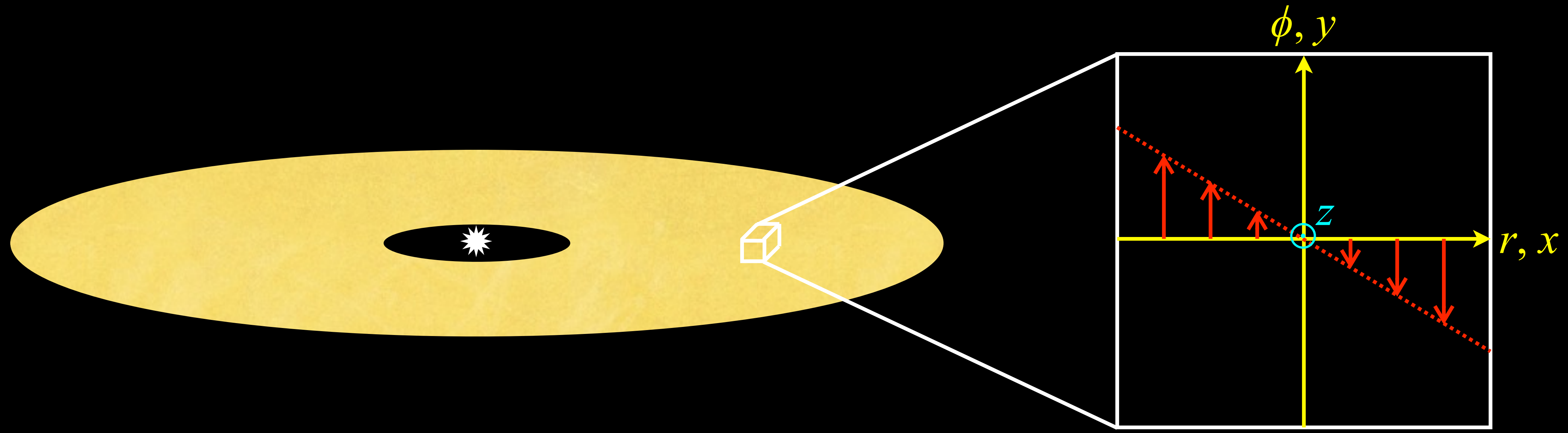
Aliasing in a Shearing Sheet

Chao-Chin Yang
Department of Astronomy and Theoretical Physics
Lund University
Sweden

Local Shearing Sheet

(Goldreich & Lynden-Bell 1965)



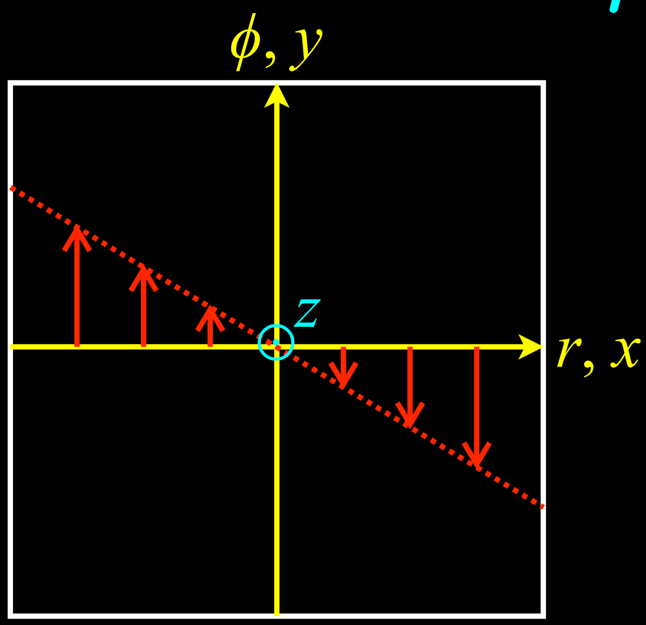
$$\frac{\partial \rho}{\partial t} - q\Omega x \frac{\partial \rho}{\partial y} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial \vec{u}}{\partial t} - q\Omega x \frac{\partial \vec{u}}{\partial y} + \vec{u} \cdot \nabla \vec{u} = q\Omega u_x \hat{e}_y - 2\vec{\Omega} \times \vec{u} - \frac{\nabla p}{\rho}$$

Shearing Waves

$$\frac{\partial \rho}{\partial t} - q\Omega x \frac{\partial \rho}{\partial y} = 0$$

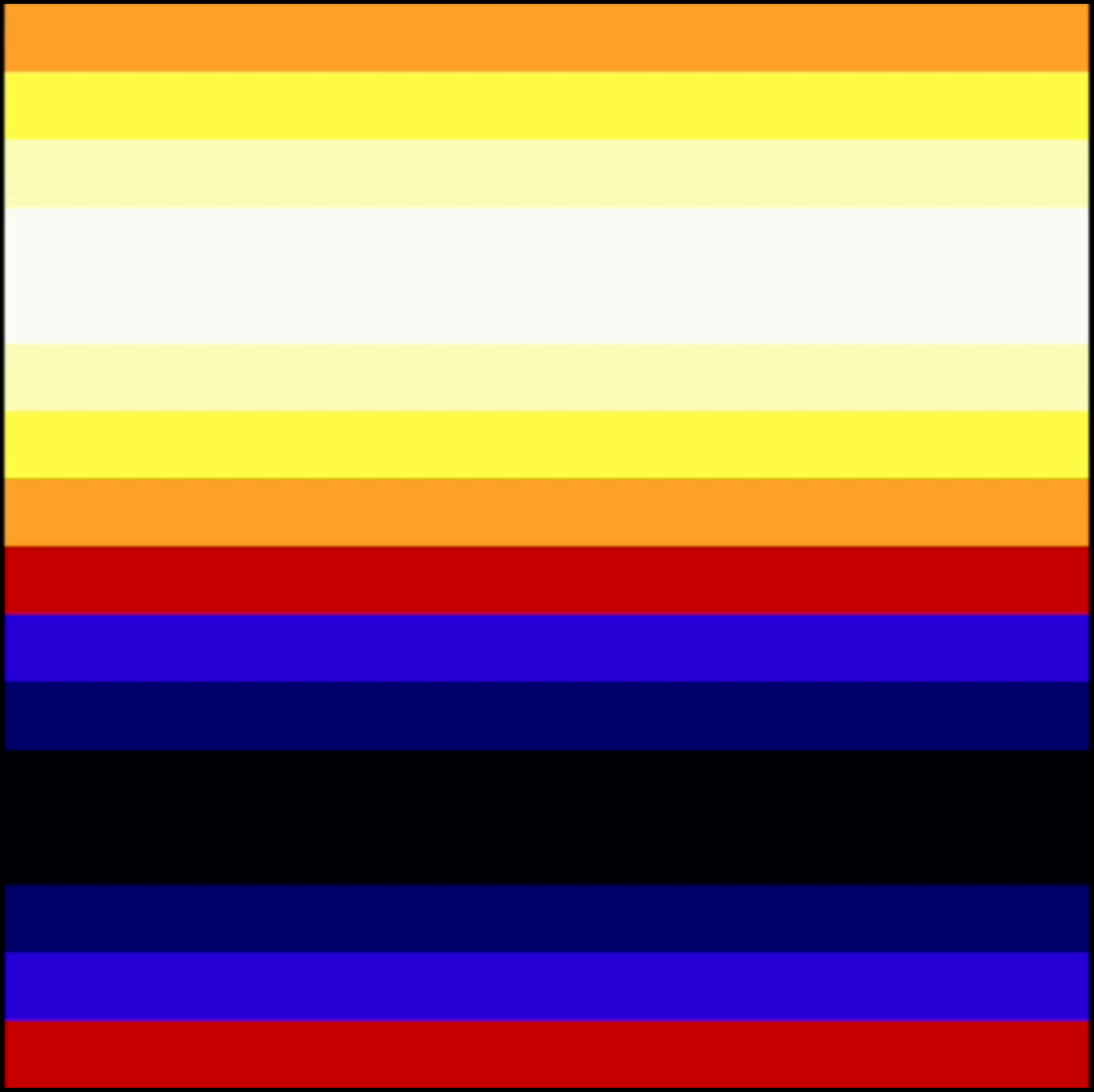
$$\rho(x, y, t) = \rho_0 e^{i[k_x(t)x + k_y y]}$$



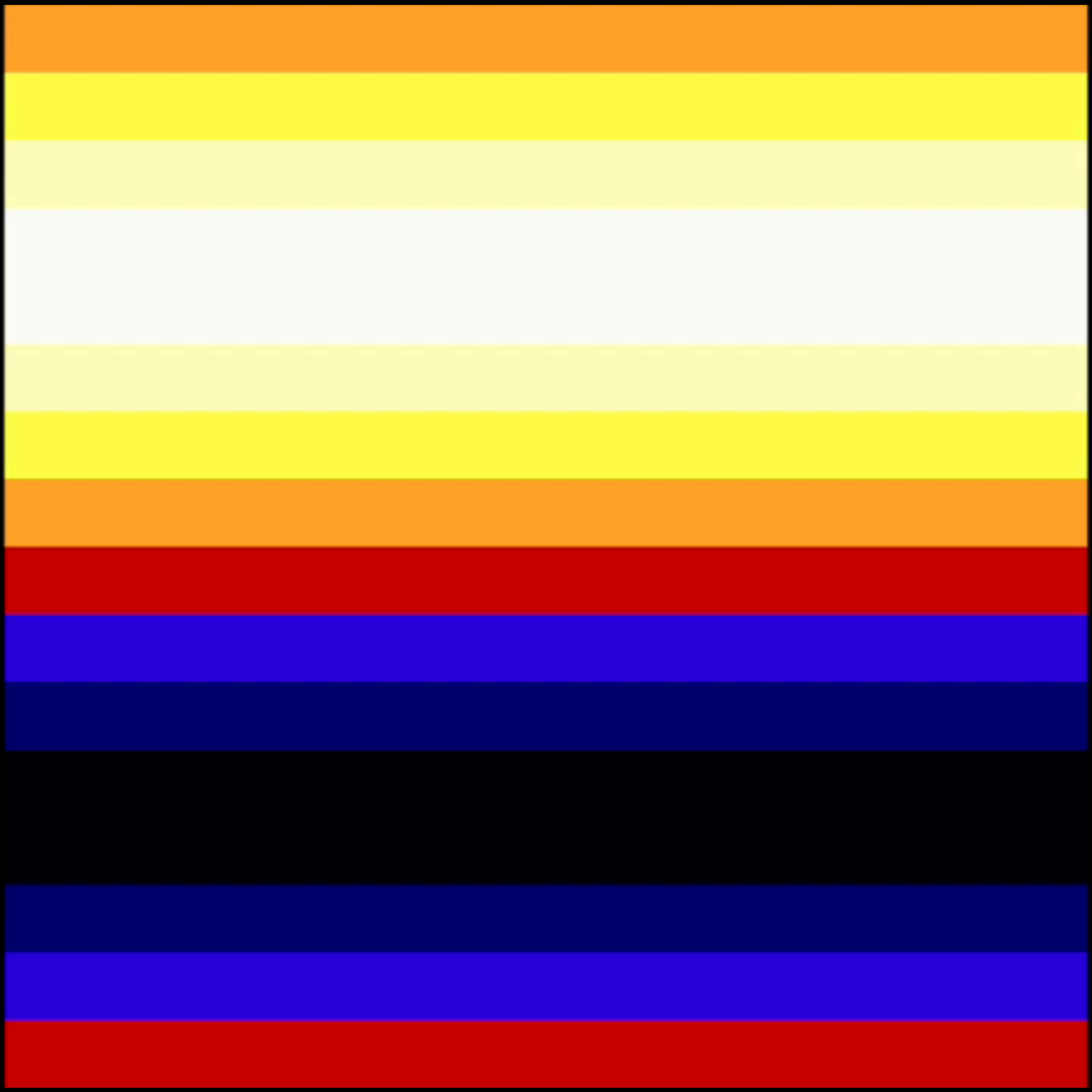
$$k_x(t) = k_{x,0} + q\Omega t k_{y,0}$$

$$k_y = k_{y,0}$$

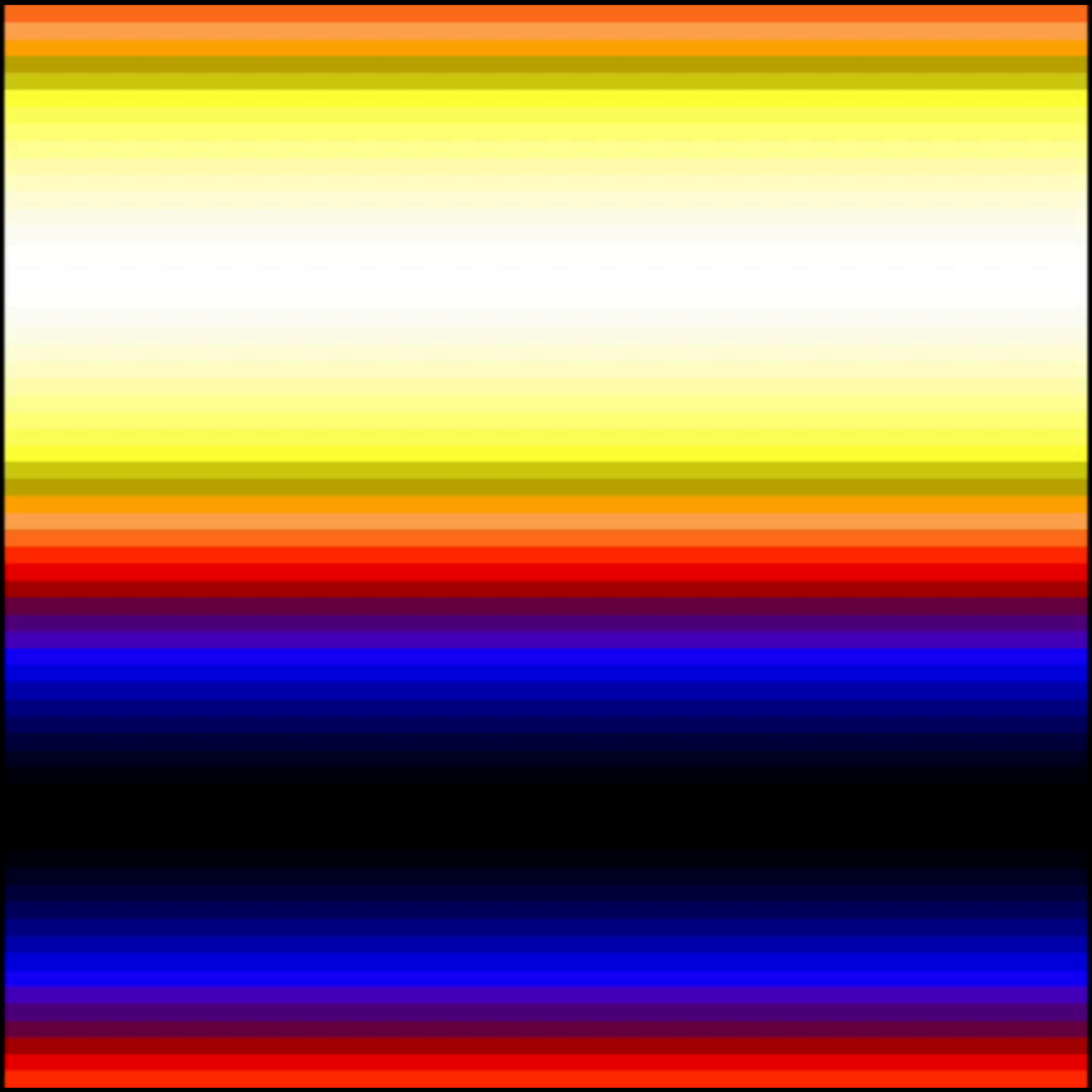
Passive Advection



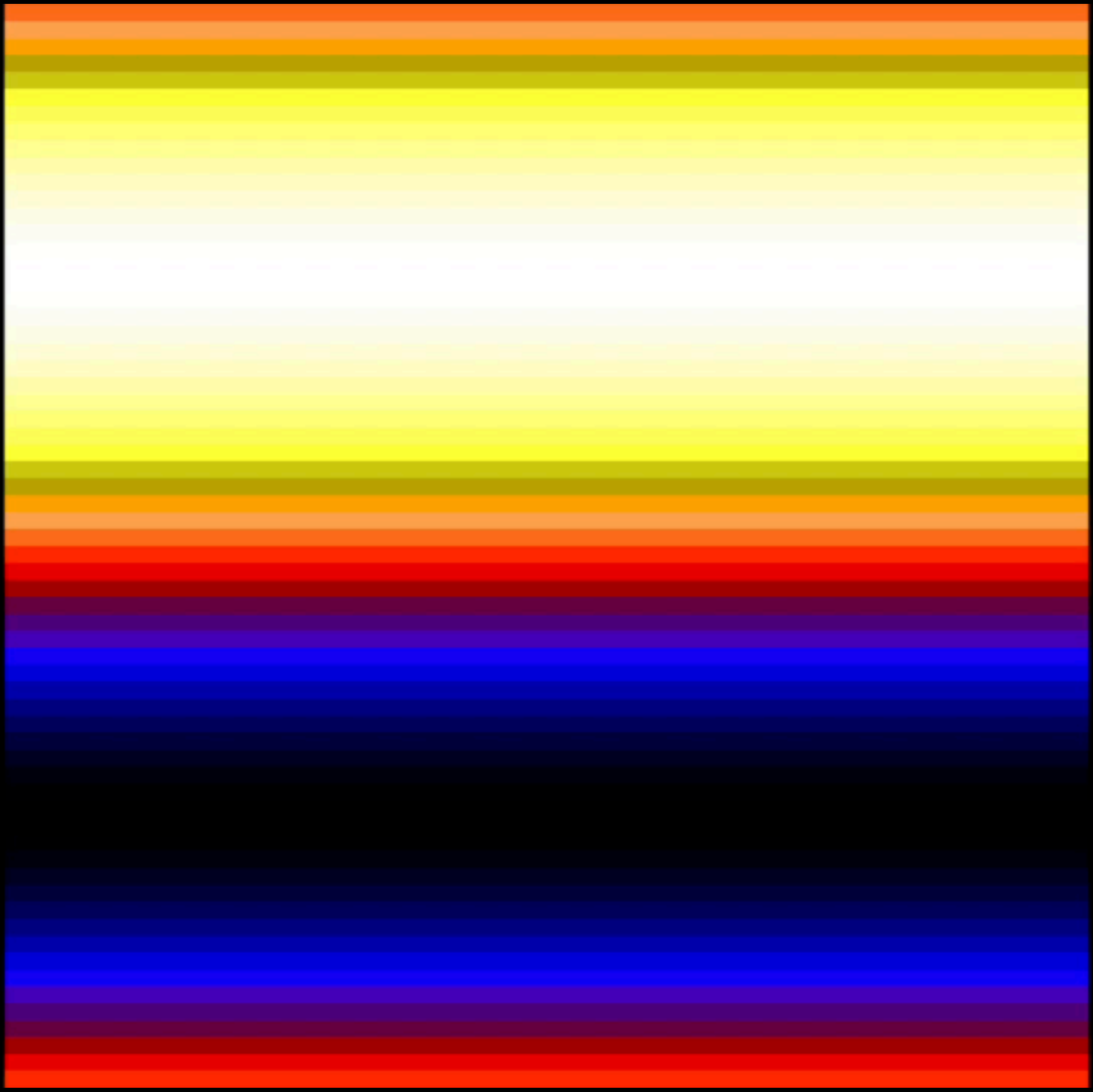
Passive Advection



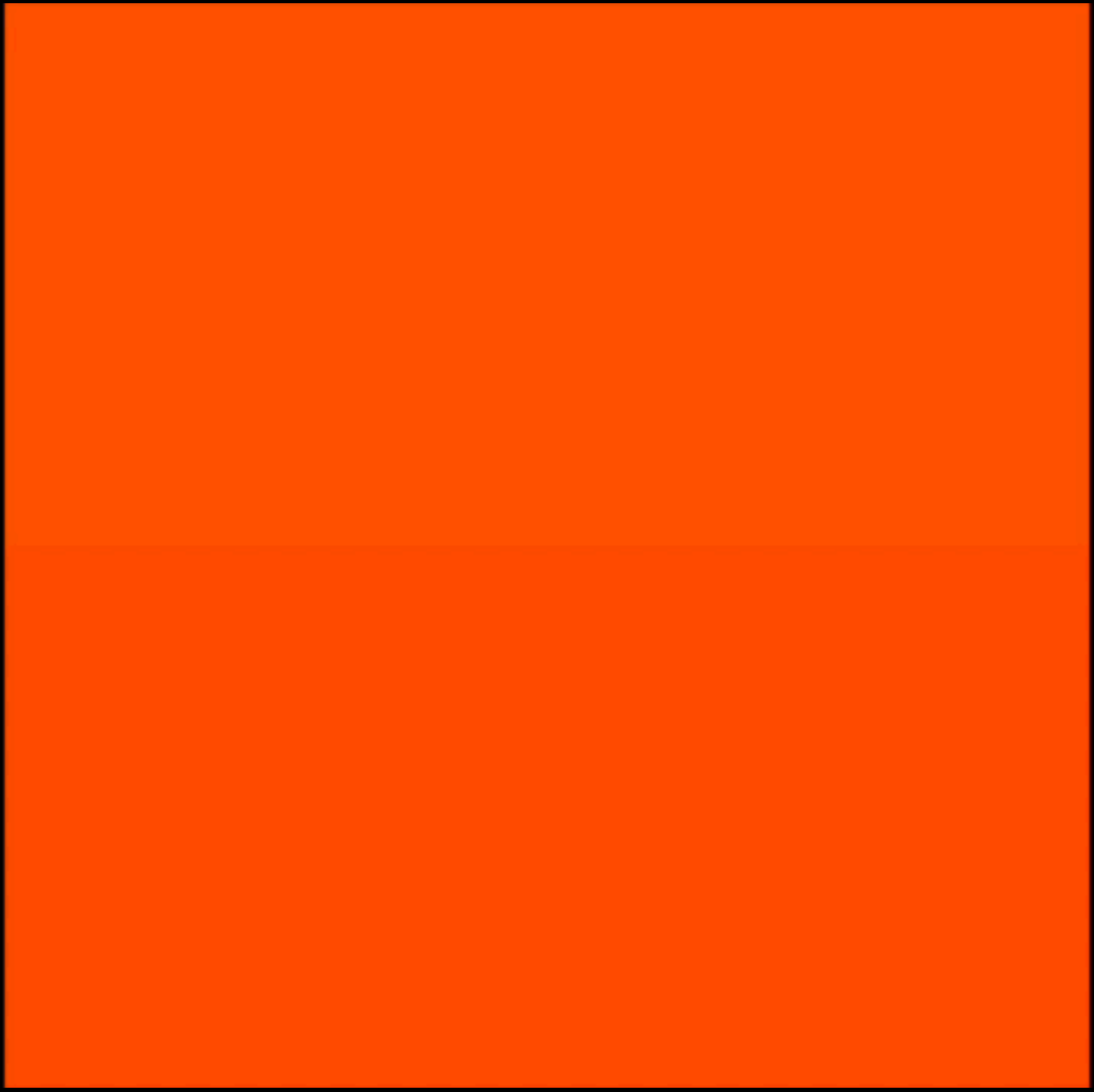
Passive Advection



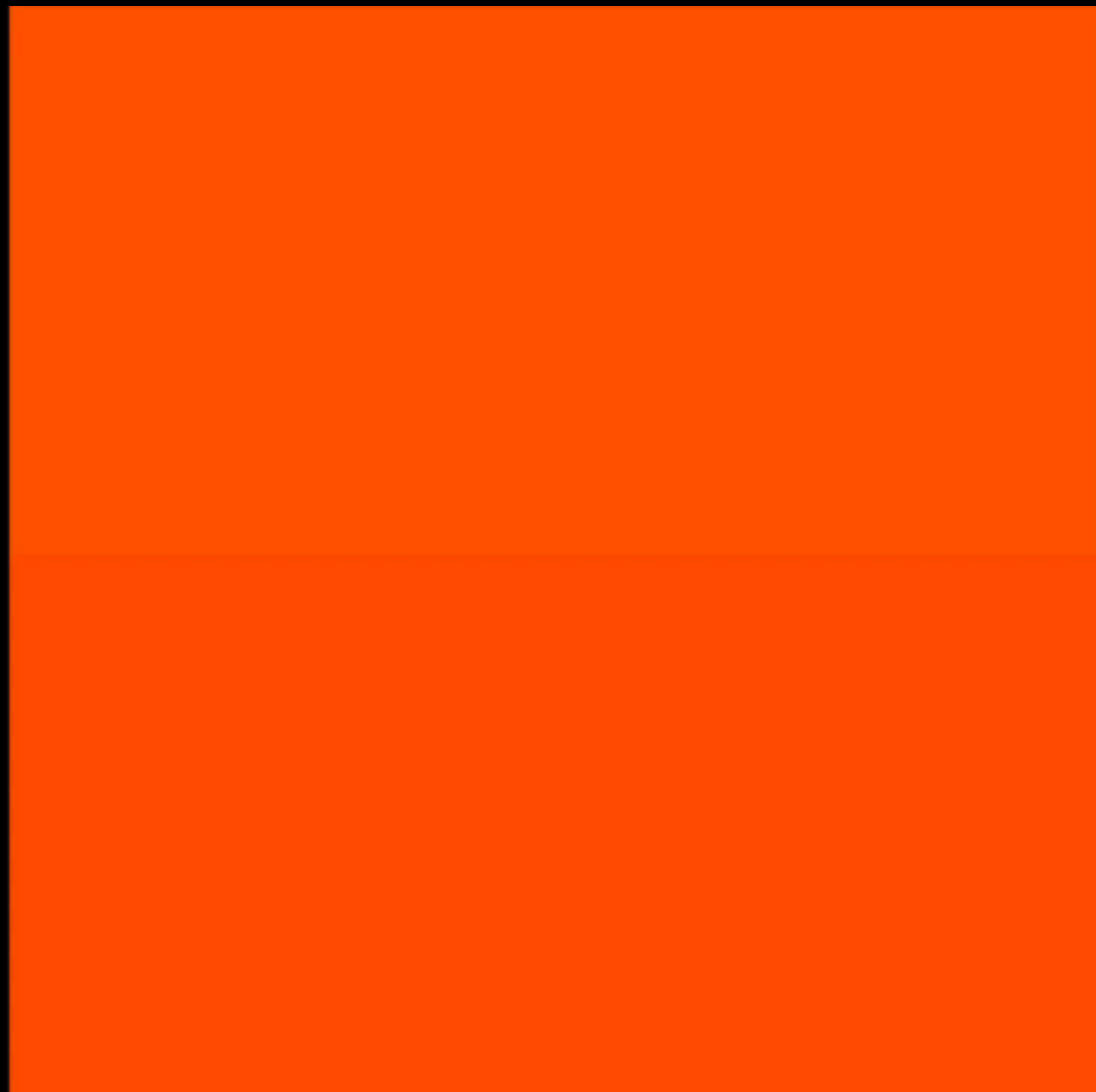
Passive Advection

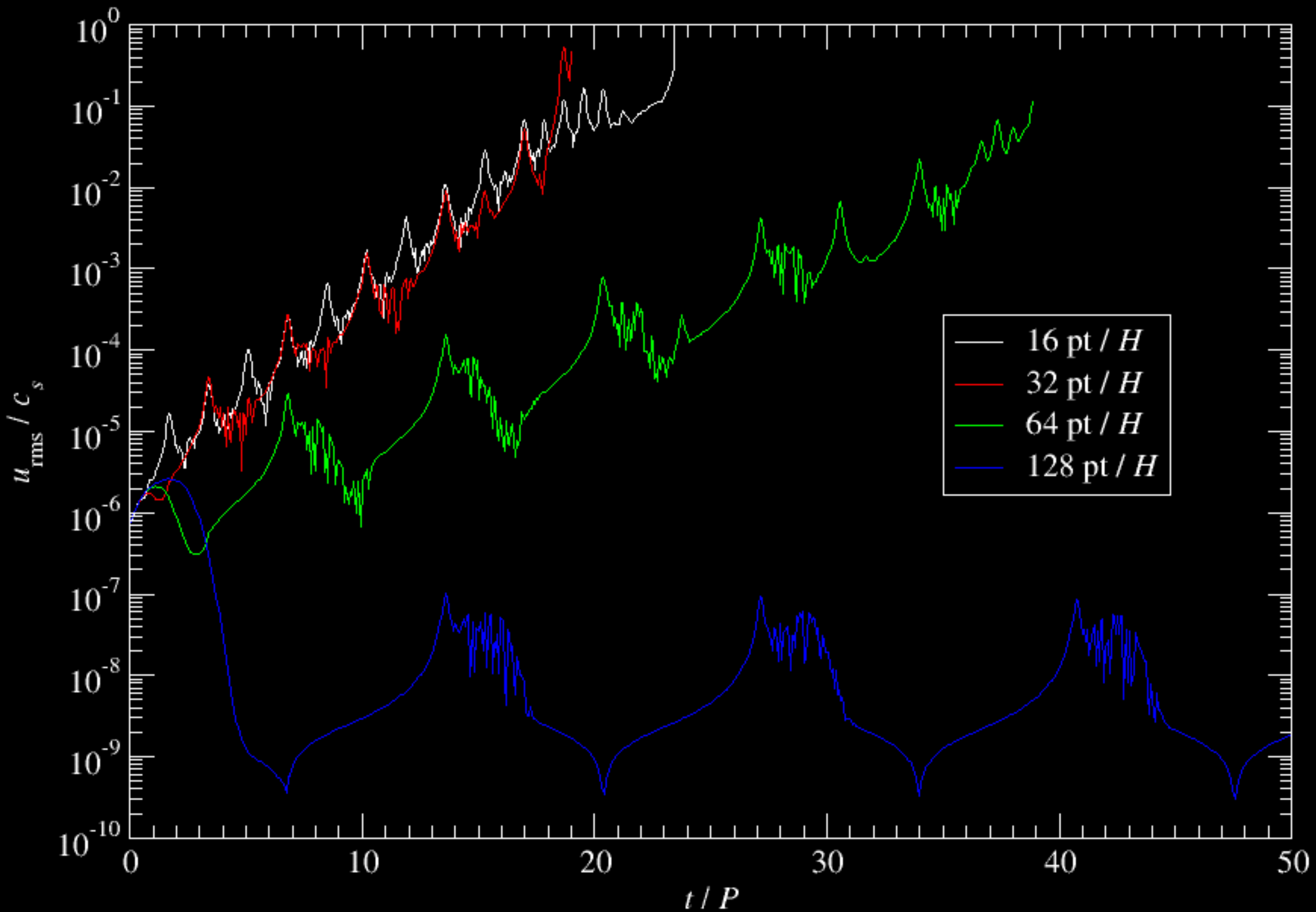


Hydrodynamical



Hydrodynamical





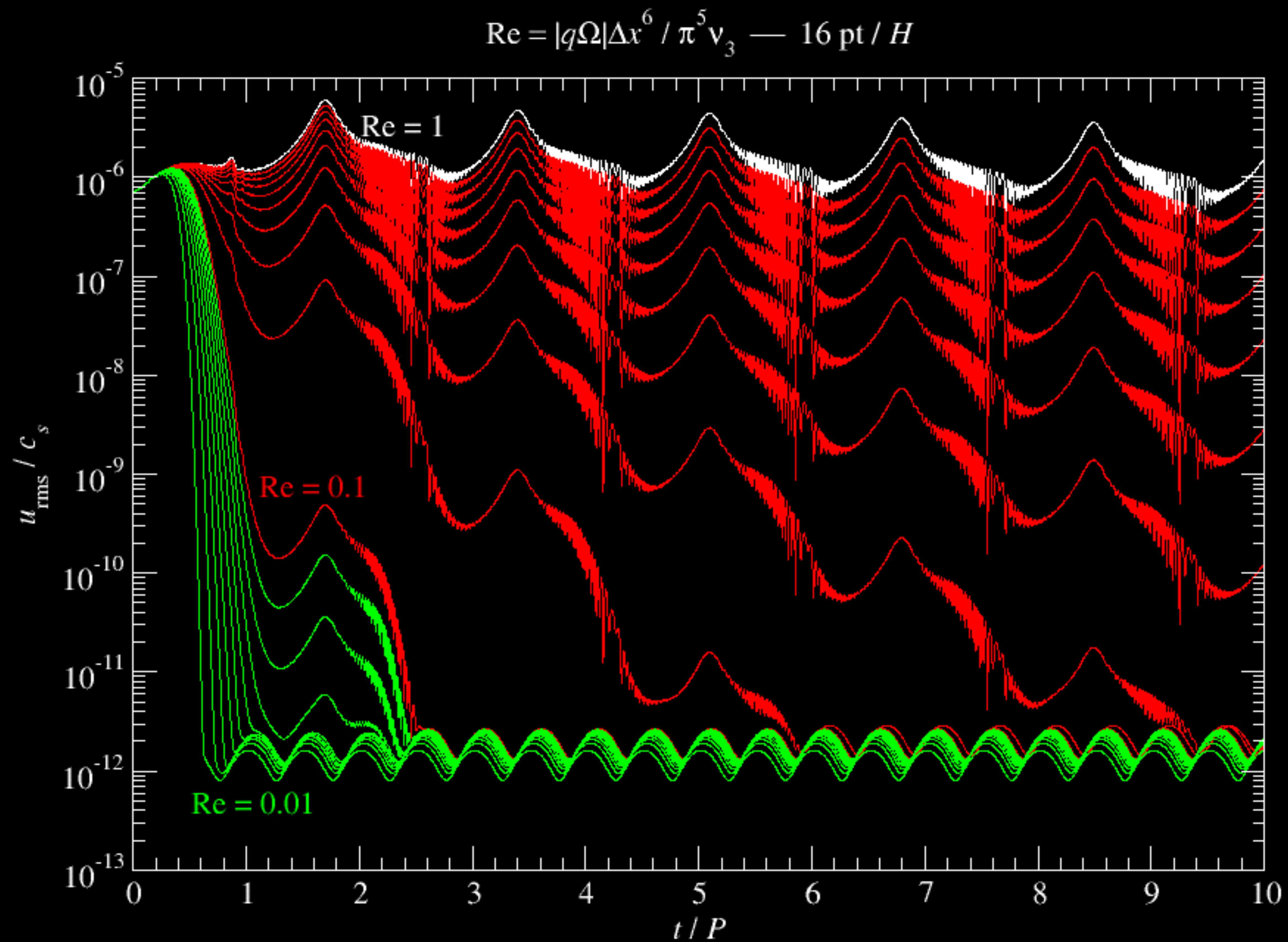
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
$u_{\rm rms} / c_s$
0
10
20
30
40
50
t / P
16 pt / H
32 pt / H
64 pt / H
128 pt / H

Hyper-diffusion to the Rescue

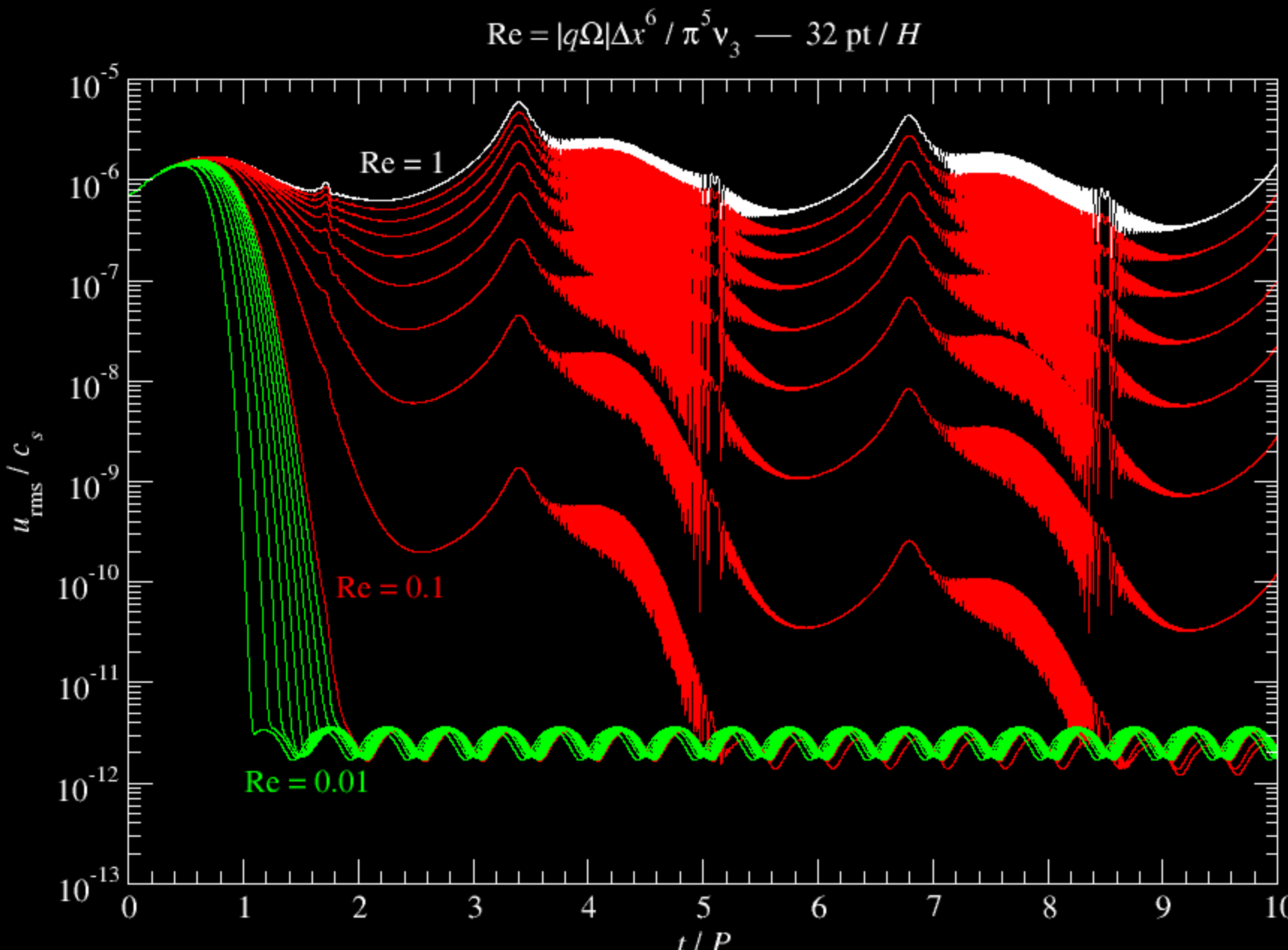
$$\frac{\partial f}{\partial t} = \nu_3 \nabla^6 f = \nu_3 \left(\frac{\partial^6}{\partial x^6} + \frac{\partial^6}{\partial y^6} + \frac{\partial^6}{\partial z^6} \right) f$$

$$f \sim e^{i(\omega t - \vec{k}\cdot\vec{x})} \;\Rightarrow\; \omega = i\nu_3 \left(k_x^6 + k_y^6 + k_z^6 \right)$$

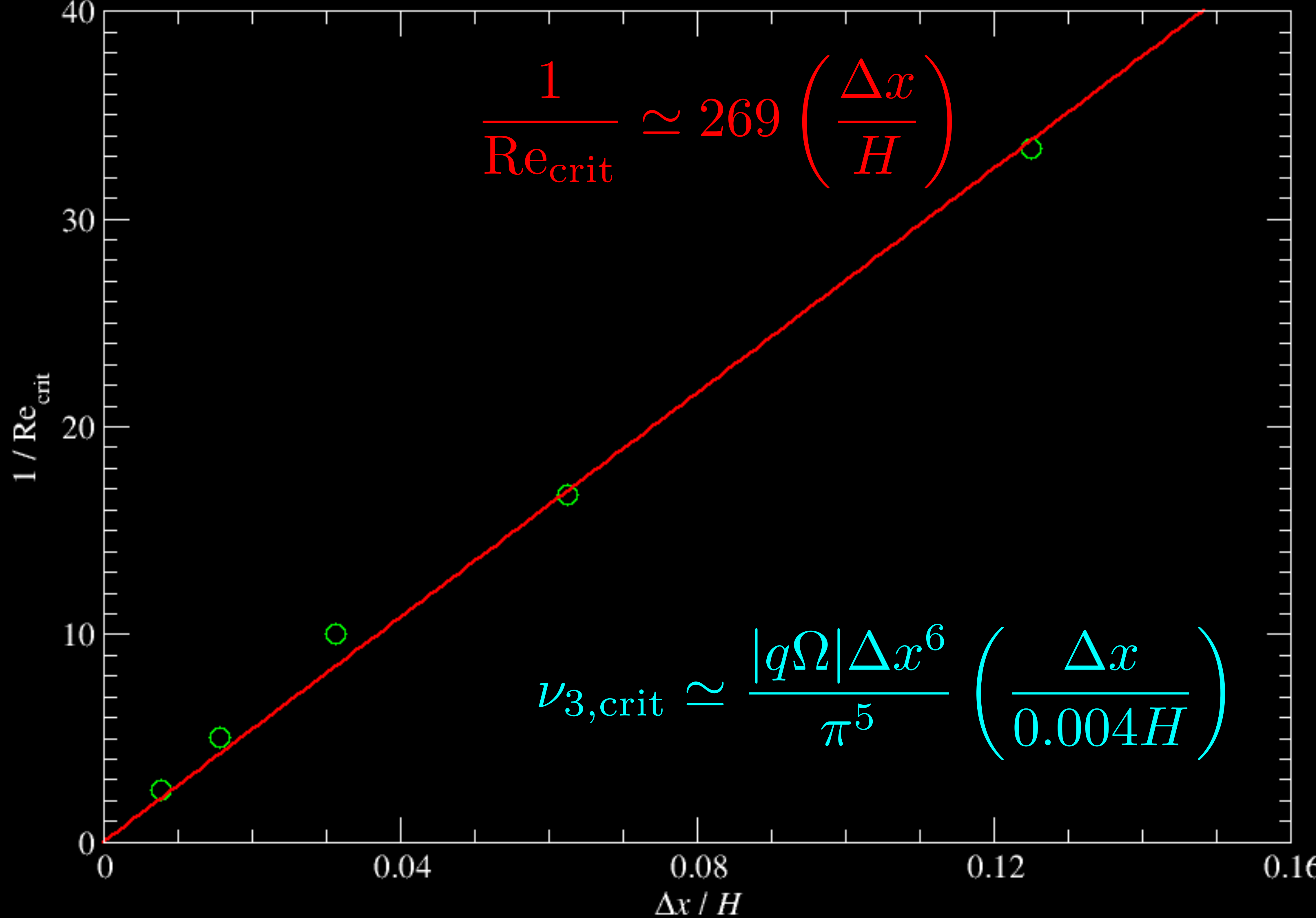
$$\mathrm{Re}_{\mathrm{mesh}} \equiv \frac{u}{\nu_3 k_{\mathrm{Nyq}}^5}, \qquad u \equiv |q\Omega|\Delta x$$



Re = |qΩ|Δx^6 / π^5ν_3 — 16 pt / H
Re = 1
Re = 0.1
Re = 0.01
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
10^-12
10^-13
u_rms / c_s
0
1
2
3
4
5
6
7
8
9
10
t / P

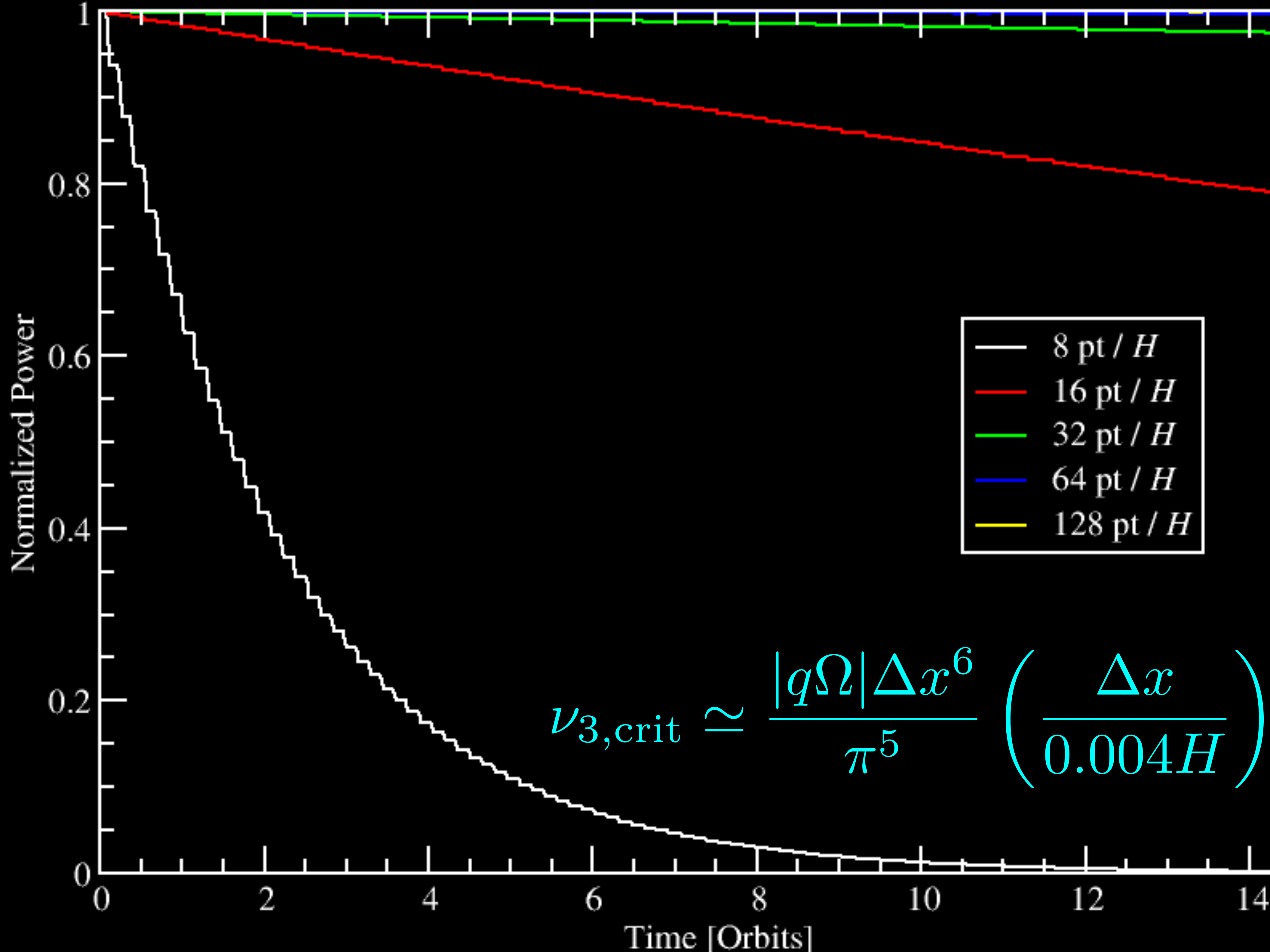


Re = |qΩ|Δx^6 / π^5ν_3 — 32 pt / H
Re = 1
Re = 0.1
Re = 0.01
u_rms / c_s
t / P
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
10^-12
10^-13
0
1
2
3
4
5
6
7
8
9
10



$\frac{1}{\mathrm{Re}_{\mathrm{crit}}} \simeq 269 \left(\frac{\Delta x}{H}\right)$
$\nu_{3,\mathrm{crit}} \simeq \frac{|q\Omega|\Delta x^6}{\pi^5} \left(\frac{\Delta x}{0.004H}\right)$
1 / Re$_{\mathrm{crit}}$
Δx / H
0
10
20
30
40
0.04
0.08
0.12
0.16

Linear Waves ($\lambda = L_x$)



Hyper-diffusion in x Only

$$\frac{\partial f}{\partial t} = \nu_3 \frac{\partial^6 f}{\partial x^6}$$

$$\nu_{3,\mathrm{crit}} \simeq \frac{|q\Omega|\Delta x^6}{\pi^5}\left(\frac{\Delta x}{0.004H}\right)$$

How to Activate It?

- makefile.local

```
SHEAR = shear
```

- run.in

```
&shear_run_pars
    lhyper3x_mesh = .true.
    diff_hyper3x_mesh = 0.03
    ! Re = 1 / pi^5 / diff_hyper3x_mesh
/
```