Multifield interactions

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FieldS coupled to dark matterS

Motivation:

- fundamental theories have multiple fields;

- e.g. neutrinos have 3 families. Dark matter could have more than one component?

- How do scalar fields, dark matter or neutrinos couple with each other?

Some field work

Assisted inflation	Liddle 1998; Copeland 1999
Assisted quintessence	Kim 2005; Tsujikawa 2006
Coupled quintessence	Amendola 2000; Holden 2000
Coupled quintessence with 2 dark matter components	Brookfield 2008; Baldi 2012
Multifield coupled quintessence with many dark matter components	This work 2014

More references in the article **arXiv:1407.2156**

Field coupled to dark matter(s)

Single field coupled quintessence:

:) Scaling accelerating solutions;

:) application to growing neutrino dark energy;

:(dark matter never dominant for scaling solutions

:(Instability problems in the matter density contrast.

Single field but 2 dark matter components (Brookfield, Baldi):

- When couplings are symmetric one obtains dark matter domination followed by scalar field domination.

Dynamical equations

$$
\begin{aligned}
\ddot{\phi}_i + 3H\dot{\phi}_i + V_{,\phi_i} &= \kappa \sum_k C_{ik}\rho_k, \\
\dot{\rho}_k + 3H\rho_k &= -\kappa \sum_i C_{ik}\dot{\phi}_i\rho_k.
\end{aligned}
$$

$$\rho_k = \rho_{k0} \exp\left(-3N - \kappa \sum_i C_{ik}(\phi_i - \phi_{i0})\right)$$

$$\rho_{\phi_i} = \sum_i \dot{\phi}_i^2/2 + V(\phi_1, ..., \phi_n)$$

$$H^2 = \frac{\kappa^2}{3}\left(\sum_k \rho_k + \sum_i \rho_{\phi_i}\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\left(\sum_k \rho_k + \sum_i \dot{\phi}_i^2\right)$$

Sum of exponentials $V(\phi_1, ..., \phi_n) = M^4 \sum_i e^{-\kappa\lambda_i\phi_i}$

$$x_i \equiv \frac{\kappa\dot{\phi}_i}{\sqrt{6}H}, \qquad y_i^2 \equiv \frac{\kappa^2 V_i}{3H^2}, \qquad z_k^2 \equiv \frac{\kappa^2 \rho_k}{3H^2},$$

$$\begin{aligned}
x_i' &= -\left(3 + \frac{H'}{H}\right) x_i + \sqrt{\frac{3}{2}} \left(\lambda_i y_i^2 + \sum_k C_{ik} z_k^2\right), \\
y_i' &= -\sqrt{\frac{3}{2}} \left(\lambda_i x_i + \sqrt{\frac{2}{3}} \frac{H'}{H}\right) y_i, \\
z_k' &= -\sqrt{\frac{3}{2}} \left(\sum_i C_{ik} x_i + \sqrt{\frac{3}{2}} + \sqrt{\frac{2}{3}} \frac{H'}{H}\right) z_k, \\
\frac{H'}{H} &= -\frac{3}{2} \left(1 + \sum_i (x_i^2 - y_i^2)\right),
\end{aligned}$$

$$\sum_i (x_i^2 + y_i^2) + \sum_k z_k^2 = 1. \qquad w_{\mathrm{eff}} = \sum_i (x_i^2 - y_i^2)$$

Critical points for sum of exponentials

1. Scalar field dominated solution

$$x_i = \frac{1}{\sqrt{6}} \frac{1}{\lambda_i \sum_j 1/\lambda_j^2},$$

$$w_{\rm eff} = -1 + \frac{1}{3}\lambda_{\rm eff}^2,$$

$$\frac{1}{\lambda_{\rm eff}^2} = \sum_i \frac{1}{\lambda_i^2}.$$

More fields => inflation easier to achieve

Critical points for sum of exponentials

2. Scaling solution (example 2 fields x 2 dark-matter)

$$x_1 = \sqrt{\frac{3}{2}}\frac{1}{\lambda_1 - \gamma_1},$$

$$x_2 = \sqrt{\frac{3}{2}}\frac{1}{\lambda_2 - \gamma_2}.$$

$$\gamma_1 = C_{11} + C_{21}\frac{\lambda_1}{\lambda_2},$$

$$\gamma_2 = C_{22} + C_{12}\frac{\lambda_2}{\lambda_1}.$$

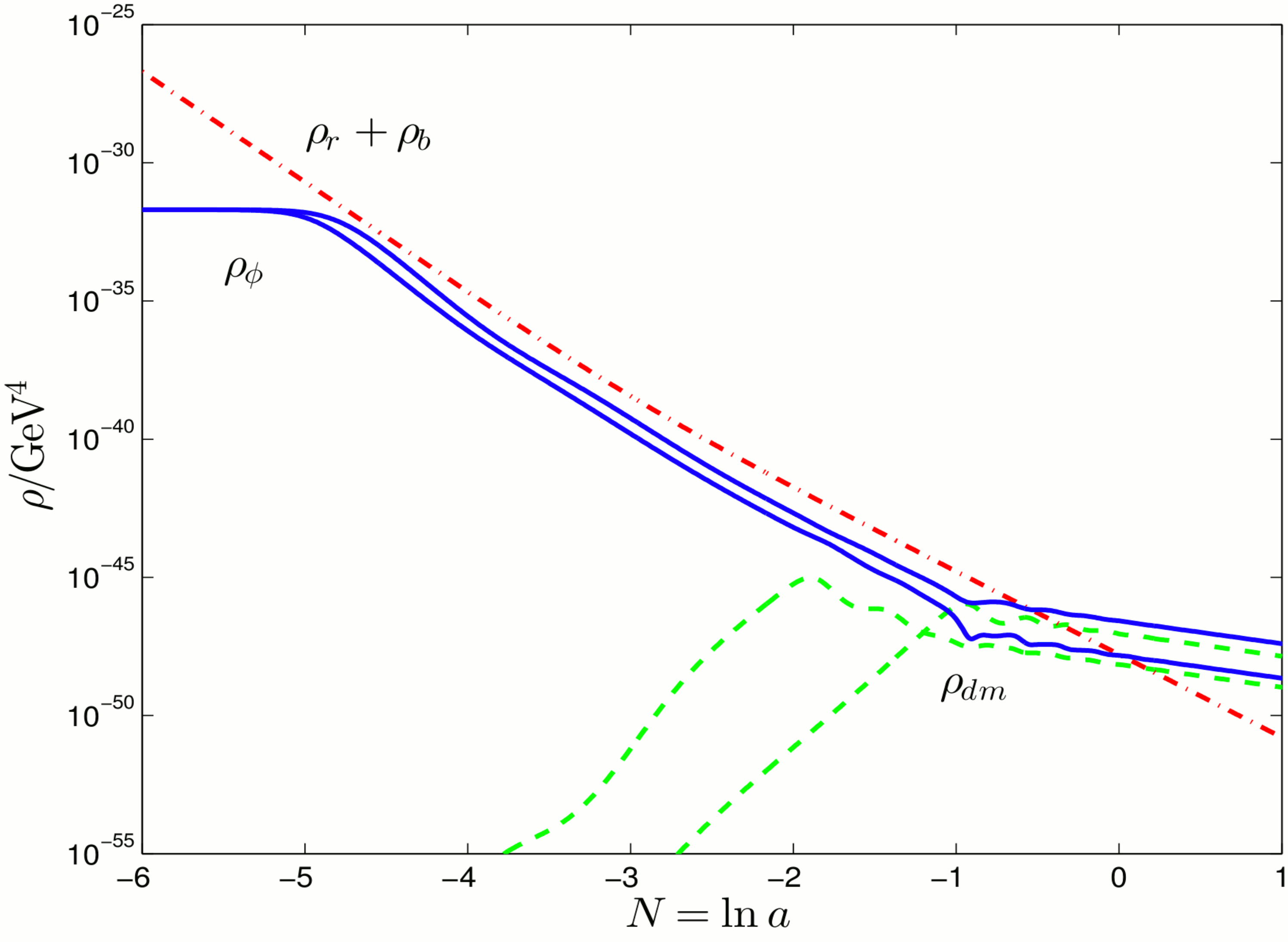
$$w_{\rm eff} = \frac{C_{\rm eff}}{\lambda_{\rm eff} - C_{\rm eff}},$$

$$\Omega_\phi = \frac{3 - \lambda_{\rm eff} C_{\rm eff} + C_{\rm eff}^2}{(\lambda_{\rm eff} - C_{\rm eff})^2},$$

$$C_{\rm eff} \equiv \lambda_{\rm eff}\frac{\gamma_i}{\lambda_i},$$

$$\frac{1}{\lambda_{\rm eff}^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}.$$

Critical points for sum of exponentials



Critical points for sum of exponentials

2. Scaling solution

n fields are copy of field ϕ_1 ,i.e., C is diagonal with the coefficients taking all the same value.

$$w_{\rm eff} = \frac{C_{11}}{\lambda_1 - C_{11}}, \qquad \Omega_\phi = \frac{3n - \lambda_1 C_{11} + C_{11}^2}{(\lambda_1 - C_{11})^2},$$

Only the field abundance has a mild dependence on n!

Exponential of sum

$$V(\phi_1, ..., \phi_n) = M^4 e^{-\sum_i \kappa\lambda_i\phi_i}$$

$$x_i \equiv \frac{\kappa\dot{\phi}_i}{\sqrt{6}H}, \qquad y^2 \equiv \frac{\kappa^2 V}{3H^2}, \qquad z_k^2 \equiv \frac{\kappa^2\rho_k}{3H^2}.$$

$$\begin{aligned}
x_i' &= -\left(3 + \frac{H'}{H}\right)x_i + \sqrt{\frac{3}{2}}\left(\lambda_i y^2 + \sum_k C_{ik} z_k^2\right), \\
y' &= -\sqrt{\frac{3}{2}}\left(\sum_i \lambda_i x_i + \sqrt{\frac{2}{3}}\frac{H'}{H}\right)y_i, \\
z_k' &= -\sqrt{\frac{3}{2}}\left(\sum_i C_{ik} x_i + \sqrt{\frac{3}{2}} + \sqrt{\frac{2}{3}}\frac{H'}{H}\right)z_k, \\
\frac{H'}{H} &= -\frac{3}{2}\left(1 + \sum_i x_i^2 - y^2\right),
\end{aligned}$$

Critical points for exponential of sum

1. Scalar field dominated solution

$$x_i = \frac{\lambda_i}{\sqrt{6}}$$

$$w_{\rm eff} = -1 + \frac{1}{3}\lambda_{\rm eff}^2,$$

$$\lambda_{\rm eff}^2 = \sum_i \lambda_i^2.$$

More fields means inflation more difficult to achieve.

Critical points for exponential of sum

2. Scaling solution (example 2 fields x 2 dark-matter)

It is useful to perform an orthogonal transformation Q, s.t.

$$
\begin{aligned}
\hat{x}_i &= Q_{ij} x_j , \\
\hat{\lambda}_i &= Q_{ij} \lambda_j , \\
\hat{C}_{ij} &= Q_{il} C_{lj} ,
\end{aligned}
$$

$$
\begin{aligned}
\hat{x}_1 &= \sqrt{\frac{3}{2}} \frac{1}{\lambda_{\text{eff}} - C_{\text{eff}}} , \\
\hat{x}_2 &= 0 ,
\end{aligned}
$$

Critical points for exponential of sum

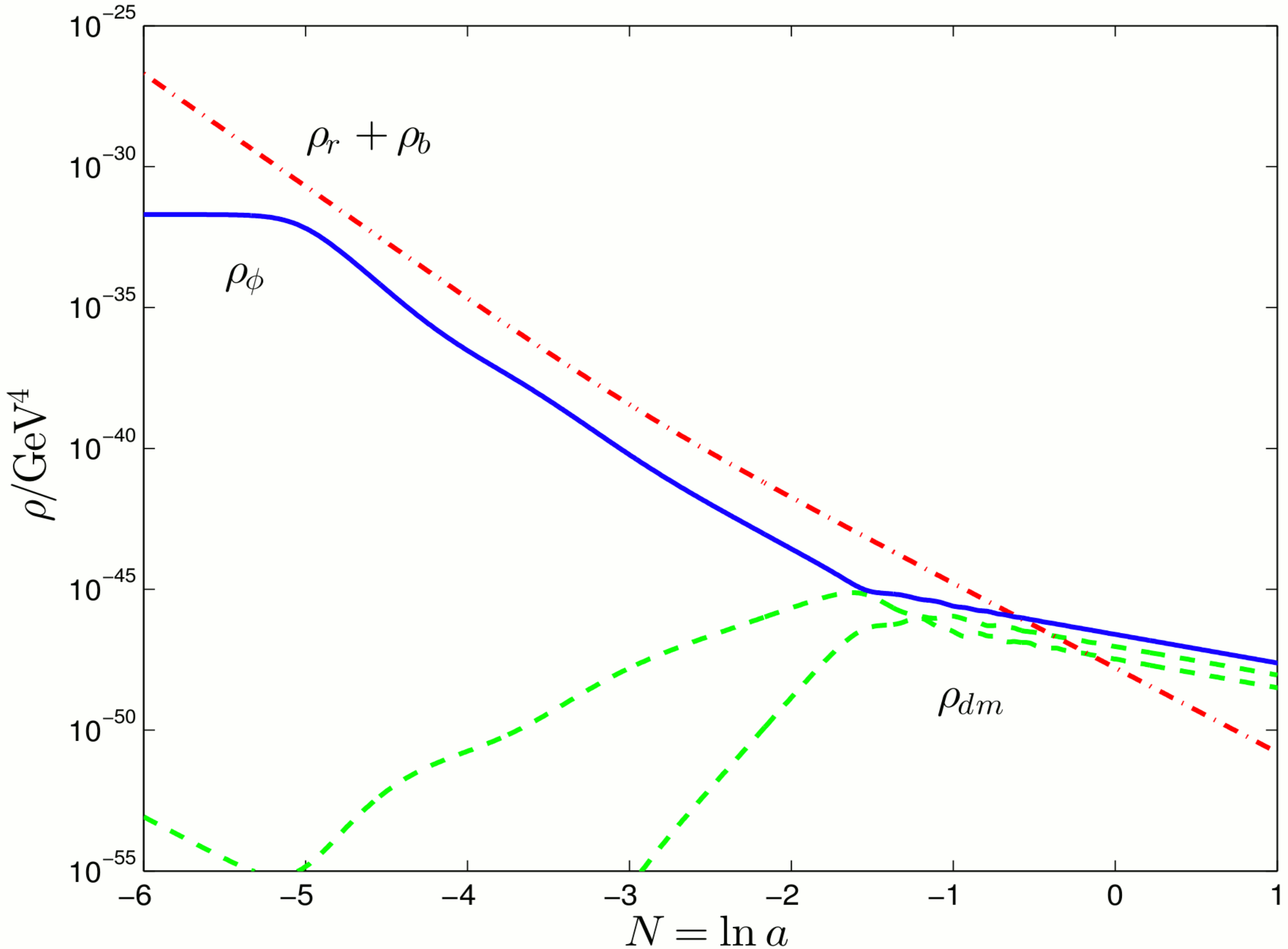
2. Scaling solution (example 2 fields x 2 dark-matter)

$$w_{\rm eff} = \frac{C_{\rm eff}}{\lambda_{\rm eff} - C_{\rm eff}}, \qquad \Omega_\phi = \frac{3 - \lambda_{\rm eff} C_{\rm eff} + C_{\rm eff}^2}{(\lambda_{\rm eff} - C_{\rm eff})^2},$$

$$\begin{aligned} C_{\rm eff} = \hat{C}_{11} &= Q_{11}C_{11} + Q_{12}C_{21} \\ &= \frac{C_{22}C_{11} - C_{21}C_{12}}{\sqrt{(C_{11} - C_{12})^2 + (C_{22} - C_{21})^2}} \end{aligned}$$

$$\begin{aligned} \lambda_{\rm eff} = \hat{\lambda}_1 &= Q_{11}\lambda_1 + Q_{12}\lambda_2 \\ &= \frac{(C_{22} - C_{21})\lambda_1 + (C_{11} - C_{12})\lambda_2}{\sqrt{(C_{11} - C_{12})^2 + (C_{22} - C_{21})^2}} \end{aligned}$$

Critical points for exponential of sum



Critical points for exponential of sum

2. Scaling solution

n fields are copy of field ϕ_1

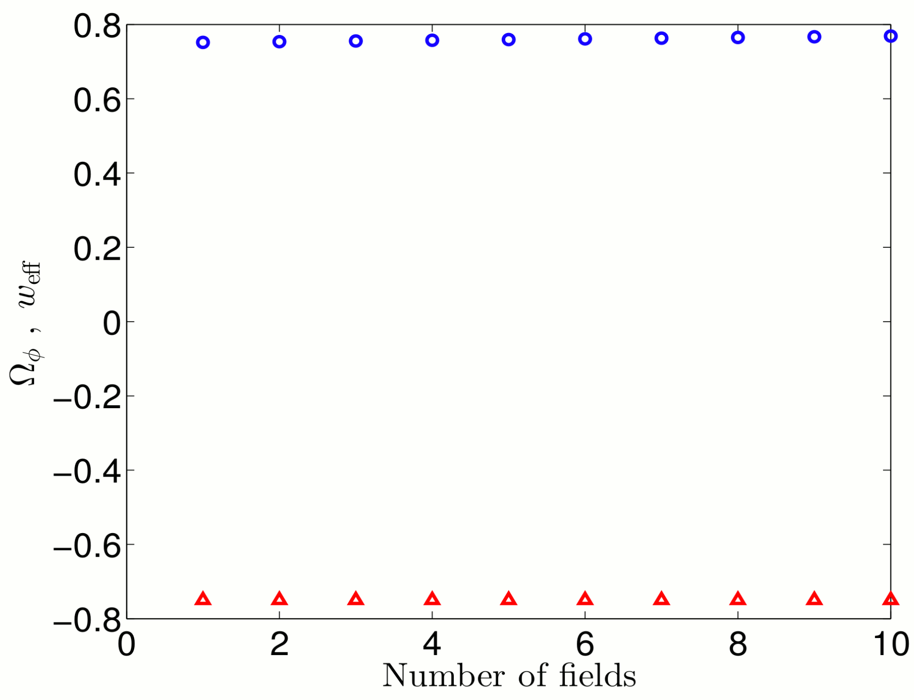
$$
\begin{aligned}
w_{\rm eff} &= \frac{C_{11}}{n\lambda_1 - C_{11}}, \\
\Omega_\phi &= \frac{3n - C_{11}\lambda_1 n + C_{11}^2}{(n\lambda_1 - C_{11})^2},
\end{aligned}
$$

Strong dependence on n!

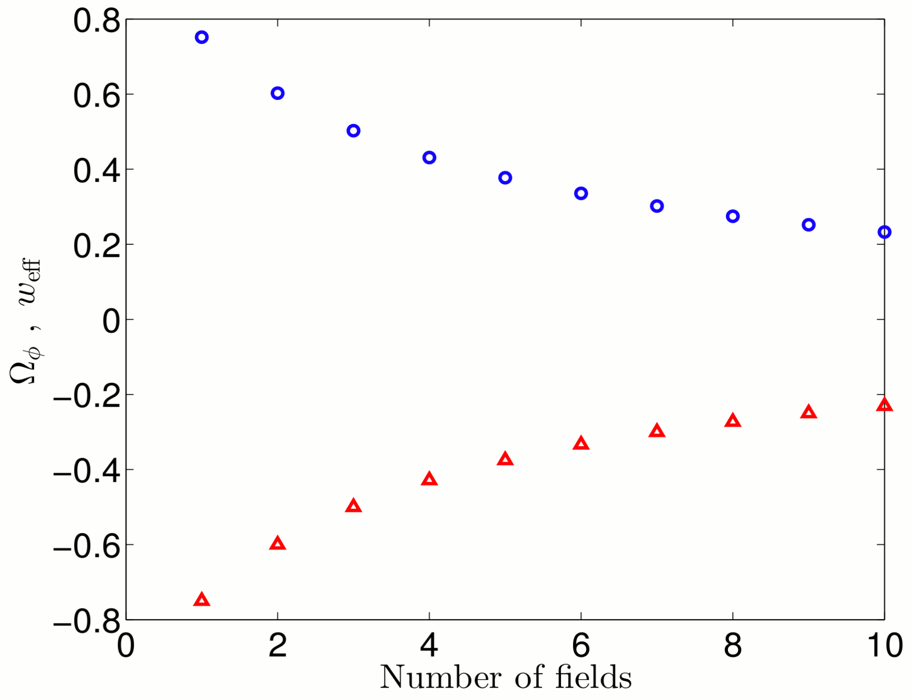
Critical points for exponential of sum

n fields are copy of field ϕ_1

$$V(\phi_1,\ldots,\phi_n) = M^4 \sum_i e^{-\kappa\lambda_i\phi_i}$$



$$V(\phi_1,\ldots,\phi_n) = M^4 e^{-\sum_i \kappa\lambda_i\phi_i}$$



Kinetic dominated solution

This solution is common to both potentials

$$w_{\mathrm{eff}} = \Omega_{\phi} = 1.$$

Subdominant potential

This solution is common to both potentials

$$x_i = \sqrt{\frac{2}{3}} C_{ik},$$

$$w_{\text{eff}} = \Omega_\phi = \sum_i x_i^2 = \frac{2}{3} \sum_i C_{ik}^2.$$

Matter dominated solution

When this relation is satisfied,

$$\sum_k C_{ik} z_k^2 = 0,$$

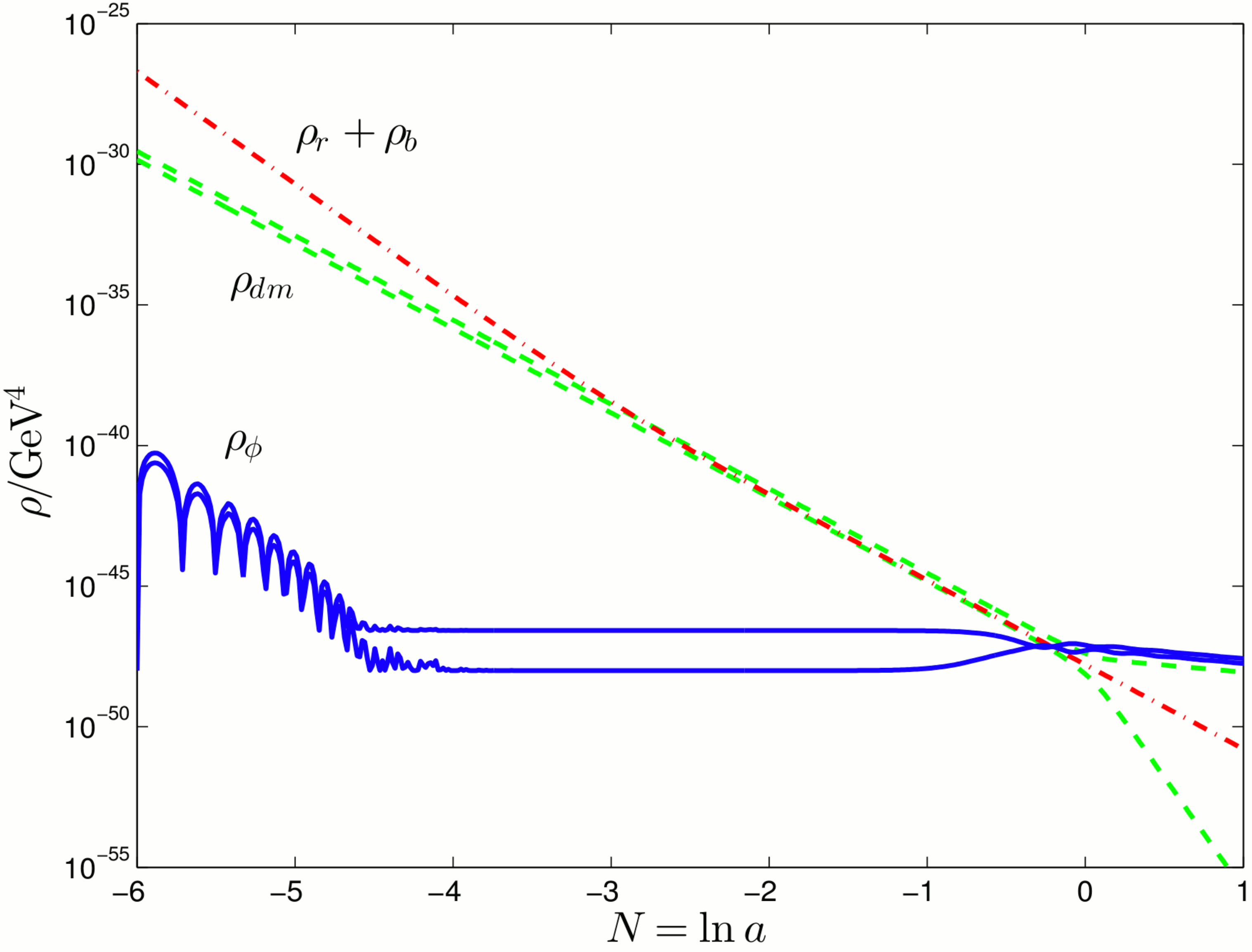
and the couplings obey,

$$\frac{C_{11}}{C_{12}} = \frac{C_{21}}{C_{22}},$$

Fields settle at the bottom of the effective potential along a flat direction defined by,

$$\begin{aligned} (C_{11} - C_{12})(\phi_1 - \phi_{10}) + (C_{21} - C_{22})(\phi_2 - \phi_{20}) &= \frac{1}{\kappa} \ln \left(-\frac{C_{11}}{C_{12}} \frac{\rho_{10}}{\rho_{20}} \right) \\ &= \frac{1}{\kappa} \ln \left(-\frac{C_{21}}{C_{22}} \frac{\rho_{10}}{\rho_{20}} \right) \end{aligned}$$

Matter dominated solution



Matter density contrast

Density contrast for dark matter component k:

$$\delta_k'' + \left[2 - \frac{3}{2}\sum_l \Omega_l - \sum_i \left(3x_i^2 + \sqrt{6}C_{ik}x_i\right)\right]\delta_k' - \frac{3}{2}\sum_l \left(1 + \sum_i C_{ik}C_{il}\right)\Omega_l\delta_l = 0.$$

Beware of excessive growth or damping which may arise from the third term.

2 fields x 2 dark matter solutions

$$\begin{aligned} \delta_1'' + A_1\delta_1' - B_1\delta_1 - B_2\delta_2 &= 0, \\ \delta_2'' + A_2\delta_2' - C_1\delta_1 - C_2\delta_2 &= 0, \end{aligned}$$

Where A, B, C are constants for the scaling solutions, and then $\delta_1 \propto e^{\xi N} \propto a^{\xi}$ $\delta_2 = b\delta_1$

When one of the dark matter components is much smaller than the other:

$$\xi \approx -\frac{1}{2}\left(A_1 \pm \sqrt{A_1^2 + 4B_1}\right),$$ << can be compared to obs.

$$b \approx \frac{1}{2}\frac{C_1}{B_1^2 + B_1A_2(A_1 - A_2)}\left[2B_1 + (A_1 - A_2)\left(A_1 \pm \sqrt{A_1^2 + 4B_1}\right)\right]$$

Summary

- The scalar field dominated solution: easier to obtain inflation for sum of exponentials;

- Scaling solution: Effective coupling depends on Cs and λs for sum of exponentials but only depends on Cs for the exponential of sum potential;

- Matter dominated epoch: couplings must obey relation for early dust behavior. Fields settle at the bottom of the effective potential which is a flat direction;

- Matter density contrast: Source term in the density contrast equation => excessive growth for large Cs.