Convection simulations with reduced sound speed method

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Outline

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Mach number issue in simulations of the Sun

The maximum sound speed in the convection zone of the Sun is of the order of $200\ \mathrm{km\ s^{-1}}$ **whereas the maximum flow speed is** $2\ \mathrm{km\ s^{-1}}$**.**

In numerical simulations the time step is determined by the CFL-condition using the propagation speed of information (flow speed, sound speed, Alfven speed etc.).

Solution I: anelastic approximation

The idea is to filter out sound waves so that they no longer enter the CFL-condition (e.g. the ASH code, Brun et al. 2004).

The time derivative is omitted in the continuity equation:

$$\frac{\partial\rho}{\partial t} - \boldsymbol{\nabla}\cdot(\rho\boldsymbol{u}) = 0 \quad \longrightarrow \quad \boldsymbol{\nabla}\cdot(\rho\boldsymbol{u}) = 0$$

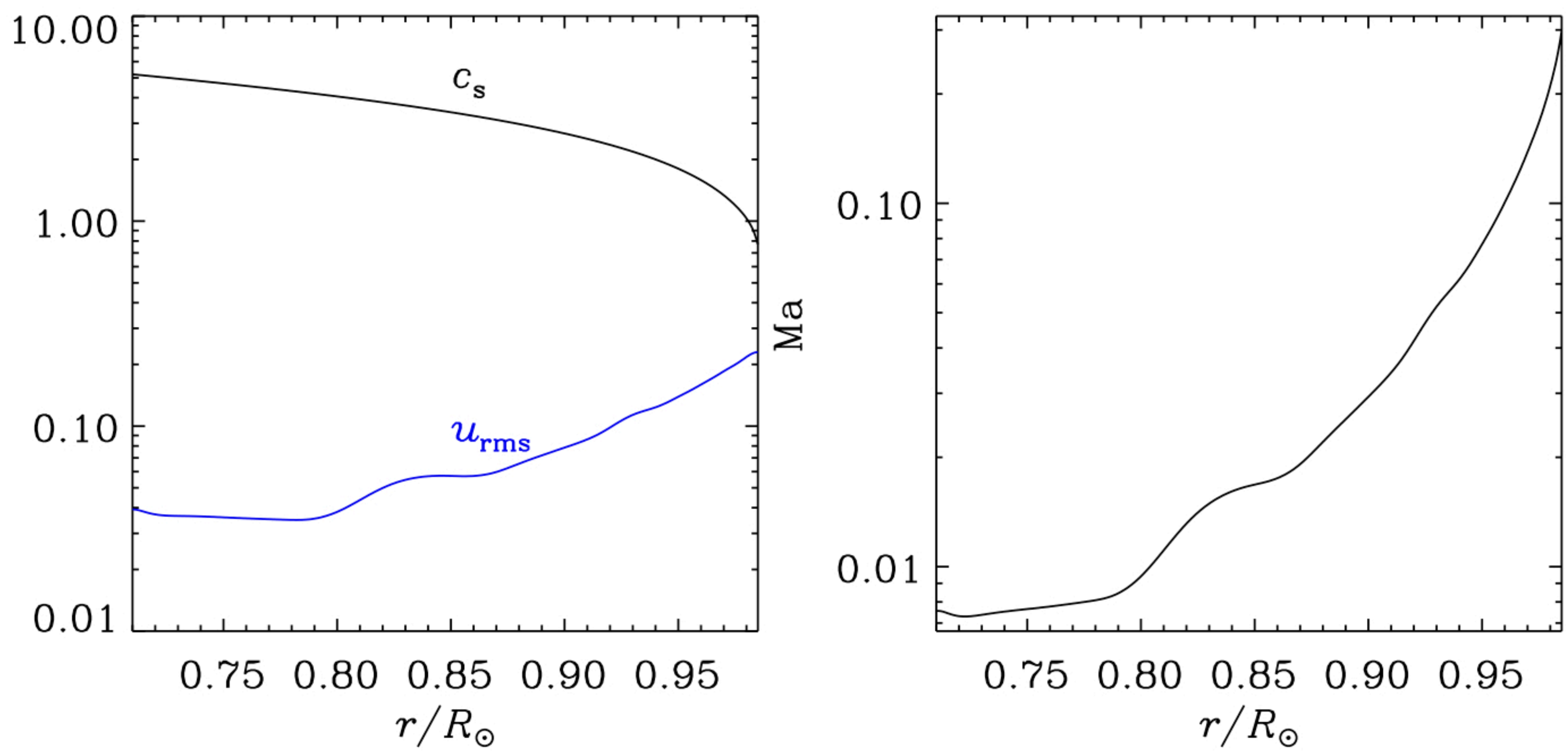
The drawback is that a Poisson problem needs to be solved for the pressure. This cannot be done locally and global communication is detrimental to the scalability of the code.

Solution II: increased luminosity

Another way is to increase the luminosity of the star in the simulation by a substantial amount and scale the relevant quantities accordingly:

$$u \propto L^{1/3} \quad L_{\rm sim}/L_{\odot} = 10^6 \longrightarrow \begin{array}{l} u_{\rm sim}/u_{\odot} = 10^2 \\ \Omega_{\rm sim}/\Omega_{\odot} = 10^2 \end{array}$$

However, even in this case the sound speed becomes the obstacle.



Solution III: reduced sound speed method

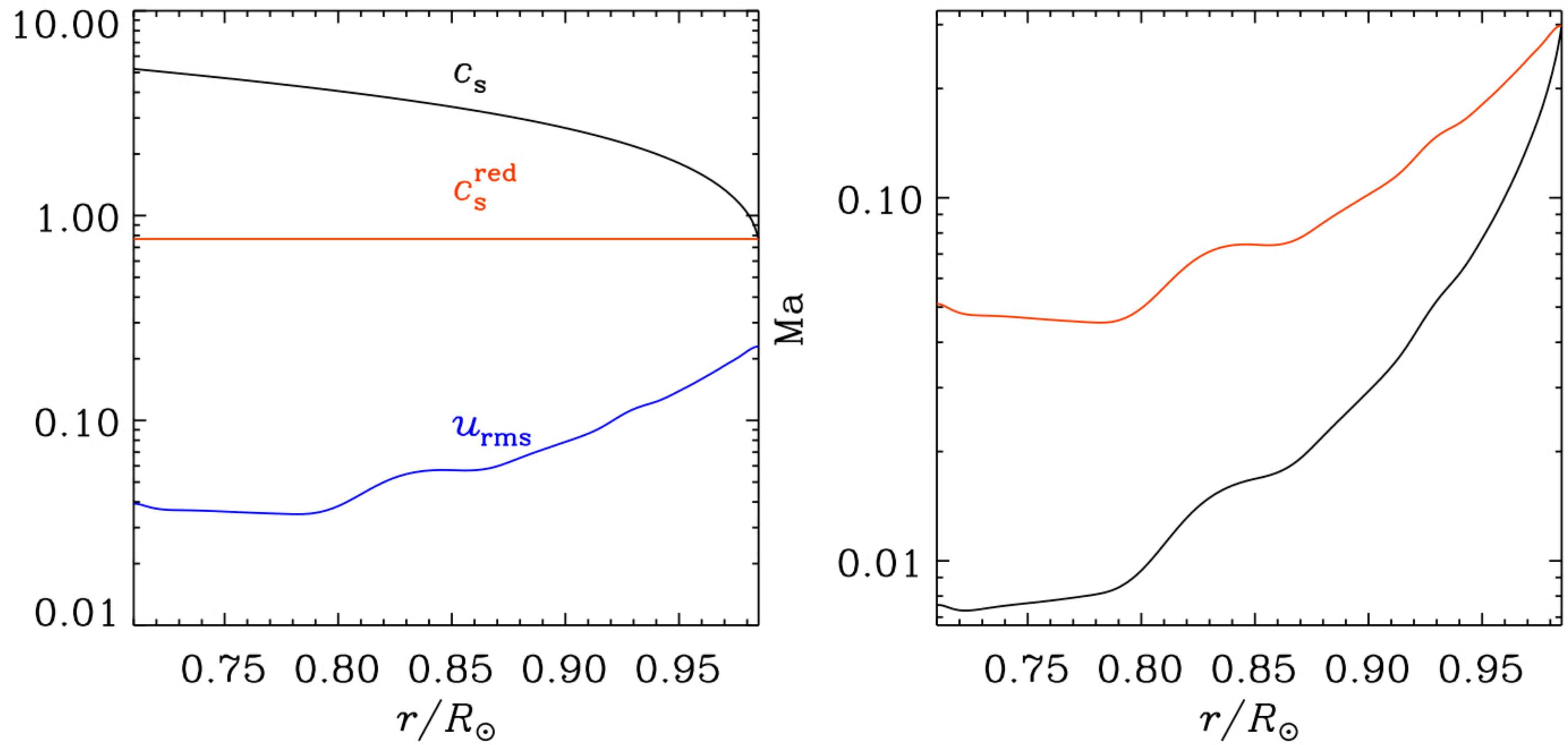
Instead of increasing the luminosity the sound speed can be artificially lowered. This is achieved by introducing a factor in front of the divergence term in the continuity equation (Rempel 2005, ApJ, 622, 1320)**:**

$$\frac{\partial\rho}{\partial t} - \frac{1}{\xi^2}\boldsymbol{\nabla}\cdot(\rho\boldsymbol{u}) = 0$$

This changes the propagation speed of all waves in the same way and the reduced sound speed $c_{\mathrm{s}}^{\mathrm{red}} = c_{\mathrm{s}}/\xi$ **enters the CFL-condition.**

The factor can also depend on spatial coordinates, i.e. $\xi = \xi(r)$ **so in solar simulations we can scale so that** $c_{\mathrm{s}}^{\mathrm{red}}(r) = c_{\mathrm{s}}(\mathrm{surface})$**.**

Solution III: reduced sound speed method



The RSS allows fully compressible simulations of solar convection that do not require global communication (but at the cost of mass conservation).

Solution III: reduced sound speed method