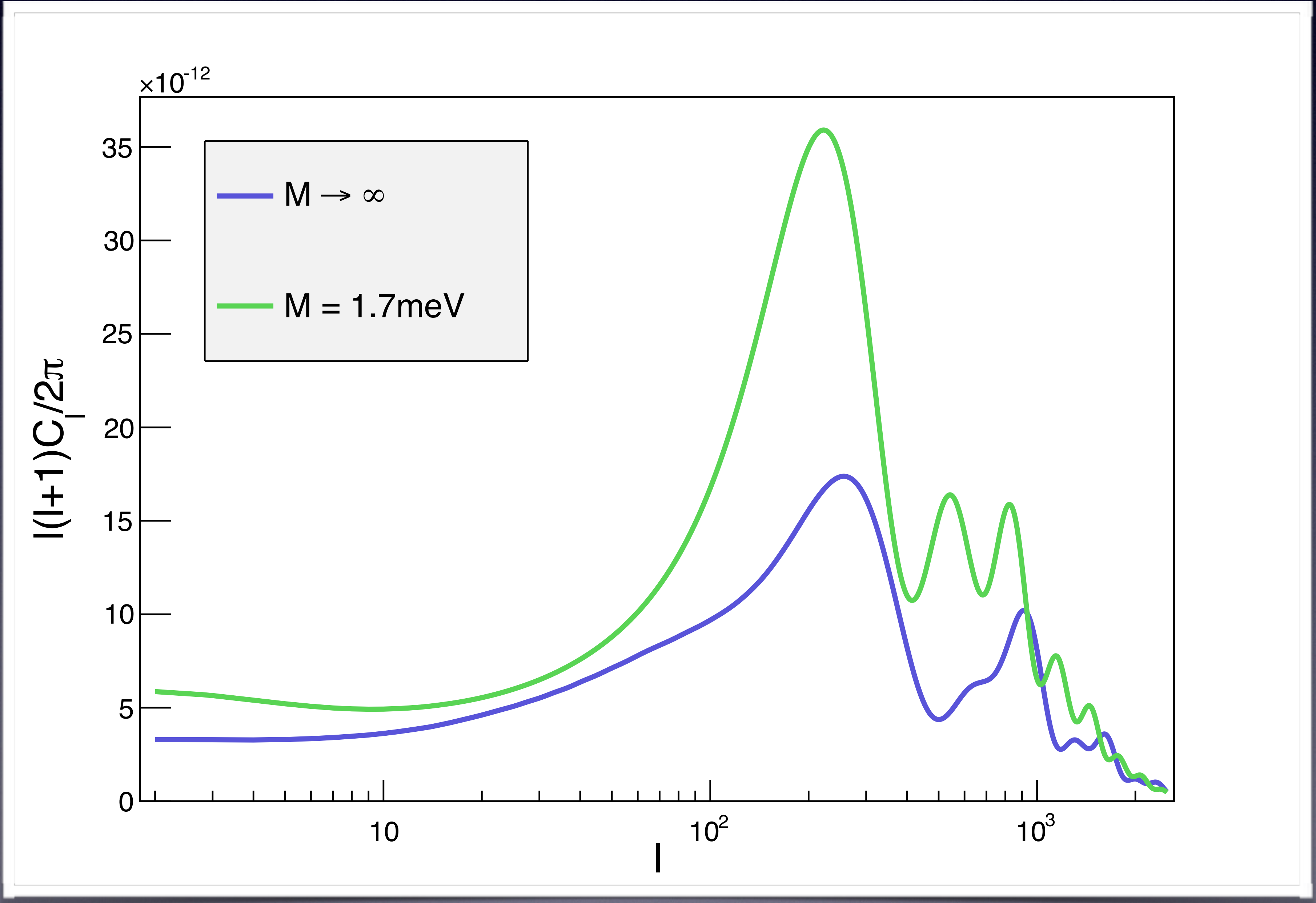

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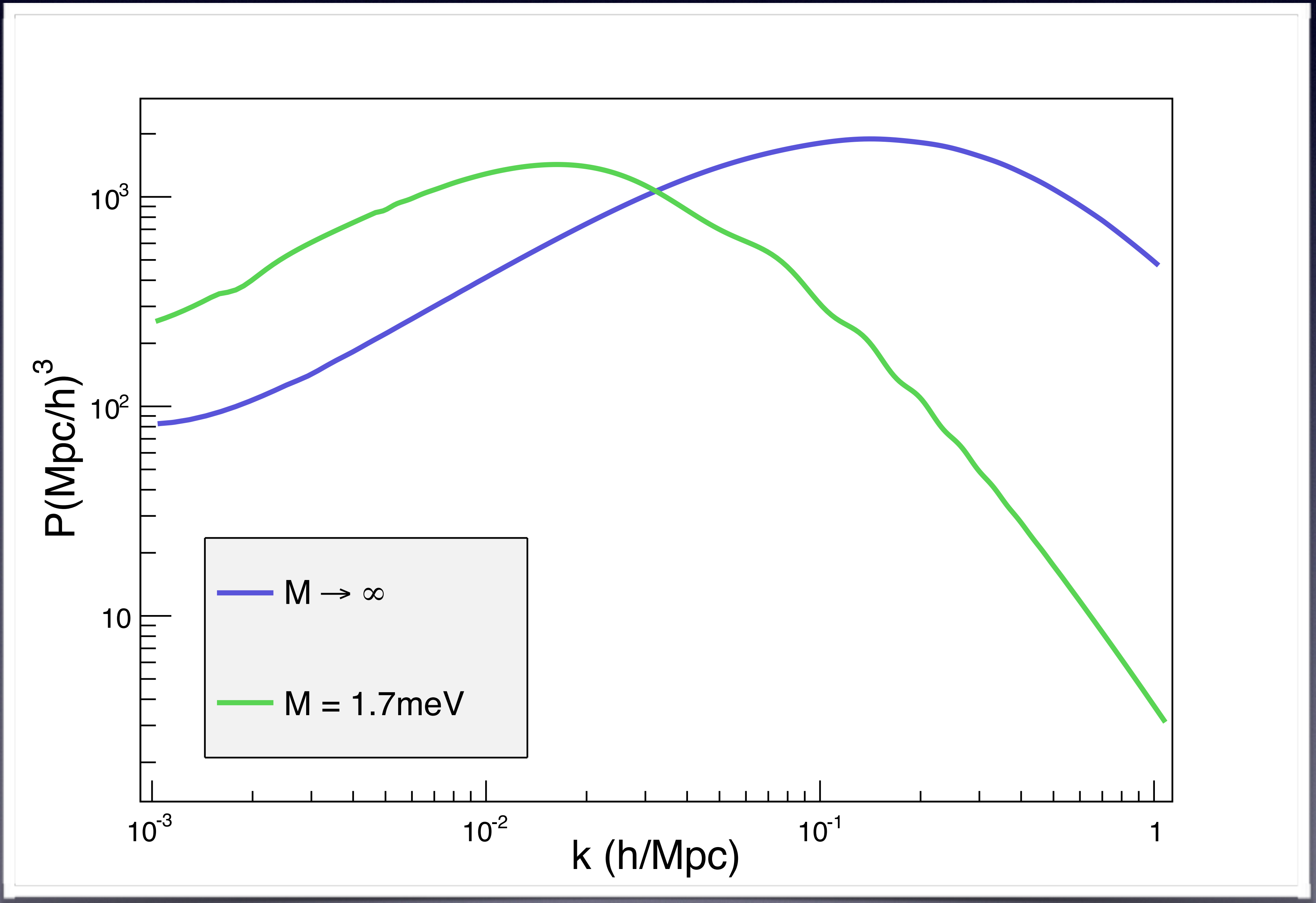


Cosmological Perturbations:

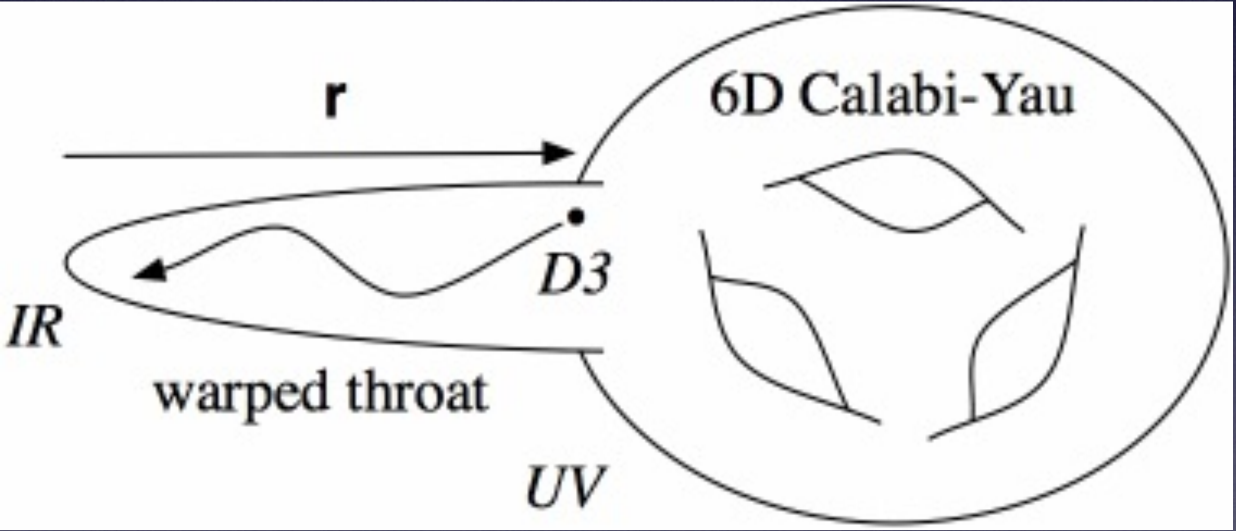
Disformal coupling *suppresses* conformal coupling contribution.

$$a^2Q = -\frac{\rho_{\rm dm}}{2(C + D(\rho_{\rm dm} - \dot{\phi}^2/a^2))}\left[a^2\frac{dC}{d\phi} - 2D\left(3\mathcal{H}\dot{\phi} + a^2\frac{dV}{d\phi} + \frac{d\log C}{d\phi}\dot{\phi}^2\right) + \frac{dD}{d\phi}\dot{\phi}^2\right]$$

Coupling to matter becomes important in the later history of structure formation at z<10 (details depend on value of *M* and β_c).

Coupled Quintessence: a brane model

Dark matter coupled both conformally and disformally to a scalar field can be motivated from a string theory setup: dark matter lives on a brane, moving in an extra-dimensional space along an AdS throat. The induced metric provides a conformal and disformal coupling:



$$g^{\text{ind}}_{\mu\nu} = h^{-1/2}(r) g_{\mu\nu} + h^{1/2}(r) r_{,\mu} r_{,\nu}$$

Dark energy scalar field is proportional to r, of DBI nature.

Koivisto, Wills, Zavalla (2014)

Coupled Quintessence: a brane model

We assume that the scalar is slowly moving, we ignore DBI form here. The conformal and disformal couplings are specified by the warp factor:

$$h(\phi) \propto \frac{\kappa^{4-m}}{\phi^m}$$

$$C(\phi) \propto h(\phi)^{-1/2} \qquad D(\phi) \propto h(\phi)^{1/2}$$

Potential assumed to have the form

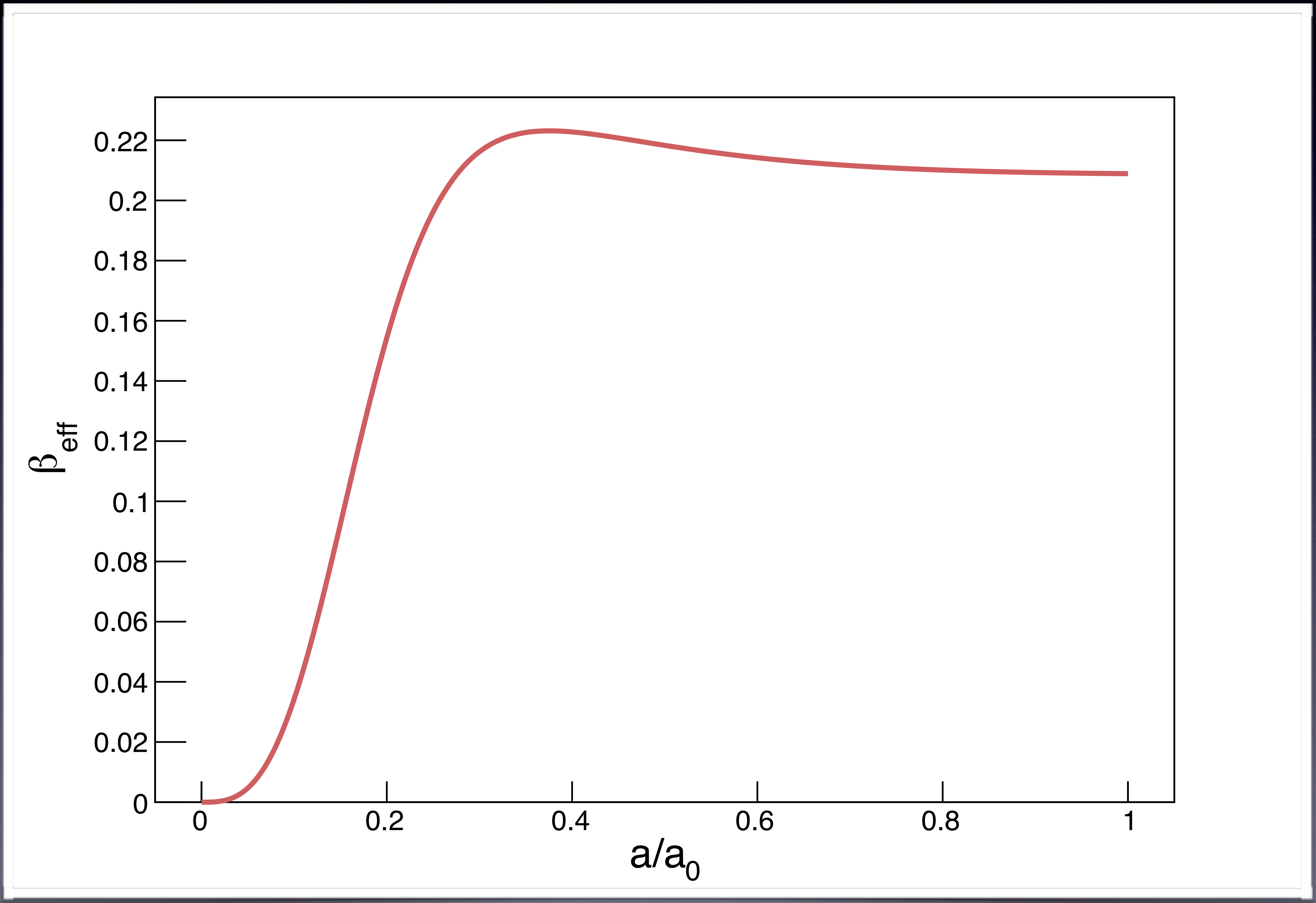
$$V(\phi) \propto \kappa^{n-4}\phi^n$$

In the following: $n = 2, m = 4$

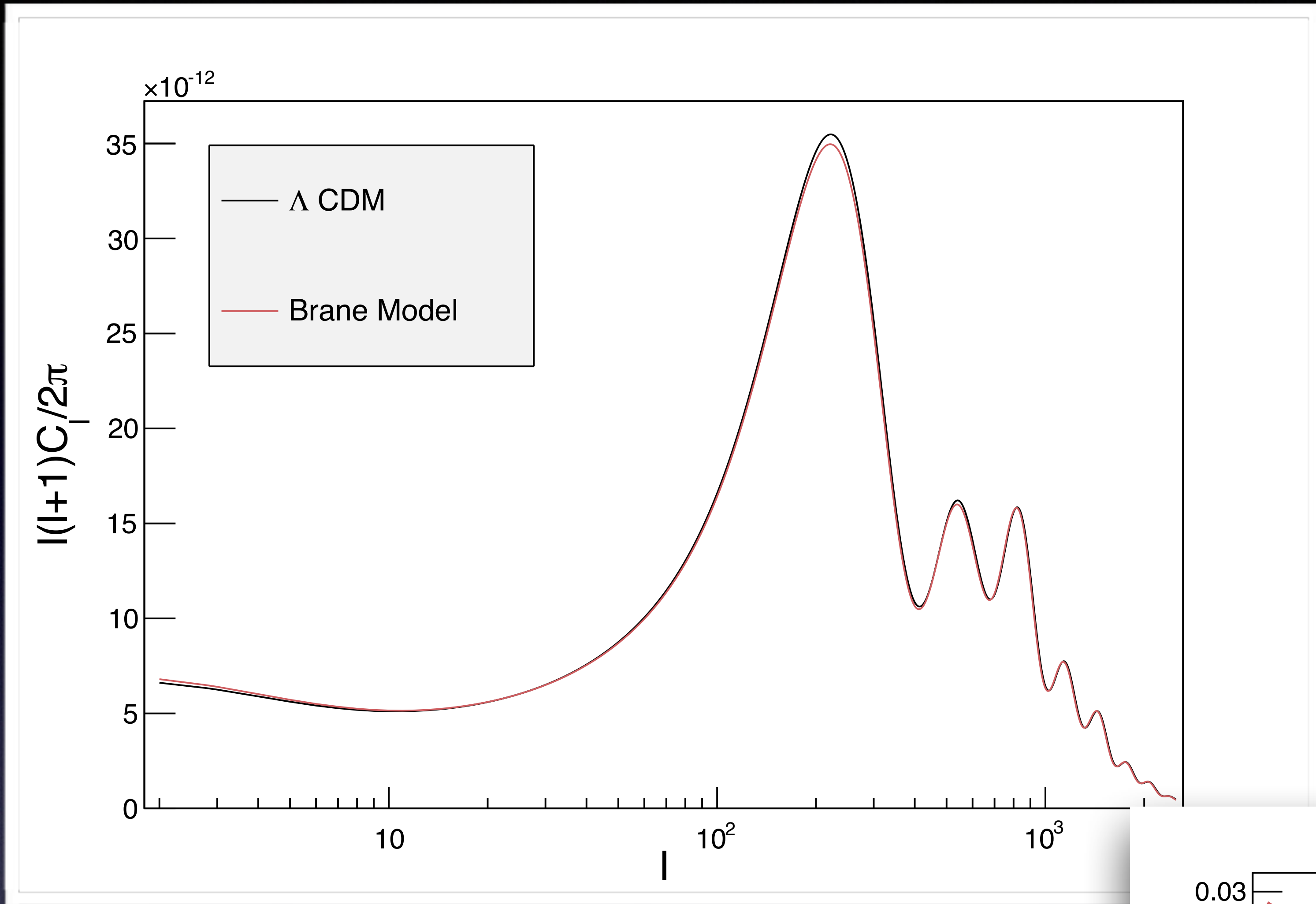
vdB, Koivisto, Morrice (2014)

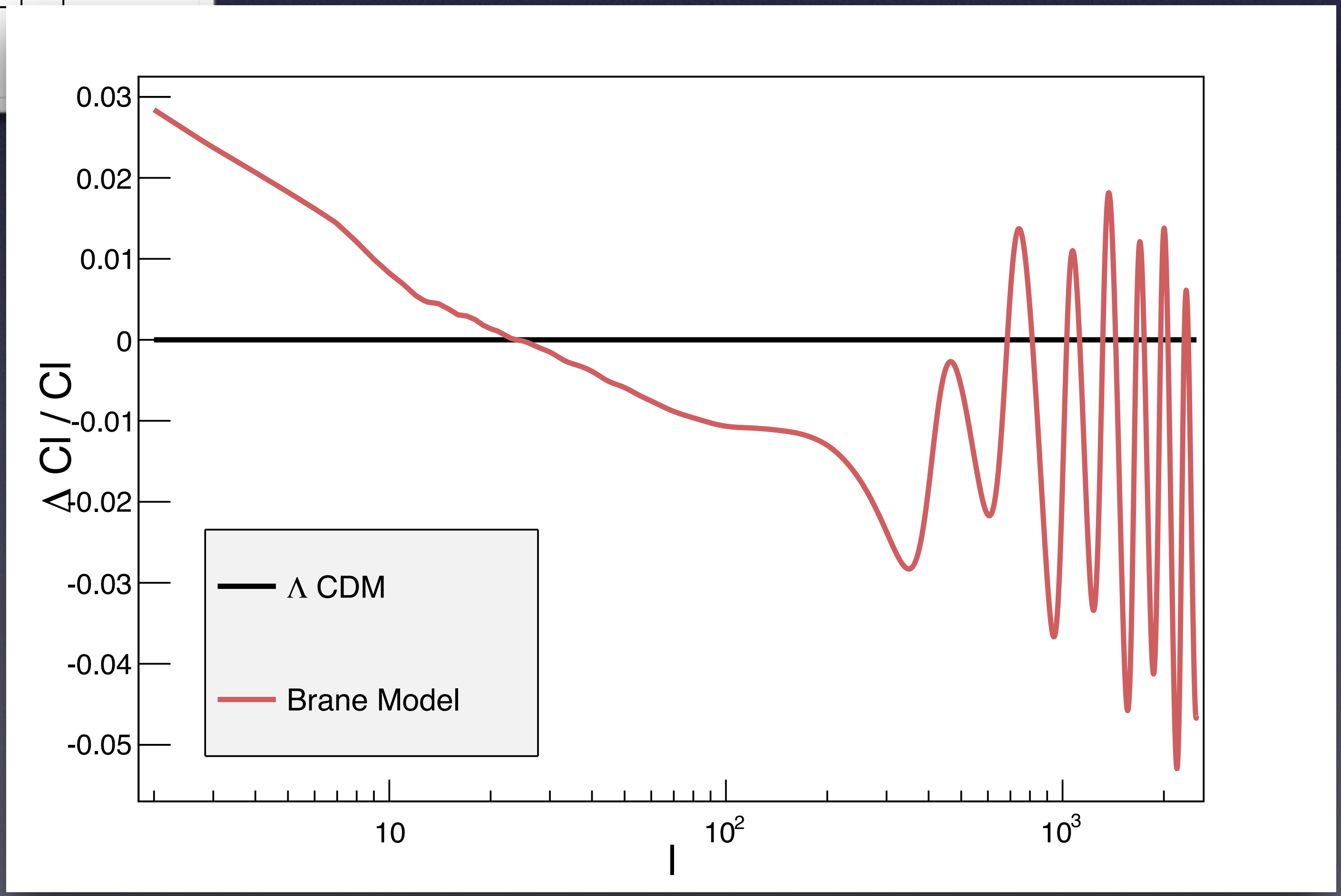
Coupled Quintessence: a brane model

Evolution of effective coupling:

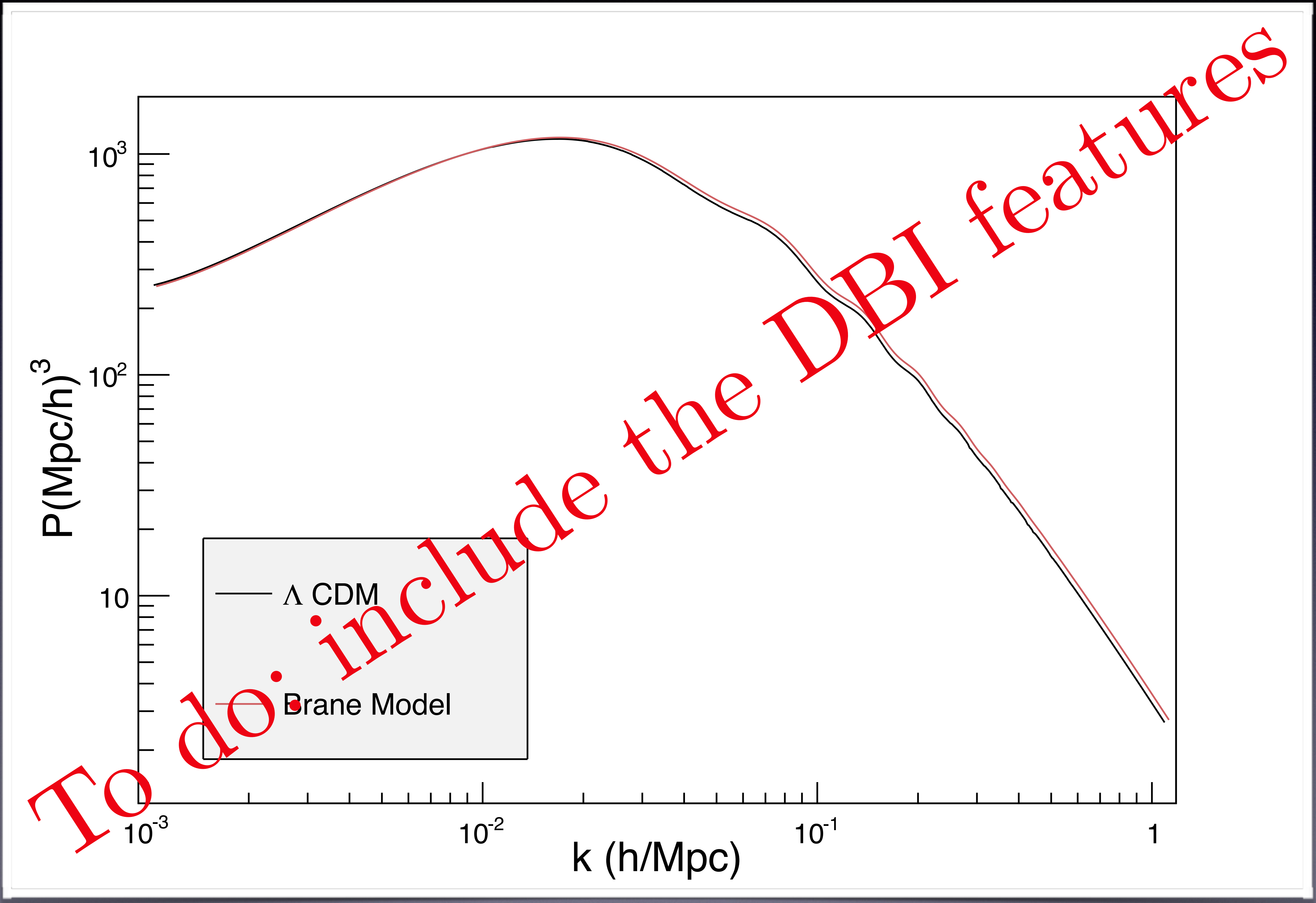


Coupled Quintessence: a brane model





Coupled Quintessence: a brane model



3. Multiple Species & Couplings

Scalar could couple differently to different matter species, or there could be different species of dark matter and/or neutrinos. Consider couplings to two species with different equation of state:

$$\tilde{g}^i_{\mu\nu} = C_i(\phi) g_{\mu\nu} + D_i(\phi)\partial_\mu\phi\partial_\nu\phi$$

$$Q_i = \frac{C'_i}{2C_i} T_i + \frac{D'_i}{2C_i}\phi,_\mu \phi,_\nu T_i^{\mu\nu} - \nabla_\mu\left(\frac{D_i}{C_i}\phi_\nu T_i^{\mu\nu}\right)$$

vdB, Morrice, Vu (2013) vdB, Nunes, Mimoso (2014)

Multiple couplings - 2 species:

$$\ddot{\phi} + 3H\dot{\phi} + V' = \sum Q_i \qquad \dot{\rho}_i + 3H(\rho_i + p_i) = -Q_i\dot{\phi}$$

$$Q_1 = \frac{\mathcal{A}_2}{\mathcal{A}_1\mathcal{A}_2 - D_1 D_2 \rho_1 \rho_2}\left(\mathcal{B}_1 - \frac{\mathcal{B}_2 D_1 \rho_1}{\mathcal{A}_2}\right)$$

$$Q_2 = \frac{\mathcal{A}_1}{\mathcal{A}_1\mathcal{A}_2 - D_1 D_2 \rho_1 \rho_2}\left(\mathcal{B}_2 - \frac{\mathcal{B}_1 D_2 \rho_2}{\mathcal{A}_1}\right)$$

$$\mathcal{A}_i = C_i + D_i(\rho_i - \dot{\phi}^2)$$

$$\mathcal{B}_i = \left[\frac{C_i'}{2}\left(-1 + 3\frac{p_i}{\rho_i}\right) - \frac{D_i'}{2}\dot{\phi}^2 + D_i\left\{3H\left(1 + \frac{p_i}{\rho_i}\right)\dot{\phi} + V' + \frac{C_i'}{C_i}\dot{\phi}^2\right\}\right]\rho_i$$

vdB, Morrice, Vu (2013)

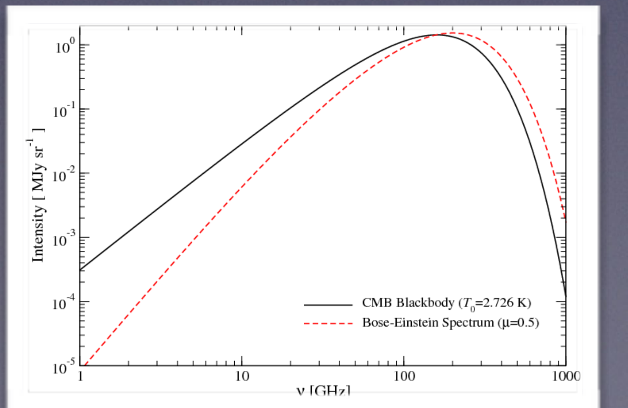
Multiple couplings - 2 species:

Equation of state not frame invariant!
E.g. case of pure disformal couplings:

$$\frac{p}{\rho} = \frac{\tilde{p}}{\tilde{\rho}}\left(1 - \frac{\dot{\phi}^2}{M^4}\right)$$

To be seen whether there is interesting phenomenology for warm dark matter cosmology.

Interesting phenomenology when considering coupling to photons (temperature redshift relation, spectral distortions, varying speed of light).



4. Summary

Both from the model building point of view as well as phenomenology of structure formation, disformal couplings are interesting. Viable models can be found.

Questions to be addressed:

- What the observationally distinctive signatures of disformal couplings? The background dynamics can be mapped onto a purely conformal theory. Is this degeneracy broken at the level of linear perturbations (so that it can be actually observed)?

4. Summary

Both from the model building point of view as well as phenomenology of structure formation, disformal couplings are interesting. Viable models can be found.

Questions to be addressed:

- It is essential to study non-linear behaviour. The perturbation of coupling is $\mathbf{k}$-dependent, but not really observable at the linear level. Does this remain valid for highly non-linear scales?