# Constraining the Disformal Coupling

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Matter couples to a metric which can differ from the Einstein metric involved in the Einstein-Hilbert term with a constant Newton constant:

$$S = \int d^4x \sqrt{-g} \frac{R(g)}{16\pi G_N} + S_m(\psi^i, \tilde{g}_{\mu\nu})$$

Bekenstein (1992) showed that causality and the weak equivalence principle restricts the form of the auxiliary metric:

$$\tilde{g}_{\mu\nu} = A(\phi, X) g_{\mu\nu} + B(\phi, X) \partial_\mu \phi \partial_\nu \phi \qquad X = \frac{1}{2}(\partial\phi)^2$$

The scalar field φ is motivated by dark energy/modified gravity and is in general very light on large scales and screened on smaller scales. The conformal factor A(ϕ,X) is crucial for the screening properties.

What is the physics associated with the disformal coupling B(ϕ,X) ?

Simplified forms of this coupling appear in brane-world models and in the decoupling limit of "new" models of massive gravity. Phenomenologically, we shall restrict our investigation to:

$$\tilde{g}_{\mu\nu} = A(\phi) g_{\mu\nu} + \frac{2}{M^4} \partial_\mu \phi \partial_\nu \phi$$

The conformal factor must be almost constant cosmologically between Big Bang Nucleosynthesis and now as particle masses are proportional to $A(\phi)$, and they cannot vary by more than 10% since BBN. We normalise $A(\phi)=1$ in this talk. The disformal coupling could affect:

Gravity tests

Laboratory Physics: Casimir effect, scattering experiments…

Astrophysics: the burning of stars, the propagation of light…

Expanding the effect of the disformal coupling to leading order, we obtain a direct coupling with the energy-momentum tensor:

$$\mathcal{L} \supset \frac{1}{M^4}\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$$

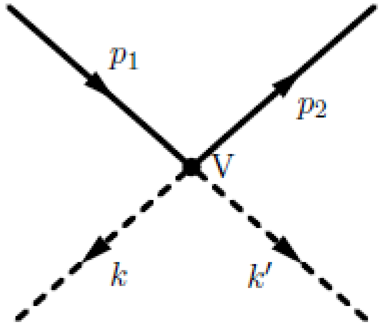
Gravity tests involving static objects are not affected by the disformal coupling.

$$T^{00} = \rho \quad \to \quad \frac{\dot{\phi}^2}{M^4}\rho \equiv 0$$

For mildly time-dependent situations, see J. Sakstein's talk. The static case is true to all order in 1/M.

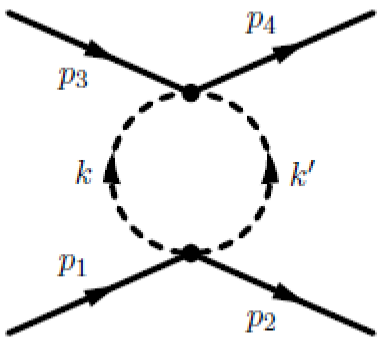
The only interaction that the disformal coupling induces is a quantum effect. The disformal coupling to the energy momentum tensor is a higher order operator involving two scalars, they can form a loop and lead to a loop induced interaction. In static situations, especially at short distances, this is the leading effect of the disformal coupling.

For matter made out of fermions, the interaction vertex is given by:




$$V = -\frac{1}{4M^4}\bar{u}(p_2)[(k.p_1)\not{k}' + (k'.p_1)\not{k} + (k.p_2)\not{k}' + (k'.p_2)\not{k}]u(p_1)$$

This leads to a one loop interaction between two particles, proportional to their masses:



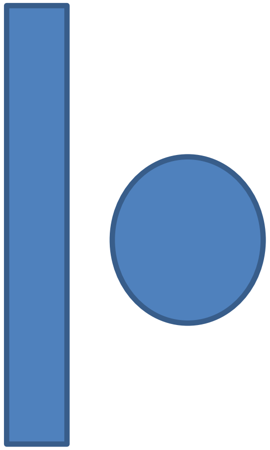

$$V(r) = -\frac{1}{32\pi^3 M^8}\frac{m_1 m_2}{r^7}$$

The calculation and the result is similar to the relativistic Van der Waals interaction between polarised molecules.

The Eot-wash experiment constrains the extra interaction compared to Newton's law down to a distance of 55 µm:

$$M \geq 0.06 \text{ MeV}$$

The Casimir force between a plate and a sphere can also be affected. Best tested at the 5% level for d=0.5µm with a sphere of radius R=11.3 cm and a plate of width a=0.5 cm:



$$\frac{F}{F_C} = \frac{315}{8\pi^4}\frac{\rho_1\rho_2 a R^4}{M^8 d^5}$$

$$M \geq 0.1 \text{ GeV}$$

One must go to shorter distance scales to feel the full effect of the disformal interaction. The first obvious place is atomic physics where new interactions displace the energy levels of atoms.

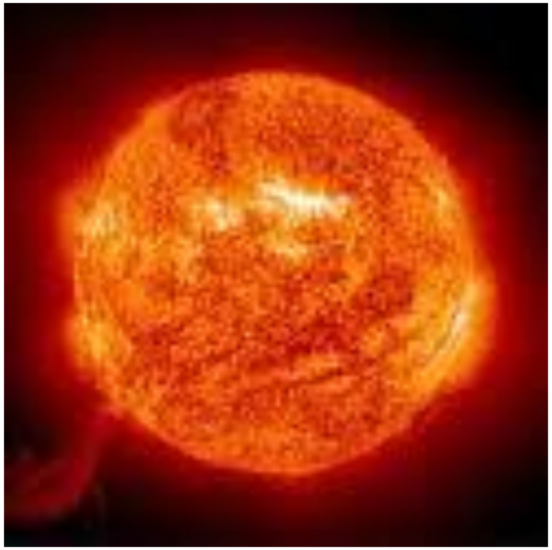
$$\delta E = -\frac{m_e m_N}{32\pi^3 M^8} < E|r^{-7}|E >$$

One of the most sensitive tests is the 1s-2s gap for hydrogen at the one billionth of eV level:

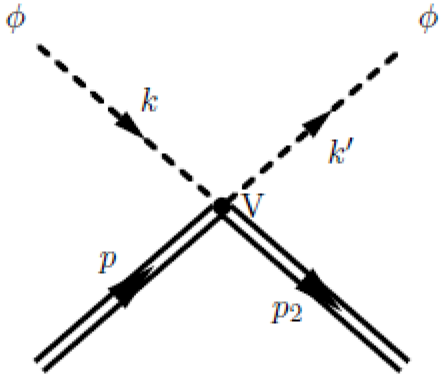
$$M \geq 0.17 \text{ GeV}$$

The disformal interaction could also play a role in the proton radius puzzle… One can test it to even lower scales in neutron scattering experiments between one neutron and neutral gases like Argon where one measures the scattering asymmetry between the scattered and back scattered neutrons with an angle of 45 degrees. The bound is weaker than the atomic bound.

As the axions, the disformally coupled scalars can have an effect at high density and high temperature inside the inferno at the core of stars. From the gentle burning of main sequence stars, to helium burning stars and then the explosion of supernovae, the processes involve higher energies (hence shorter distances) and electromagnetic  to strong interaction processes.

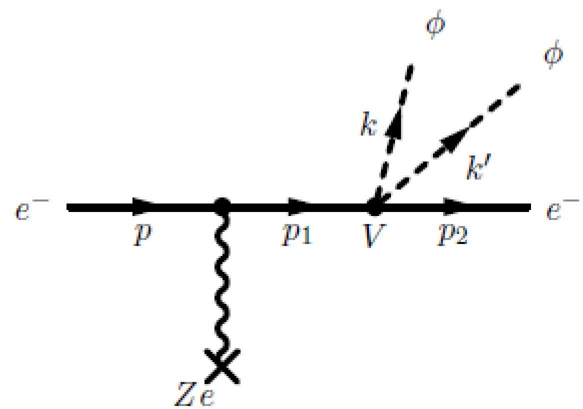


Scalars increase the burning rate provided they can escape the star itself. This can be hampered if the scalar have a mean free path due to their interactions with nucleons which is smaller than the size of the star. We will see that the mean free path is always large for the values of M compatible with the absence of too much burning inside a star.




$$l = \frac{16\pi(N+Z)}{Z} \frac{M^8}{\rho T^4 m_p}$$

Bremsstrahlung is one of the most common processes in stars. Here two scalars can be emitted from one electron in the electric field of a nucleus.



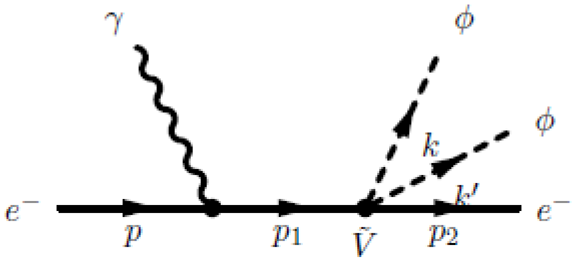

$$\epsilon = \frac{Z^2\alpha^2 m_e}{210\pi^3 A m_p}\rho\left(\frac{T}{M}\right)^8 \frac{1}{(2\pi m_e T)^{3/2}} g\left(\frac{m_D^2}{2m_e T}\right),$$

$$g(x) = \int_0^\infty du \frac{u^9 e^{-u}}{(u+x)^2}.$$

The strongest constraint comes from stars for which helium burns: $\epsilon_{HB} \leq 10\ \mathrm{erg/s \cdot g}$

$$M \geq 173\ \mathrm{MeV}$$

Compton Scattering is only a very important process. Here photons interact with a proton and emit two scalars as the dominant channel.




$$\epsilon = \frac{3dh}{8\pi(2\pi)^3}\frac{\alpha Z m_p}{(Z+N)}\left(\frac{T}{M}\right)^8 ,$$

$$h = \int_0^\infty \frac{u^7 du}{e^u - 1} .$$

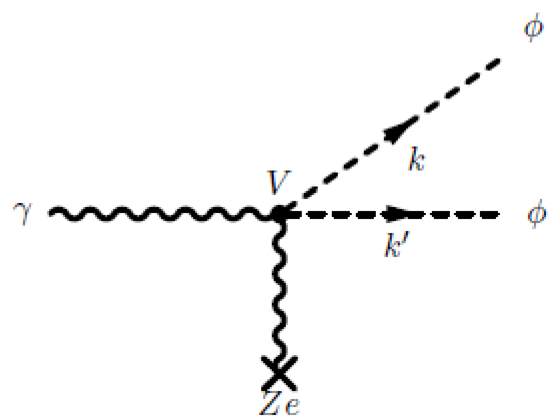
Again, the strongest bound comes from helium burning stars:

$$M \geq 811 \text{ MeV}$$

Finally, the Primakoff effect where one photon in the electric field of a nucleus emits two scalars.

$$\epsilon_\gamma = \frac{UhZ^2\alpha}{4\pi(2\pi)^3}\frac{T^{14}}{m_p m_D^4 M^8},$$

$$h = \int_0^\infty dx \frac{x^{13}}{e^x - 1}.$$

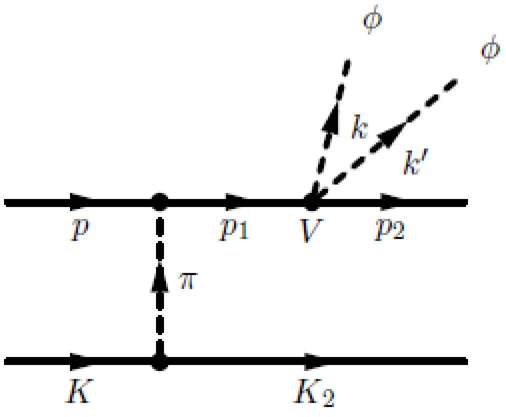



$$V_\gamma = \frac{1}{M^4}\Big((p.k)(p'.k')(\epsilon.\epsilon') - (p.k)(k'.\epsilon')(p'.\epsilon) - (p.\epsilon')(k.\epsilon)(p'.\epsilon') + (p.p')(k.\epsilon)(k'.\epsilon') - \frac{(k.k')}{2}[(\epsilon.\epsilon')(p.p') - (p'.\epsilon)(p.\epsilon')]\Big)$$

The outcome is a bound which is weaker than for Compton scattering:

$$M \geq 346\ \mathrm{MeV}$$

In the core of type II supernovae, the main nuclear process involves the exchange of one pion between nucleon. This is a dirty process as the coupling between pions and nucleons is >1 ! Still, if you believe in nuclear physics, you can calculate the tree level diagram corresponding to the emission of two scalars.

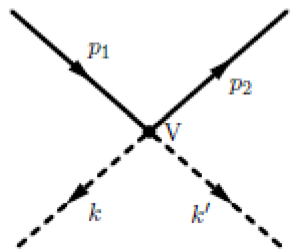



$$\epsilon = \frac{A^4 \alpha_\pi^2}{105\pi^3}\rho \left(\frac{T}{M}\right)^8 \frac{1}{(2\pi m_N T)^{3/2}} g\left(\frac{m_\pi^2}{2 m_N T}\right) .$$

Due to the high density and 30 MeV temperature, the bound is much tighter:

$$M \geq 92 \text{ GeV}$$

The most stringent constraints on the disformal coupling come from particle physics at the LHC. Signals coming from the annihilation of two fermions into invisible particles are difficult to track, they can be analysed when one W is emitted by the parent quark, decaying into a lepton and a neutrino. The bound for the scattering cross section of these events translates into a bound on the associated fermion annihilation:



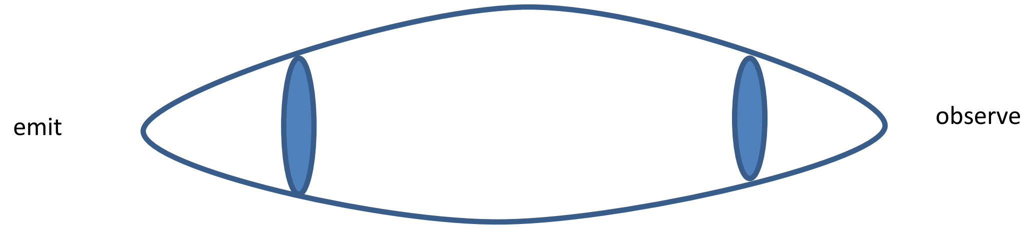

$$\sigma = \frac{m^2 E^4}{\pi M^8 \sqrt{1 - \frac{m^2}{E^2}}} < 0.2 \text{ pb}$$

This results in a tight bound on the disformal coupling:

$$M > 120 \text{ GeV}$$

The disformal coupling has also interesting effects in cosmology as it modifies the propagation of light: the speed of light becomes time dependent. This changes the distances used in cosmology:

$$d_A^2 = \frac{dS_{\rm emit}}{d\Omega_{\rm obs}} \qquad d_L^2 = \frac{dL_{\rm emit}}{4\pi dF_{\rm obs}}$$




$dS_{\rm emit}\ ,\ d\Omega_{\rm emit}$ $\qquad$ $dS_{\rm obs}\ ,\ d\Omega_{\rm obs}$

$$d_L = \frac{c_{\rm obs}^2}{c_{\rm emit}^2}(1+z)^2 d_A$$

This can be constrained by the observation of clusters in X-ray and the SZ effect, between z=0.35 and now, we must have:

$$|\frac{\delta c}{c}| \leq 0.06$$

The disformal coupling modifies the CMB spectrum as the angular distances are changed:

$$I_{\text{obs}} = (\frac{c_{\text{emit}}}{c_{\text{obs}}})^4 I_{\text{LS}}$$

The observed spectrum is distorted if the initial one is a pure black body. The effective chemical potential is:

$$\mu = 2(e^{-k/T_0} - 1)\frac{\delta c}{c}$$

This is constrained to be very small between last scattering and now:

$$|\frac{\delta c}{c}| < 2.6\ 10^{-4}$$

The speed of light is determined by:

$$c^2 = 1 - \frac{2\dot{\phi}^2}{M^4}$$

It depends on the cosmological model for the scalar field ϕ.

# GALILEONS

Galileons are  fundamental examples as they arise in the decoupling limit of “new” massive gravity and can describe dark energy even in the absence of a bare cosmological constant.

$$\mathcal{L} = -\frac{c_2}{2}(\partial\phi)^2 - \frac{c_3}{\Lambda^3}D^2\phi(\partial\phi)^2 - \frac{c_4}{\Lambda^6}\mathcal{L}_4 - \frac{c_5}{\Lambda^9}\mathcal{L}_5 + \frac{c_0\phi}{m_{\rm Pl}}T - \frac{c_G}{\Lambda^3 m_{\rm Pl}}\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$$

$$\mathcal{L}_4 = (\partial\phi)2[2(\Box\phi)^2 - 2D_\mu D_\nu\phi D^\nu D^\mu\phi - R\frac{(\partial\phi)^2}{2}]$$
$$\mathcal{L}_5 = (\partial\phi)^2[(\Box\phi)^3 - 3(\Box\phi)D_\mu D_\nu\phi D^\nu D^\mu\phi + 2D_\mu D^\nu\phi D_\nu D^\rho\phi D_\rho D^\mu\phi - 6D_\mu\phi D^\mu D^\nu\phi D^\rho\phi G_{\nu\rho}]$$

For the cosmological Galileons, acceleration sets:

$$\Lambda^3 = H_0^2 m_{\rm Pl}$$

The coupling c0 corresponds to the conformal coupling of the model. When c0 is zero, the Galileon is simply a dark energy model coupled only disformally to matter. When c0 is non-vanishing, the Vainshtein mechanism screens the gravitational effects of the massless Galileons in gravity experiments.

The cosmological equations for the model only involve:

$$x = \frac{\phi'}{m_{\rm Pl}}, \; y = x', H \qquad {}' = d/d\ln a$$

They can be written in terms of:

$$\bar{x} = \frac{x}{x_0}, \; \bar{y} = \frac{y}{y_0}, \; \bar{H} = \frac{H}{H_0}, \; \bar{c}_i = c_i x_0^i, \; \bar{c}_G = c_G x_0^2, \bar{c}_0 = c_0 x_0$$

**Table 6.** Disformally-coupled Galileon model best-fit values from growth rate measurements combined with JLA+BAO+WMAP9 data.

| Probe | $\Omega_m^0$ | $\bar{c}_2$ | $\bar{c}_3$ | $\bar{c}_4$ | $\bar{c}_G$ | $h$ | $\Omega_b^0 h^2$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|
| JLA+BAO+WMAP9 | $0.282^{+0.015}_{-0.009}$ | $-4.811^{+1.427}_{-1.990}$ | $-1.525^{+0.637}_{-1.073}$ | $-0.531^{+0.209}_{-0.275}$ | $0.183^{+0.188}_{-0.133}$ | 0.689 | 0.0228 | 693.2 |
| All | $0.279^{+0.013}_{-0.008}$ | $-3.401^{+0.315}_{-0.565}$ | $-1.043^{+0.195}_{-0.252}$ | $-0.614^{+0.087}_{-0.076}$ | $0.147^{+0.077}_{-0.060}$ | 0.719 | 0.0220 | 714.8 |

Neveu et al. (2014)

As cG >0, this corresponds to an increase of the speed of light. Not compatible with the original hypothesis on the disformal metric concerning causality as the speed of light in a cosmological background is larger than in vacuum.

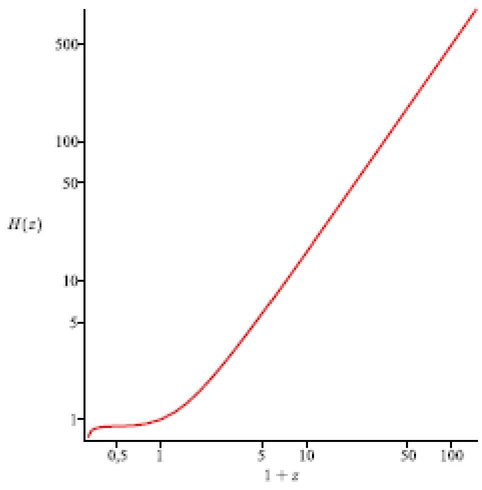
As c2<0, this corresponds to a ghost in Minkowski space. As c0=0, no conformal coupling and no Vainshtein effect, this implies instabilities of the laboratory vacuum.

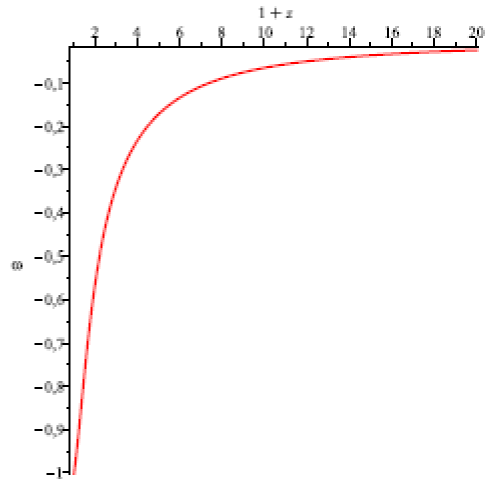
Reassess the cosmological vacua for Galileons with the theoretical priors:

$$c_2 > 0, \quad c_G < 0$$

In the absence of a conformal coupling, the background is either very far from w=-1 (dark energy) or plagued with instabilities with negative speed of sound squared and singularities in the recent past of the Universe. To get a sensible background, with w=-1 or so, one must take c0>0. We normalise the initial conditions such that:

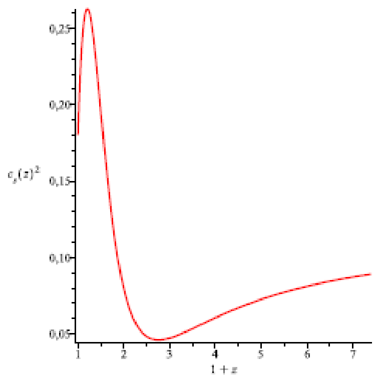
$$\bar{c}_2 = 1$$

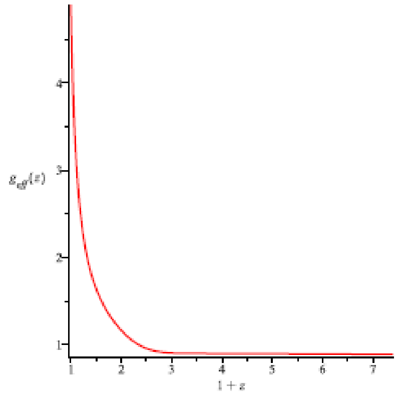



$$\bar{c}_3 = 1.2, \;\; \bar{c}_0 = 0.32, \;\; \bar{c}_G = 0.02$$

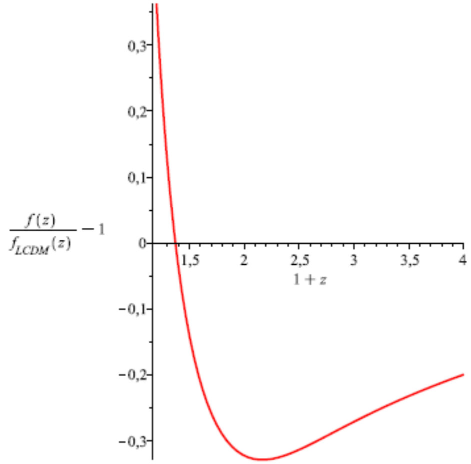
The background converges to a quasi de Sitter attractor now, destabilised in our future by c0>0. The disformal coupling can be put to zero and w=-1 can be achieved by increasing c0. The effect of c5 is to induce singularities and instabilities, set to zero.



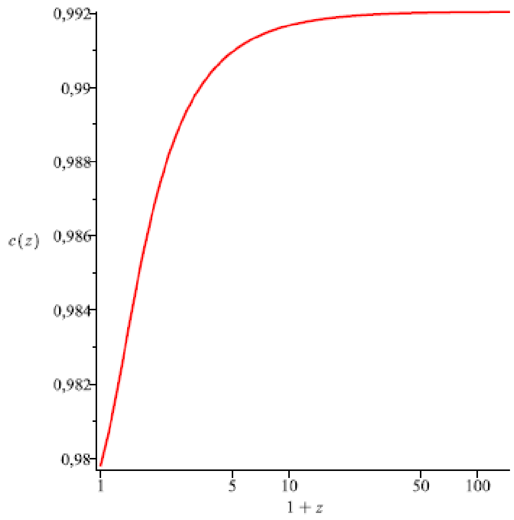

The growth of structure is also affected by the Galileon:

$$\delta'' + (1 + 2\frac{H'}{H})\delta' - \frac{3}{2}g_{eff}\delta = 0$$



This might be compatible with present data, could be ruled out by EUCLID, but does it make sense in terms of the variations of the speed of light and Newton's constant?

$$c^2 = 1 + 2\bar{c}_G \bar{H}^2 \bar{x}^2$$




This is compatible with the duality relation but not compatible with the CMB distorsions:

$$|\bar{c}_G| < 10^{-4}$$

The disformal coupling cannot play any cosmological role for Galileons.

The disformal coupling is determined by:

$$c_G = -\frac{H_0^2 m_{\rm Pl}^2}{M^4}$$

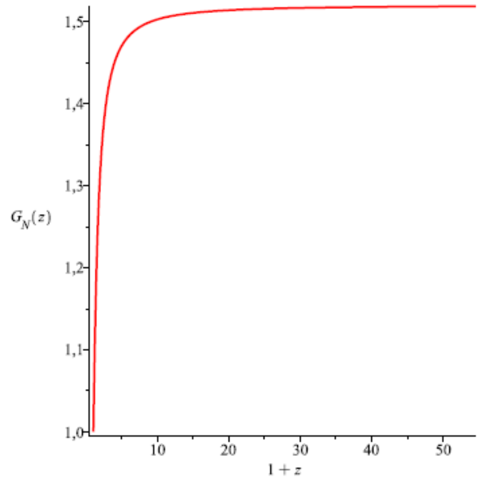
This is a tiny number as M>120 GeV!   cG is suppressed by at least 56 orders of magnitude… barring unnaturally large initial values of x0>>1, this is way tighter than the bound from the variation of c. When c0 is not zero, cG is modified by the Vainshtein effect:

$$c_G \to Z_{\oplus} c_G$$

Gravitational tests of Galileons on earth require Z>400, certainly not enough to compensate the 56 order of magnitude.

When c0 is non-zero, the local Newton constant is the bare one in the Lagrangian due to the screening of the scalar field deep inside the Vainshtein radius while the cosmological value in the unscreened cosmological vacuum is:

$$G_N = G_N^0(1 - 2\bar{c}_0\bar{y})$$




Too large a variation between now and BBN.

$$\bar{c}_0 < 0.1$$

$$\bar{c}_G = 0 \ , \bar{c}_0 < 0.1 \rightarrow w_0 > -0.7$$

Difficult to reduce the variation of Newton's constant and act as dark energy…

Disformal couplings are tightly constrained by high energy physics. They are “non-renormalisable” operators sensitive to UV properties. Colliders give:

$$M > 120\mathrm{GeV}$$

They also induce a variation of the speed of light cosmologically.

For cosmological Galileon models, the disformal coupling is essentially zero. Difficult to reconcile dark energy and a small variation of Newton’s constant between now and BBN.