Capacity Analysis of Collaborative Wireless Transmission

Olav Tirkkonen

Department of Communications and Networking
Helsinki University of Technology (TKK)
and
Nokia Research Center, Helsinki

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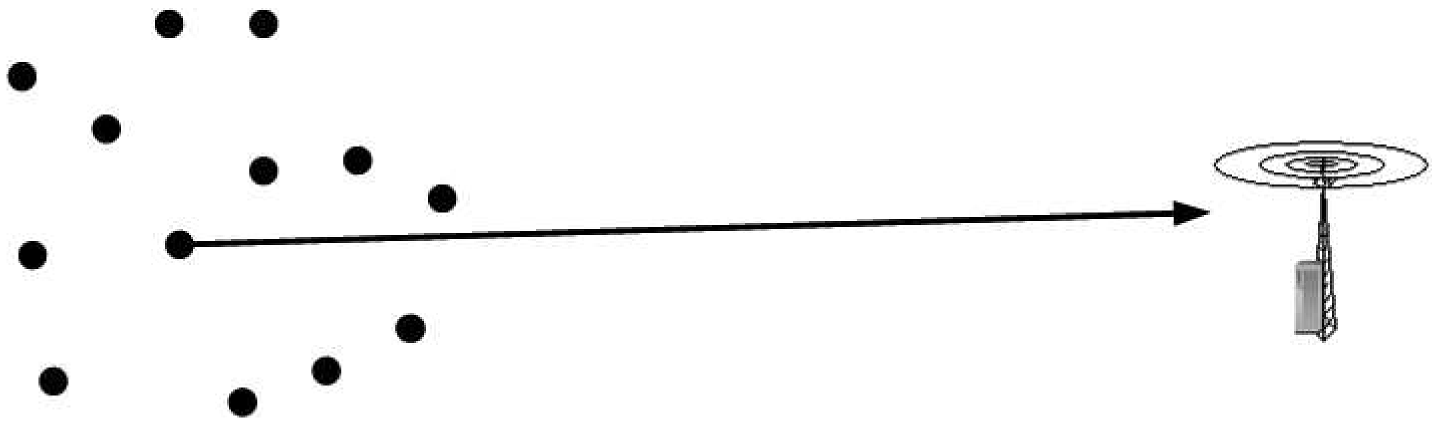
Outline

- system model
- random matrices
- results

Single-hop Network



- The source (user) transmits, the destination (BS) receives
- The BS has N_r receive antennas, performs optimum antenna processing
- user at cell edge not too happy

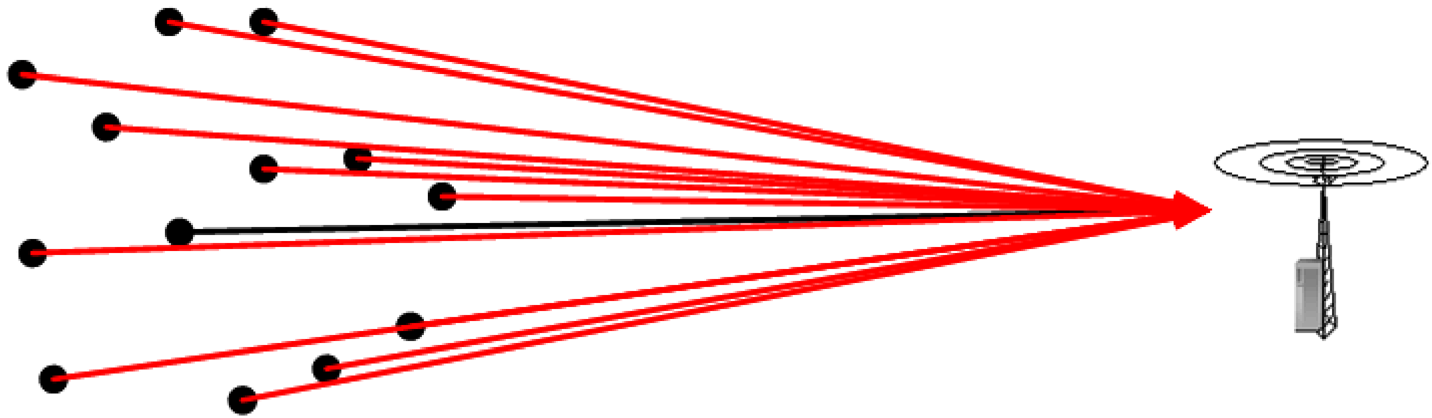
Collaborative Transmission

- [Laneman & al 2000, Sendonaris & al 2003, etc, etc]
- clusters of users collaborate to transmit messages
- formation of a virtual antenna array giving rise to matrix (MIMO) channels
- branch of widely investigated relaying field

Two-hop Network, 1:st Hop

- There are N_{t} users, one is the source
- The $N_{\mathrm{t}} - 1$ other users act as relay nodes
- The source *broadcasts* content to relays
- Assumptions:
  - source knows channel gains $\tilde{\gamma}_j$ between source and relays
  - distance to destination $>>$ distance to relays, direct link in 1:st hop discarded

Two-hop Network, 2:nd Hop



- two-hop decode and forward relaying
- Relays (and source) transmit content to destination
- Matrix channel, collaborative array transmission from N_t nodes:

$$\underset{N_\mathrm{r}\times 1}{\mathbf{y}} = \underset{N_\mathrm{r}\times N_\mathrm{t}}{\mathbf{H}} \underset{N_\mathrm{t}\times 1}{\mathbf{w}} x + \underset{N_\mathrm{r}\times 1}{\mathbf{n}}$$

Two-hop Network, 2:nd Hop, II

- matrix channel SVD:

$$\underset{N_\mathrm{r}\times N_\mathrm{t}}{\mathbf{H}} = \underset{N_\mathrm{r}\times N_\mathrm{r}}{\mathbf{U}} \underset{N_\mathrm{r}\times N_\mathrm{t}}{\Sigma} \mathbf{V}^\mathrm{H}$$

- collective knowledge of optimum beamforming vector: $\mathbf{w}$ maximum eigenvalue vector ot $\mathbf{H}^\mathrm{H}\mathbf{H}$
  - relays phase and weight transmissions—signals combine coherently at destination, power optimally distributed over channel
  - each relay node knows w_j
  - effective channel becomes $\mathbf{H}\mathbf{w} = \mathbf{U}\begin{bmatrix}\sigma_\mathrm{max}\\ 0\\ \vdots\\ 0\end{bmatrix} = \mathbf{u}_1\sigma_\mathrm{max}$
    - σ_max largest singular value of channel
  - receiver sees channel gain $\lambda_\mathrm{max} = \sigma^2_\mathrm{max}$

Capacity Analysis

- Capacity of hop j is C_j [nats/s]
- Distribution of C_j is $p_j(C_j)$
- Source decides division of resources between 1:st and 2:nd hop:

$$T_1 C_1 = T_2 C_2$$

- Capacity of relay link is

$$C = \frac{T_2 C_2}{T_1 + T_2} = \frac{C_1 C_2}{C_1 + C_2}$$

- distribution of C is

$$p(C) = \int \mathrm{d}C_2 \left(\frac{C_2}{C_2 - C} \right)^2 p_1 \left(\frac{C C_2}{C_2 - C} \right) p_2(C_2)$$

Capacity: 1:st Hop

- broadcast channel capacity: capacity of worst link

$$C_2 = \min_j \log\left(1 + \tilde{\gamma}_j\right)$$

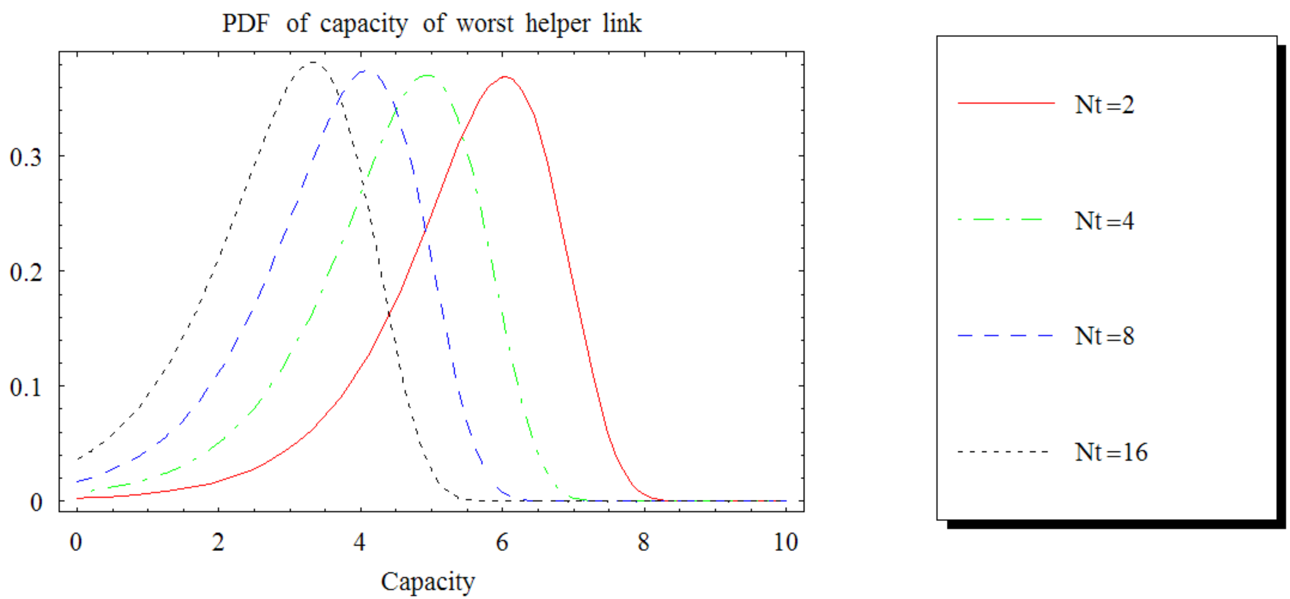
- order statistics, worst of $N_\mathrm{t} - 1$

$$f\left(\gamma_{\min}\right) = (N_\mathrm{t} - 1) F(\gamma)^{(N_\mathrm{t}-2)} f(\gamma)$$

- channels distributed as $\sqrt{g_1}\times$ i.i.d complex Gaussian, variance 1: $\tilde{\gamma}_j = g_1 \gamma_j$ where $p(\gamma_j) = \mathrm{e}^{-\gamma_j}$

- $\Rightarrow$ capacity distribution in closed form

First Hop Capacity PDF



- The more friends you want to have, the more likely a bad friend is, the more you have to pay

Second Hop Capacity

- $\mathbf{H}$ distributed as $\sqrt{g_2}$ times correlated Gaussian

$$p(\mathbf{H}) = \frac{\mathrm{e}^{-\mathrm{Tr}\left[\mathbf{R}_r^{-1}\mathbf{H}\mathbf{H}^{\mathrm{H}}\right]}}{\pi^{N_\mathrm{t}N_\mathrm{r}}\det^{N_\mathrm{t}}\mathbf{R}_r} \tag{1}$$

  - correlation only at destination end (realistic)
  - assume $N_\mathrm{r} \geq N_\mathrm{t}$: $\mathbf{H}\mathbf{H}^{\mathrm{H}}$ is Wishart

- eigendecompose

$$p(\Lambda, U) \sim \frac{\mathrm{e}^{-\mathrm{Tr}\left[\mathbf{R}_r^{-1}\mathbf{U}^{\mathrm{H}}\Lambda\mathbf{U}\right]}\det^{N_\mathrm{r}-N_\mathrm{t}}(\Lambda)\Delta^2(\Lambda)}{\det^{N_\mathrm{t}}\mathbf{R}_r}$$

- now integrate out $\mathbf{U}$
  - an *exactly solvable* partition function on compact phase space
  - Integrability theory: Duistermaat-Heckman can be applied [Morozov 1995]

Determinant Formulae for Correlated Matrices

- for correlation in smaller dimension [Harish-Chandra 1957, Itzykson-Zuber 1992]

- for correlation in larger dimension [Gao-Smith 2000]

- here HCIZ:

$$p(\Lambda) \sim \frac{\det \mathbf{A} \det^{N_\mathrm{r}-N_\mathrm{t}}(\Lambda)\Delta(\Lambda)}{\Delta\left(\mathbf{R}_r^{-1}\right)\det^{N_\mathrm{t}}\mathbf{R}_r}$$

  $\mathbf{A}$ is a matrix of exps, $a_{ij} = \mathrm{e}^{-\lambda_i/\rho_j}$

- $\det \mathbf{A}$ carries a contribution from each of the extremal points of the integrand on compact phase space

Distribution of Largest Eigenvalue

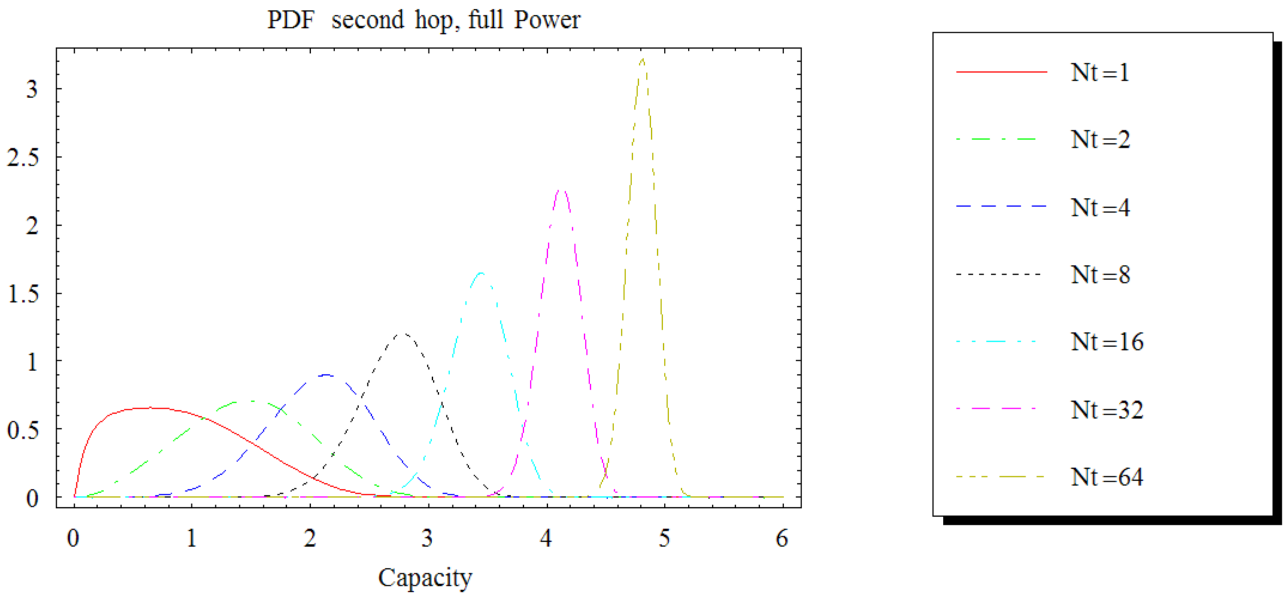
- CDF of alrgest eigenvalue is

$$F(u) = \prod_{j=1}^{N_\mathrm{t}} \left(\int_0^u \mathrm{d}\lambda_j \right) p(\Lambda)$$

- [Dighe & al 2003] integrated non-correlated case, use same trick
- $\det \mathbf{A}$ is a sum (over permutations of correlation eigenvalues) of integration measures. Treat one-by-one.
- expression to integrate is $\det \mathbf{E} \equiv \det \left(\Lambda^{N_\mathrm{r}-N_\mathrm{t}} \mathbf{D}(\Lambda) \right)$
  - $\mathbf{D}(\Lambda)$ is the VanDermonde matrix, $d_{mn} = \lambda_m^{n-1}$.
  - elements of $\mathbf{E}$ are thus $e_{mn} = \lambda_m^{N_\mathrm{r}-N_\mathrm{t}+n-1}$.
  - each row of $\mathbf{E}$ depends only on one λ_j
  - integration decouples to product of trivial 1D integrals

$\Rightarrow \lambda_{\max}$ distribution in closed form

Second Hop Capacity PDF

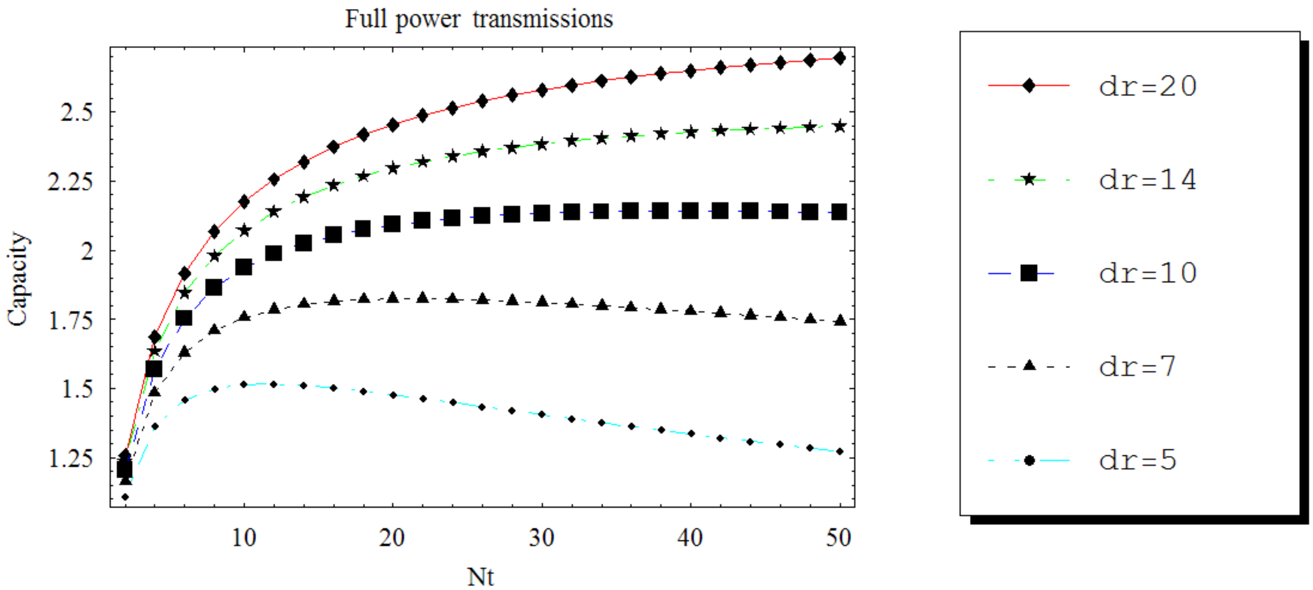


- $C_2 = \log(1 + g_2 \lambda_{\max})$
- closed form distriburtion $p_2(C_2)$
- The more friends you have, the more they may help you
- with numerous friends, mean-field bahavior

Physical Parameters

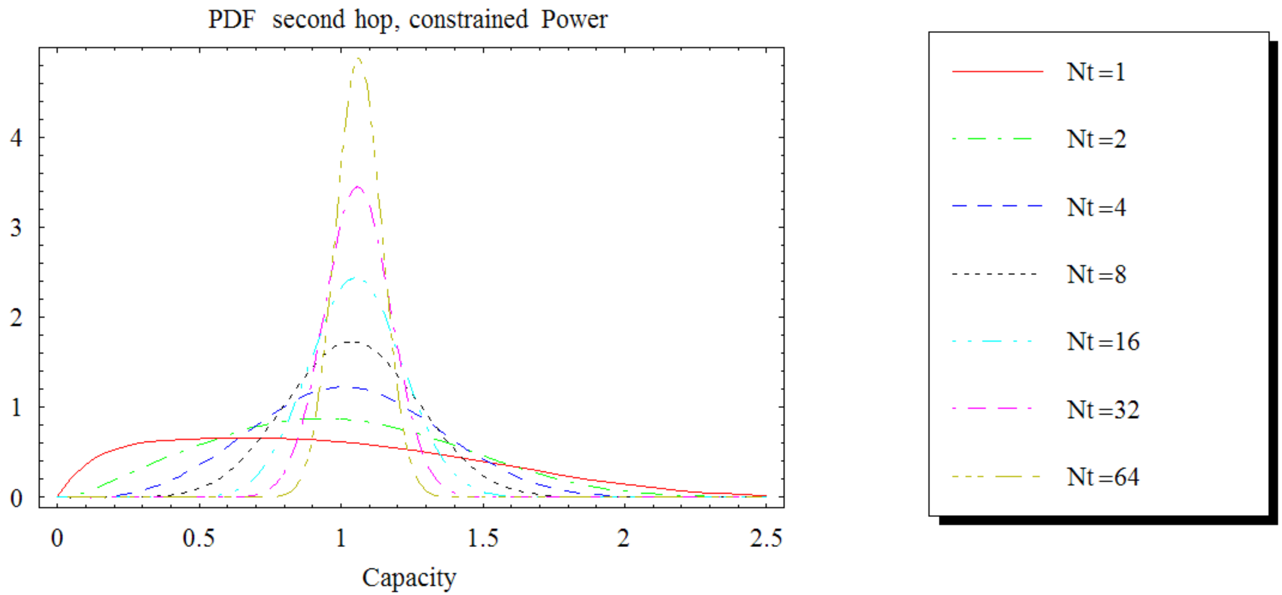
- $g_2 = 1$ (Signal-to-Noise Ratio on relay link)
  - Relay links are weak, cluster far from destination
- same power used by the source in the first hop, and on the average by the source and relays in the second hop
  - the more relay nodes, the more power radiated into air
- $g_2 = d_r^\alpha / G_r$
  - Path-loss exponent $\alpha = 3.75$
  - destination antenna gain $G_r = 10$
  - d_r ratio between radius of cluster and distance b/w cluster and destination

Collaborative Ergodic Capacity



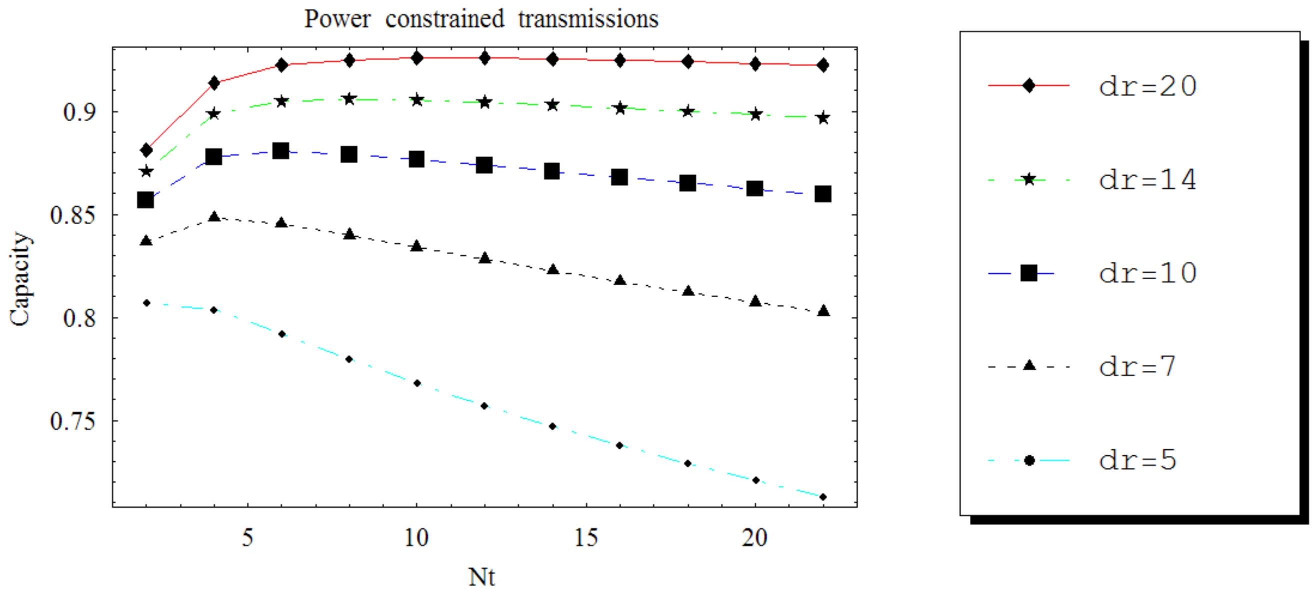
- without relaying, the capacity is 0.94
- cost vs benefit tradeoff
- the furhter away you search for friends, the more you pay

2:nd Hop, Constrained Power



- Strict control on interference: constrained total power usage in cell
- some array gain visible (optimum usage of differnet channels)

Collaborative Capacity, Strict Power Laws



- without relaying, the capacity is 0.94
- The cost outweights the gains
- outage analysis (reliability) would look different

Summary

- Analyzed cost-benefit tradeoff on collaborative relaying for cell-edge users
- No gain if not allowed to radiate more power into cell
- this would improve with instantaneous selection of relay cluster
- gains come at a cost of the batteries of friends
- allowing increased power usage, significant iprovements to cell-edge service