Dark Energy Interactions – the Nordic Cosmology Workshop

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DARK ENERGY ELECTRODYNAMICS

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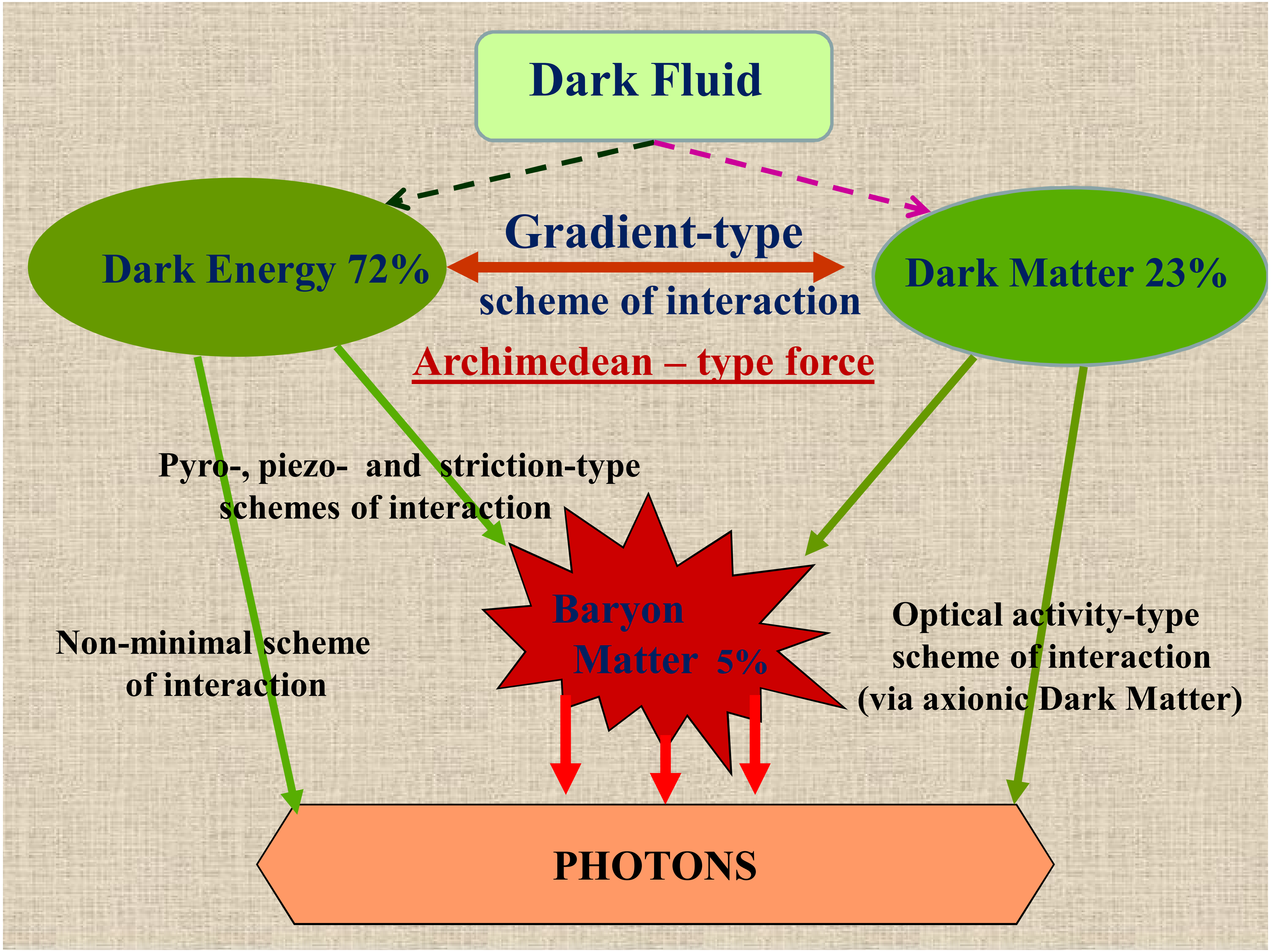
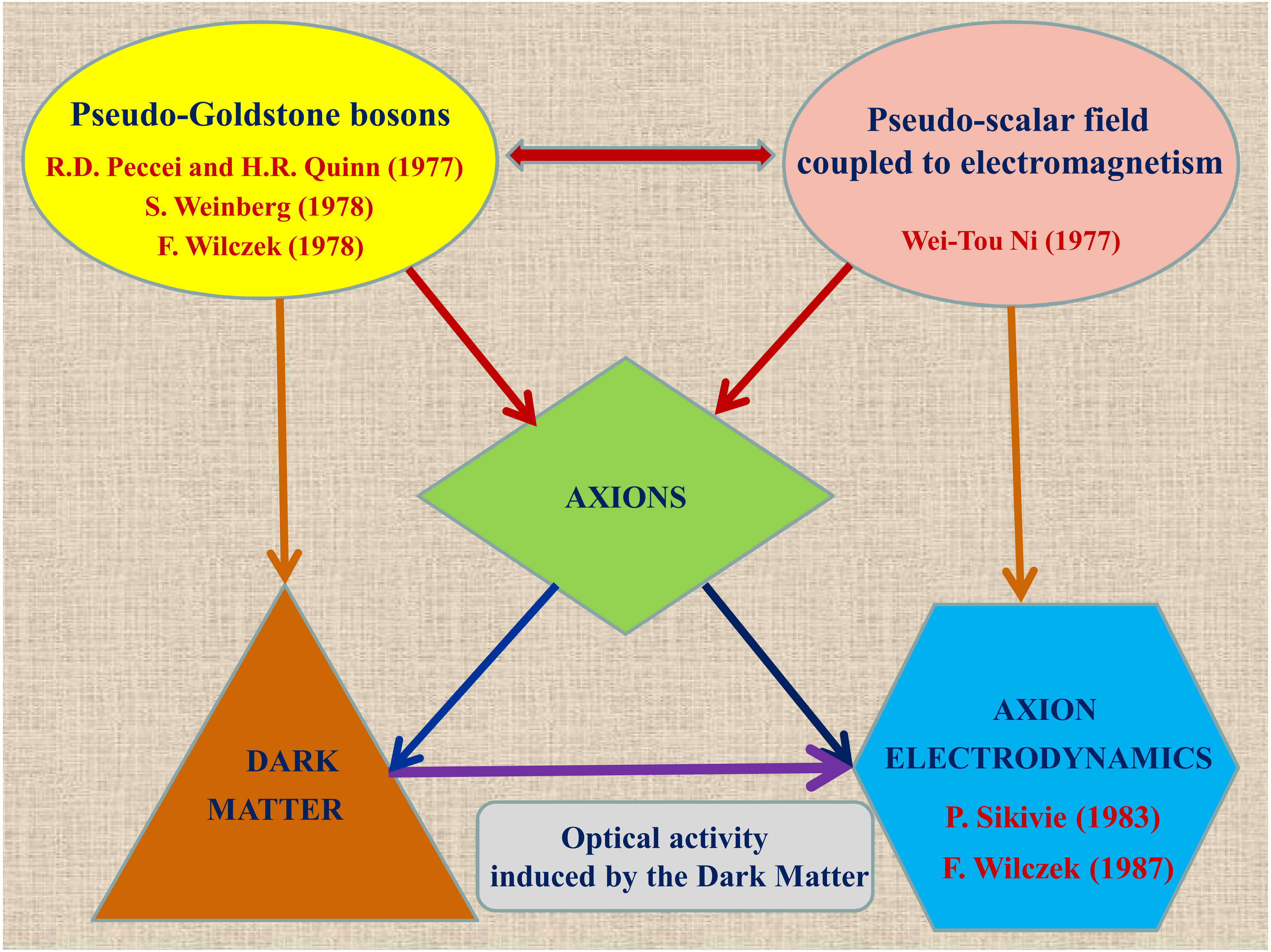
Plan of the talk

- **Motivation and analogies**
- **Mathematical formalism**
- **Exact analytical solutions**
- **Qualitative analysis and numerical simulation**
- **Cosmological applications: Unlighted epochs**

***Main idea*: although Dark Energy seems to be an electrically neutral substratum, it can be considered as a cosmic medium, within which electromagnetic waves propagate; and we can search for fingerprints of a non-stationary Dark Energy in cosmic electrodynamic systems…**


Dark Fluid
Dark Energy 72%
Gradient-type
scheme of interaction
Dark Matter 23%
Archimedean – type force
Pyro-, piezo- and striction-type
schemes of interaction
Baryon
Matter 5%
Optical activity-type
scheme of interaction
(via axionic Dark Matter)
Non-minimal scheme
of interaction
PHOTONS


Pseudo-Goldstone bosons
R.D. Peccei and H.R. Quinn (1977)
S. Weinberg (1978)
F. Wilczek (1978)
Pseudo-scalar field
coupled to electromagnetism
Wei-Tou Ni (1977)
AXIONS
DARK
MATTER
AXION
ELECTRODYNAMICS
P. Sikivie (1983)
F. Wilczek (1987)
Optical activity
induced by the Dark Matter

The talk is based on the results published in the papers

- *Balakin A.B., Bochkarev V.V.* Archimedean-type force in a cosmic dark fluid.

I. Phys. Rev. D 83, 024035 (2011) arXiv:1012.2431
II. Phys. Rev. D 83, 024036 (2011) arXiv:1012.2433
III. Phys. Rev. D 87, 024006 (2013) arXiv:1212.4094

- *Balakin A.B., Bochkarev V.V. and Lemos J.P.S.* Non- minimal coupling…

I. Phys. Rev. D 85, 064015 (2012) arXiv:1201.2948
II. Phys. Rev. D 77, 084013 (2008) arXiv:0712.4066

- *Balakin A.B., Dolbilova N.N.* Electrodynamic phenomena induced by a dark fluid: Analogs of pyromagnetic, piezoelectric, and striction effects.
  Phys. Rev. D , 89, 104012 (2014) arXiv:1404.5058

- *Balakin A.B., Ni Wei-Tou*. Non- minimal coupling of photons and axions

I. Classical and Quantum Gravity, 27, 055003 (2010) arXiv:0911.2946
II. Classical and Quantum Gravity, 31, 105002 (2014) arXiv:1403.6711

Mathematical formalism:

Action functional describing mentioned indirect schemes of interactions between Dark Energy, Dark Matter and Electromagnetic field

Pseudoscalar (axion) field

DE energy density evolution rate

Linear response tensor including nonminimal cross-terms

Maxwell tensor

Pyro coefficients

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2\kappa} + L_{(\mathrm{DE})} + \frac{1}{4} C^{ikmn}_{(0)} F_{ik} F_{mn} + L_{(\mathrm{m})} + \frac{1}{2} \Psi_0^2 \left[-\nabla_k \phi \nabla^k \phi + \mathcal{V}(\phi^2) \right] + \frac{1}{4} \phi F^*_{mn} F^{mn} + \frac{1}{2} \left(\pi^{ik} + \frac{1}{2} \lambda^{ikmn} F_{mn} \right) F_{ik} DW + \frac{1}{2} \left(\mathcal{D}^{ikpq} + \frac{1}{2} Q^{ikmnpq} F_{mn} \right) F_{ik} \mathcal{P}_{pq} \right\},$$

Piezo coefficients

Striction coefficients

DE pressure tensor

Electrodynamic equations

$$\nabla_k H^{ik} = -\frac{4\pi}{c} I^i$$

$$\nabla_k F^{*ik} = 0,$$

Excitation tensor

$$H^{ik} = \mathcal{H}^{ik} + \phi F^{*ik} + \mathcal{C}^{ikmn} F_{mn}$$

Tensor of spontaneous polarization-magnetization

$$\mathcal{H}^{ik} \equiv \pi^{ik} DW + \mathcal{D}^{ikpq} \mathcal{P}_{pq}$$

Total linear response tensor

$$\mathcal{C}^{ikmn} \equiv C^{ikmn}_{(0)} + \lambda^{ikmn} DW + Q^{ikmnpq} \mathcal{P}_{pq}$$

DE stress-energy tensor

$$T^{(\mathrm{DE})}_{ik} \equiv W U_i U_k + \mathcal{P}_{ik}$$

Convective derivative

$$D \equiv U^i \nabla_i .$$

Extended electric, magnetic permittivities and tensor of magneto-electric cross-effects

$$\varepsilon^{im} = 2\mathcal{C}^{ikmn} U_k U_n = \varepsilon^{im}_{(0)} + \sigma^{im} DW + \alpha^{im(pq)} \mathcal{P}_{pq}$$

$$(\mu^{-1})^{ab} = -\frac{1}{2}\eta^{a}{}_{ik}\mathcal{C}^{ikmn}\eta_{mn}{}^{b} = (\mu^{-1})^{ab}_{(0)} + \rho^{ab} DW + \beta^{ab(pq)} \mathcal{P}_{pq}$$

$$\nu^{am} = \eta^{a}{}_{ik}\mathcal{C}^{ikmn} U_n = \nu^{am}_{(0)} + \omega^{am} DW + \gamma^{am(pq)} \mathcal{P}_{pq}$$

based on the standard decomposition of the linear response tensor

$$\mathcal{C}^{ikmn} = (\varepsilon^{i[m} U^{n]} U^k - \varepsilon^{k[m} U^{n]} U^i) - \frac{1}{2}\eta^{ikl}(\mu^{-1})_{ls}\eta^{mns} + \eta^{ikl} U^{[m} \nu_l{}^{n]} + \eta^{lmn} U^{[i} \nu_l{}^{k]}$$

Model of spatially isotropic non-stationary Dark Energy

$$\mathcal{P}_{ik} = -P\Delta_{ik}$$

$$\varepsilon^{im} = \Delta^{im}\varepsilon$$

$$(\mu^{-1})_{ab} = \frac{1}{\mu}\Delta_{ab}$$

$$\nu^{am} = 0$$

$$C^{ikmn}_{(0)} = \frac{1}{2\mu_{(0)}}[g^{ikmn} + (\varepsilon_{(0)}\mu_{(0)} - 1)(g^{ikmn} - \Delta^{ikmn})]$$

Reduced standard part of linear response tensor

$$\begin{aligned} Q^{ikmnpq} = \frac{1}{2}[&\alpha_{(1)}\Delta^{pq}(g^{ikmn} - \Delta^{ikmn}) \\ &+ \alpha_{(2)}U_l U_s(g^{iklp}g^{mnsq} + g^{iklq}g^{mnsp}) \\ &+ \beta_{(1)}\Delta^{pq}\Delta^{ikmn} - \beta_{(2)}(\eta^{ikp}\eta^{mnq} + \eta^{ikq}\eta^{mnp})] \end{aligned}$$

Reduced tensor of striction coefficients

Auxiliary tensors

$$g^{ikmn} \equiv g^{im}g^{kn} - g^{in}g^{km}$$

$$\Delta^{ikmn} \equiv \Delta^{im}\Delta^{kn} - \Delta^{in}\Delta^{km}$$

$$\Delta^l_i \equiv \delta^l_i - U^l U_i$$

$$\eta^{ikl} \equiv \epsilon^{ikls}U_s$$

Dark Energy fingerprints in a cosmic electrodynamics: Unlighted epochs in the Universe history

[epochs for which the square of an effective refraction index of the cosmic medium is negative, and thus propagation of electromagnetic waves is stopped]

Phase velocity

$$V_{\rm ph} \equiv \frac{\omega}{k} = \frac{1}{n(t)}$$

$$V_{\rm gr} = \frac{2n}{n^2 + 1}$$

Group velocity

Refraction index

$$n^2(t) \equiv \varepsilon(t)\mu(t)$$

$$n^2(t) < 0$$

How does the non-stationary Dark Energy influence the refraction index ?
We elaborated in detail two schemes of indirect interactions.

I. Archimedean (gradient – type) scheme with non-minimal coupling

FLRW–type space-time

$$ds^2 = dt^2 - a^2(t)[(dx^1)^2 + (dx^2)^2 + (dx^3)^2].$$

Gravity field equations

$$H^2 = \frac{8\pi G}{3}(\rho + E),$$

Dark Energy **Dark Matter**

energy densities

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[(\rho + E) + 3(\Pi + P)],$$

Dark Energy **Dark Matter**

pressures

Compatibility condition

$$\dot{\rho} + \dot{E} + 3H(\rho + E + \Pi + P) = 0$$

Cosmological constant is included into the DE energy density and pressure

Kinetic description of the Dark Matter

Kinetic equation

$$\frac{p^i}{m_{(a)}}\left(\frac{\partial}{\partial x^i} - \Gamma^k_{il} p^l \frac{\partial}{\partial p^k}\right) f_{(a)} + \frac{\partial}{\partial p^i}\left[\mathcal{F}^i_{(a)} f_{(a)}\right] = 0$$

Relativistic four-gradient-type force (Archimedean-type force), which acts on the DM particles in a DE reservoir

$$\mathcal{F}^{\,i}_{(a)} = m_{(a)} \mathcal{V}_{(a)} \left[g^{ik} - \frac{p^i p^k}{p^l p_l}\right] \nabla_k \Pi$$

New coupling constants

Classical 3-dimensional analog: the standard Archimedean force

$$\vec{F}_{(\mathrm{Arch})} = -V_0 \vec{\nabla} P_{(\mathrm{Pascal})}$$

Macroscopic moments of distribution functions of the DM particles

DM energy density

$$E(x) = \sum_{(a)} \frac{4\pi m^4_{(a)}}{x^3} \int_0^\infty q^2 dq f^0_{(a)}(q^2) \sqrt{1 + q^2 F_{(a)}(x)}$$

DM pressure

$$P(x) = \sum_{(a)} \frac{4\pi m^4_{(a)} F_{(a)}(x)}{3x^3} \int_0^\infty \frac{q^4 dq f^0_{(a)}(q^2)}{\sqrt{1 + q^2 F_{(a)}(x)}}$$

Coupling functions

$$F_{(a)}(x) = \frac{1}{x^2} \exp\{2\mathcal{V}_{(a)}[\Pi(1) - \Pi(x)]\}$$

$$x = \frac{a(t)}{a(t_0)}$$

DM balance equations

$$\dot{E} + 3H(E + P) = -\mathcal{Q}$$

$$\mathcal{Q} \equiv 3\dot{\Pi} \sum_{(a)} \mathcal{V}_{(a)} P_{(a)}$$

Hydrodynamic description of Dark Energy

Extended constitutive equation for DE

$$\rho(t) = \rho_0 + \sigma\Pi + \frac{\xi}{H(t)}\dot{\Pi}$$

Rheologic–type contribution

Balance equations for DE

$$\dot{\rho} + 3H(\rho + \Pi) = \mathcal{Q}$$

$$\mathcal{Q} \equiv 3\dot{\Pi}\sum_{(a)} \mathcal{V}_{(a)} P_{(a)}$$

Key equation for the DE pressure

$$\xi x^2 \Pi''(x) + x\Pi'(x)(4\xi + \sigma) + 3(1 + \sigma)\Pi + 3\rho_0 = \mathcal{J}(x)$$

Nonlinear Archimedean source in the key equation of DE evolution describing dark matter backreaction on the DE

$$\mathcal{J}(x) \equiv -\sum_{(\mathrm{a})} E_{(\mathrm{a})} \frac{[x^2 F_{(\mathrm{a})}(x)]'}{2x^4} \int_0^\infty \frac{q^4 dq e^{-\lambda_{(\mathrm{a})}\sqrt{1+q^2}}}{\sqrt{1+q^2 F_{(\mathrm{a})}(x)}}$$

Analytic example I: the case of massless (or ultrarelativistic) DM

$$\mathcal{J}_{(0)}(x) = E_{(0)} \frac{\mathcal{V}_{(0)}\Pi'(x)}{x^3} \exp\{\mathcal{V}_{(0)}[\Pi(1) - \Pi(x)]\}$$

Analytic example II: the case of cold DM

$$\mathcal{J}_{(\mathrm{C})}(x) = \frac{3N_{(\mathrm{C})}T_{(\mathrm{C})}\mathcal{V}_{(\mathrm{C})}\Pi'(x)}{x^4} e^{2\mathcal{V}_{(\mathrm{C})}[\Pi(1)-\Pi(x)]}$$

Exact solution of the anti-Gaussian type

$$\sigma = -1$$

Exact logarithmic solutions for DE pressure and energy density

$$\Pi(x) = \Pi(1) - \frac{4}{\mathcal{V}_{(0)}} \log x$$

$$\rho(x) = \rho(1) + \frac{4}{\mathcal{V}_{(0)}} \log x$$

$$E_{(0)} = \left[\frac{3\xi - 1}{\mathcal{V}_{(0)}} - \frac{3}{4}\rho_0\right]$$

$$E_{(a)} \equiv \frac{N_{(a)} m_{(a)} \lambda_{(a)}}{K_2(\lambda_{(a)})}$$

$$\lambda_{(a)} = \frac{m_{(a)}}{k_{(B)} T_{(a)}}$$

Hubble function

$$H^2(x) = \frac{8\pi G}{3}\left[\rho(1) + E_{(0)} + \frac{4}{\mathcal{V}_{(0)}} \log x\right]$$

Scale factor is regular

$$a(t) = a^* \exp\left\{\frac{8\pi G}{3\mathcal{V}_{(0)}}(t - t^*)^2\right\}$$

$$H(t) = \frac{16\pi G}{3\mathcal{V}_{(0)}}(t - t^*)$$

Acceleration parameter

$$-q(t) \equiv \frac{\ddot{a}}{aH^2} = 1 + \frac{3\mathcal{V}_{(0)}}{16\pi G(t - t^*)^2} \geq 1$$

Perpetually accelerated Universe

Explicit example of the Little Rip – type solution

Exact super-exponential solution

$\sigma = -1$ $\xi = \frac{1}{3}$ $\mathcal{V} = 0$

$$\Pi(x) = \Pi(1) + 3[\rho(1) + \Pi(1) - \rho_0]\log x - \frac{9}{2}\rho_0 \log^2 x,$$

$$\rho(x) = \rho(1) - 3[\rho(1) + \Pi(1)]\log x + \frac{9}{2}\rho_0 \log^2 x,$$

Scale factor is super-exponential

$$\frac{a(t)}{a(t_0)} = \exp\left\{\sqrt{\frac{2\rho(1)}{9\rho_0}}\sinh[\sqrt{12\pi G\rho_0}(t - t_0)]\right\}$$

Hubble function

$$H(t) = \sqrt{\frac{8\pi G\rho(1)}{3}}\cosh[\sqrt{12\pi G\rho_0}(t - t_0)]$$

Acceleration parameter

$$-q(t) = 1 + \sqrt{\frac{9\rho_0}{2\rho(1)}}\frac{\sinh[\sqrt{12\pi G\rho_0}(t - t_0)]}{\cosh^2[\sqrt{12\pi G\rho_0}(t - t_0)]}$$

Explicit example of the Little Rip – type solution

Unlighted epochs in the Universe history

Refraction index calculated using nonminimal interaction scheme

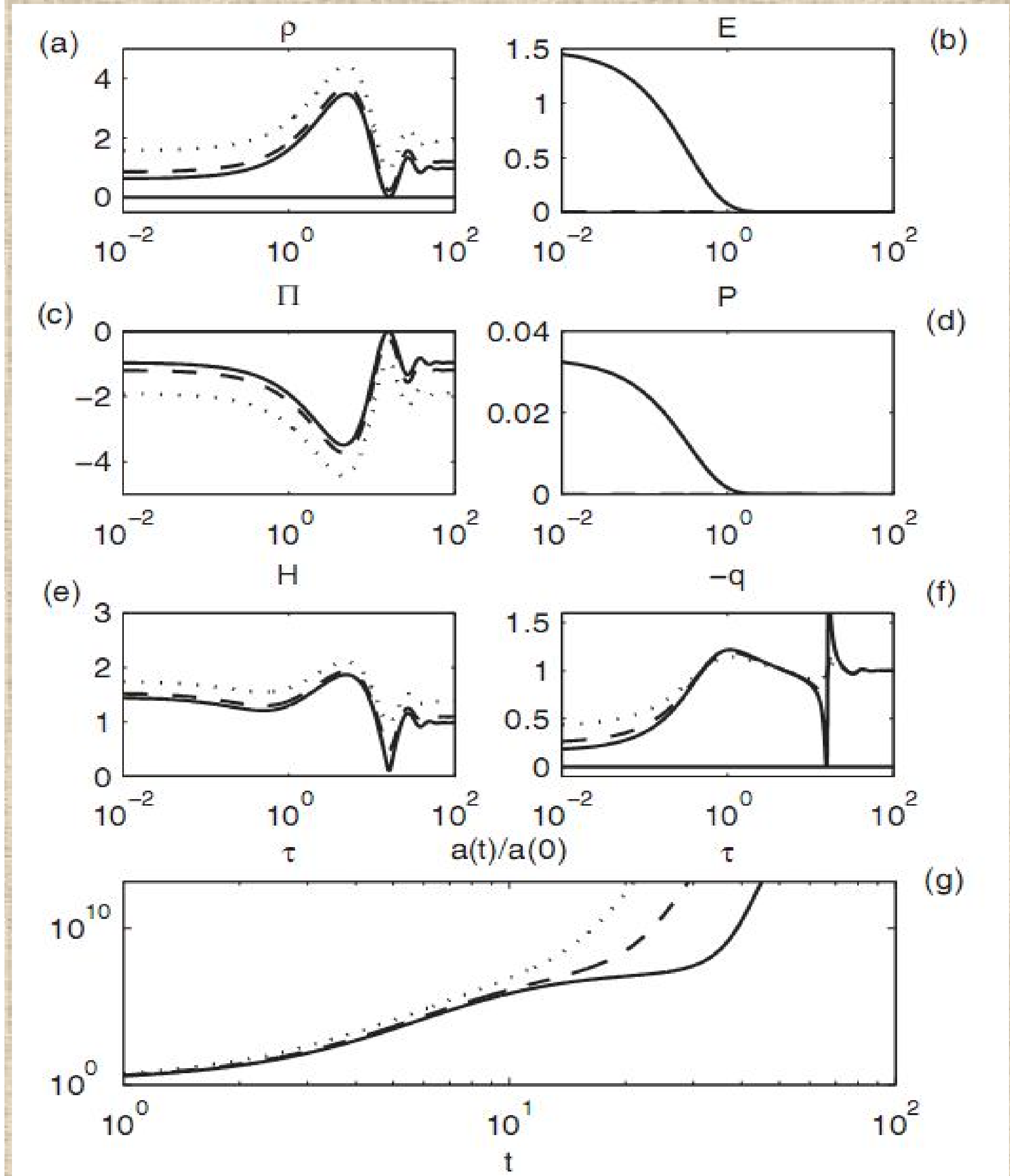
$$n^2(t) = \frac{1 - QH^2(t)[1 + q(t)]}{1 + QH^2(t)[1 + q(t)]} = \frac{1 + Q\dot{H}(t)}{1 - Q\dot{H}(t)}$$

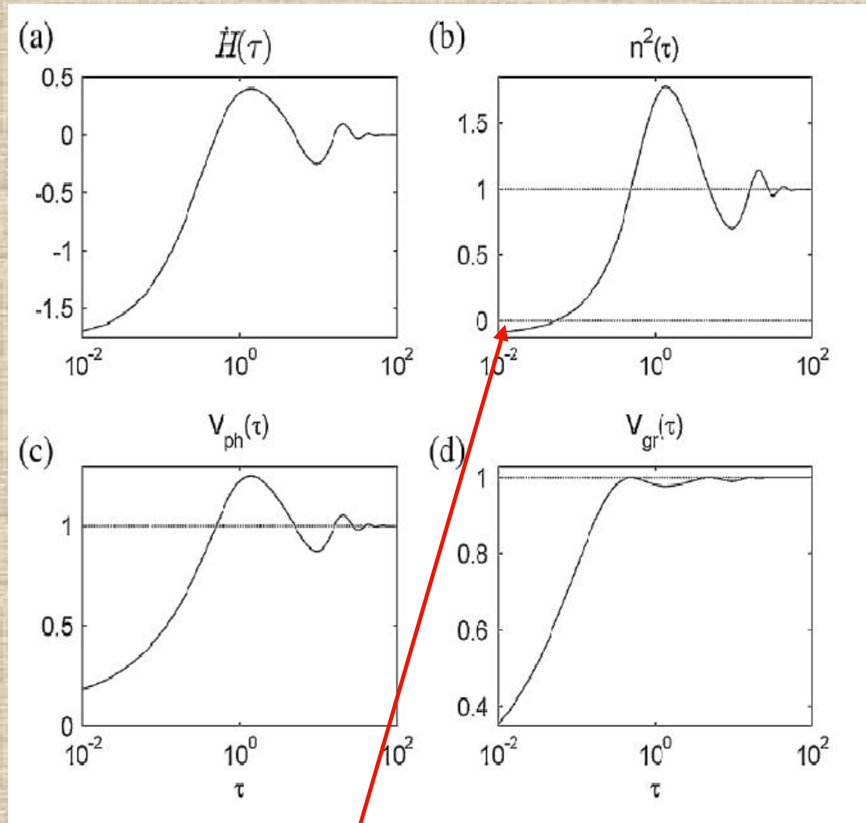
The model operates with one nonminimal coupling parameter and time derivative of the Hubble function

For more details see, e.g.,

Balakin A.B., Bochkarev V.V., Lemos J.P.S. Phys. Rev. D 85, 064015 (2012)

Numerical study of the Archimedean-type model: Solution without transition points (perpetually accelerated Universe)

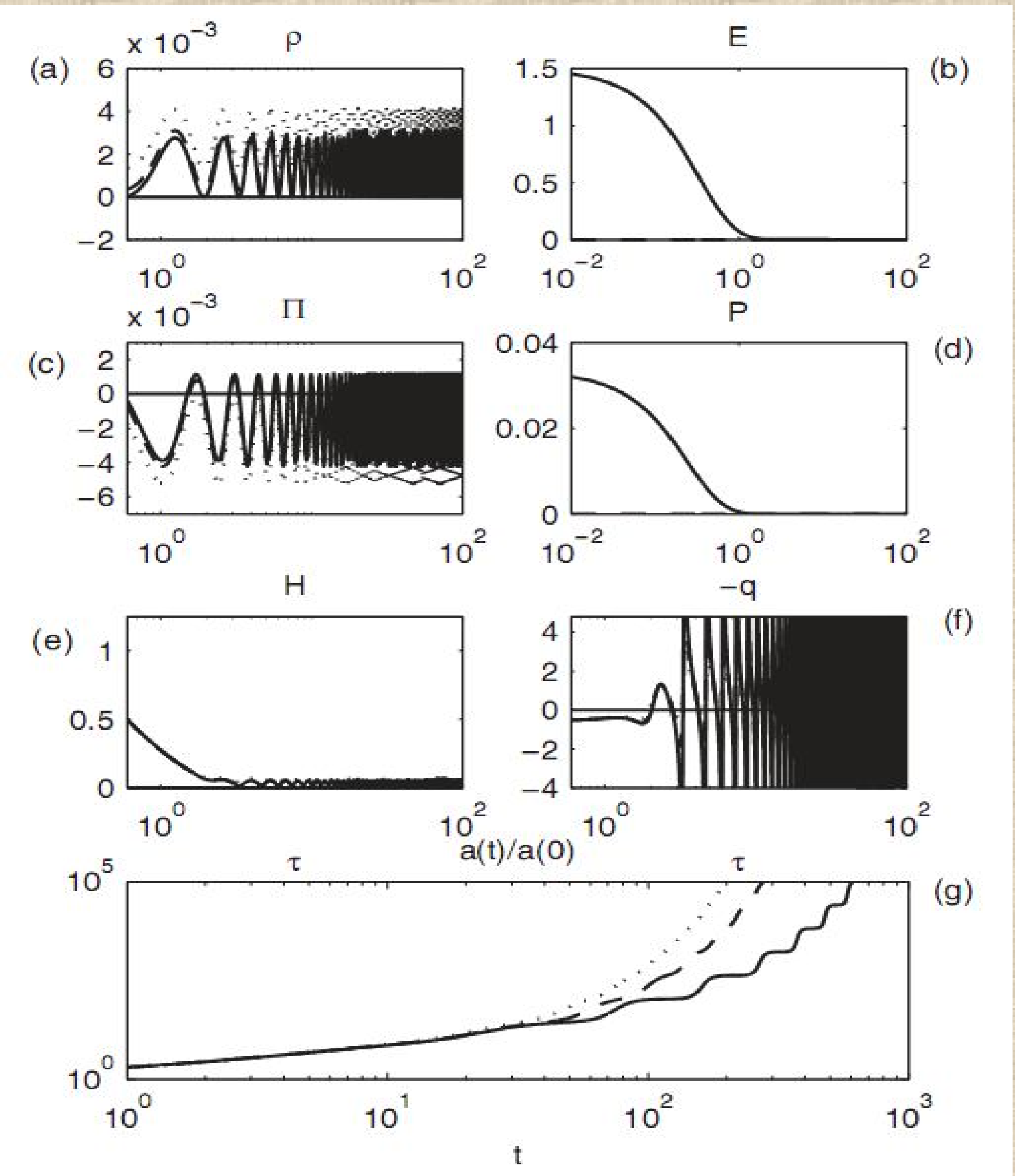


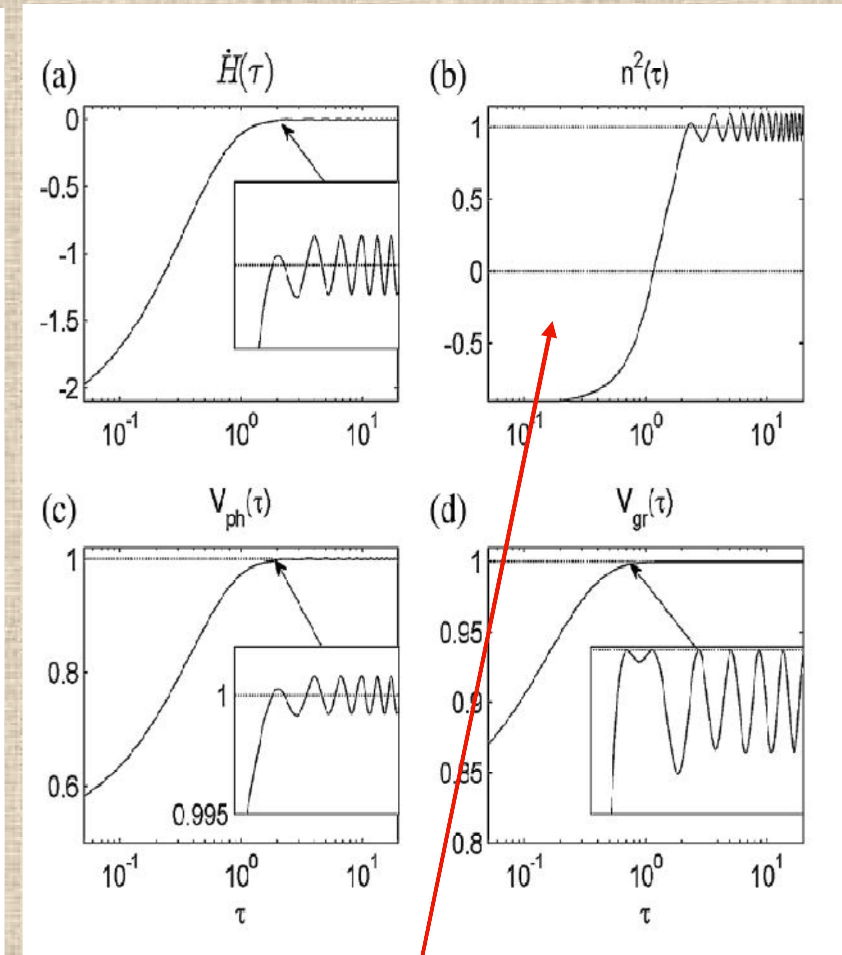


Unlighted epoch of the first type

$$n^2(t) < 0$$

Numerical study of the Archimedean-type model: Periodic solution with infinite number of transition points

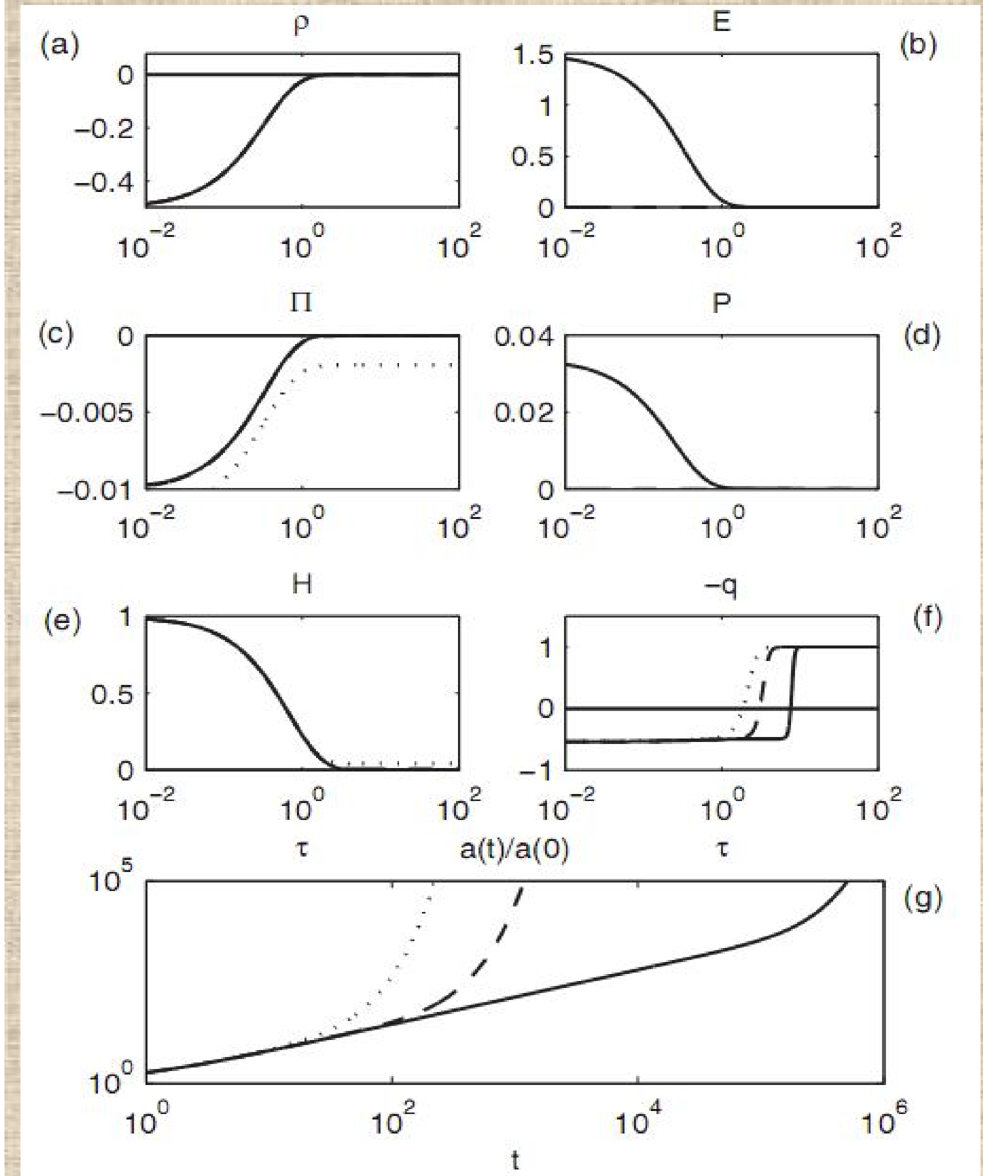


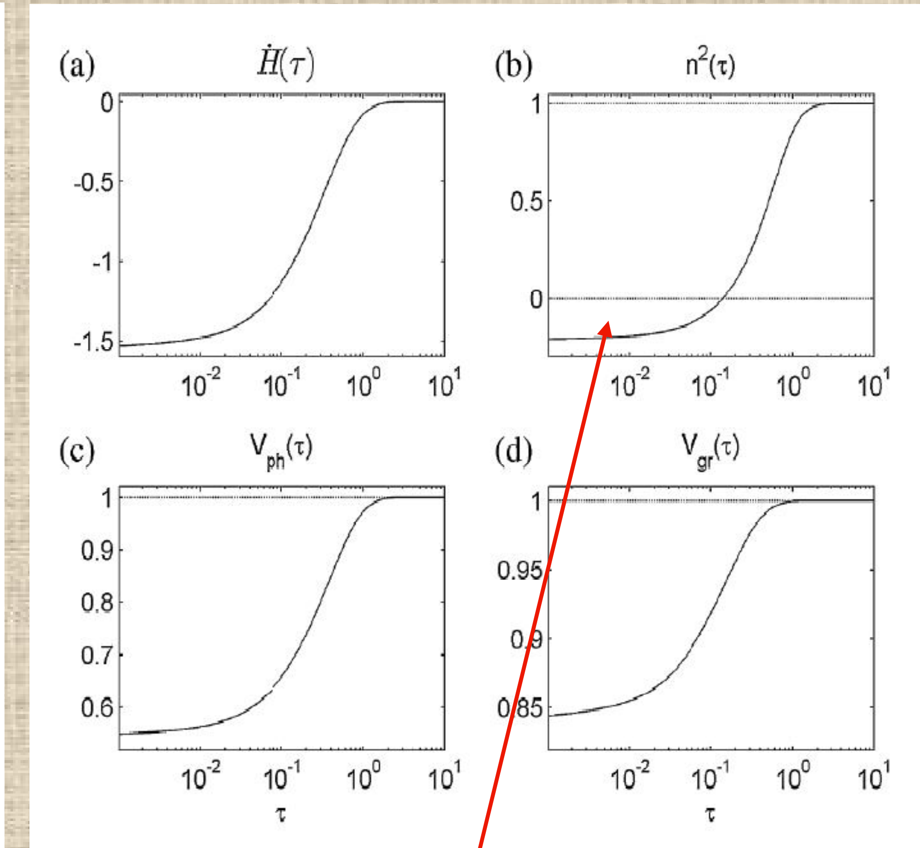


Unlighted epoch of the first type

$$n^2(t) < 0$$

Numerical study of the Archimedean-type model: Solution with one transition point

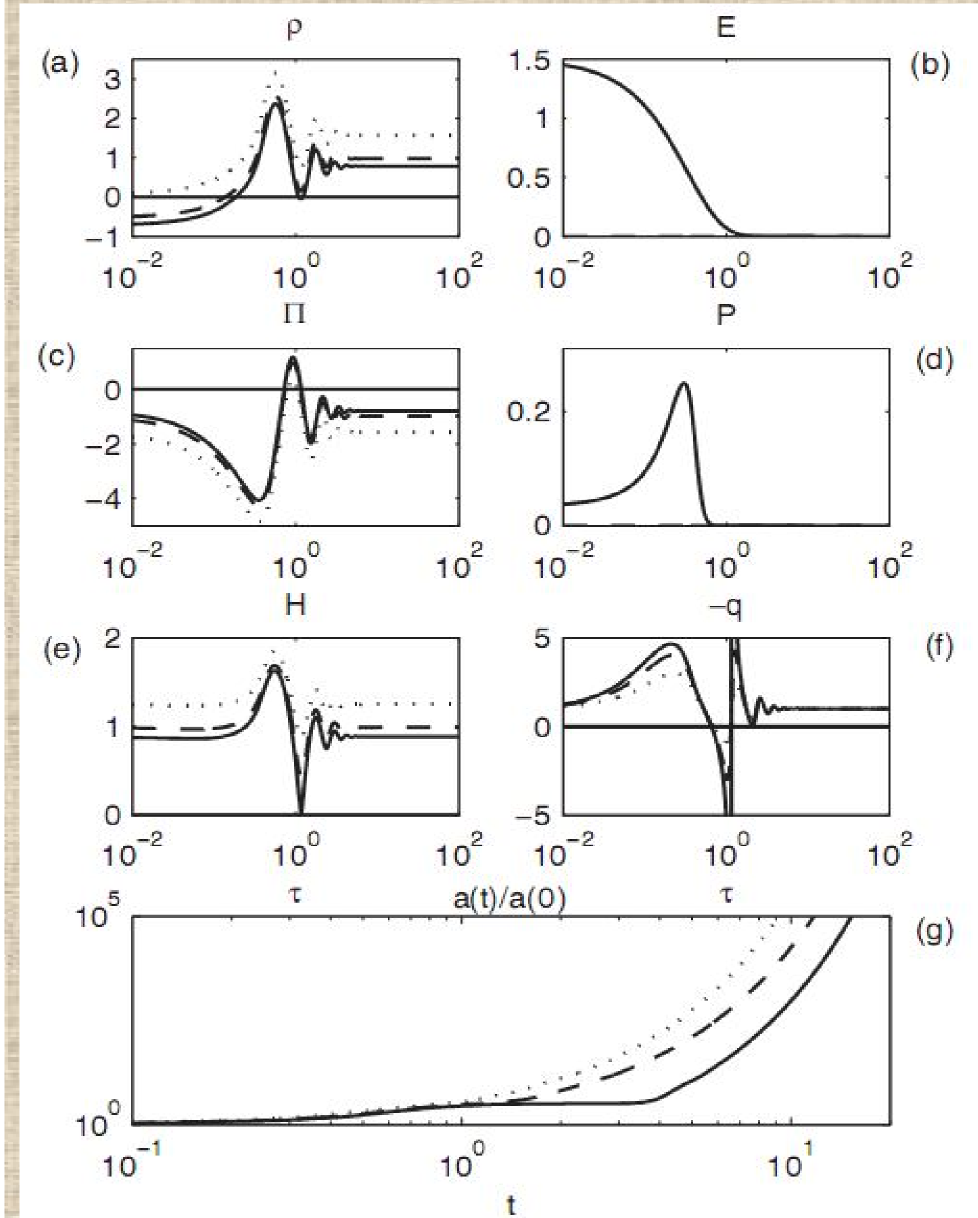


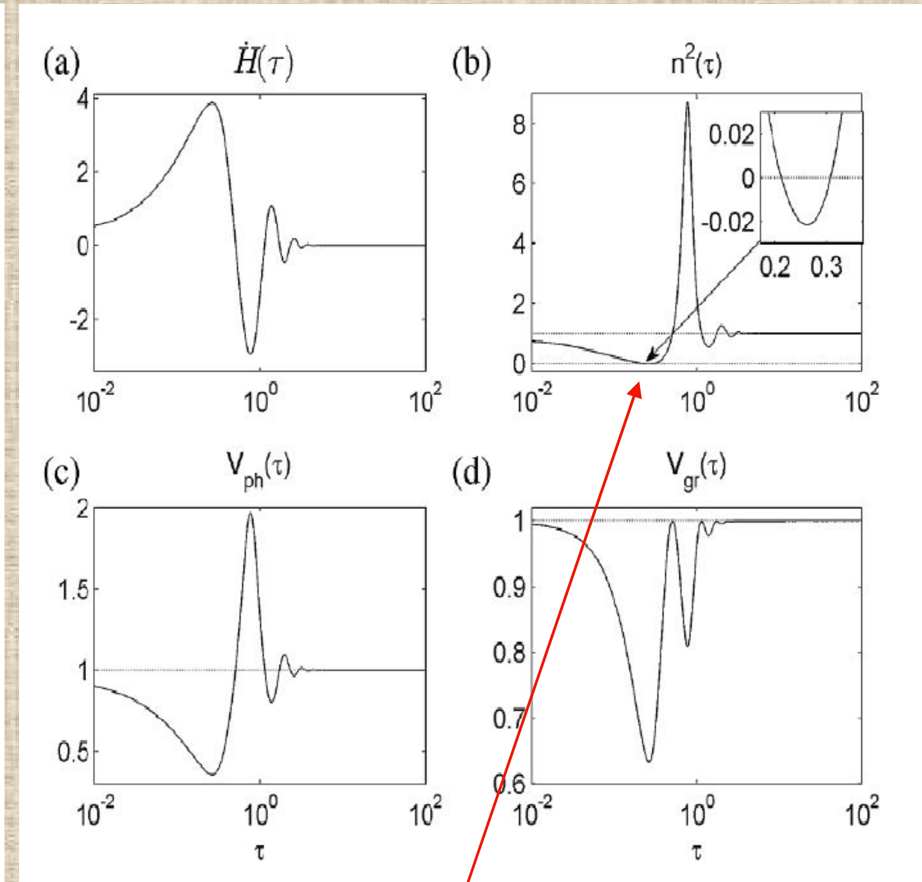


Unlighted epoch of the first type

$$n^2(t) < 0$$

Numerical study of the Archimedean-type model: Solution with two transition points

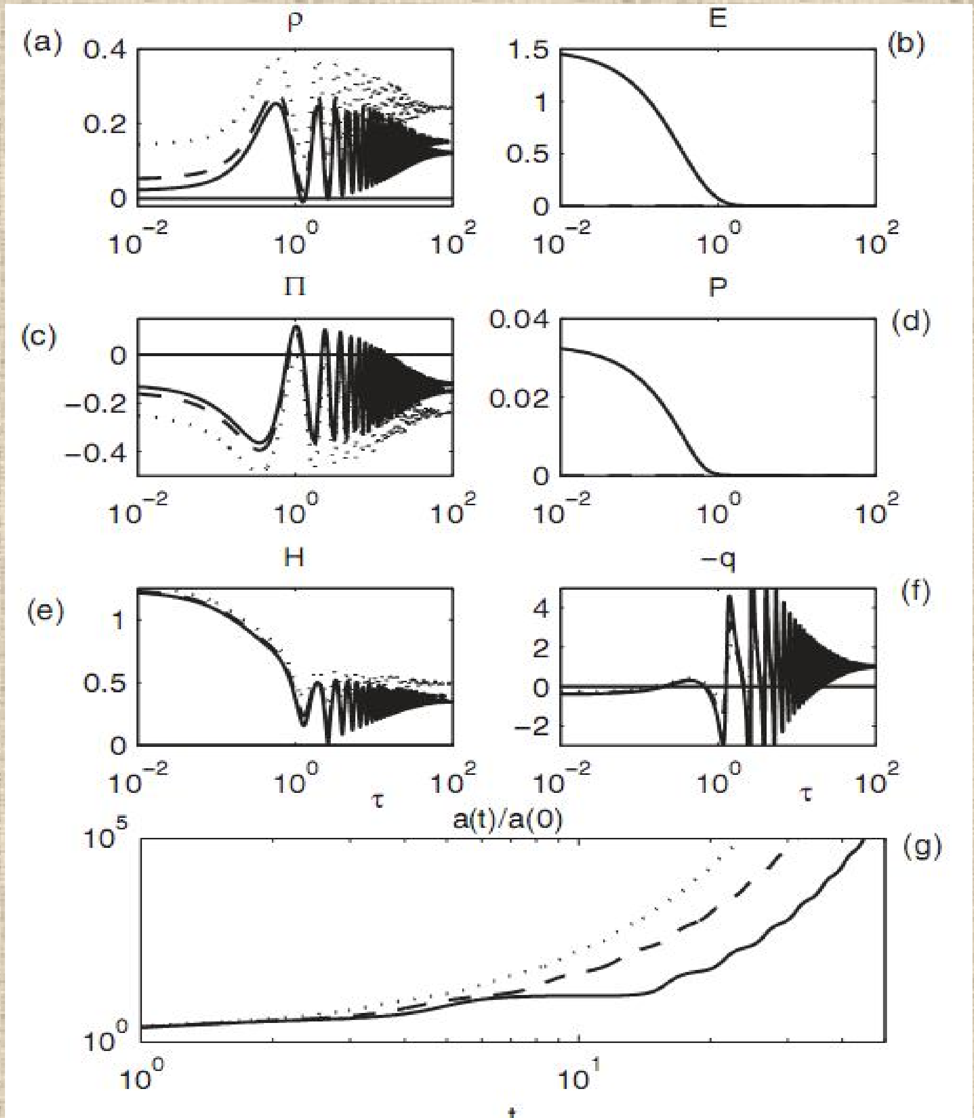


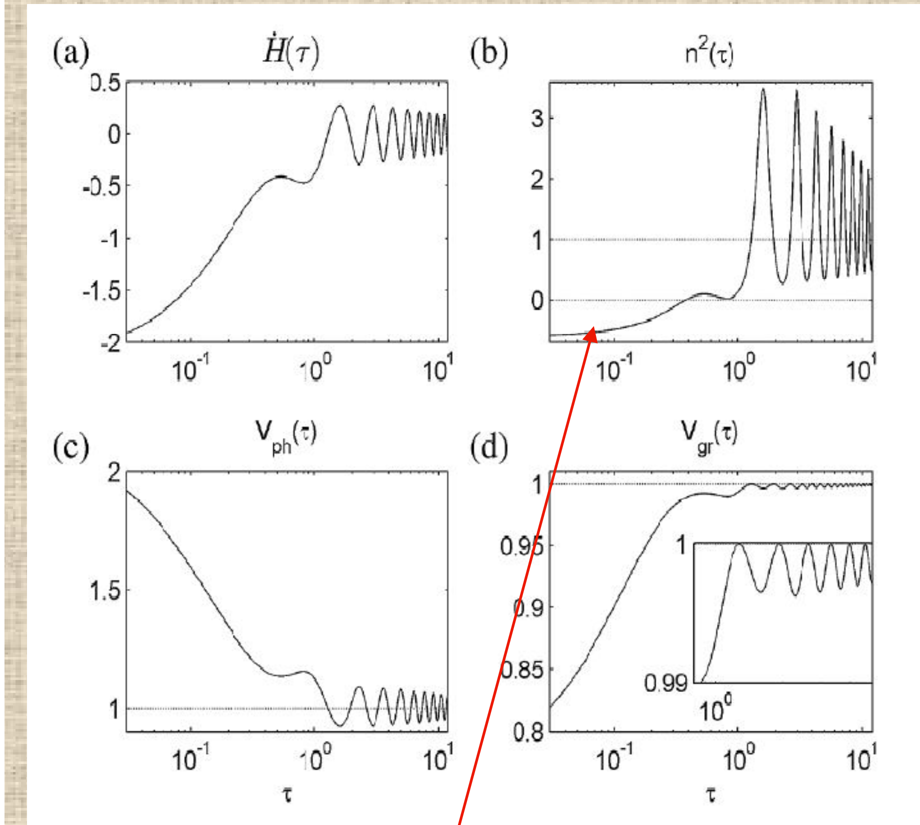


Unlighted epoch of the first type

$n^2(t) < 0$

Numerical study of the Archimedean-type model: Quasi-periodic solution with finite number of transition points





Unlighted epoch of the first type

$$n^2(t) < 0$$

Refraction index calculated using the interaction scheme involving analogy with electro- and magneto-striction

Balakin A.B., Dolbilova N.N.   Phys. Rev. D , 89, 104012 (2014)

$$\varepsilon = \varepsilon_{(0)} + \lambda_1 DW - \alpha P$$

$$\frac{1}{\mu} = \frac{1}{\mu_{(0)}} + \lambda_2 DW - \beta P$$

Convective derivative of DE energy density

DE pressure

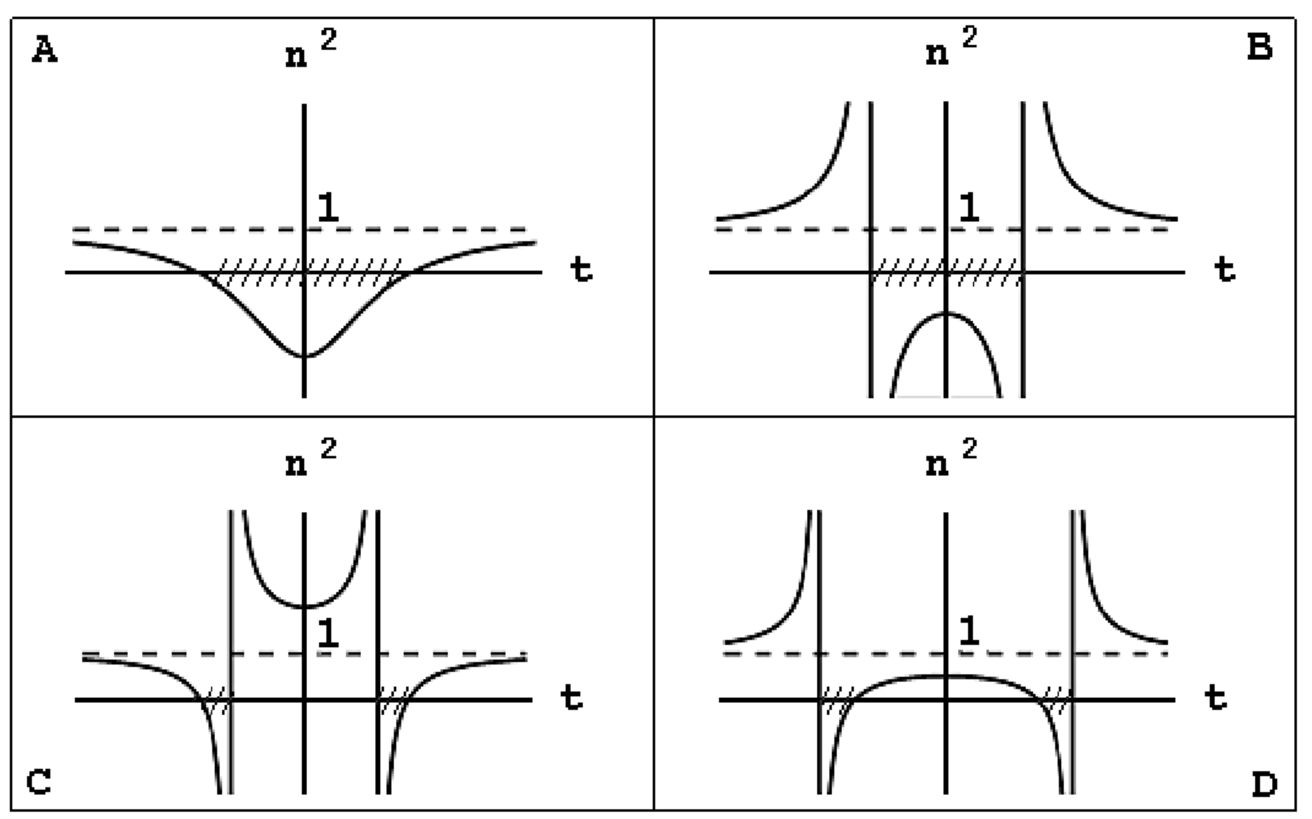
Coupling parameters describing striction-type interaction of DE with spatially isotropic electrodynamic system

$$n^2 \equiv \varepsilon\mu = \frac{n^2_{(0)} + \mu_{(0)}(\lambda_1 DW - \alpha P)}{1 + \mu_{(0)}(\lambda_2 DW - \beta P)}$$

Typical behavior of the squared effective refraction index for two exact analytic solutions: anti-Gaussian and super-exponential

I type

II type



III type

III type

Unlighted epochs (periods with negative squared refraction index) of the I type (continuous), II type (discontinuous simply connected) and III type (discontinuous double-connected)

Conclusions

- **The main idea of the work is to try to find Dark Energy fingerprints in the cosmic electrodynamic systems; we discussed the DE fingerprints associated with the so-called unlighted (dark) epochs in the Universe history, during which electromagnetic waves cannot propagate and thus cannot scan Universe interior.**

- **The unlighted epochs could be formed due to non-minimal coupling of photons and gravitons, and due to a striction-type coupling of an electrodynamic system with the Dark Energy, when the Universe expansion is non-stationary and the Universe history contains epochs of accelerated and decelerated expansion.**

- **Specific non-stationary Universe evolution displaying the unlighted epochs can be provided by an Archimedean-type interaction between Dark Energy and Dark Matter, which plays a role of effective energy-momentum re-distributor between DM and DE components of the Dark Fluid.**

Thank you for the attention!