ESPCI ECOLE SUPÉRIEURE DE PHYSIQUE ET DE CHIMIE INDUSTRIELLES

ParisTech
INSTITUT DES SCIENCES ET TECHNOLOGIES
PARIS INSTITUTE OF TECHNOLOGY

Consequences of Phase Transitions in Random Constraint Satisfaction Problems: *Arkless strategy for flood victims !*

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NORDITA, Stockholm Sweden

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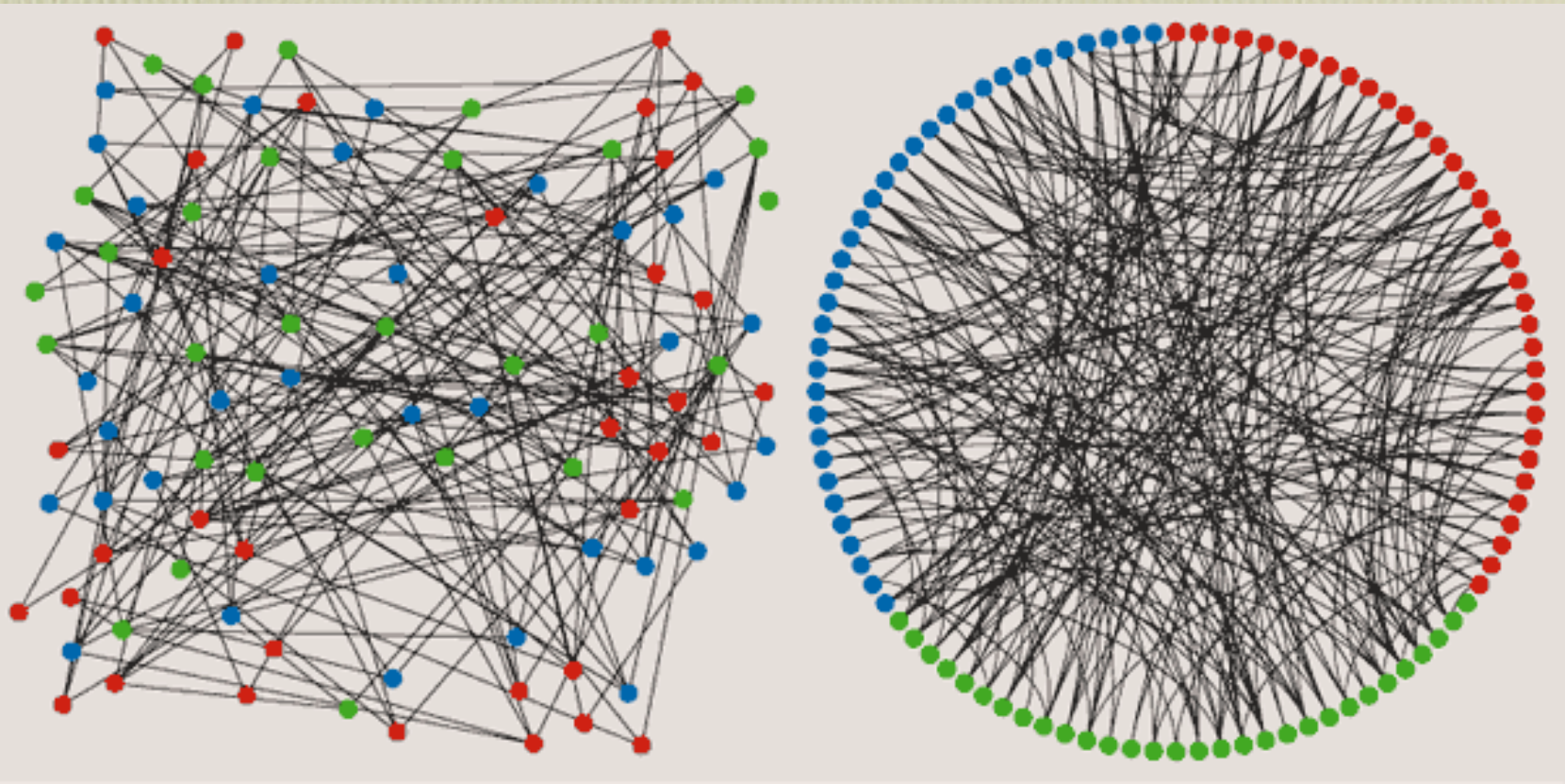
Coloring random graphs

- Graph q-coloring is NP-complete (for q>=3)
- Random constraint satisfaction problem: coloring random graph



Ex: N=100 vertices, M=218 edges, average degree c=2M/N=4.36

Questions ?

- How is the set of solutions organized ?
- What are the consequences of phases transition for (some) algorithms ?
- How to design very hard instances ?

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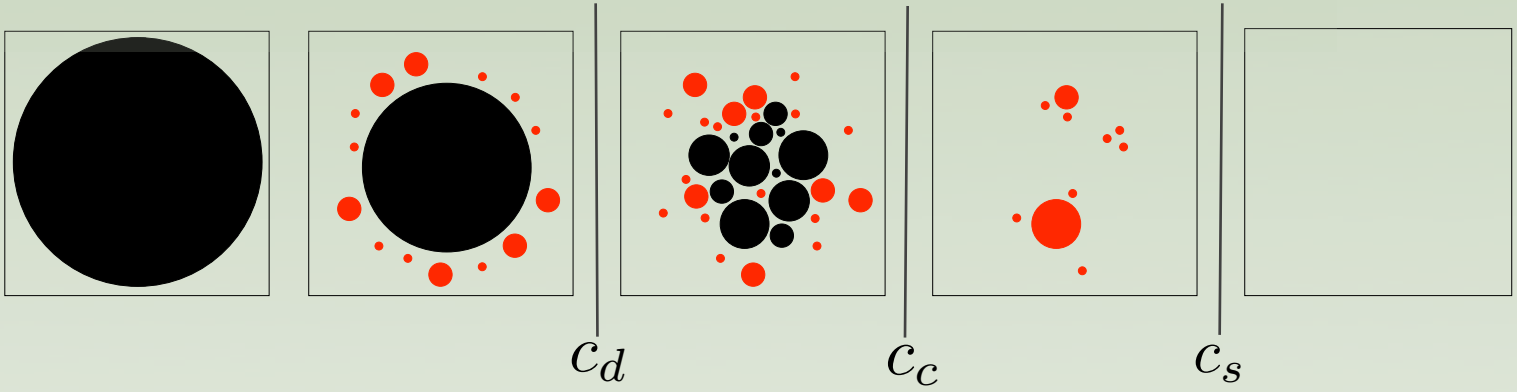
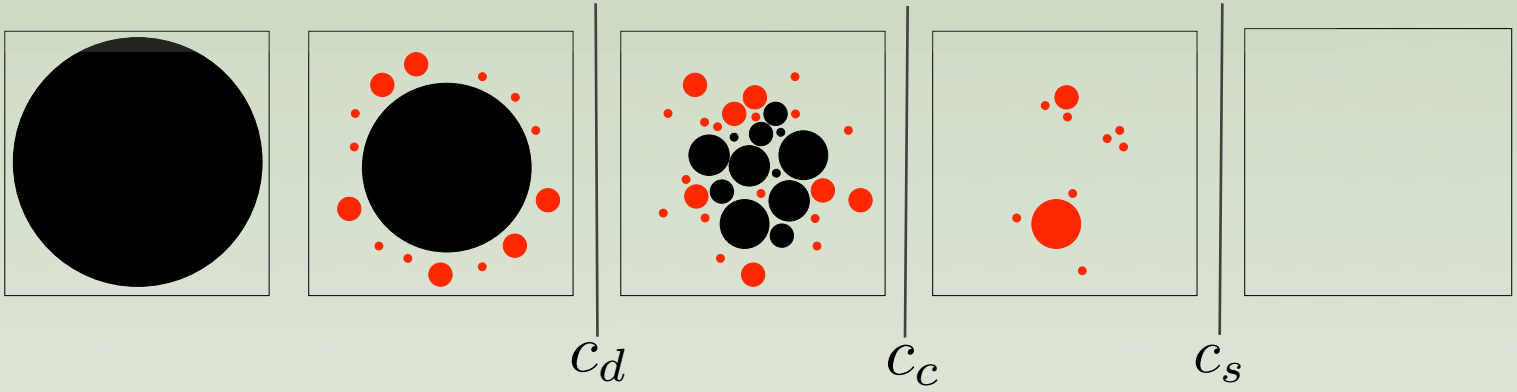
Consequences of the transitions



- Clustering/Dynamic transition
- Condensation/Static transition
- COL/UNCOL transition
- Freezing of clusters

Consequences of the transitions



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  - "Ergodicity breaking transition", equilibration time diverges
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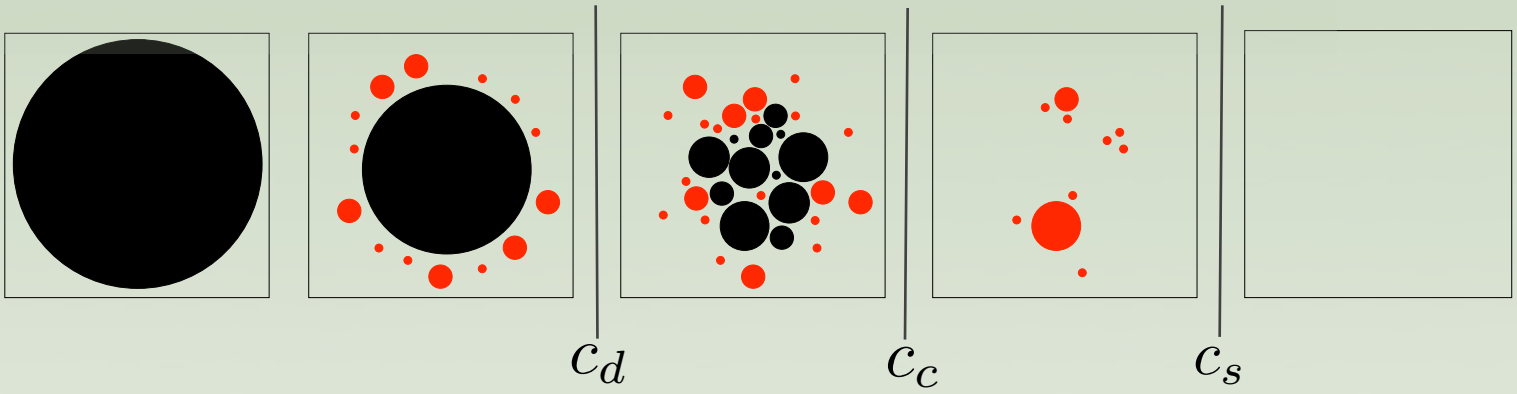

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Easy/Hard transition for the "MC sampling" problem
(but not for the "solving" problem)

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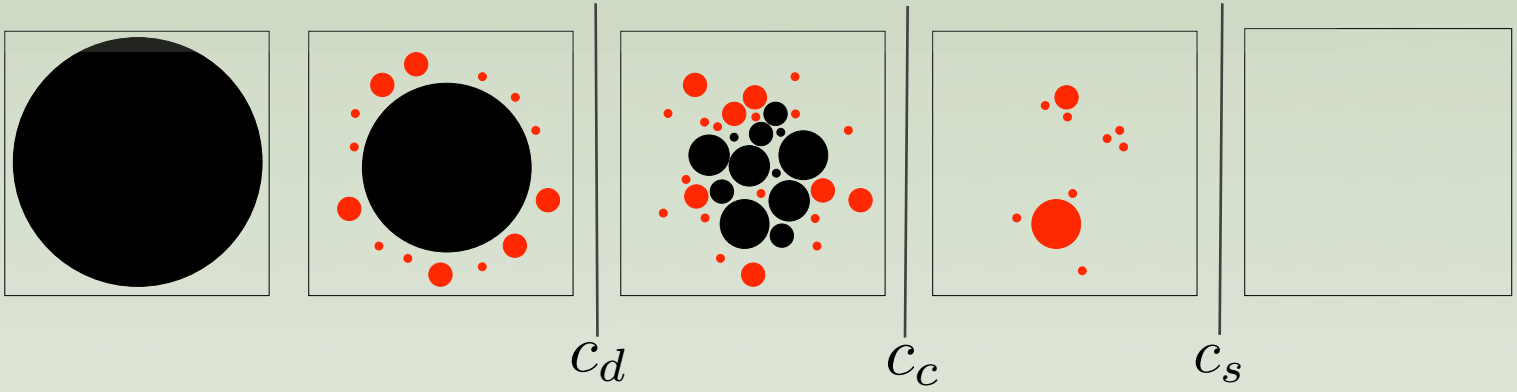

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- Many clusters exist, but a finite number of them covers almost all solutions
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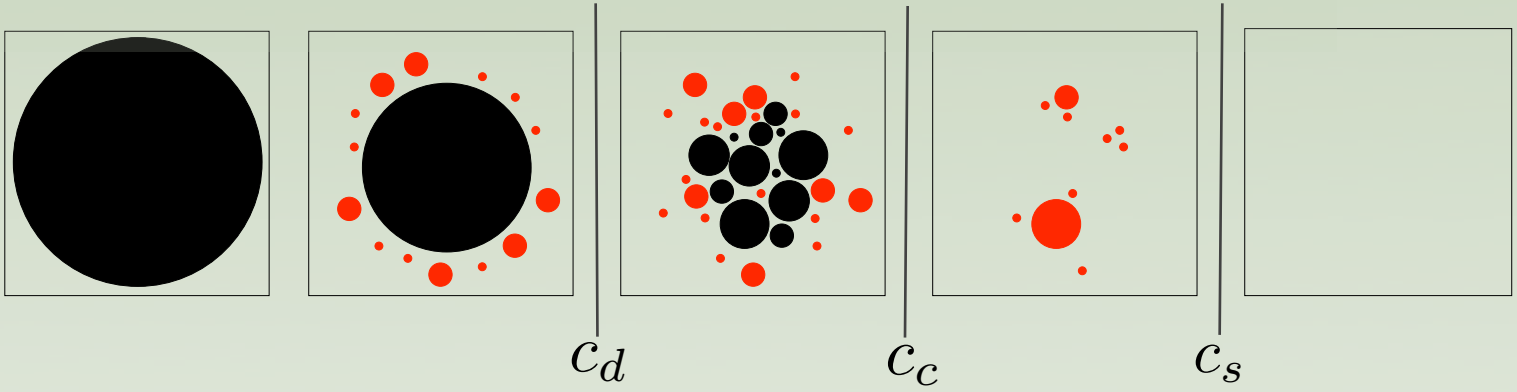

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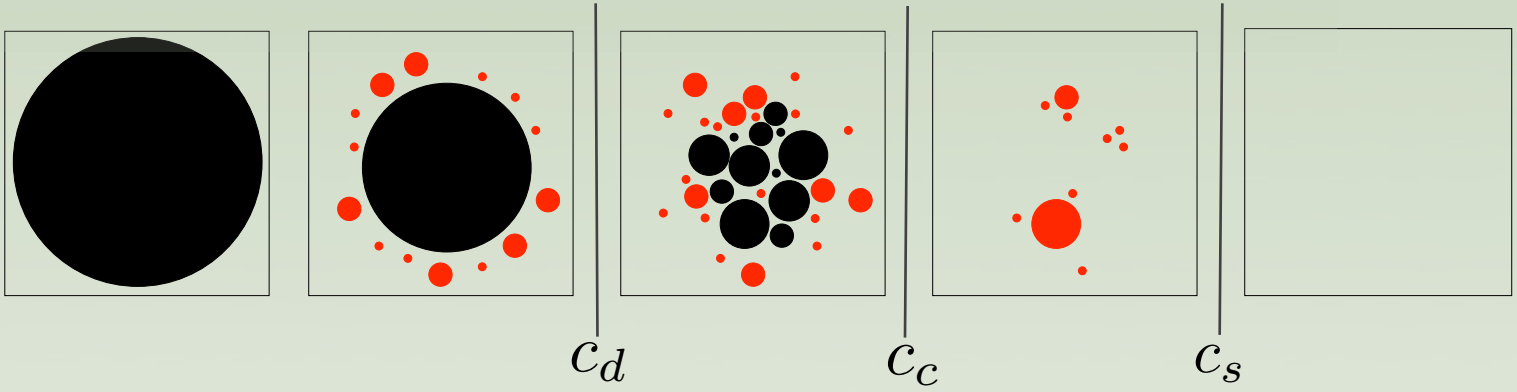

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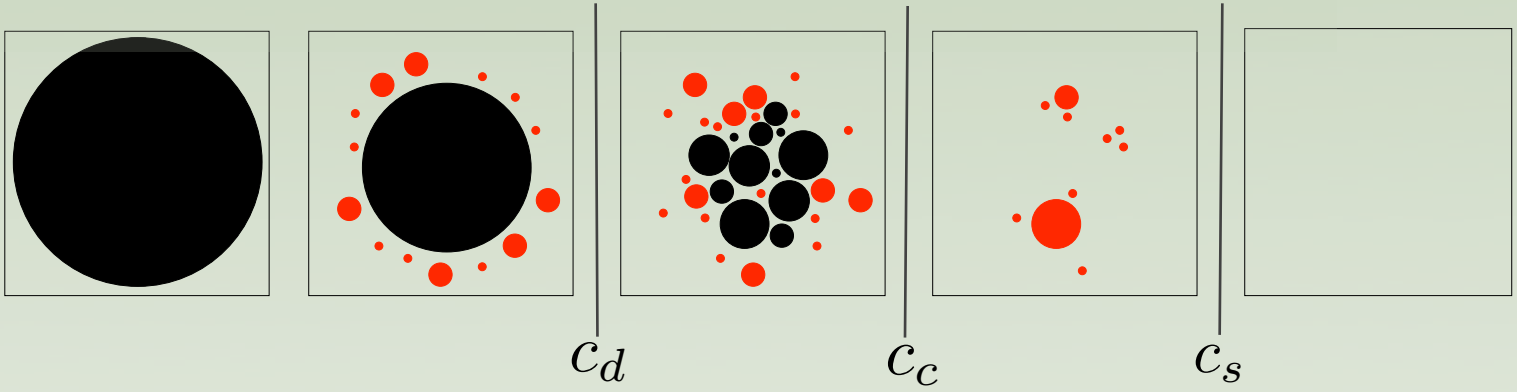

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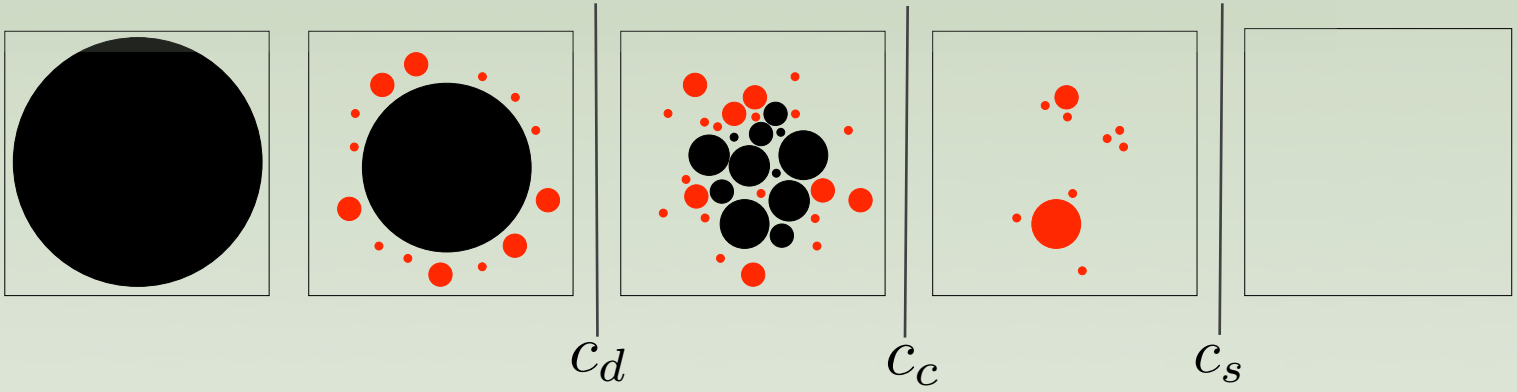

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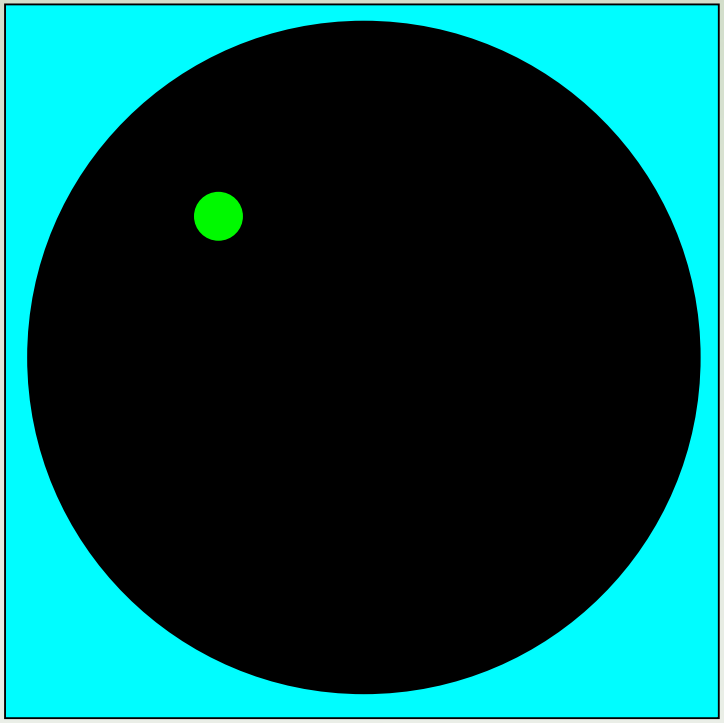
CONJECTURE

The frozen clusters are responsible for the difficulty of finding solutions
(not the clustering in itself)

Arkless strategy for flood victims



You are on a rugged landscape that is being flooded
What to do?

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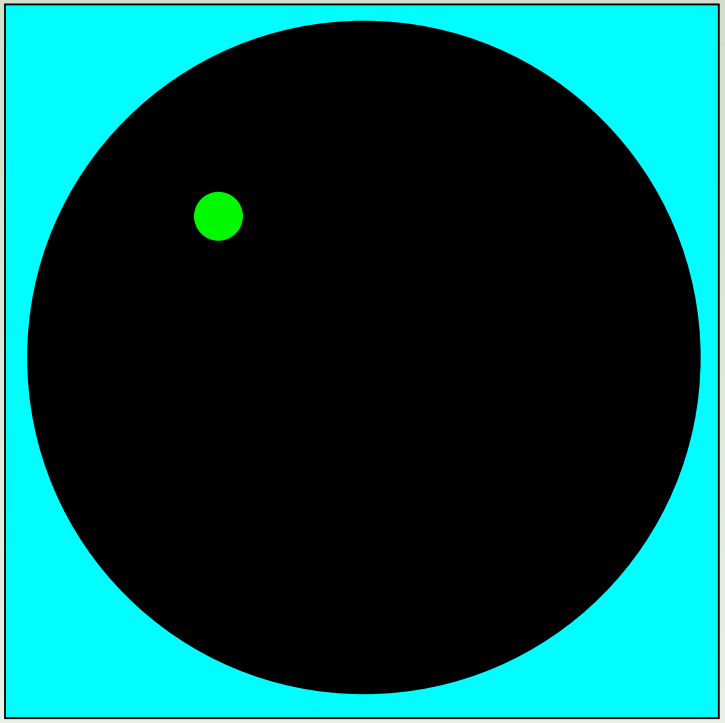

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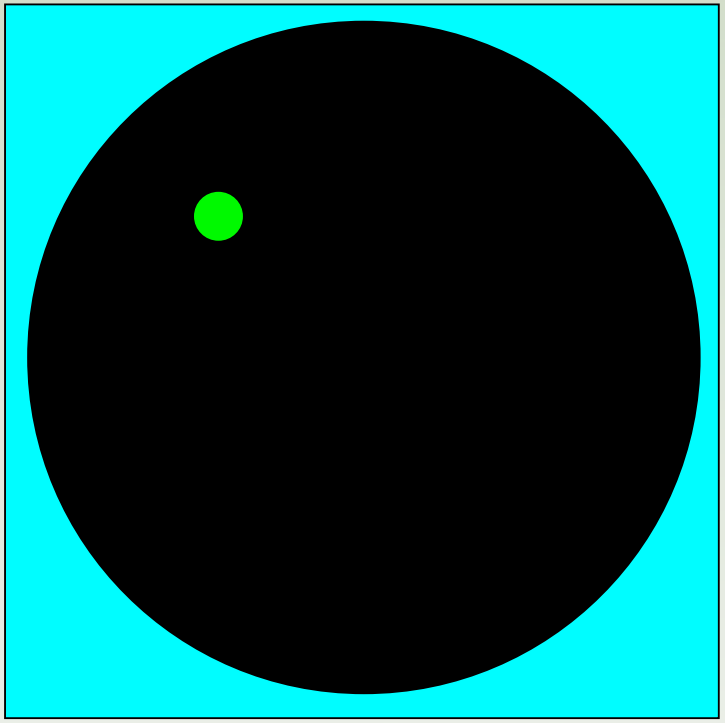

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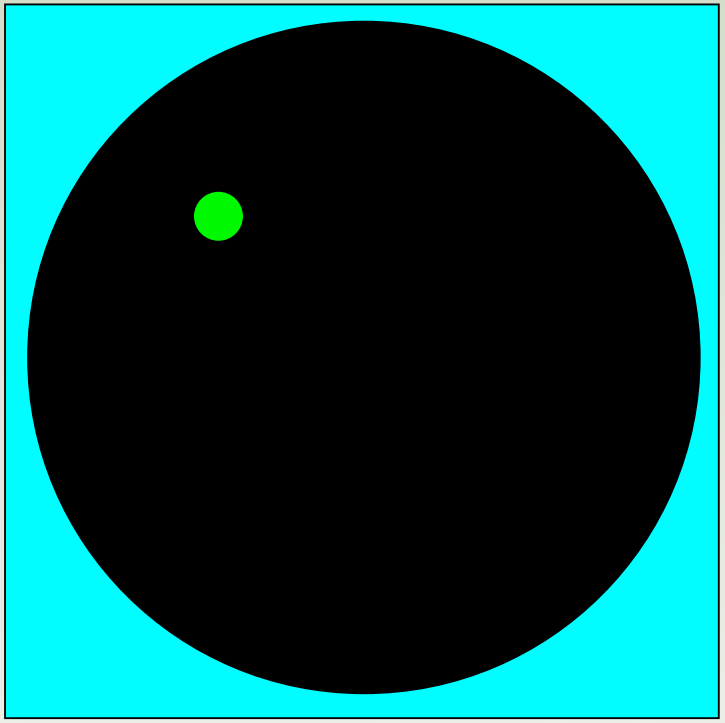

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Strategy 1: Build Noah's ark !

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Need for an ark-less strategy!

“Wet toes” algorithm

Arkless strategy for flood victims

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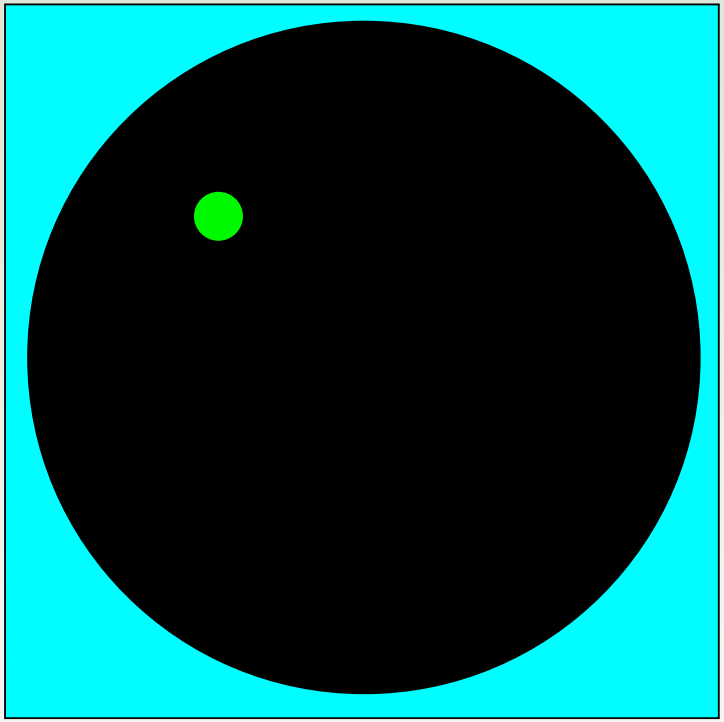
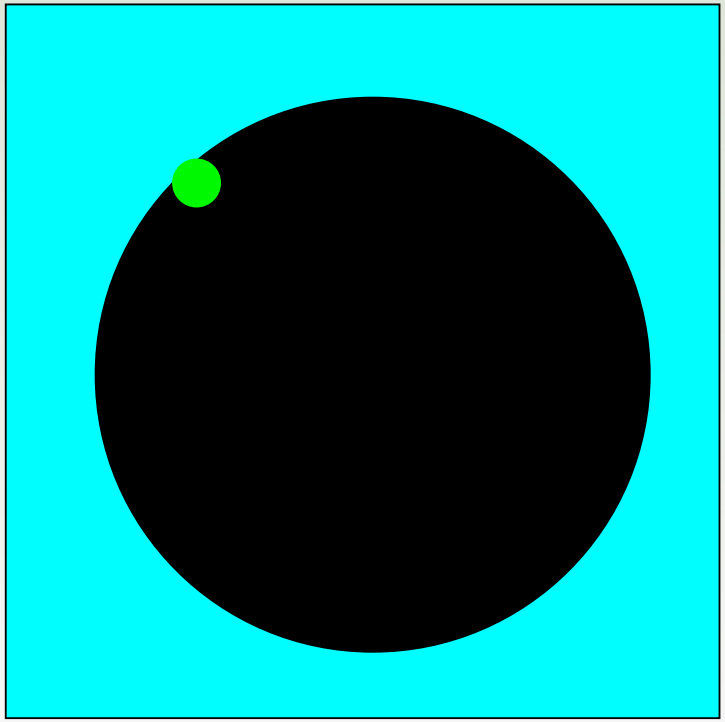


“Wet toes” algorithm

Arkless strategy for flood victims

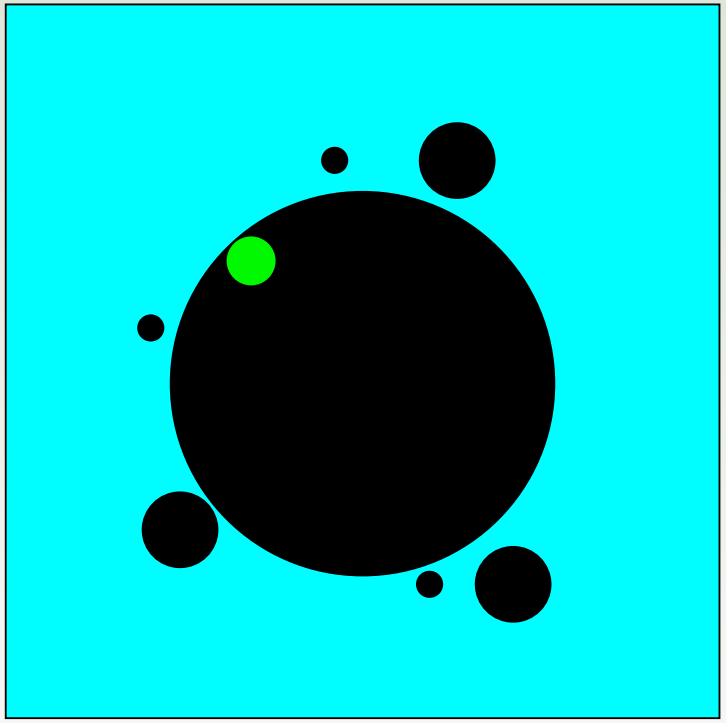
Water goes up. When your toes are wet
step back on the land!



“Wet toes” algorithm

Arkless strategy for flood victims

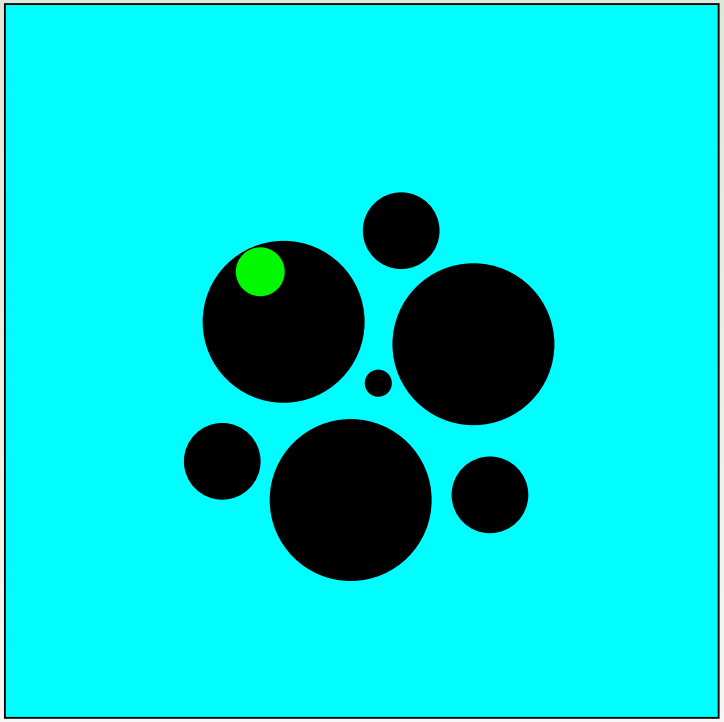
And wait until your toes get wet again...



“Wet toes” algorithm

Arkless strategy for flood victims

Sooner or later you’ll find yourself on a smaller island...



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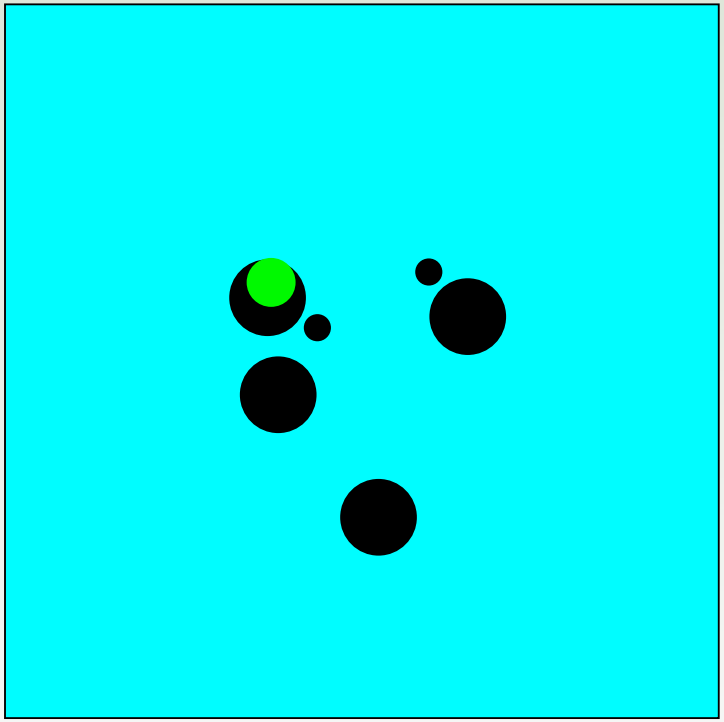
Then even a smaller one...



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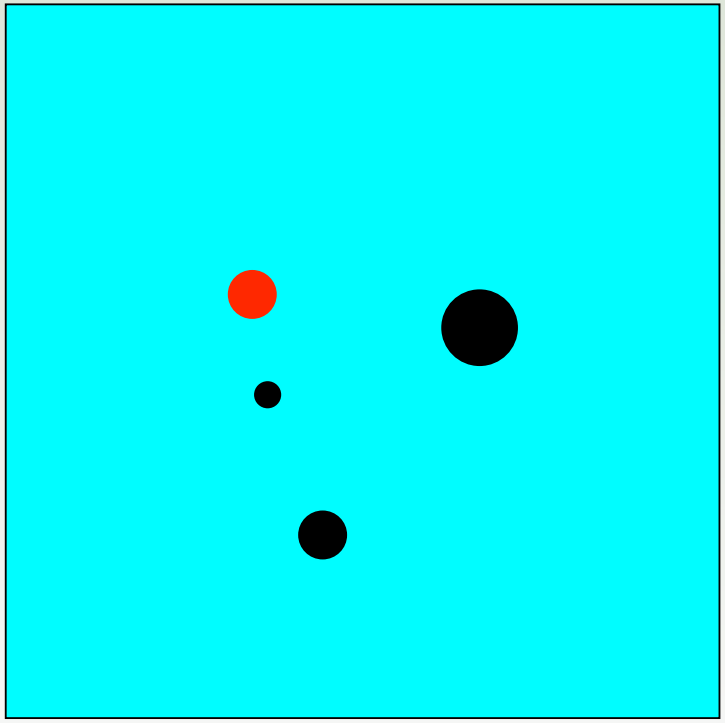
Until eventually you’ll drown (if you can’t swim!)



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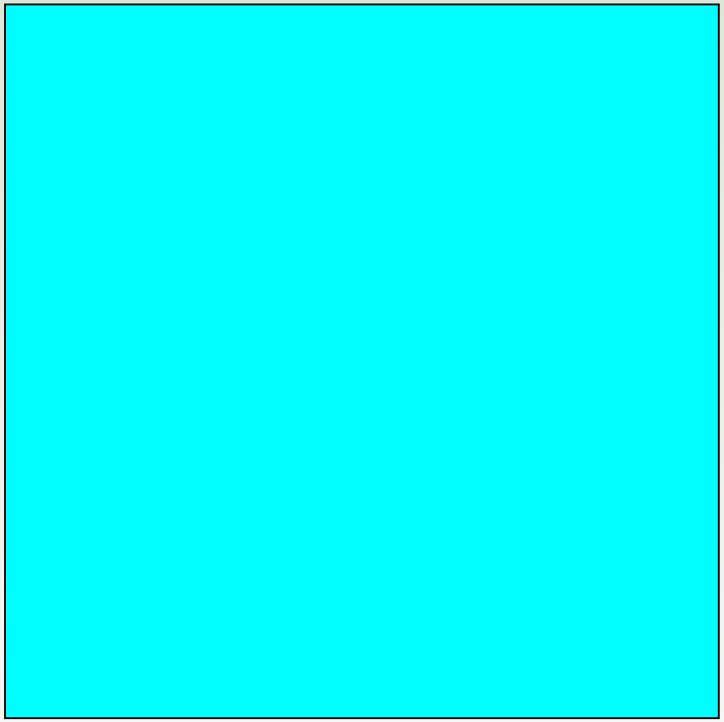
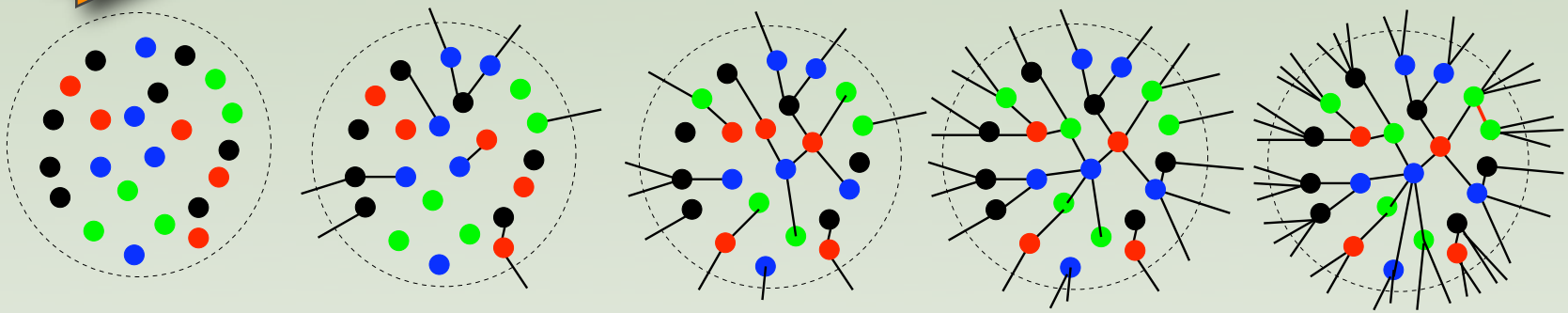
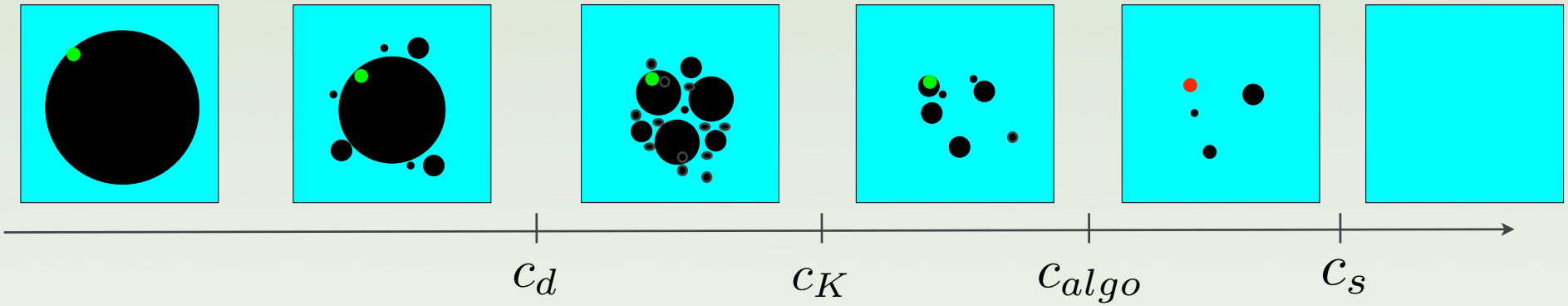
Finally, all land will be flooded!



“Wet toes” algorithm

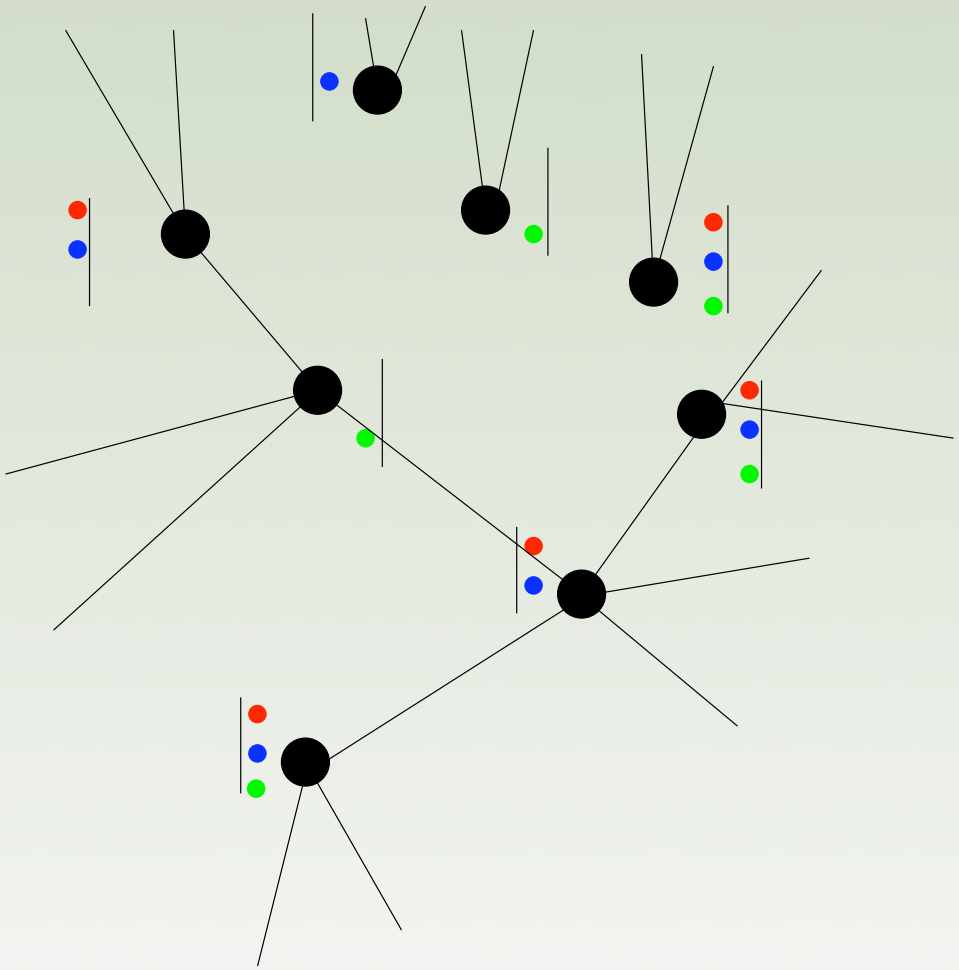
Add links one by one and use a local algorithm to solve contradictions





The algorithm works until the cluster disappears

The hard-field catastrophe



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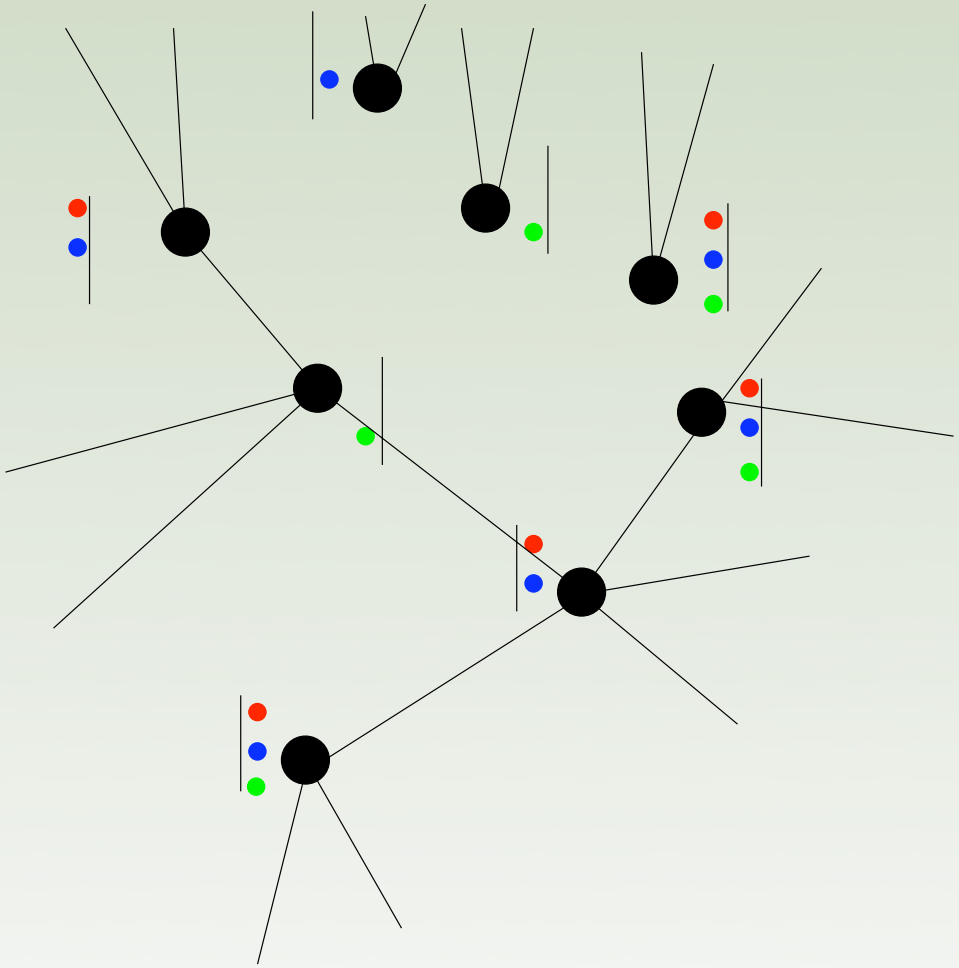

- *Consider a cluster with (a finite fraction of) frozen variables*

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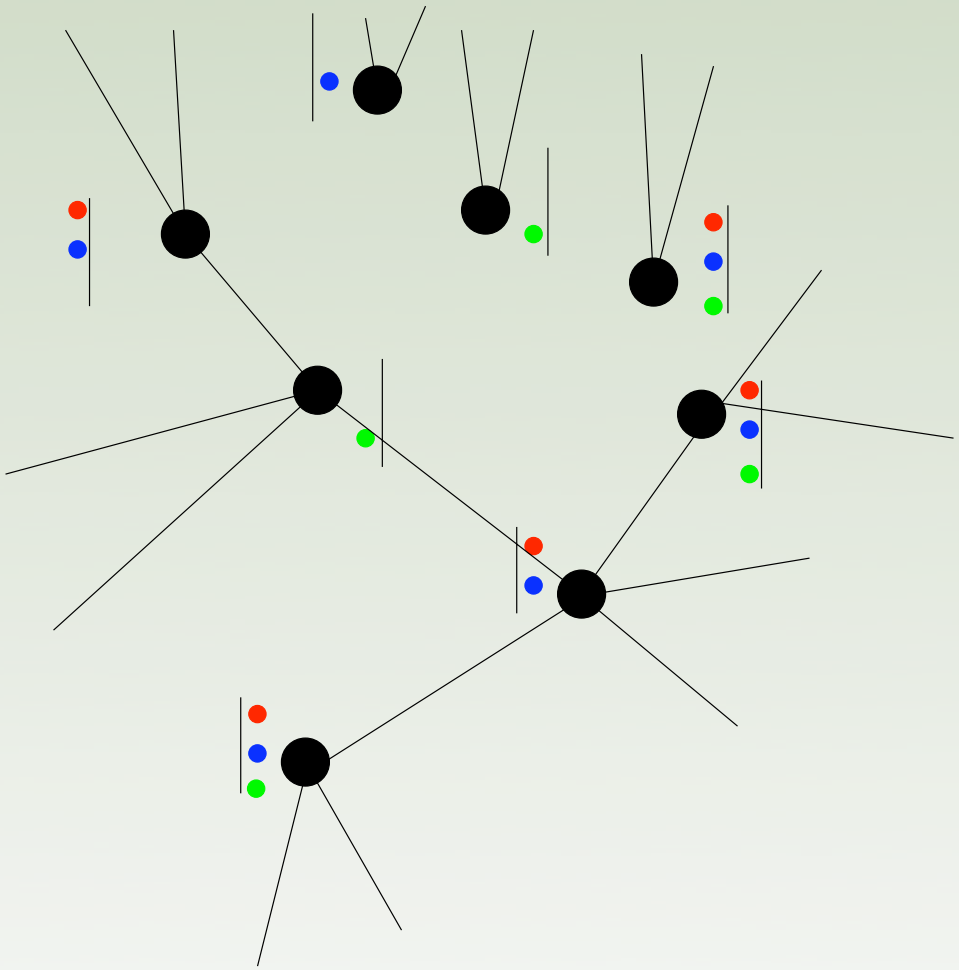

- *Consider a cluster with (a finite fraction of) frozen variables*
- *Add a link at random*

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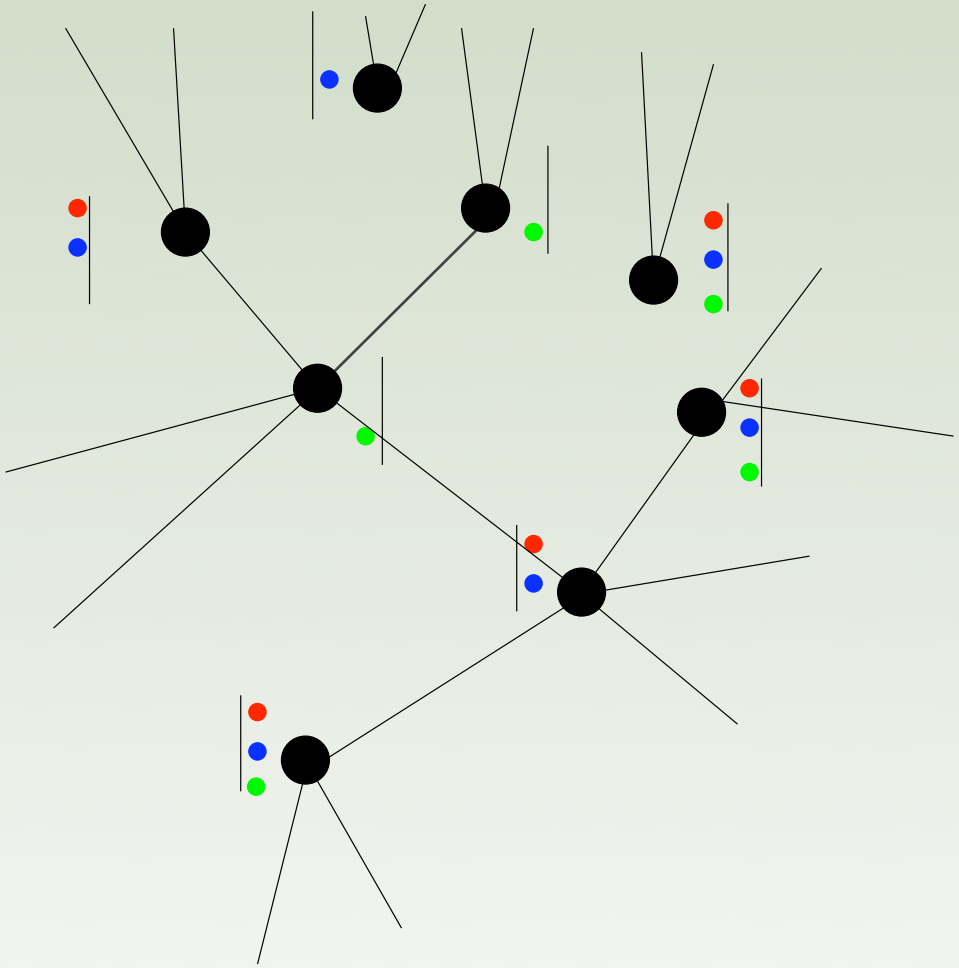

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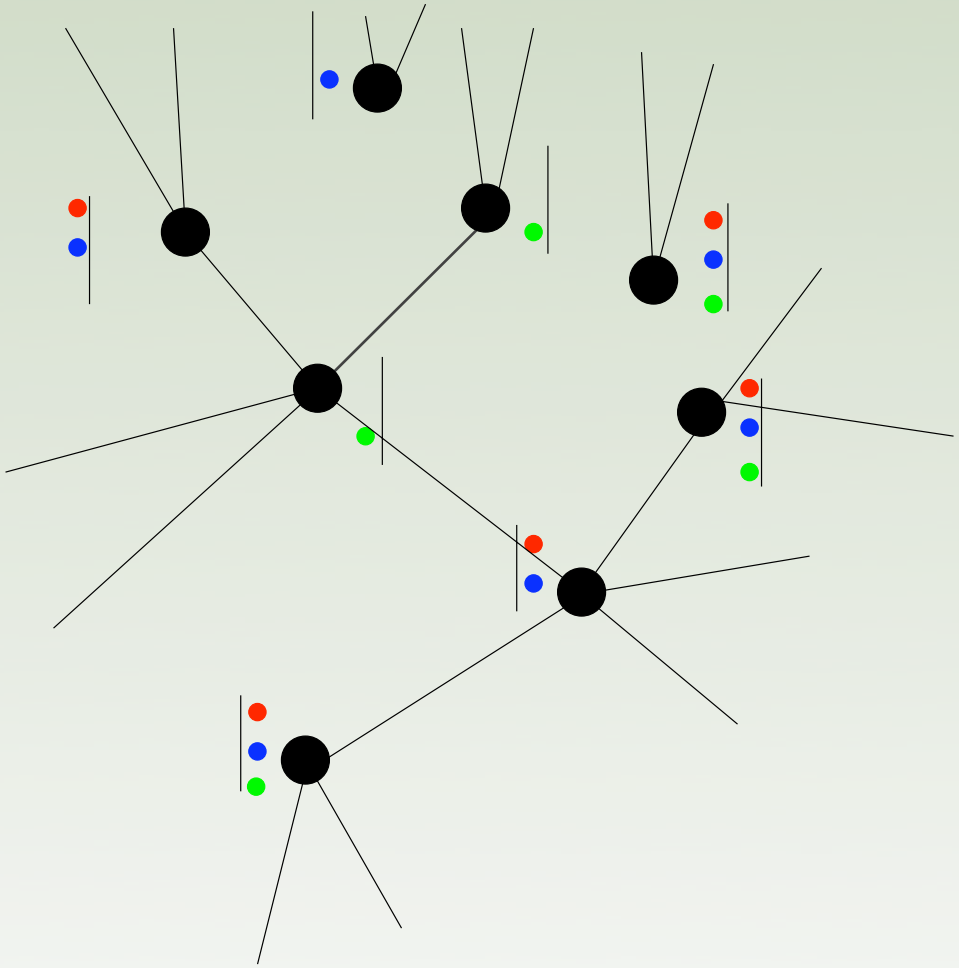

- *Consider a cluster with (a finite fraction of) frozen variables*
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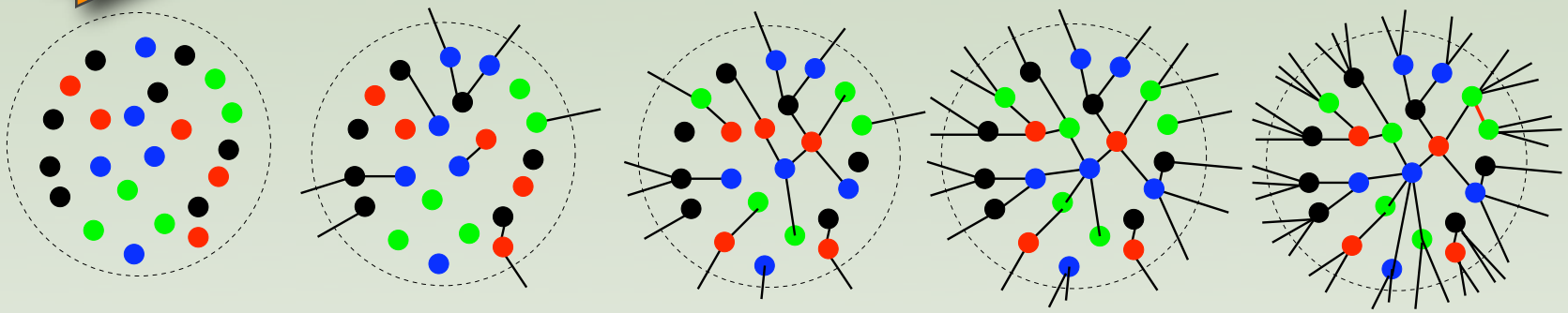
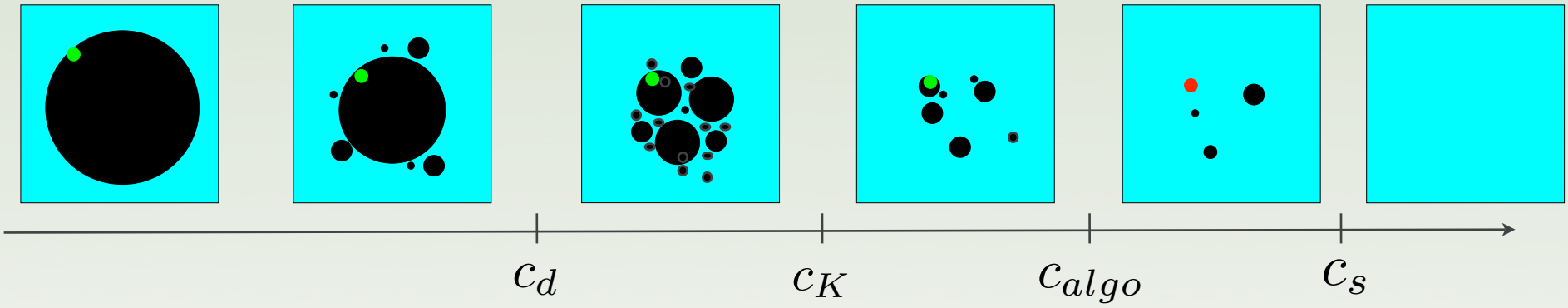
- *All solutions in the clusters are killed!*

Frozen clusters make it hard!

Add links one by one and use a local algorithm to solve contradictions

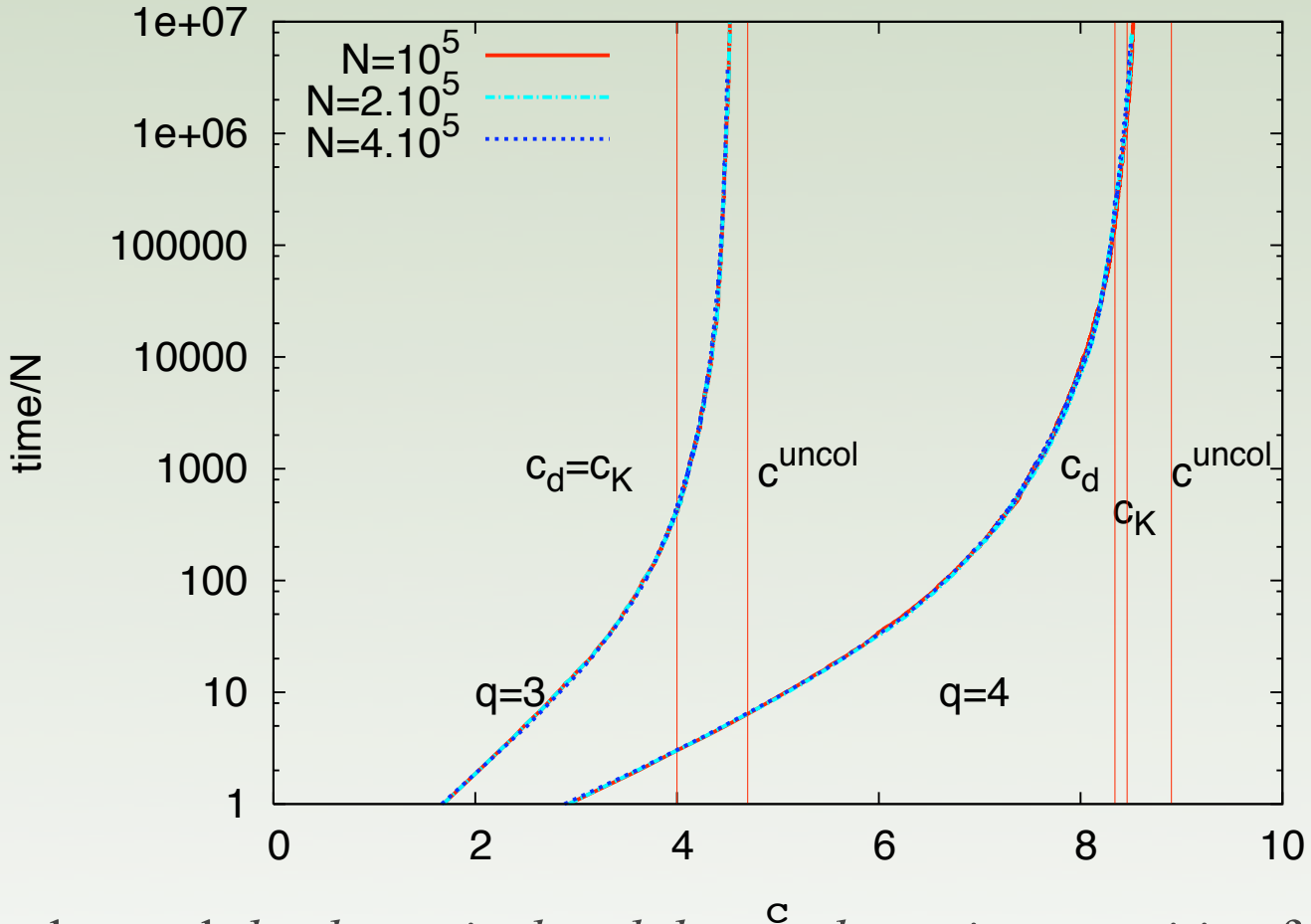




The algorithm works until the cluster disappears

and this happens when frozen variables appear

Performance of the “Wet toes” algorithm



Goes beyond *the dynamical and the condensation transition* for q=3 & 4

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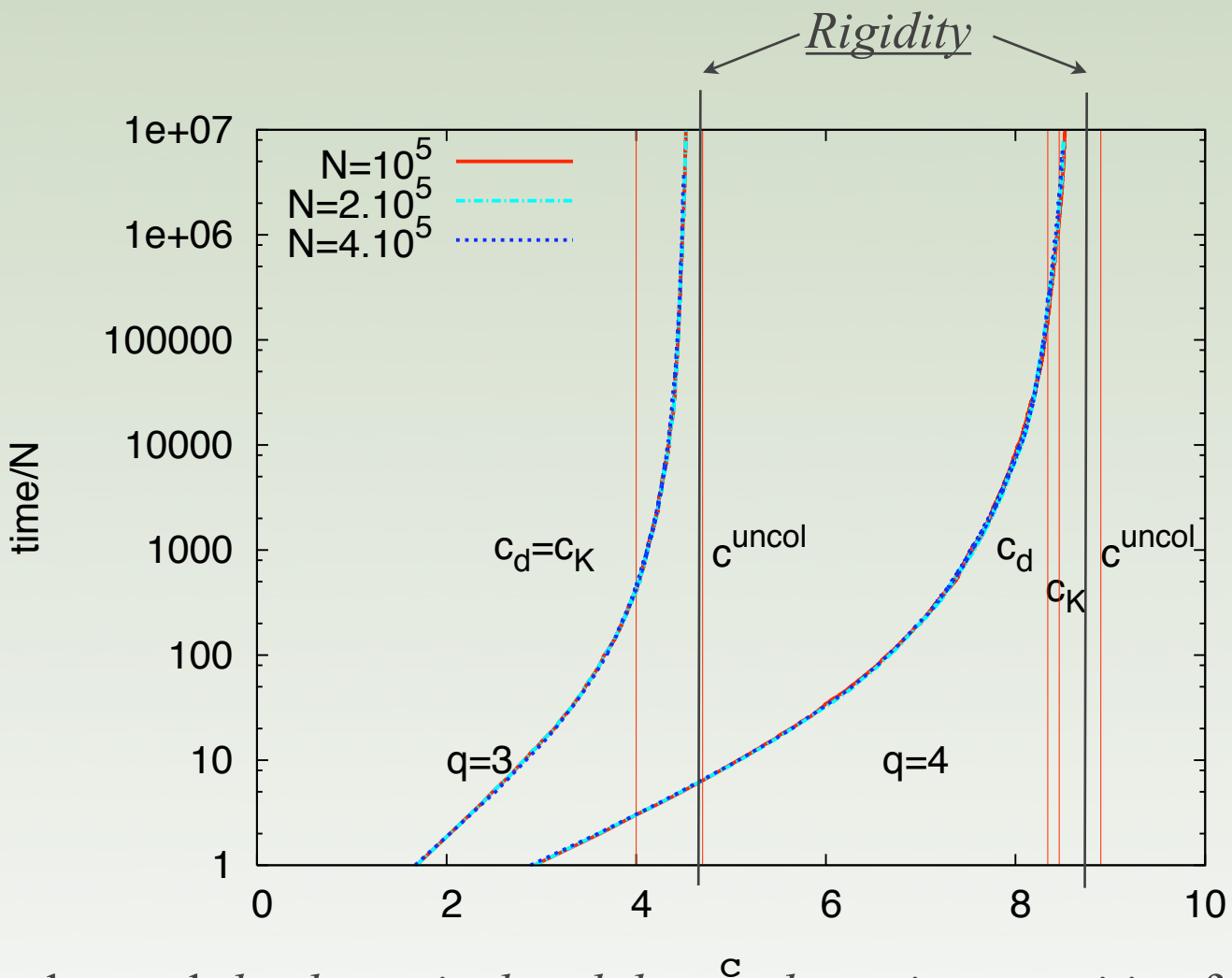
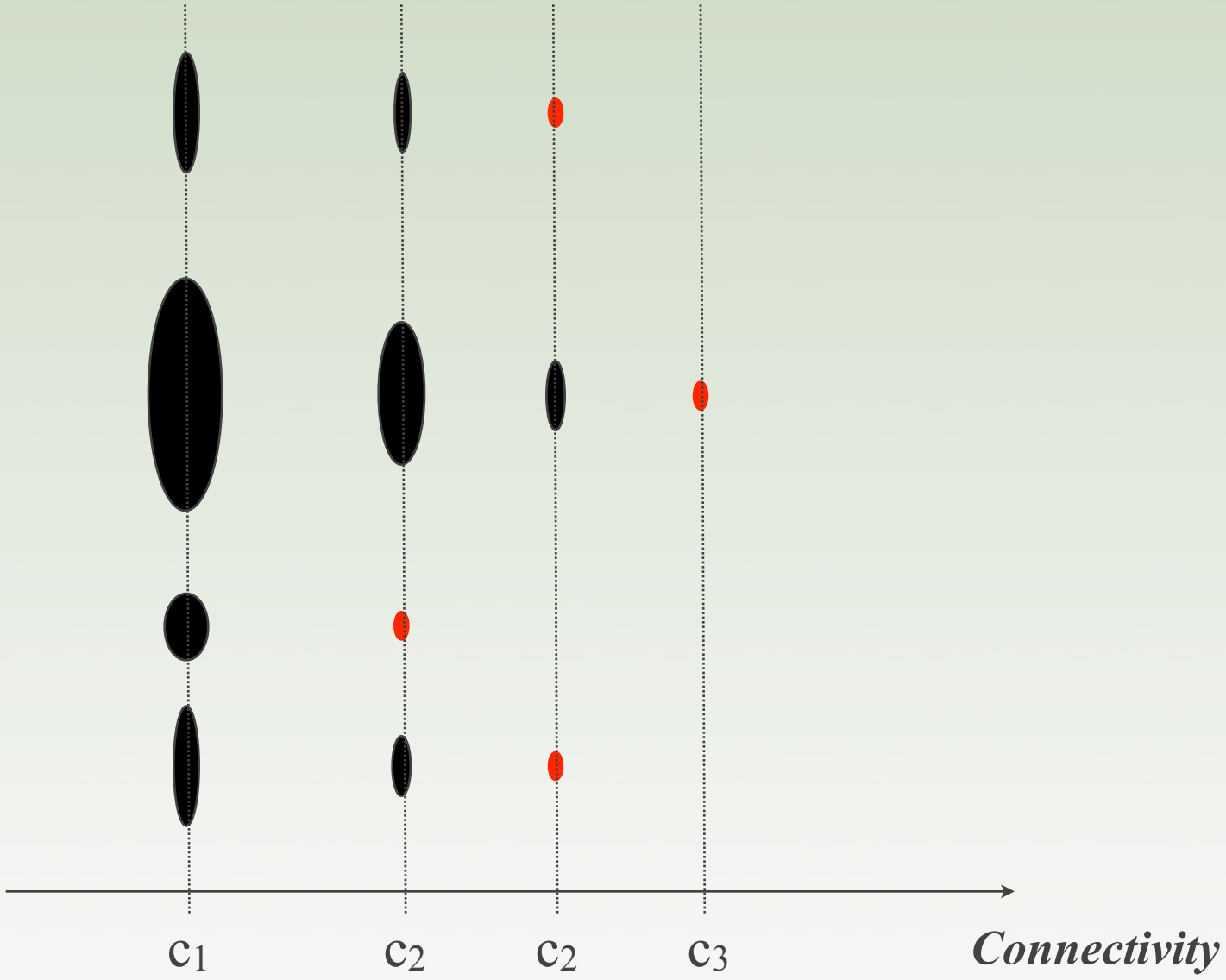


Goes beyond *the dynamical and the condensation transition* for q=3 & 4

But stops before all clusters freeze !

And the energy landscape ?



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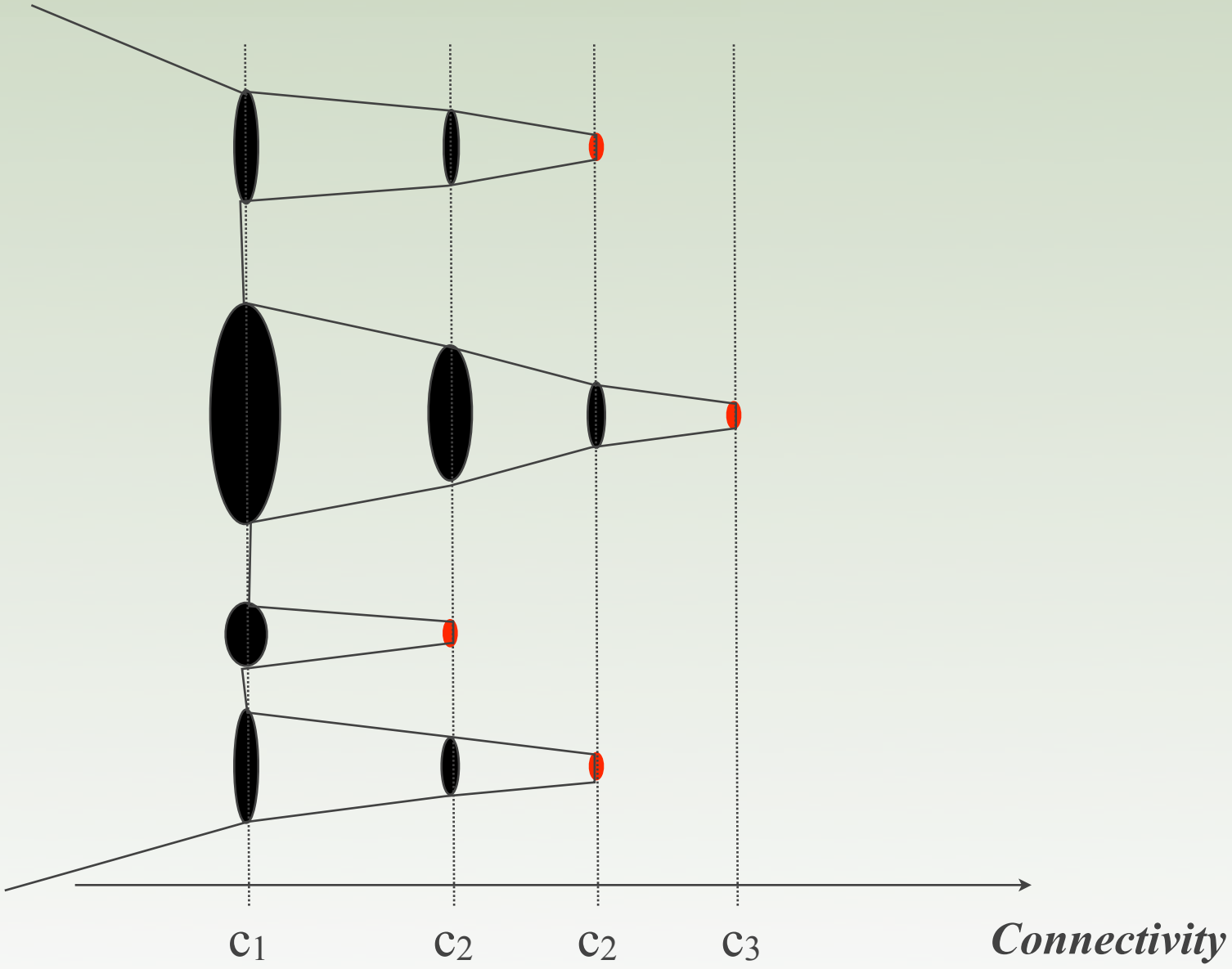

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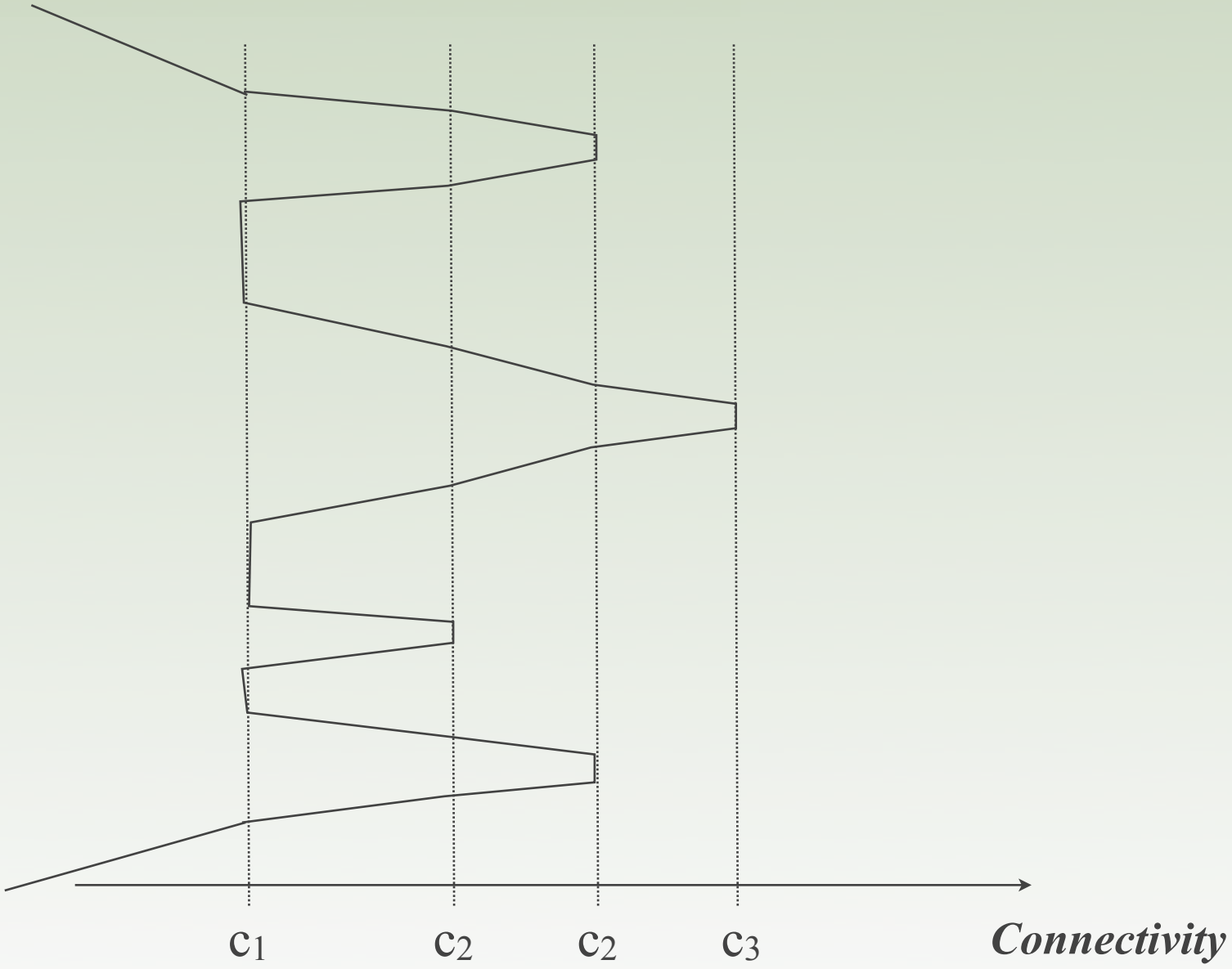

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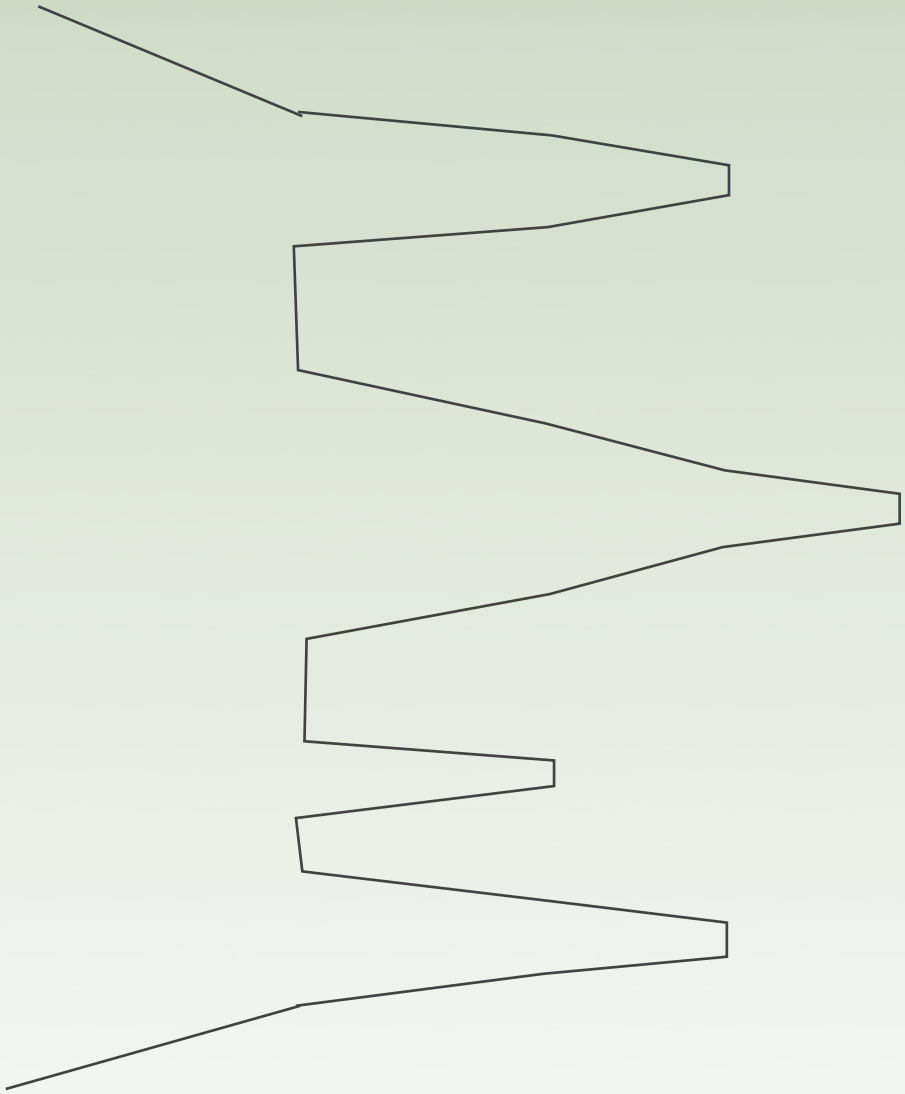

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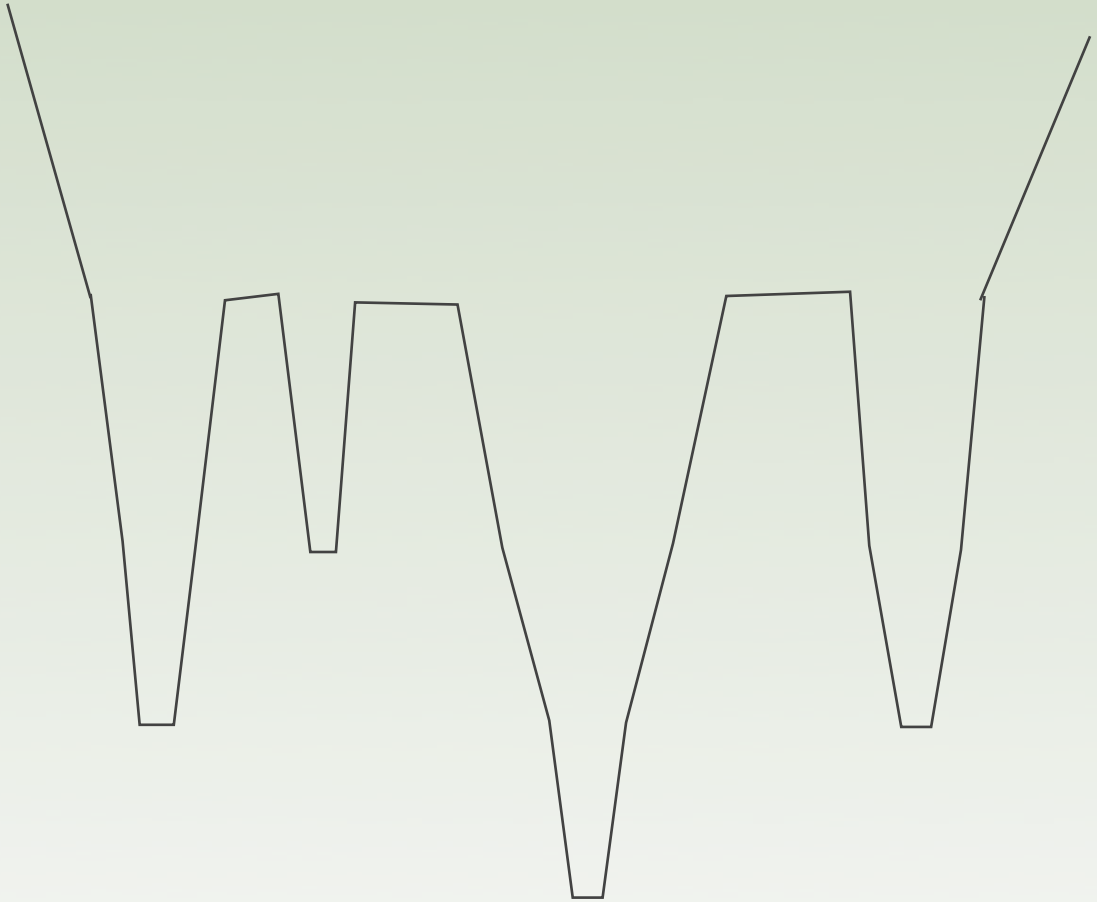

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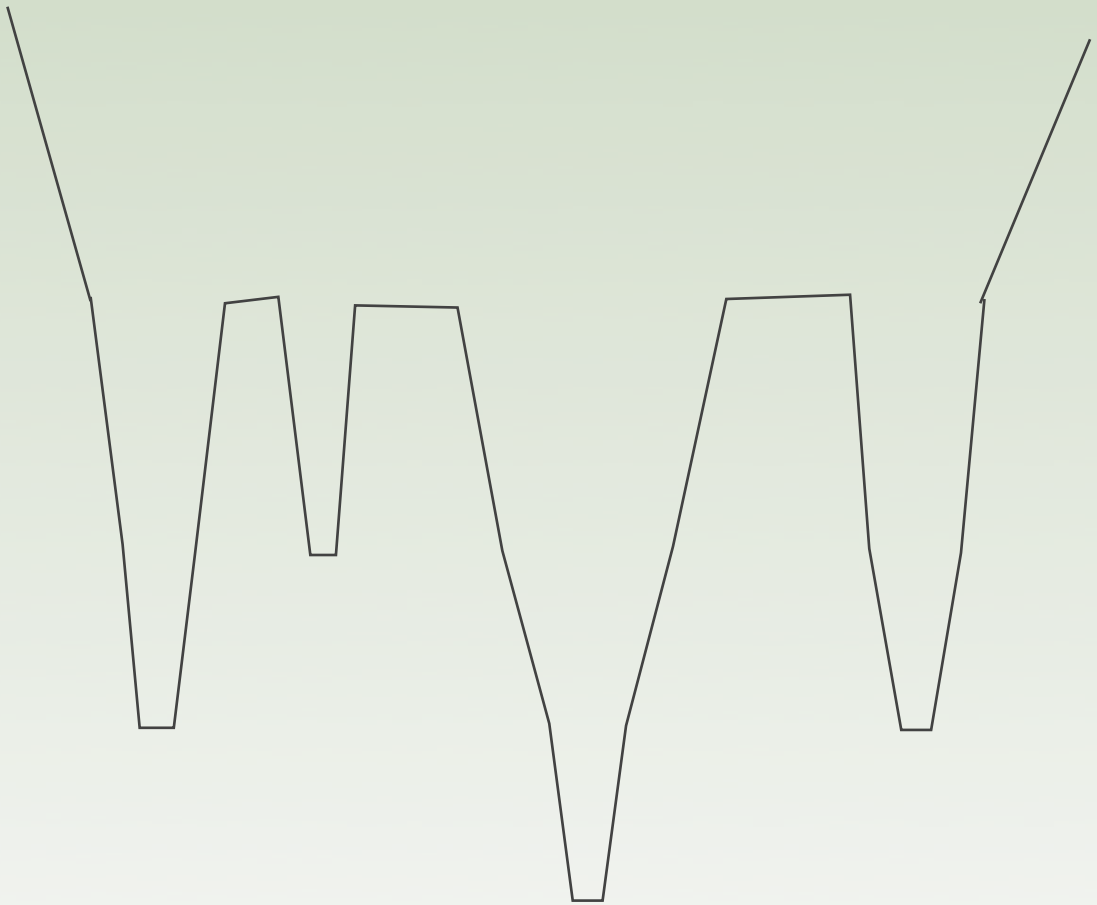

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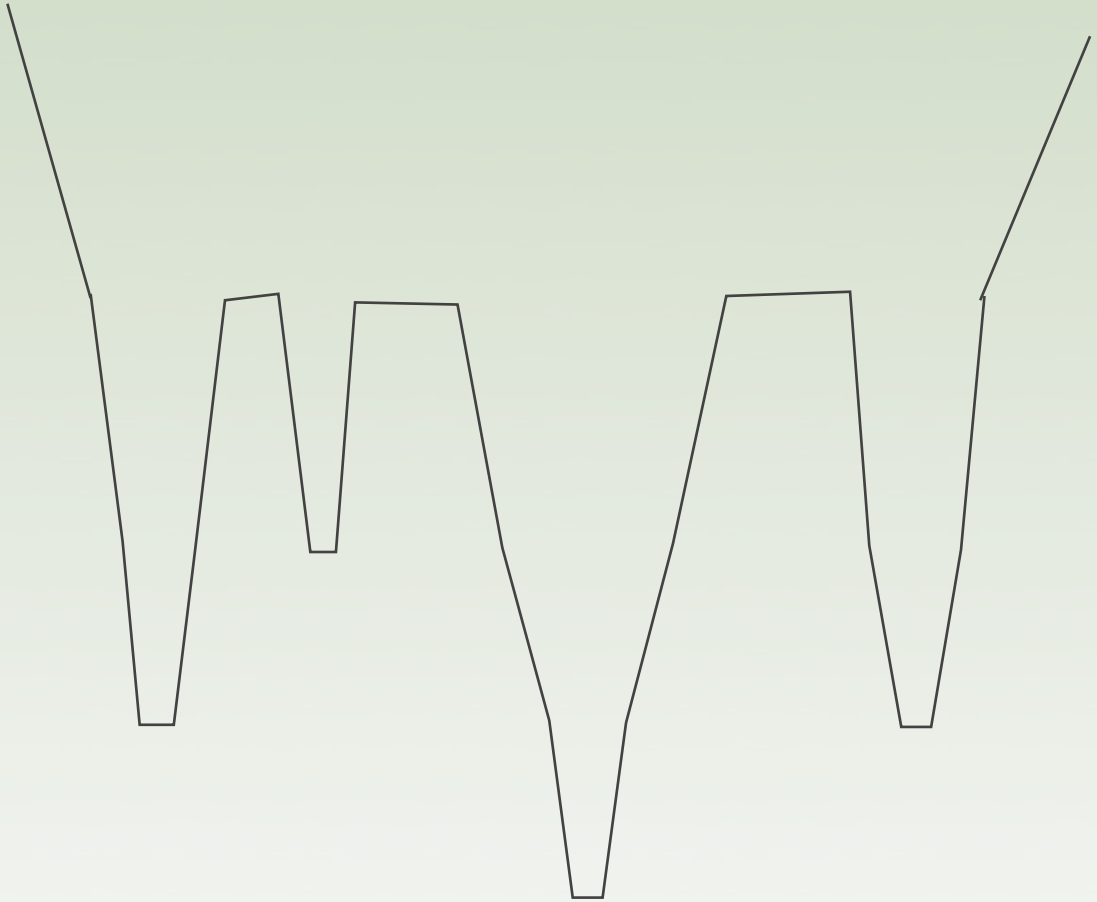

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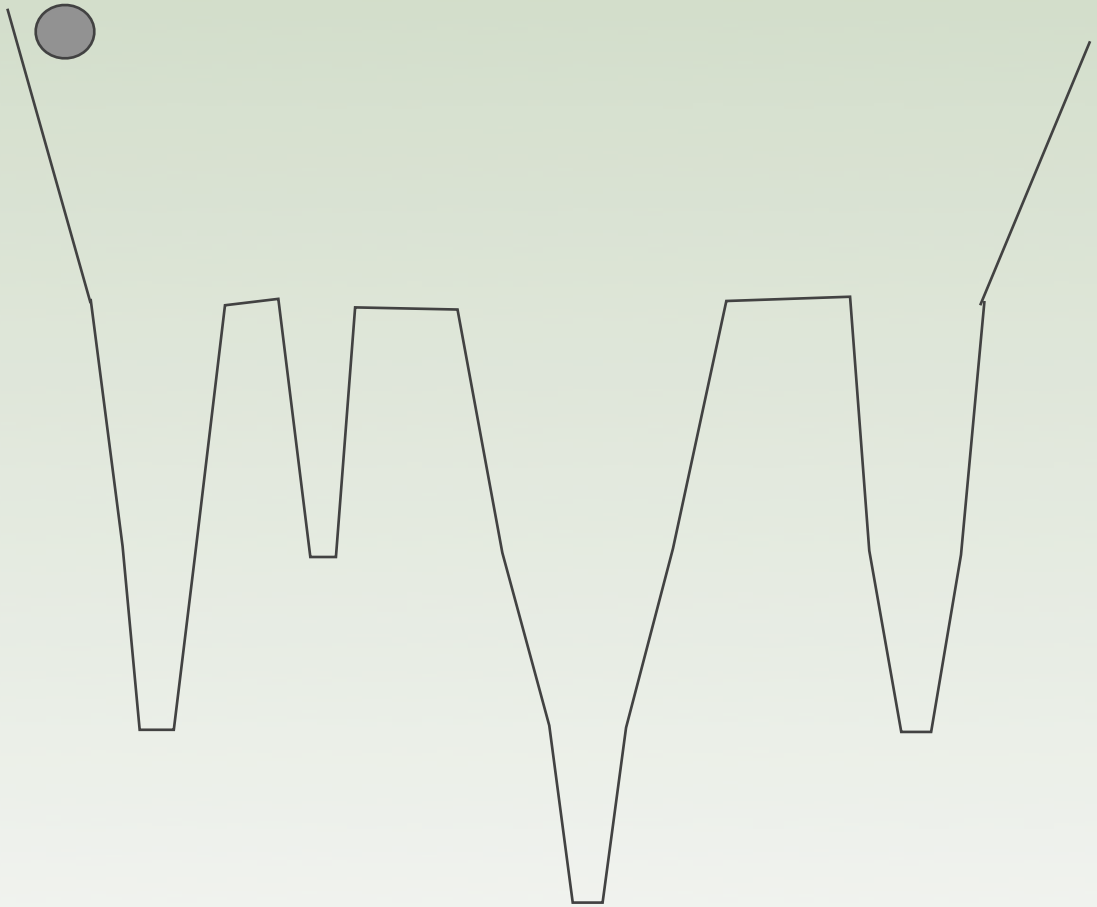

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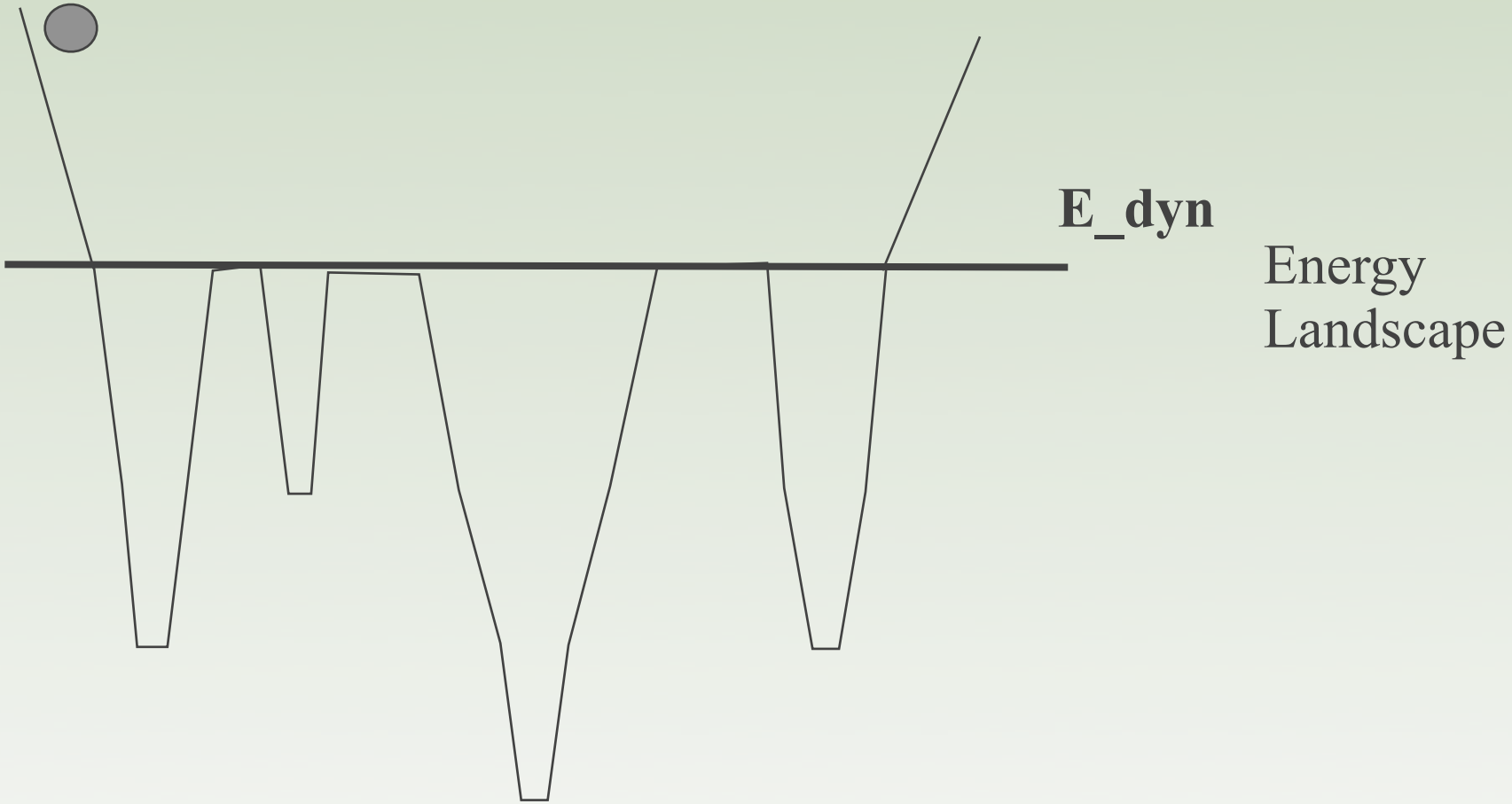

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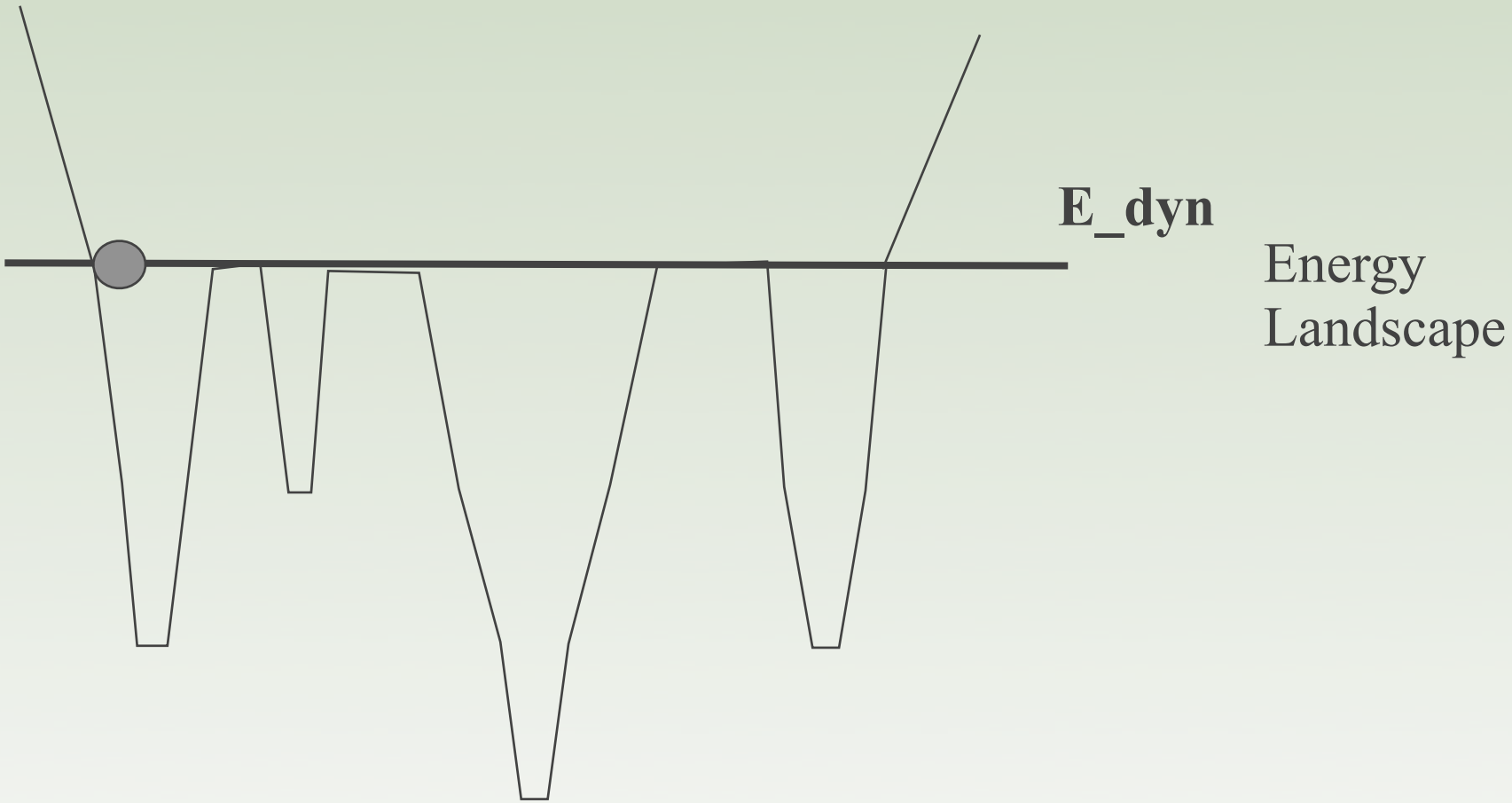

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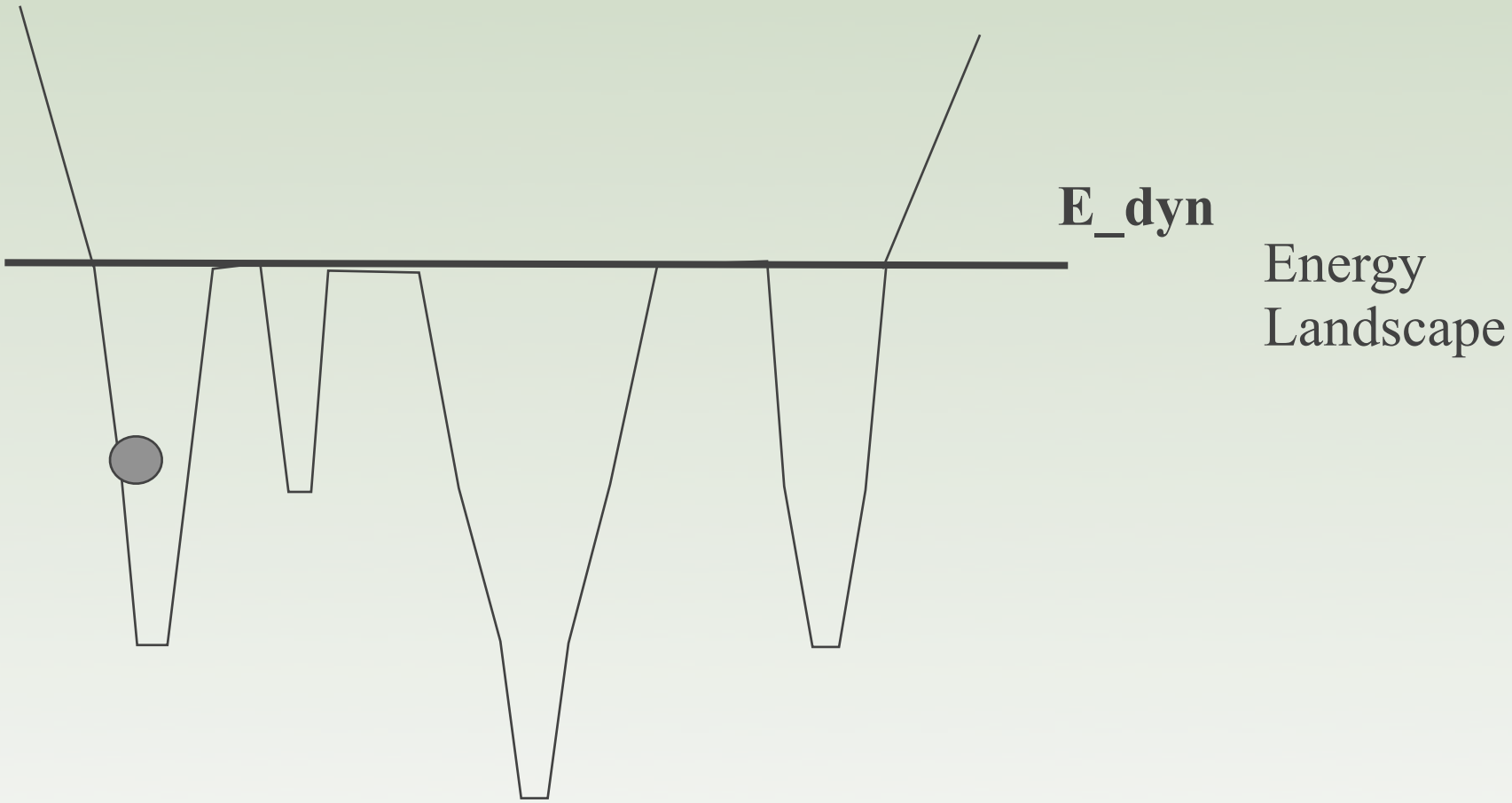

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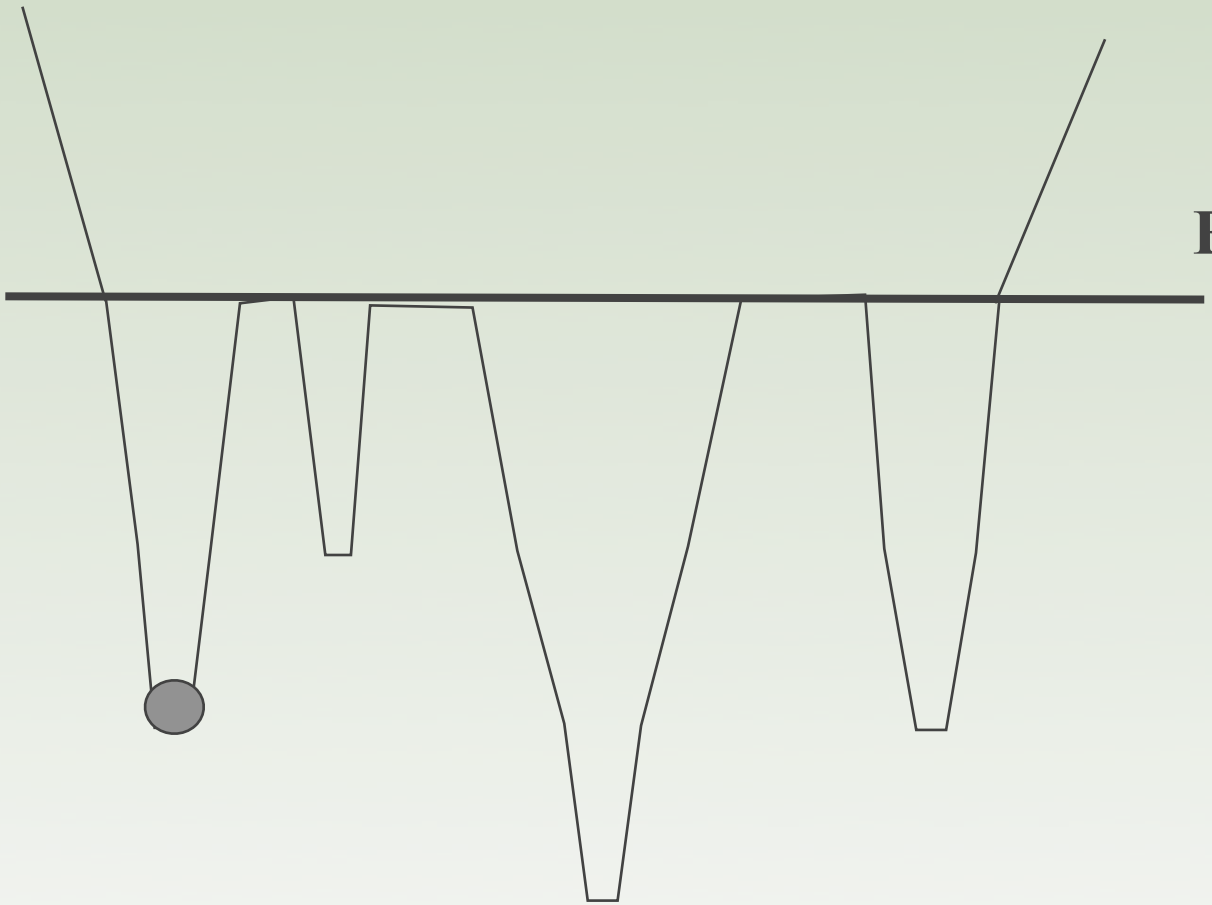

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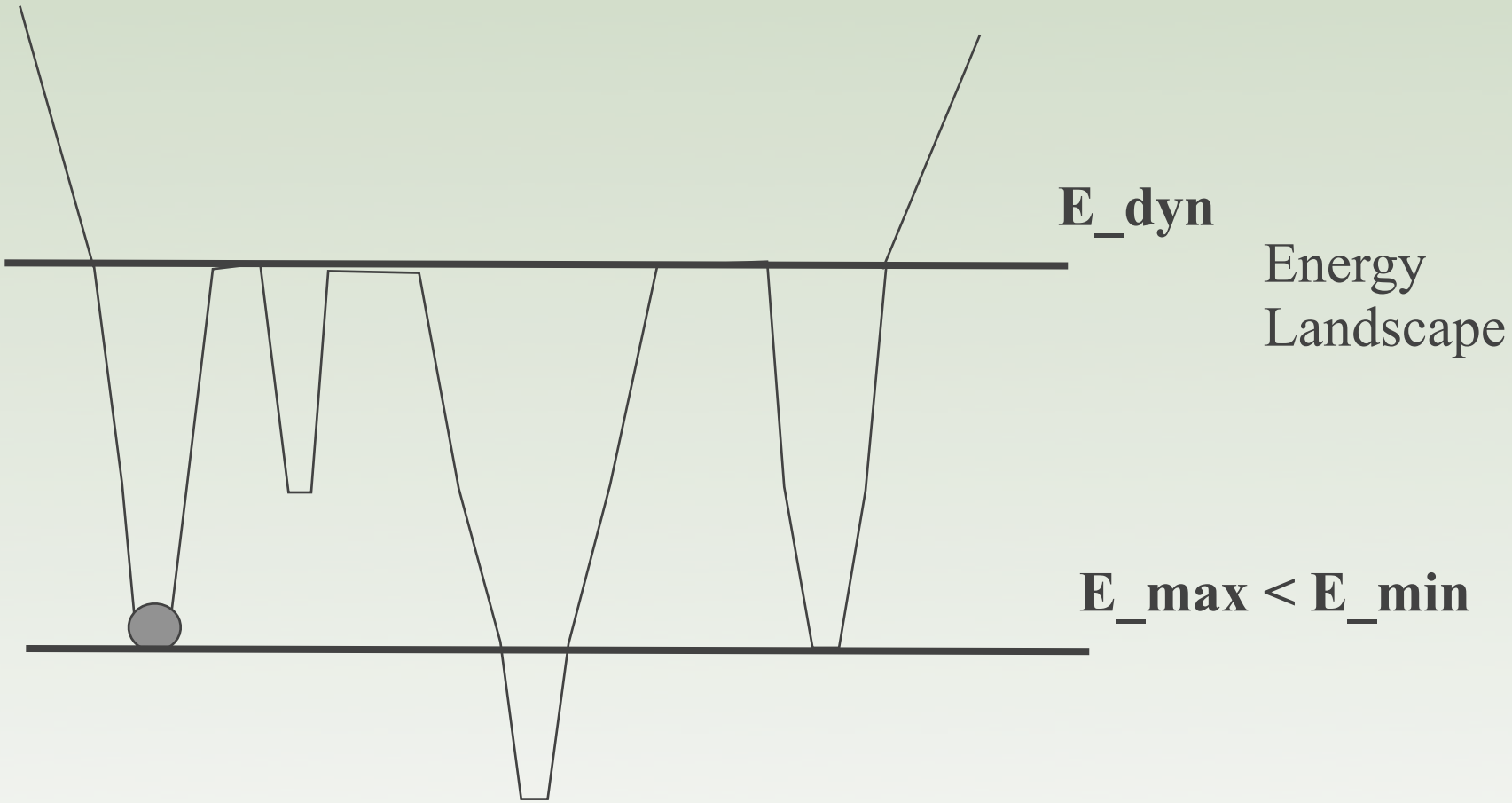


Temperature annealing goes below dynamical and static transitions !

Analytically: Spherical p-spin model

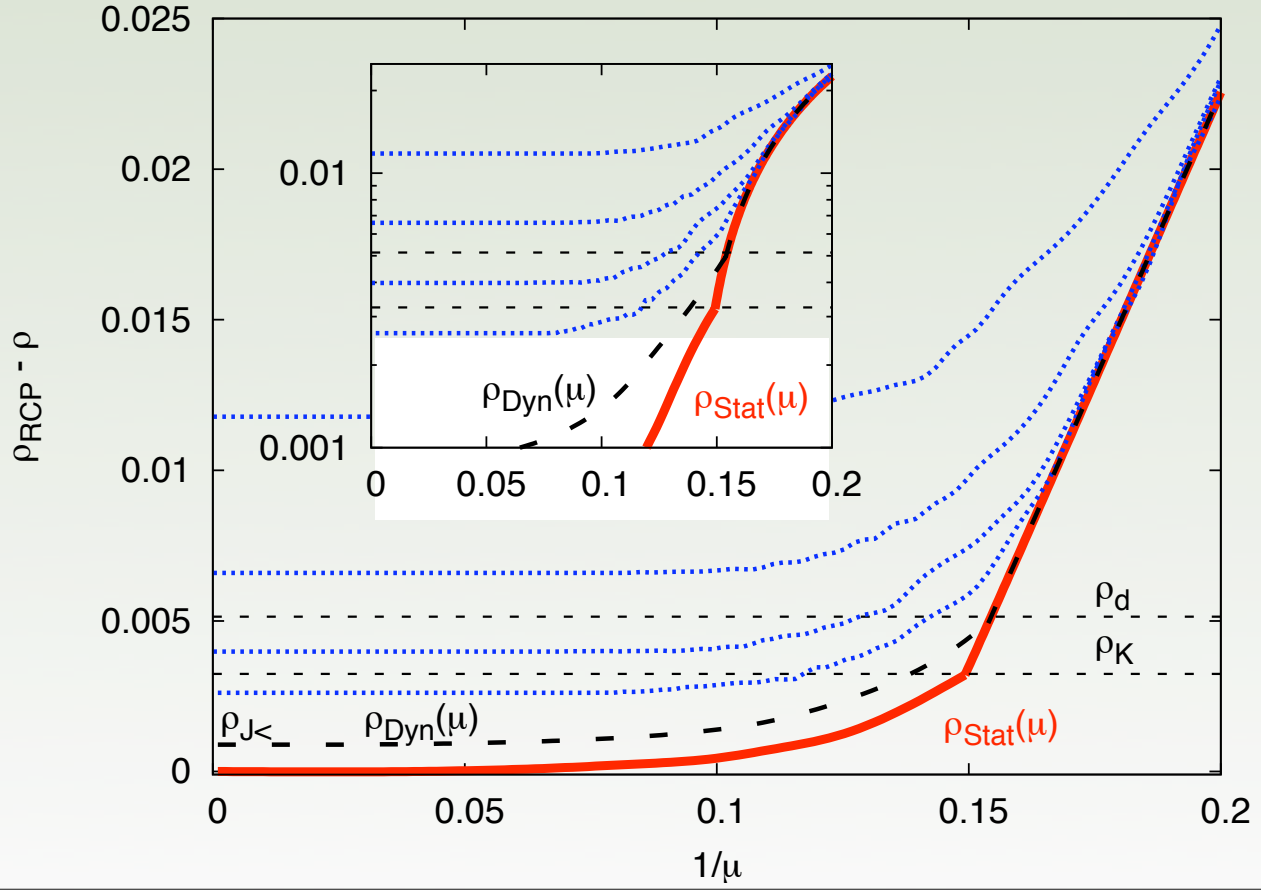
FK, Kurchan 07'

Numerically: Fully connected p-spin model

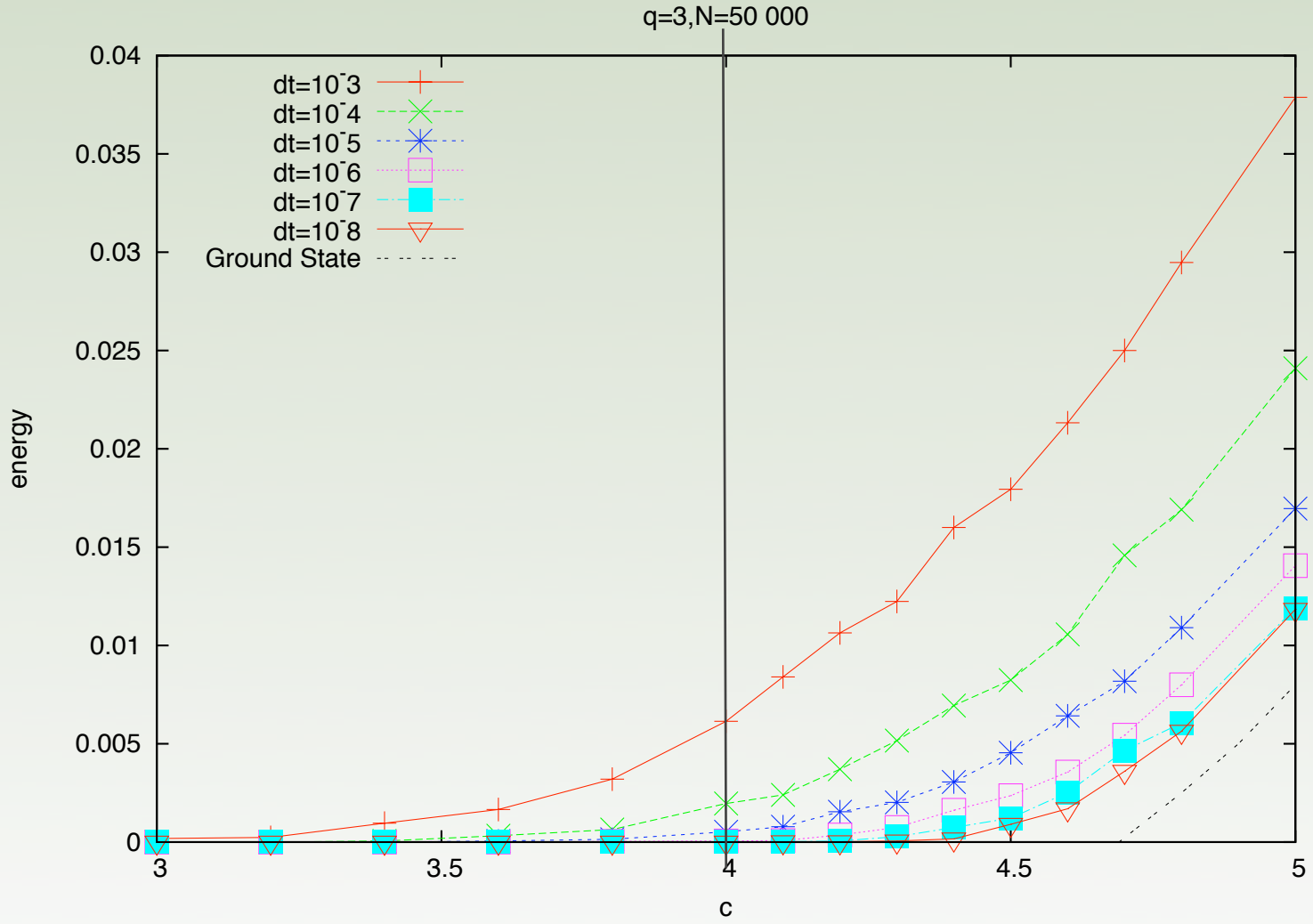
Montarani, Ricci-Tersenghi 04'

Lattice glass model for Jamming (packing problem)

FK, Tarzia & Zdeborova (to appear)



and in 3-coloring as well...



Conclusions & "Take Home" message

Dynamical transition: Easy/Hard transition for *SAMPLING*

Freezing of clusters: frozen solution are **hard** to find!

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(Some) references:

FK & Kurchan, Phys. Rev. E 76, 021122 (2007)

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FK & Zdeborova, J. Phys.: Conf. Ser. 95 012012

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Thanks to my collaborators...

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Thanks to my collaborators... ***and to the organizers!***

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