Solving K-SAT by going slowly down

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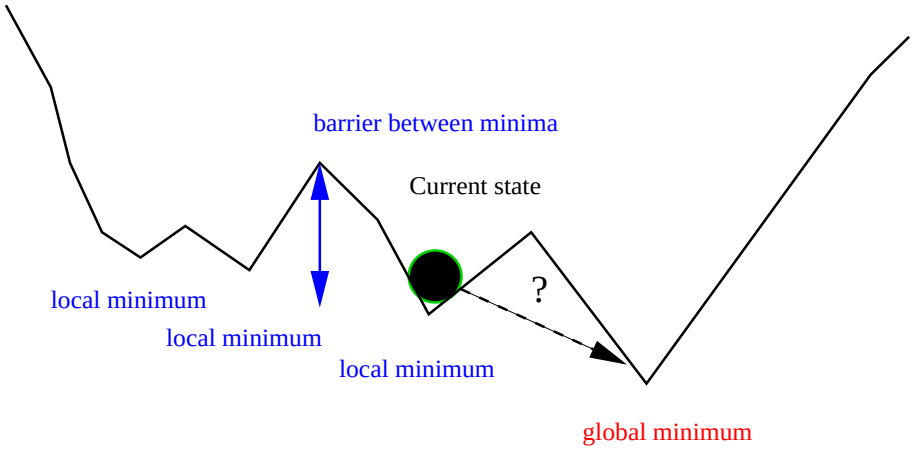
Why or background

Solving K-Satisfiability problems: why and why do not algorithms work?

Structure of "energy landscape"? Or, solution landscape?

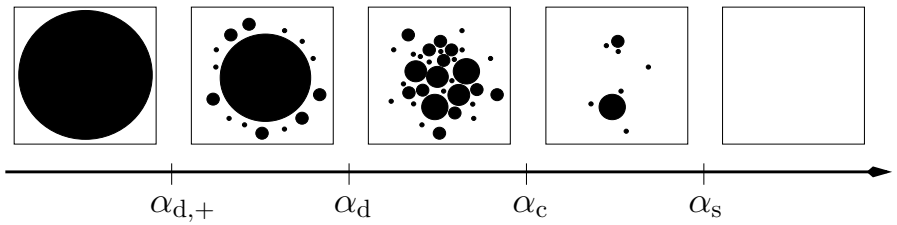
Here, a Focused Stocastic Local Search, ChainSAT.

Energy landscape?



There are local and global minima (solutions).
How does a local algorithm ("flipping spins" or variables) utilize local landscapes?
Simulated annealing, Focused Metropolis Search, ASAT, Focused Walk-SAT vs. Survey Propagation....

Phase Diagram

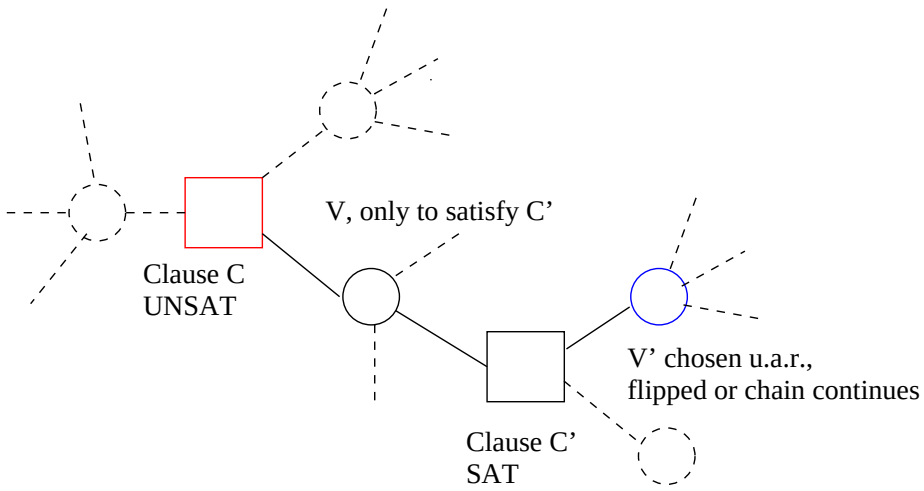


Dynamic, clustering transitions. Rigidity transition? (Krzakala et al., PNAS 104, 10318 (2007)).

Effect of these on algorithms?

ChainSAT

(Focused) ChainSAT



Idea of an algorithm:

go down - accept or

create slack.

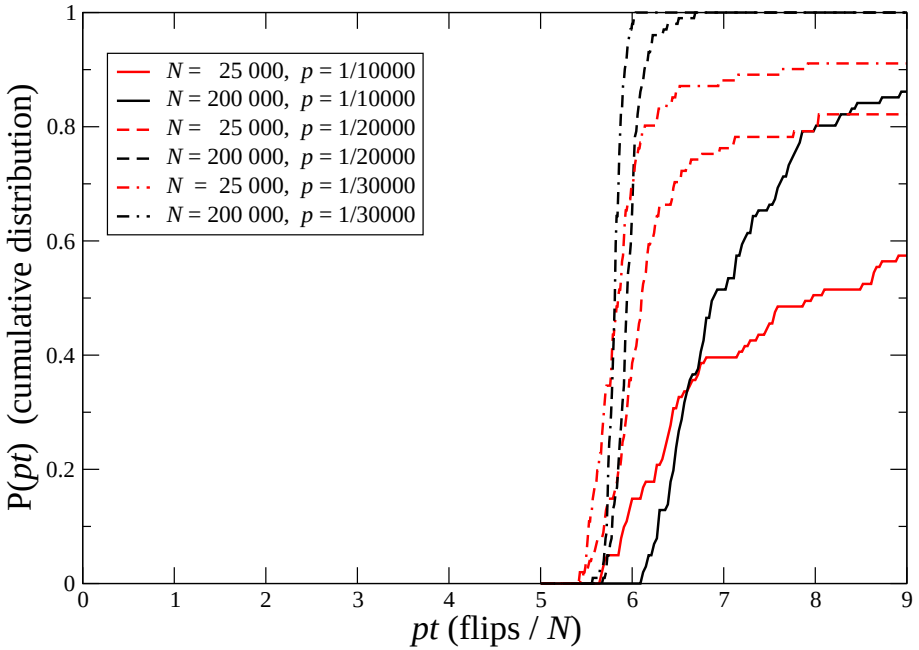
Never Goes Up in Energy

More Detail

1: S = random assignment of values to the variables
2: chaining = FALSE
3: **while** S is not a solution **do**
4: **if not** chaining **then**
5: C = a clause not satisfied by S selected uniformly at random
6: V = a variable in C selected u.a.r.
8: **end if**
9: ΔE = change in number of unsatisfied clauses if V is flipped in S
10: chaining = FALSE
11: **if** $\Delta E = 0$ **then**
12: flip V in S

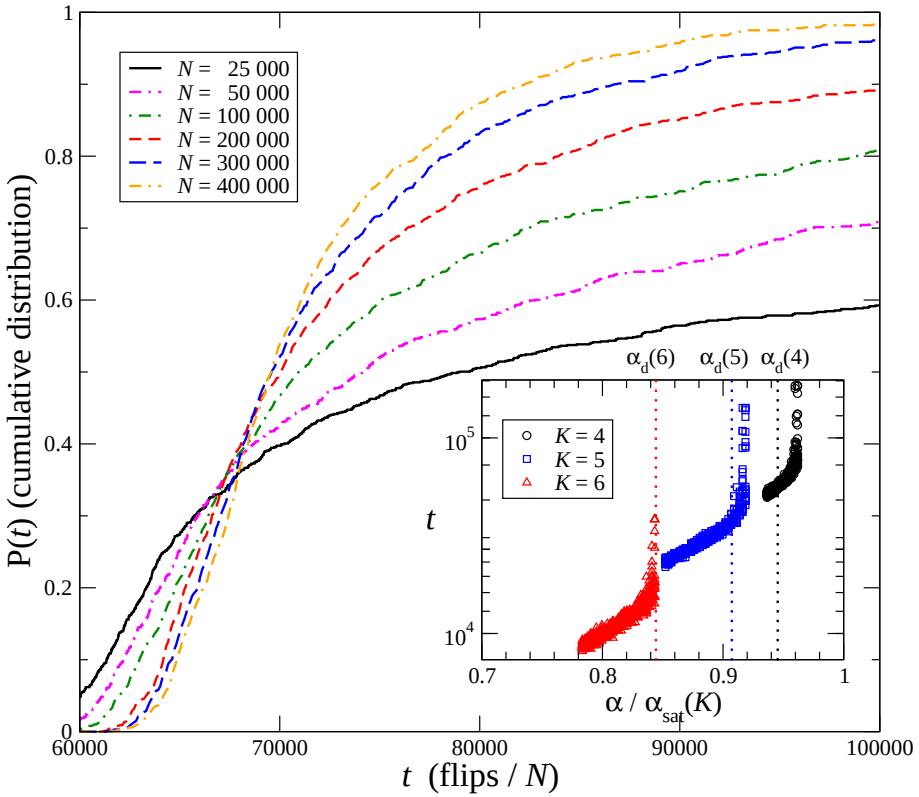
13: **else if** $\Delta E < 0$
14: **with probability** p_1
15: flip V in S
16: **end with**
17: **else**
18: **with probability** $1 - p_2$
19: $C =$ a clause satisfied only by V selected u.a.r.
20: $V' =$ a variable in C other than V selected u.a.r.
21: $V = V'$
22: chaining = TRUE
23: **end with**
24: **end if**
25: **end while**

Studying ChainSAT



What is the optimal parameter value for p_1, p_2?
ChainSAT seems to be robust... compared to ASAT, FMS?

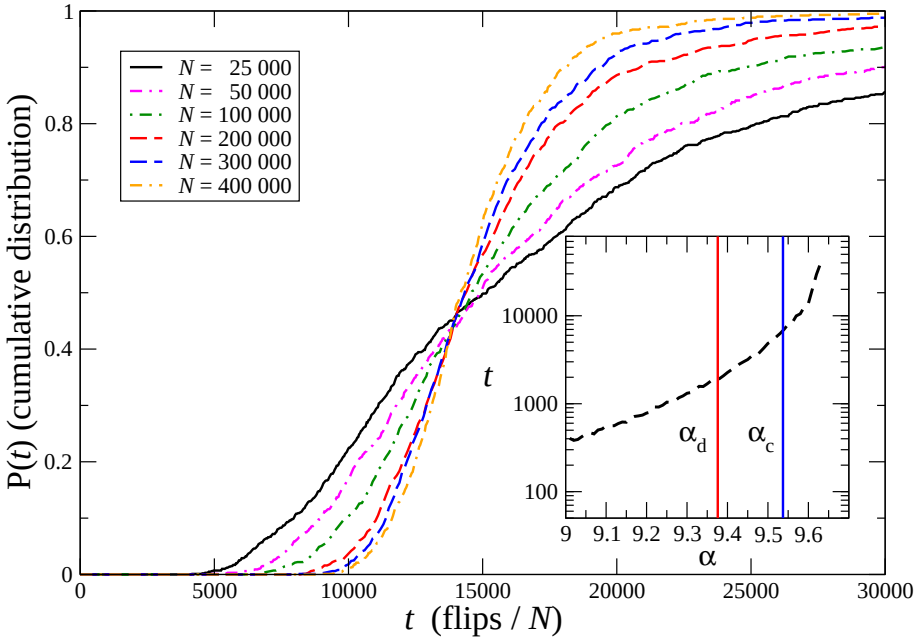
Chain/K-SAT



ChainSAT is linear in N for $K = 4 \ldots 5 \ldots 6$.

It works beyónd the dynamical and condensation transitions.

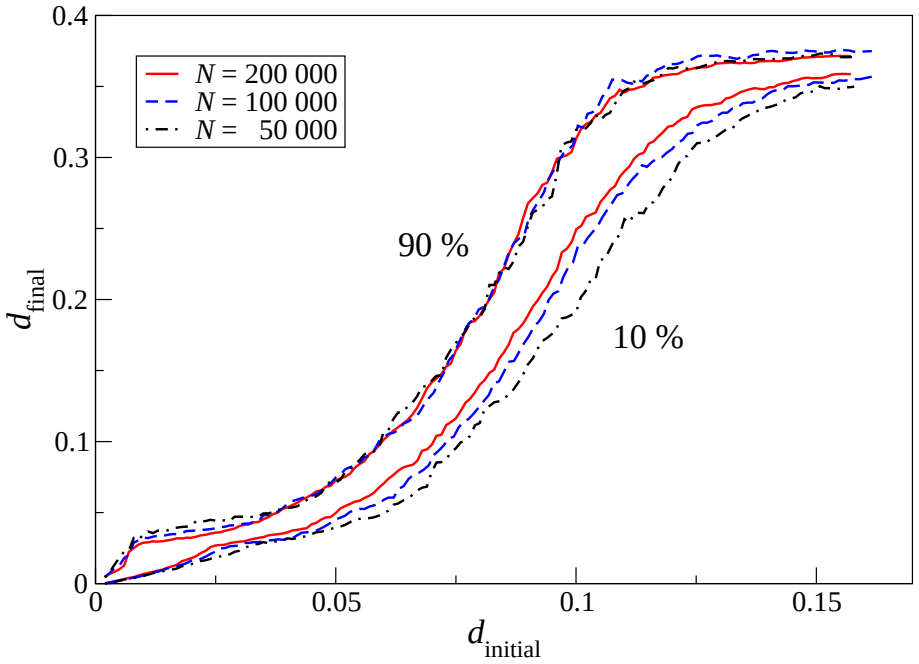
FMS comparison



For FMS (Focused Metropolis), $\alpha = 9.6$ and $K = 4$ similar behavior is found.

Here one can go over barriers.

Solution space

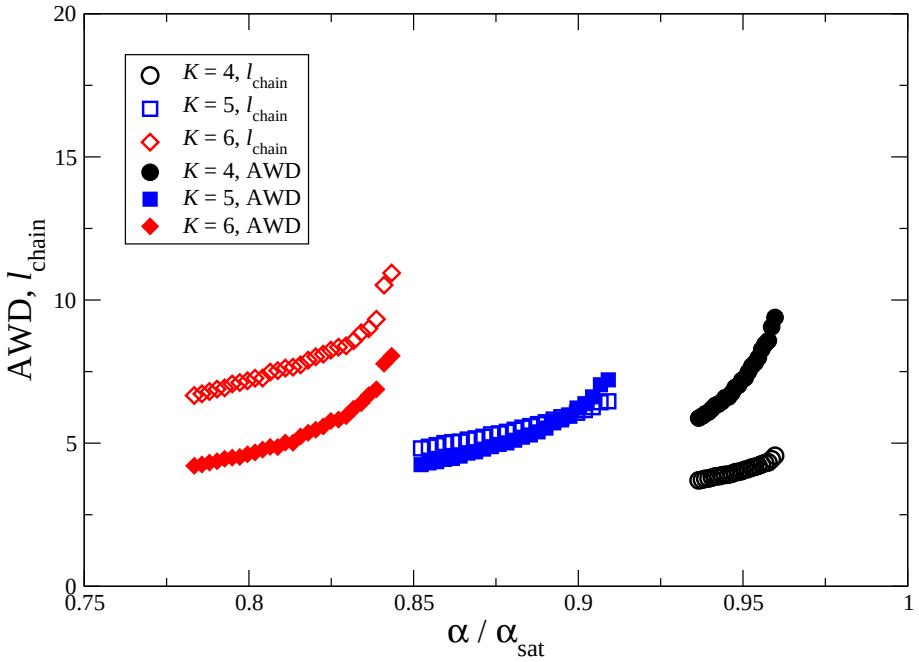


Trials: solve, flip a fraction of variables to get to $d_{initial}$ from original. Re-solve.

d_{final} continuous, concentrates.

Here $\alpha = 9.6$, $K = 4$.

Chain/K-SAT



How does the ChainSAT work?

White solutions (with slack).

Chain lengths do not diverge:

"chainable" variables found.

Conclusions

SLS algorithms find regardless of solution space structures "white" solutions.

Local minima irrelevant in K-SAT.

"rigid" solutions? Finite correlation length:
mosaic solution - linearity?

ChainSAT: extensions to other members of the CSP family. See arXiv:0711.4902.

Understanding algorithms in the average sense?
RandomWalkSAT, J. Ardelius lic. defence last Wed.