

Status of the bfield Module

Chao-Chin Yang
Department of Astronomy and Theoretical Physics
Lund University
Sweden

A vs. B

- Magnetic vector potential A
 - $\nabla \cdot \mathbf{B} = 0$ is guaranteed.
 - One *second* derivative.
- Magnetic field \mathbf{B}
 - $\nabla \cdot \mathbf{B} = 0$ is not guaranteed.
 - Two *first* derivatives.

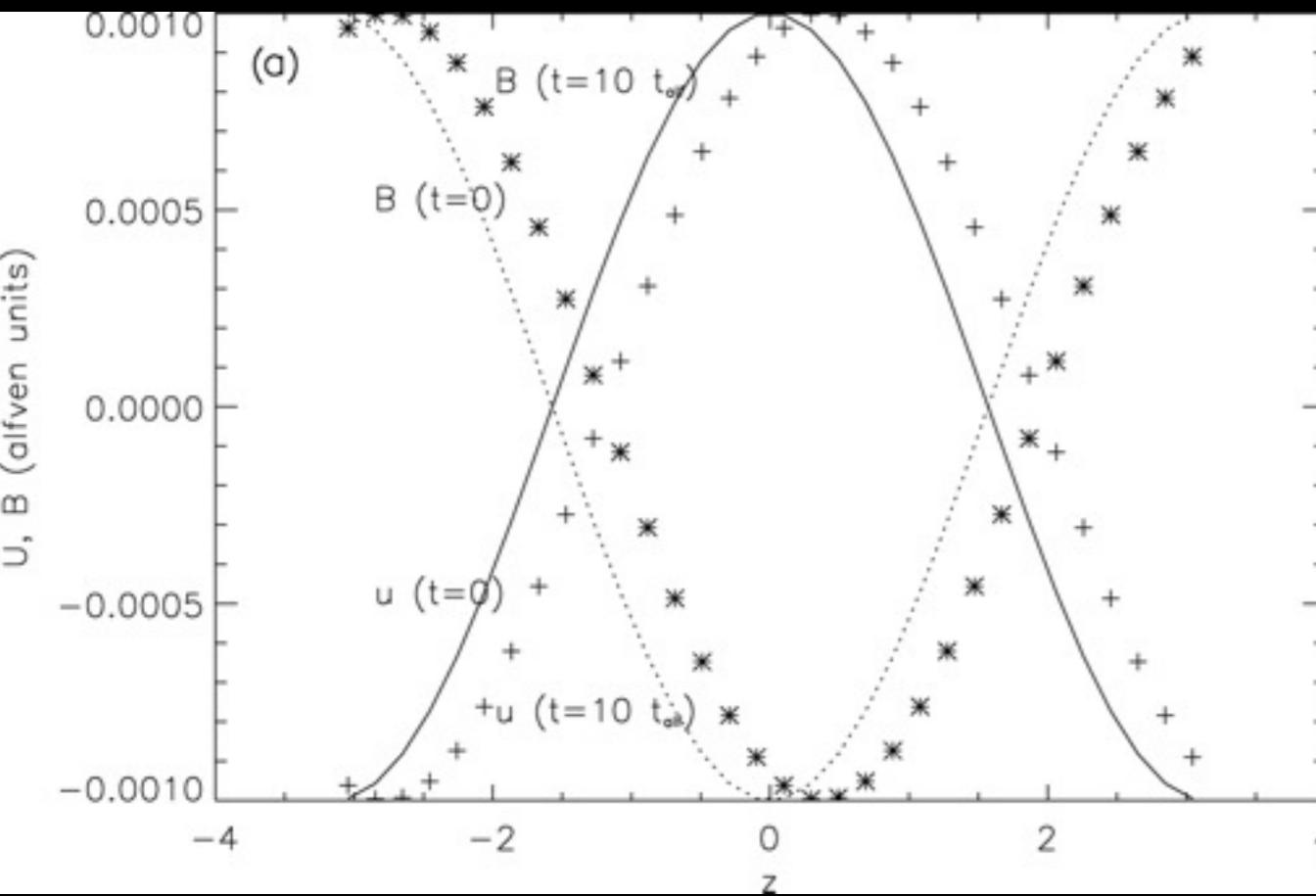
$$\nabla \cdot \mathbf{B} = 0$$

- Diffusion method: $\partial_t \mathbf{B} = \eta_D \nabla \nabla \cdot \mathbf{B}$ (*Dedner et al. 2002*)
- Projection method: $\mathbf{k} \cdot \mathbf{B} = 0$ (*Brackbill & Barnes 1980*)
 - Requires Fourier transforms.
- Divergence-free interpolation (*McNally 2011*)
 - Requires divergence-free basis.
- Face-centered constraint transport (*Evans & Hawley 1988*)
 - Requires staggered grid and interpolation.

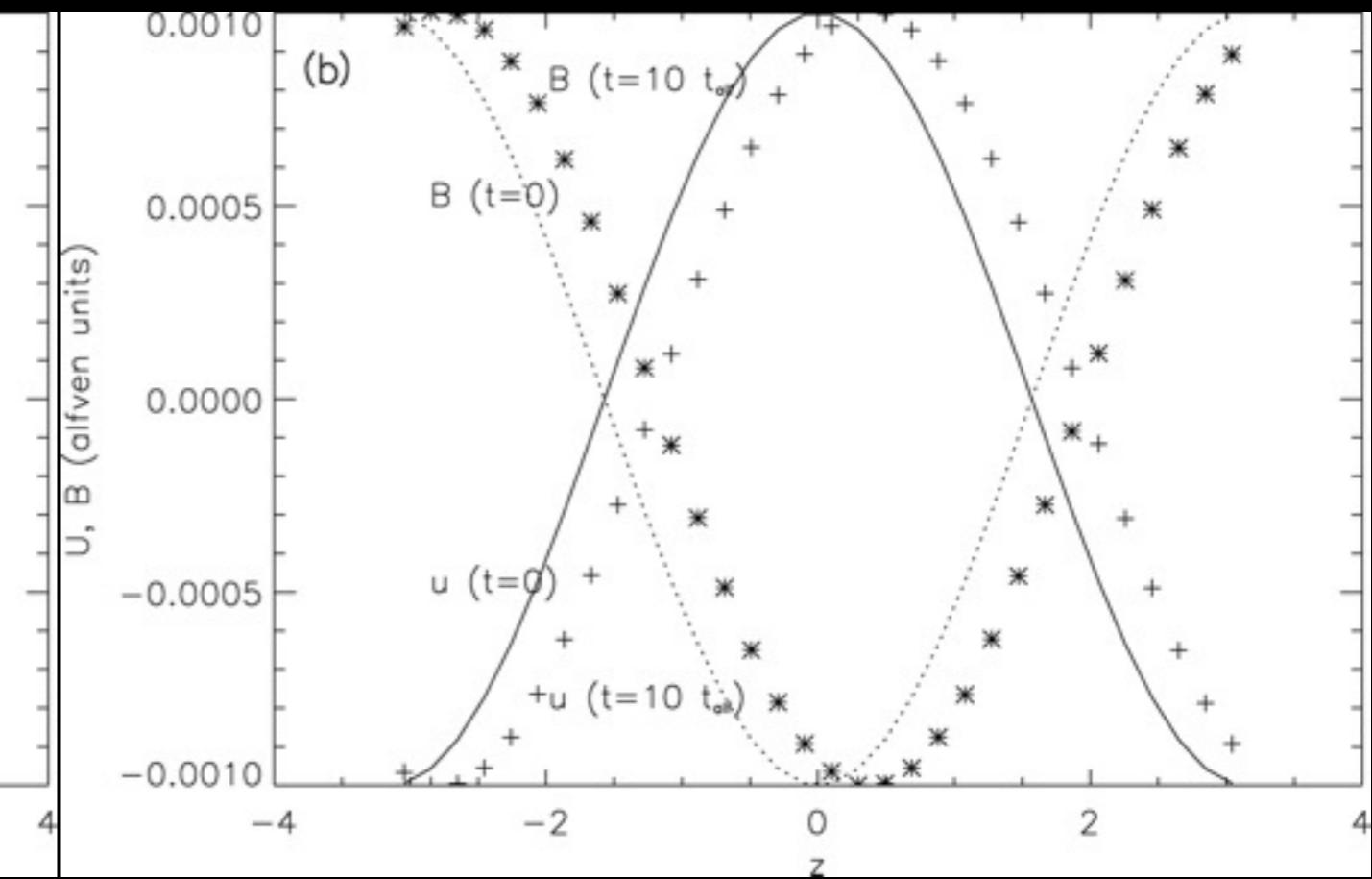
Cell-centered Constraint Transport

- This is not my / a new idea!

- *Maron, Oishi, & Mac Low (2008)*



A formulation



B formulation

Induction Equation and $\nabla \cdot \mathbf{B} = 0$

$$\partial_t \vec{B} = -\vec{\nabla} \times \vec{E}$$

$$\partial_t \vec{\nabla} \cdot \vec{B} = -\vec{\nabla} \cdot \vec{\nabla} \times \vec{E} = 0?$$

$$\partial_x f \equiv a_l f_l$$

$$\begin{aligned}\partial_x (\nabla \times \vec{E})_x &= \partial_x (\partial_y E_z - \partial_z E_y) \\&= \partial_x (a_m E_{z,0m0} - a_n E_{y,00n}) \\&= a_l (a_m E_{z,lm0} - a_n E_{y,l0n})\end{aligned}$$

$$\begin{aligned}\partial_y (\nabla \times \vec{E})_y &= \partial_y (\partial_z E_x - \partial_x E_z) \\&= \partial_y (a_n E_{x,00n} - a_l E_{z,l00}) \\&= a_m (a_n E_{x,0mn} - a_l E_{z,lm0})\end{aligned}$$

$$\begin{aligned}\partial_z (\nabla \times \vec{E})_z &= \partial_z (\partial_x E_y - \partial_y E_x) \\&= \partial_z (a_l E_{y,l00} - a_m E_{x,0m0}) \\&= a_n (a_l E_{y,l0n} - a_m E_{x,0mn})\end{aligned}$$

$$\partial_x f \equiv a_l f_l$$

$$\begin{aligned}
\partial_x (\nabla \times \vec{E})_x &= \partial_x (\partial_y E_z - \partial_z E_y) \\
&= \partial_x (a_m E_{z,0m0} - a_n E_{y,00n}) \\
&= a_l (\cancel{a_m E_{z,lm0}} - \cancel{a_n E_{y,l0n}}) \\
\partial_y (\nabla \times \vec{E})_y &= \partial_y (\partial_z E_x - \partial_x E_z) \\
&= \partial_y (a_n E_{x,00n} - a_l E_{z,l00}) \\
&= a_m (\cancel{a_n E_{x,0mn}} - \cancel{a_l E_{z,lm0}}) \\
\partial_z (\nabla \times \vec{E})_z &= \partial_z (\partial_x E_y - \partial_y E_x) \\
&= \partial_z (a_l E_{y,l00} - a_m E_{x,0m0}) \\
&= a_n (\cancel{a_l E_{y,l0n}} - \cancel{a_m E_{x,0mn}})
\end{aligned}$$

Cylindrical Coordinates

$$\begin{aligned}(\nabla \times \vec{E})_r &= r^{-1} \partial_\theta E_z - \partial_z E_\theta \\&= r_0^{-1} a_m E_{z,0m0} - a_n E_{\theta,00n} \\(\nabla \times \vec{E})_\theta &= \partial_z E_r - \partial_r E_z \\&= a_n E_{r,00n} - a_l E_{z,l00} \\(\nabla \times \vec{E})_z &= r^{-1} E_\theta + \partial_r E_\theta - r^{-1} \partial_\theta E_r \\&= r_0^{-1} E_{\theta,000} + a_l E_{\theta,l00} - r_0^{-1} a_m E_{r,0m0}\end{aligned}$$

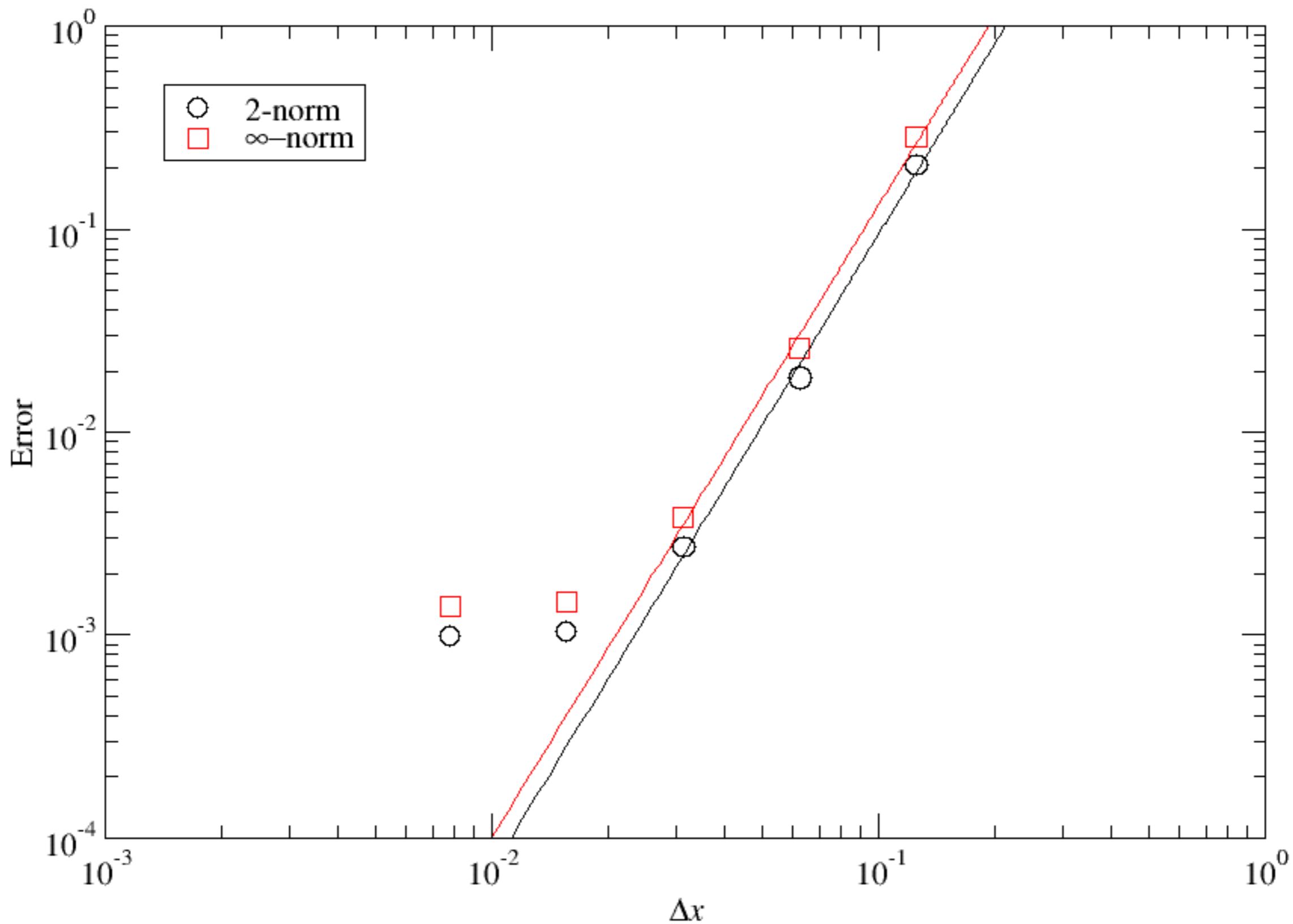
Cylindrical Coordinates

$$\begin{aligned} r^{-1} \partial_r \left[r(\nabla \times \vec{E})_r \right] &= r^{-1} (\nabla \times \vec{E})_r + \partial_r (\nabla \times \vec{E})_r \\ &= r_0^{-1} (r_0^{-1} a_m E_{z,0m0} - a_n E_{\theta,00n}) \\ &\quad + a_l (r_l^{-1} a_m E_{z,lm0} - a_n E_{\theta,l0n}) \\ r^{-1} \partial_\theta (\nabla \times \vec{E})_\theta &= r^{-1} \partial_\theta (a_n E_{r,00n} - a_l E_{z,l00}) \\ &= r_0^{-1} a_m (a_n E_{r,0mn} - a_l E_{z,lm0}) \\ \partial_z (\nabla \times \vec{E})_z &= \partial_z (r_0^{-1} E_{\theta,000} + a_l E_{\theta,l00} - r_0^{-1} a_m E_{r,0m0}) \\ &= a_n (r_0^{-1} E_{\theta,00n} + a_l E_{\theta,l0n} - r_0^{-1} a_m E_{r,0mn}) \end{aligned}$$

Cylindrical Coordinates

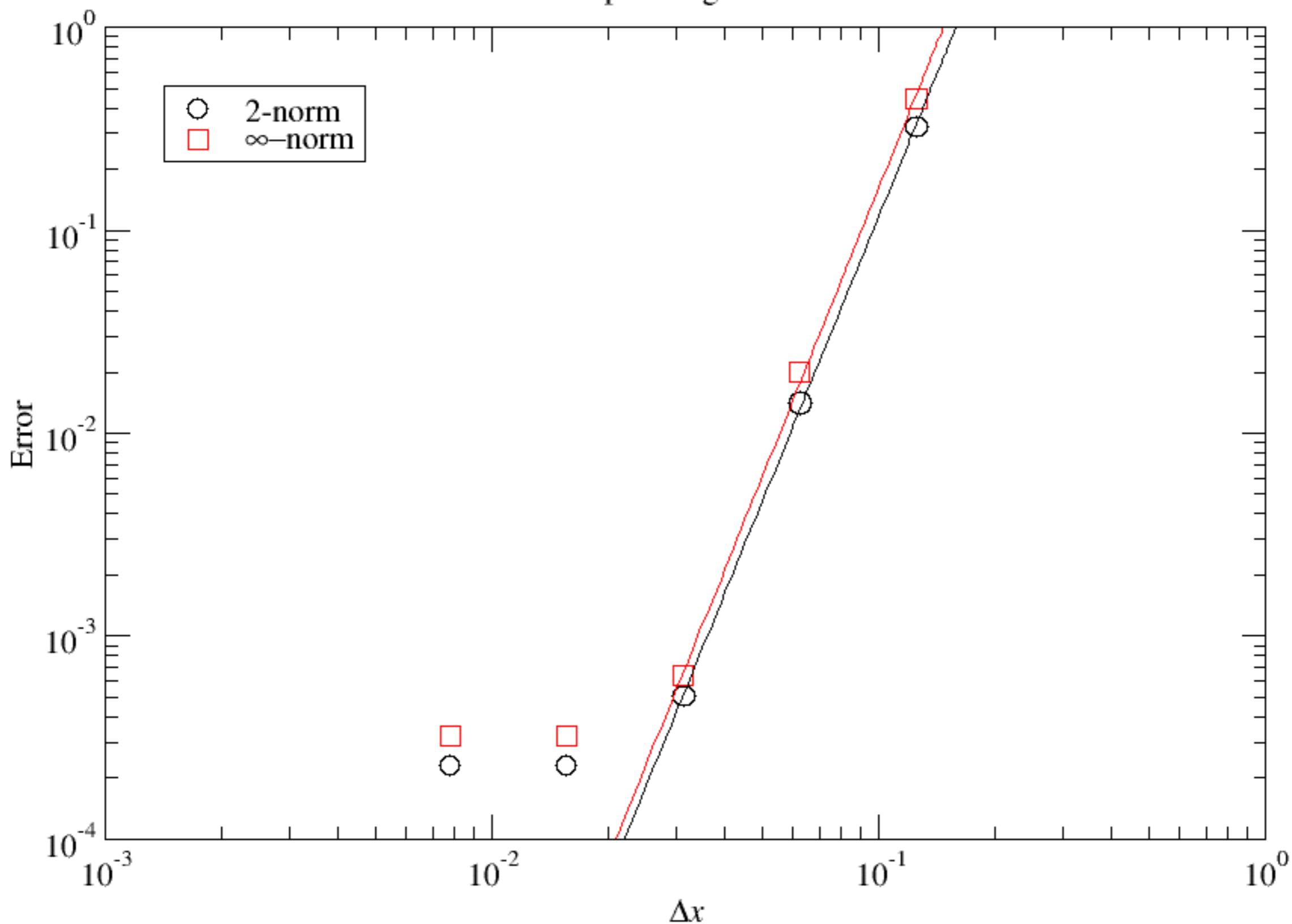
$$\begin{aligned} r^{-1} \partial_r \left[r(\nabla \times \vec{E})_r \right] &= r^{-1} (\nabla \times \vec{E})_r + \partial_r (\nabla \times \vec{E})_r \\ &= \frac{r_0^{-1} (r_0^{-1} a_m E_{z,0m0} - \cancel{a_n E_{\theta,00n}})}{a_l (r_l^{-1} a_m E_{z,lm0} - \cancel{a_n E_{\theta,l0n}})} \\ r^{-1} \partial_\theta (\nabla \times \vec{E})_\theta &= r^{-1} \partial_\theta (a_n E_{r,00n} - a_l E_{z,l00}) \\ &= \frac{r_0^{-1} a_m (a_n E_{r,0mn} - \cancel{a_l E_{z,lm0}})}{a_l (r_l^{-1} a_m E_{z,lm0} - \cancel{a_n E_{\theta,l0n}})} \\ \partial_z (\nabla \times \vec{E})_z &= \partial_z (r_0^{-1} E_{\theta,000} + a_l E_{\theta,l00} - r_0^{-1} a_m E_{r,0m0}) \\ &= a_n (r_0^{-1} \cancel{E_{\theta,00n}} + a_l \cancel{E_{\theta,l0n}} - r_0^{-1} \cancel{a_m E_{r,0mn}}) \end{aligned}$$

Ideal Alfvén Waves



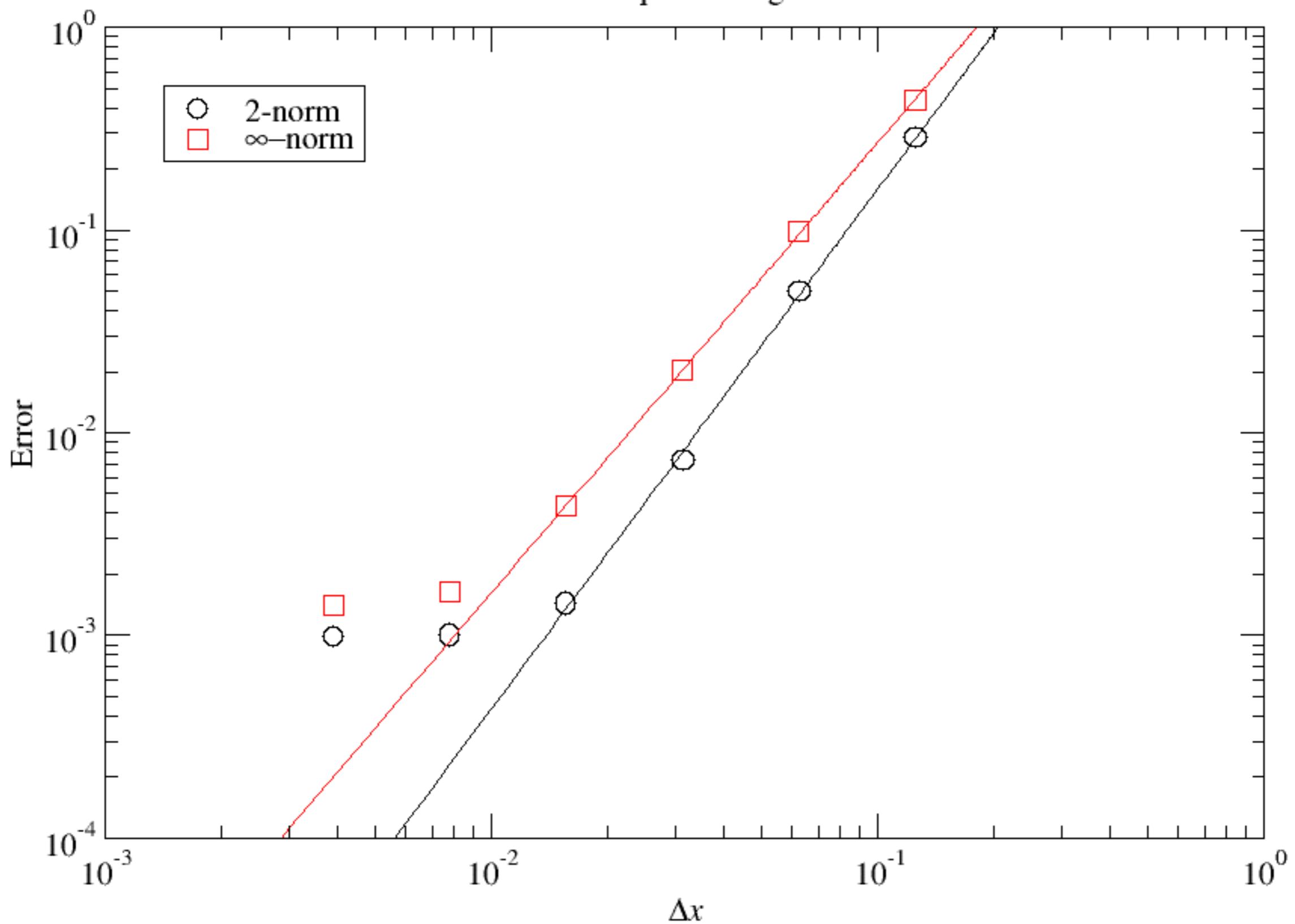
Damped Alfvén Waves

Explicit algorithm



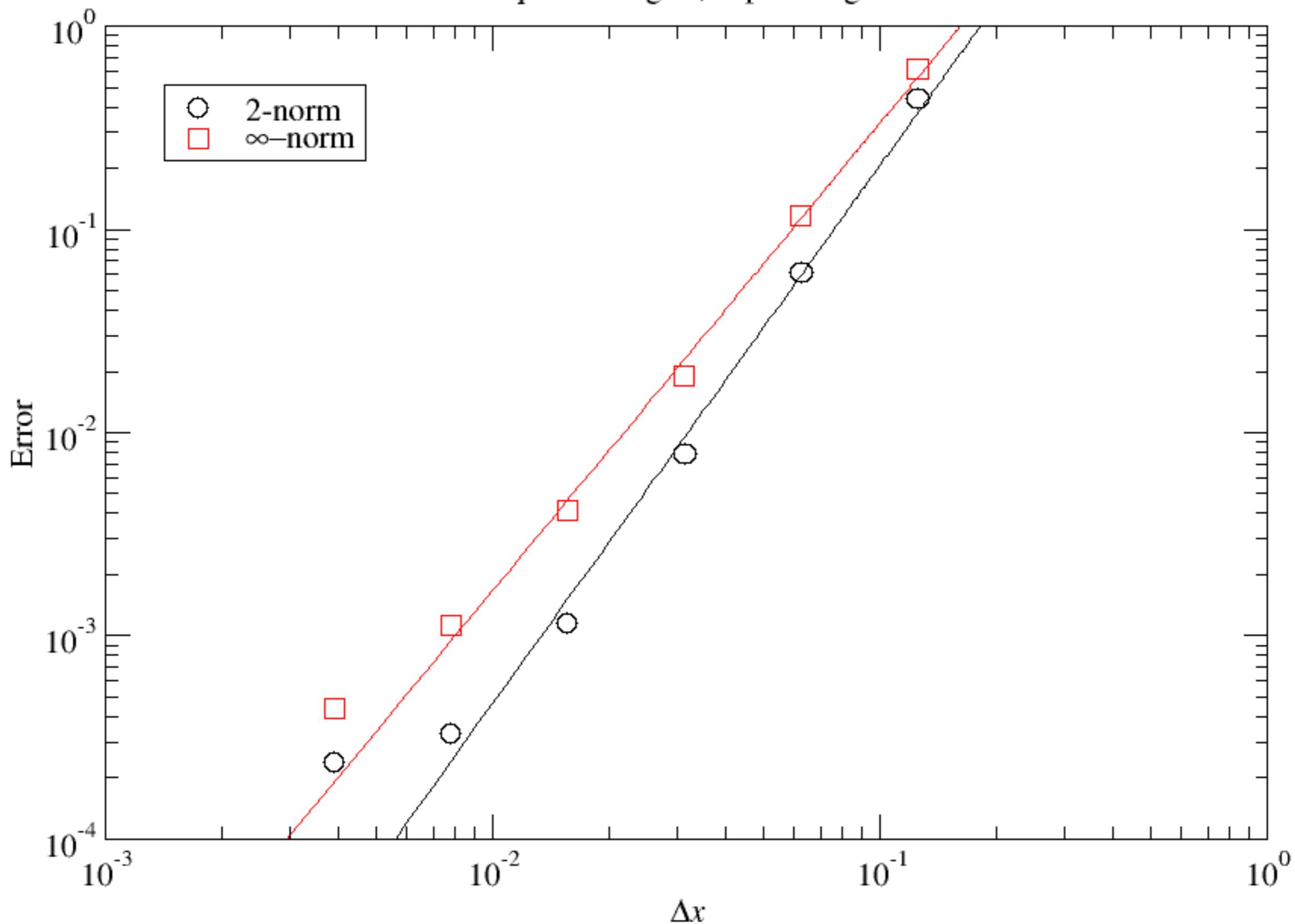
Ideal Alfvén Waves

Non-equidistant grid



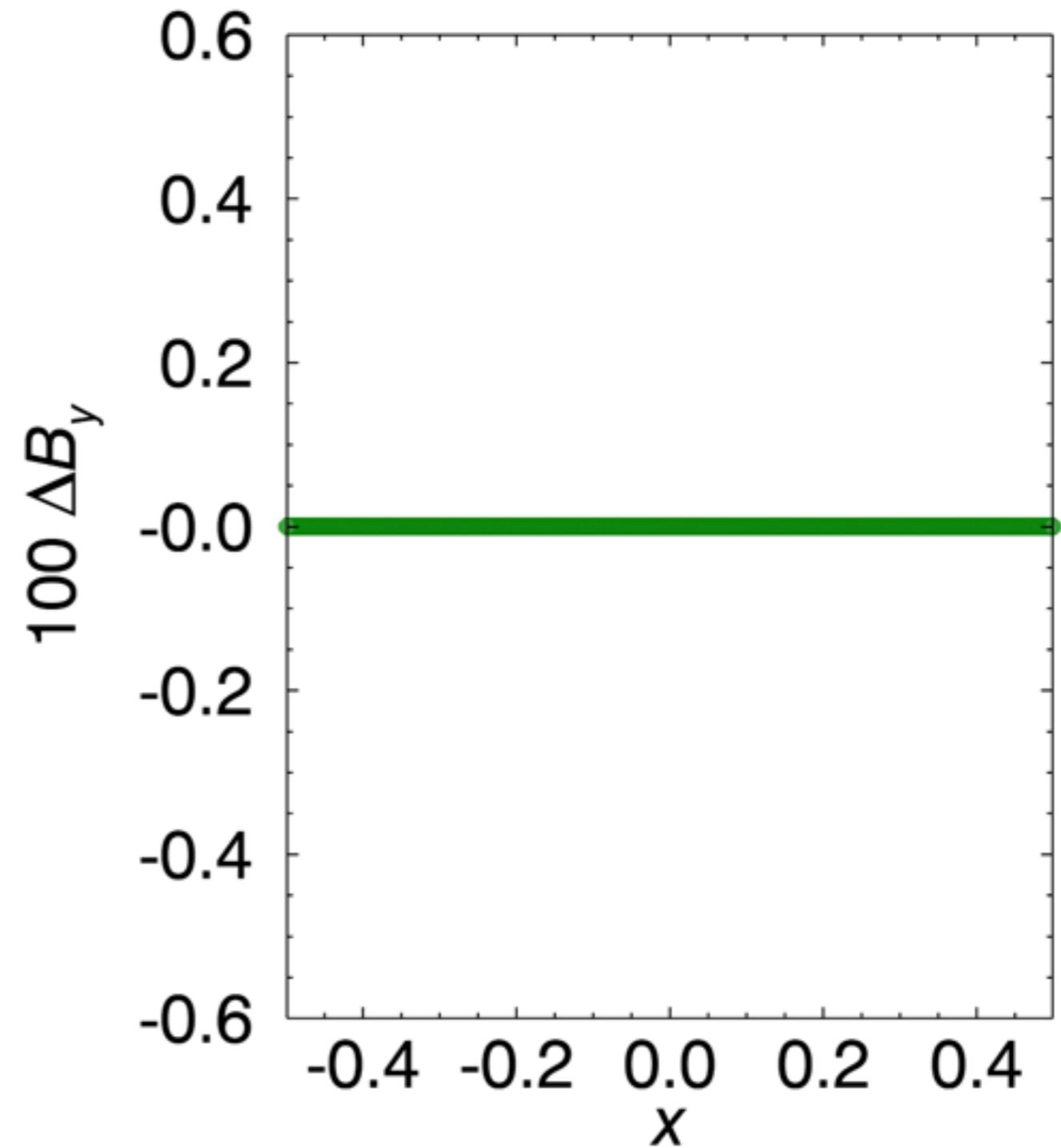
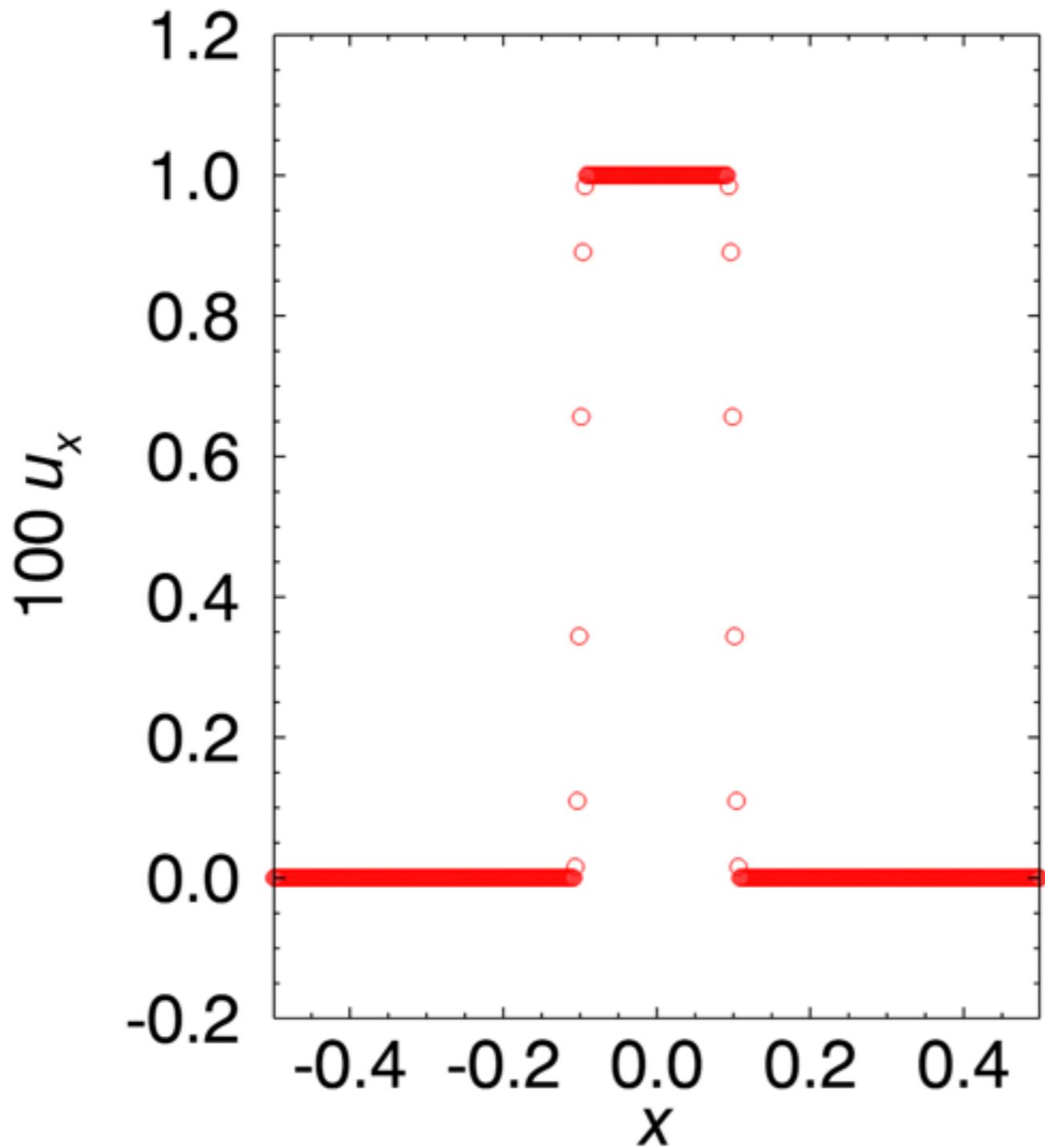
Damped Alfvén Waves

Non-equidistant grid, explicit algorithm



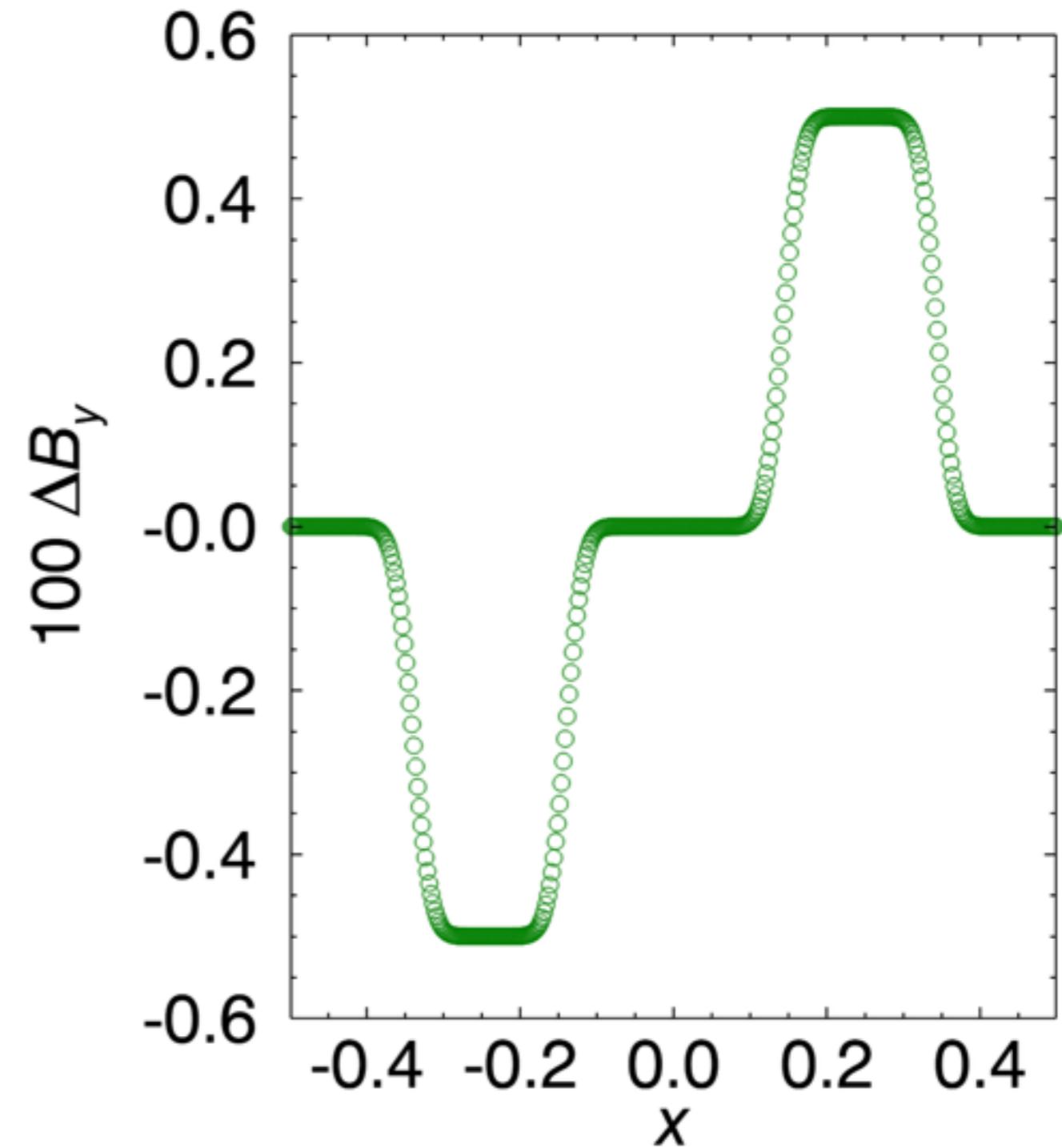
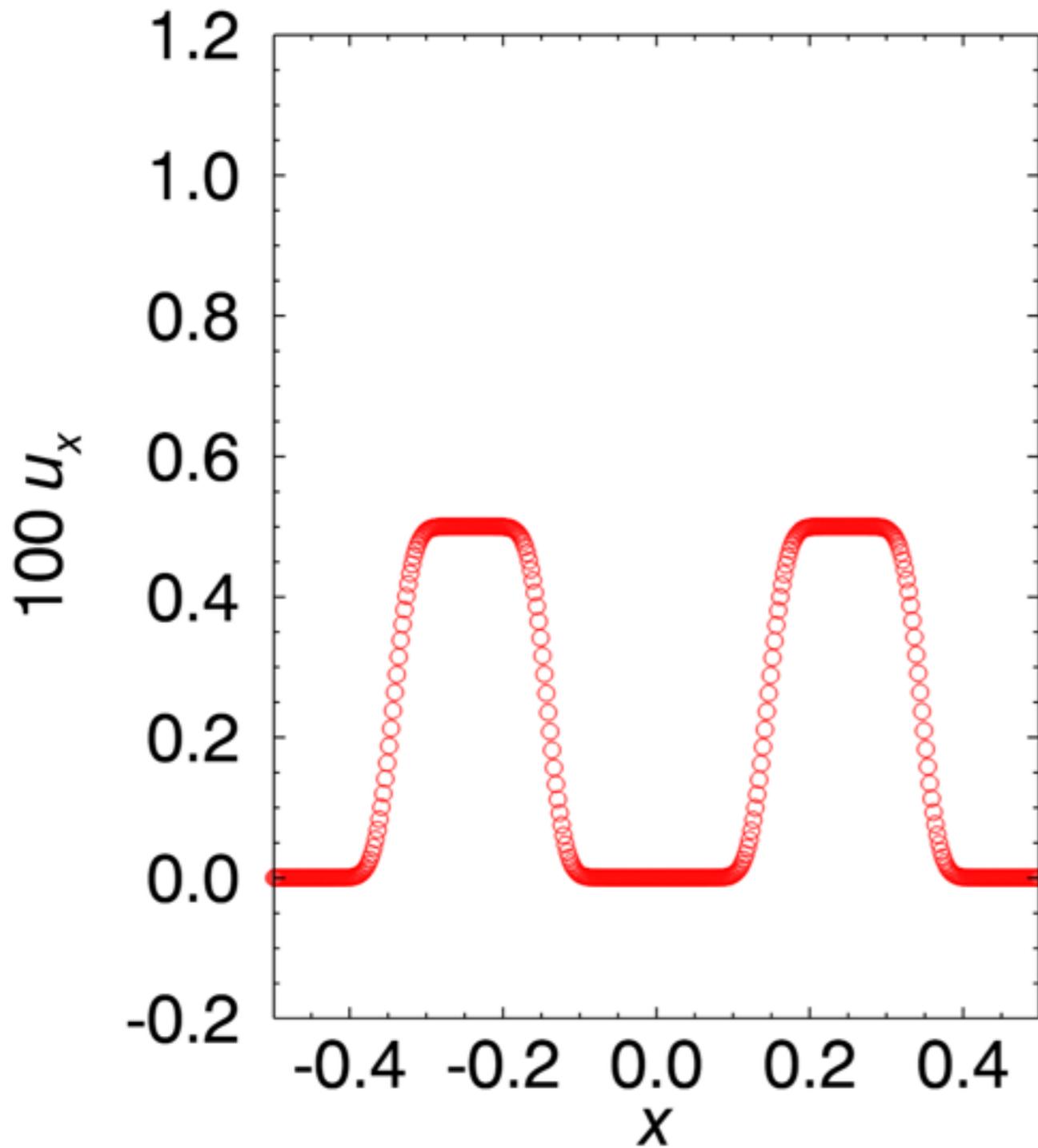
Top-hat Alfvén Wave

A form



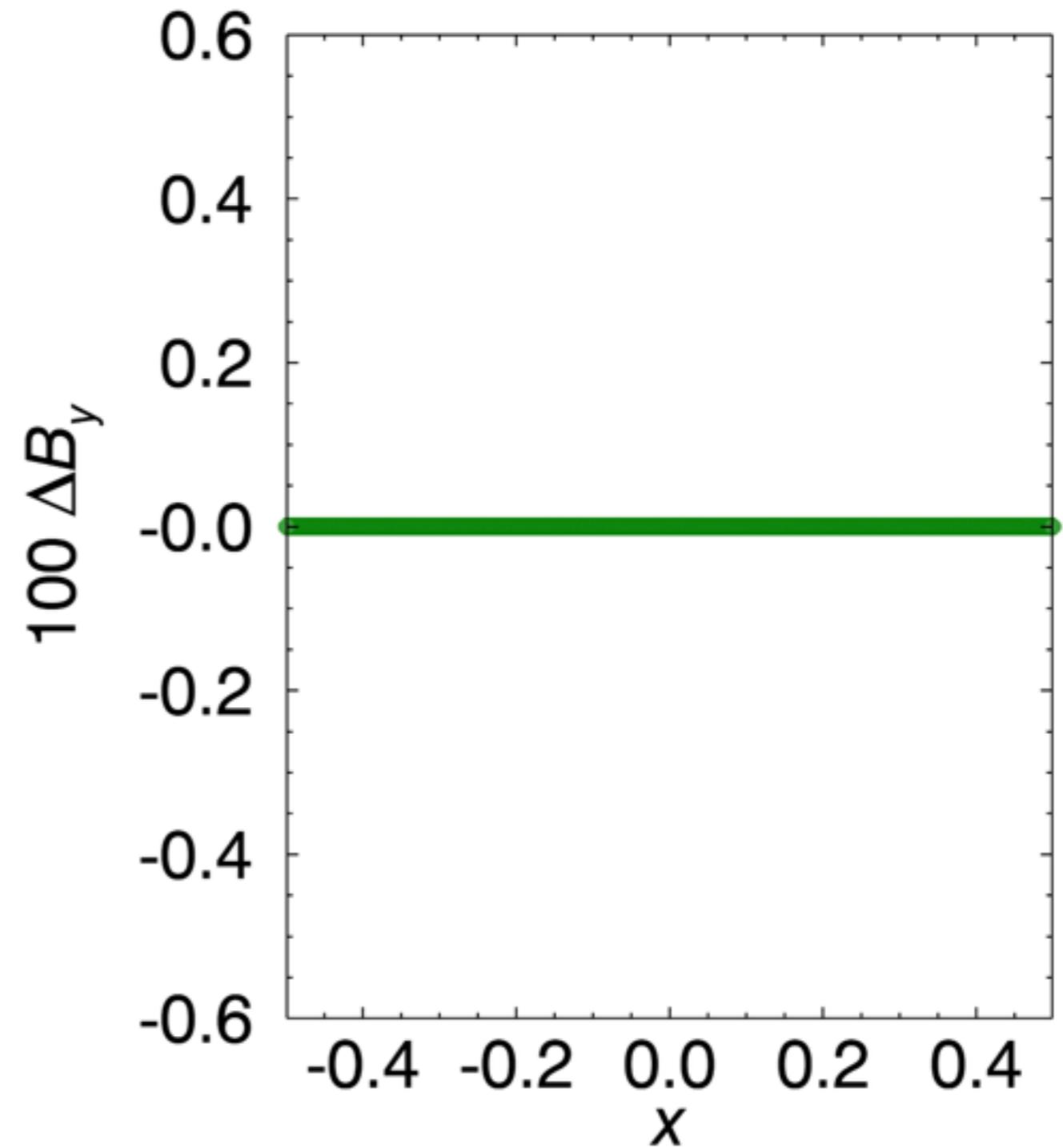
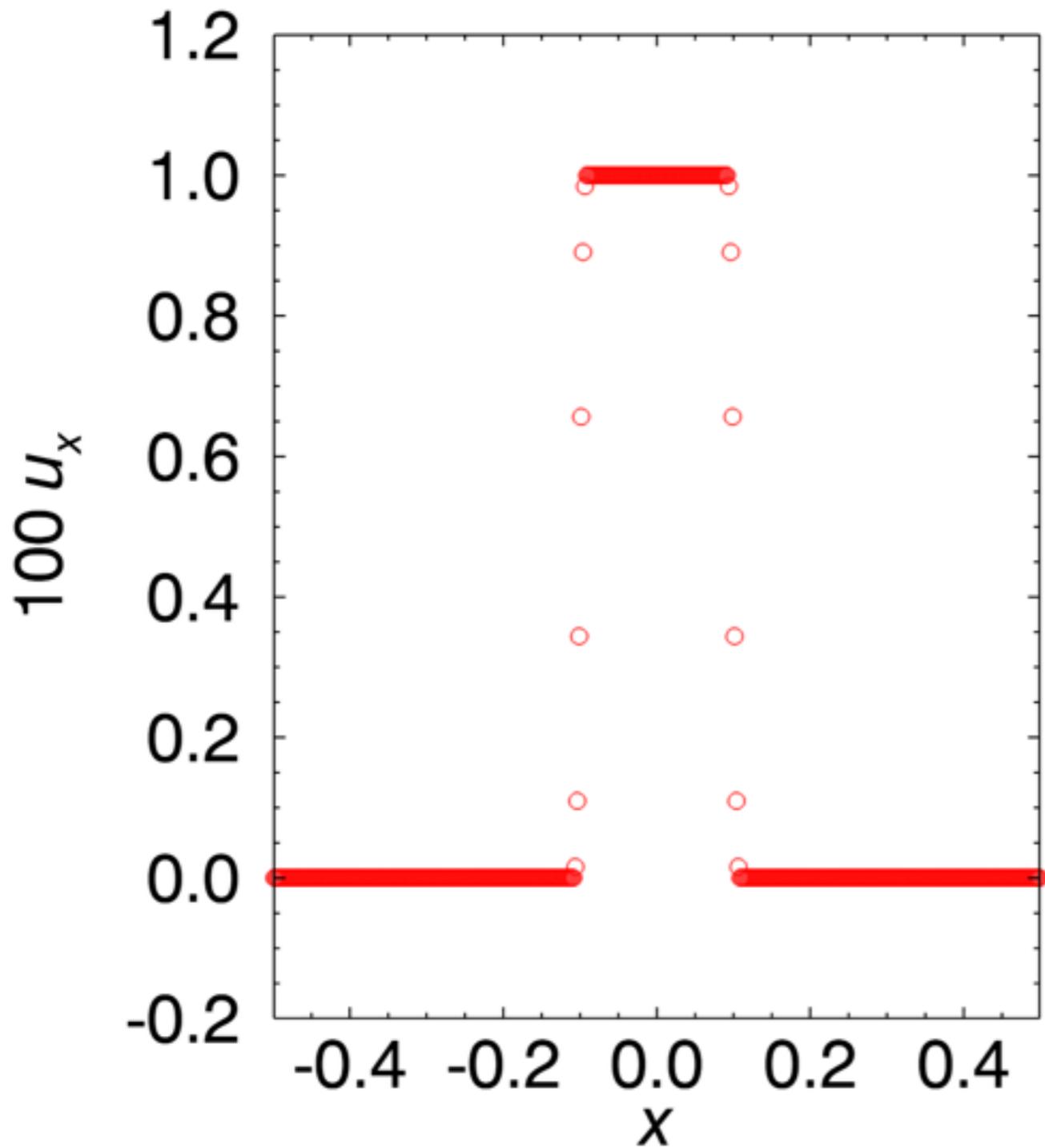
Top-hat Alfvén Wave

A form



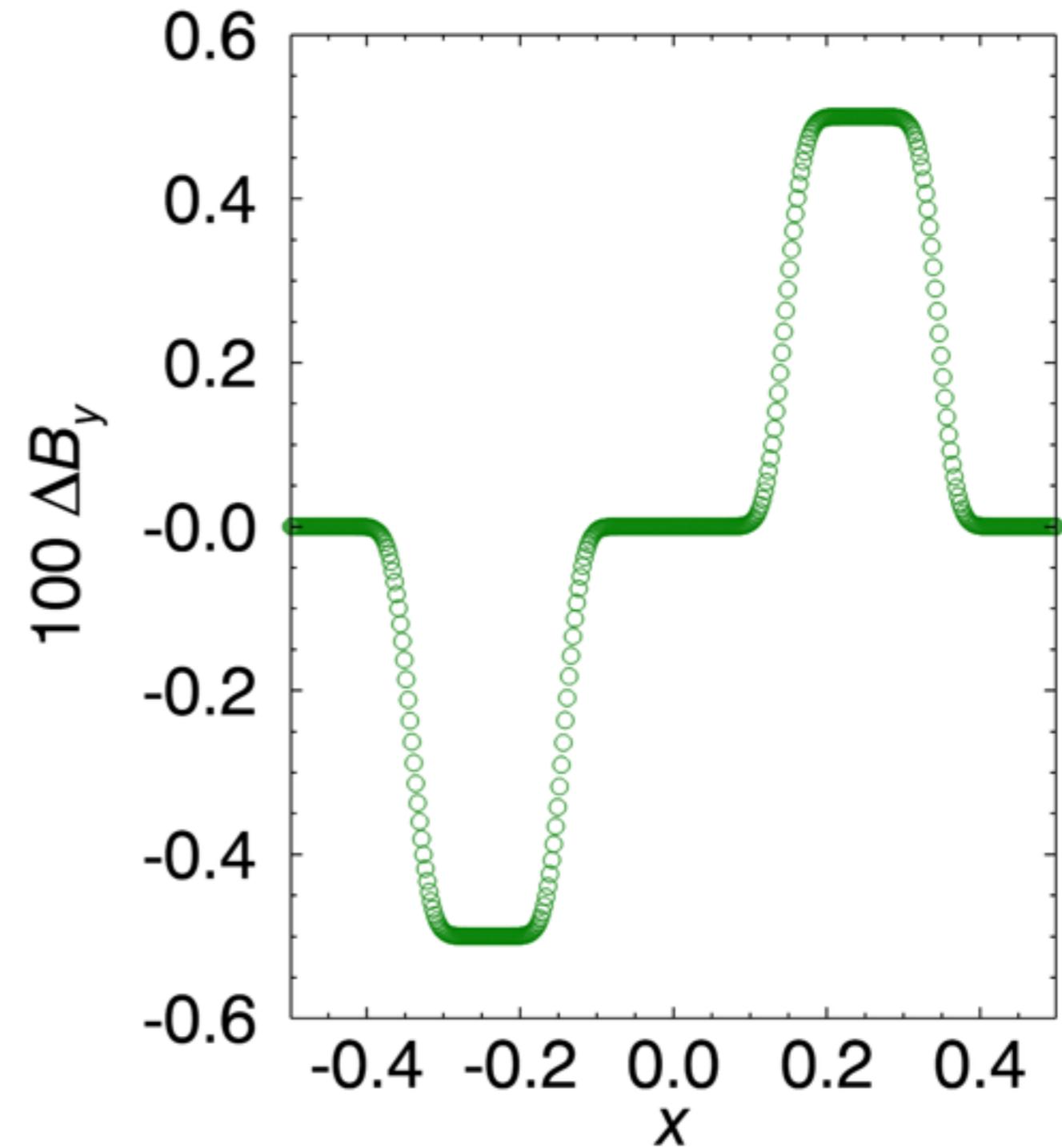
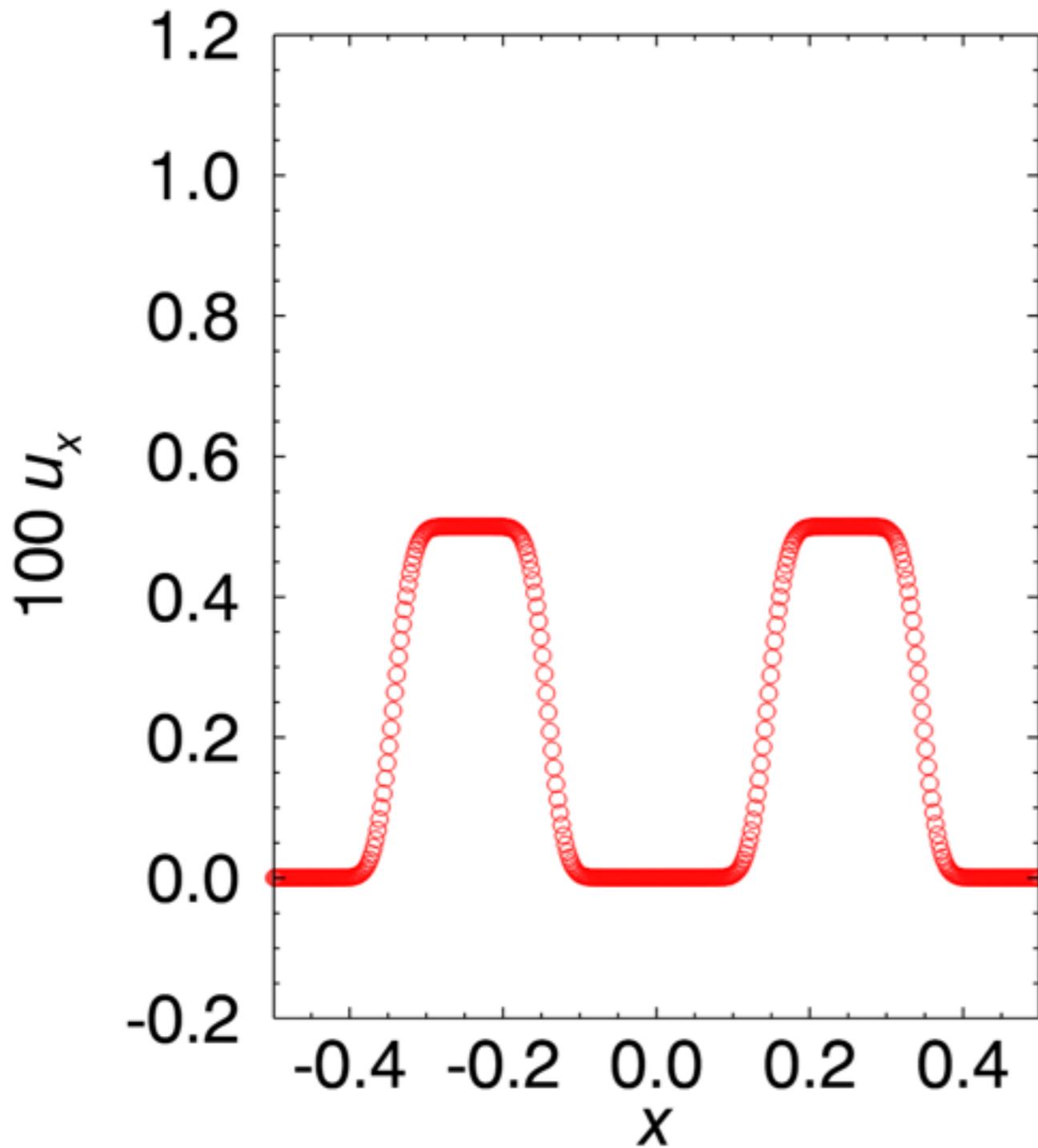
Top-hat Alfvén Wave

B form



Top-hat Alfvén Wave

B form



Status

$$\partial_t \vec{B} = \vec{\nabla} \times (\vec{v} \times \vec{B} - \mu_0 \eta \vec{J} - \mu_0 \eta_3 \nabla^4 \vec{J}), \quad \vec{J} = \vec{\nabla} \times \vec{B} / \mu_0$$

- Background uniform field \mathbf{B}_{ext}
- Resistivity
 - Constant, shock, vertically varying
 - With Ohmic heating
 - Hyper-resistivity
- Shear
 - Cannot use advection by interpolation