The effect of gravity on clustering and relative velocities in isotropic turbulence

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Motivation

- Predict the collision kernel for inertial particles embedded in turbulent flow*

$$K(\sigma) = 4\pi\sigma^2 g(\sigma) \int_{-\infty}^{0} -w_r p(w_r|\sigma) dw_r$$

$$g(\sigma) \equiv \text{Radial Distribution Function (RDF)}$$

$$p(w_r|\sigma) \equiv \text{Relative Velocity PDF}$$

- Taking into consideration gravity…

Bec et al. 2014
Gustavsson et al. 2014

* Sundaram & Collins 1997

DNS - Fluid

$$\frac{\partial u_i}{\partial t} + \epsilon_{ijk}\omega_j u_k = -\frac{\partial p^\star}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + F_i$$

$$\frac{\partial u_j}{\partial x_j} = 0 \qquad p^\star = \frac{p}{\rho} + \frac{1}{2} u_i u_i$$

Particles

$$\frac{dXi}{dt} = v_i(\mathbf{x} = \mathbf{X}) \qquad \frac{dv_i}{dt} = \frac{u_i(\mathbf{x} = \mathbf{X}) - v_i}{\tau_p} \quad {\color{red}+g_i}$$

$$St \equiv \frac{\tau_p}{\tau_\eta}$$

DNS - Fluid

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Particles Relative Motion

$$\frac{dr_i}{dt} = w_i \qquad\qquad \frac{dw_i}{dt} = \frac{\Delta u_i(\mathbf{x} = \mathbf{X}) - w_i}{\tau_p}$$

$$St \equiv \frac{\tau_p}{\tau_\eta}$$

Parameters

U'	turbulence intensity	d	diameter
ϵ	dissipation rate	ρ_p	density
ν	kinematic viscosity	n	number density
g	gravity		

$$R_\lambda \equiv {U'}^2 \sqrt{\frac{15}{\nu\epsilon}}$$

$St \equiv \dfrac{\tau_p}{\tau_\eta}$ Stokes number

d/η size parameter

Φ volume fraction

$S_v \equiv \dfrac{v_T}{u_\eta}$ settling parameter

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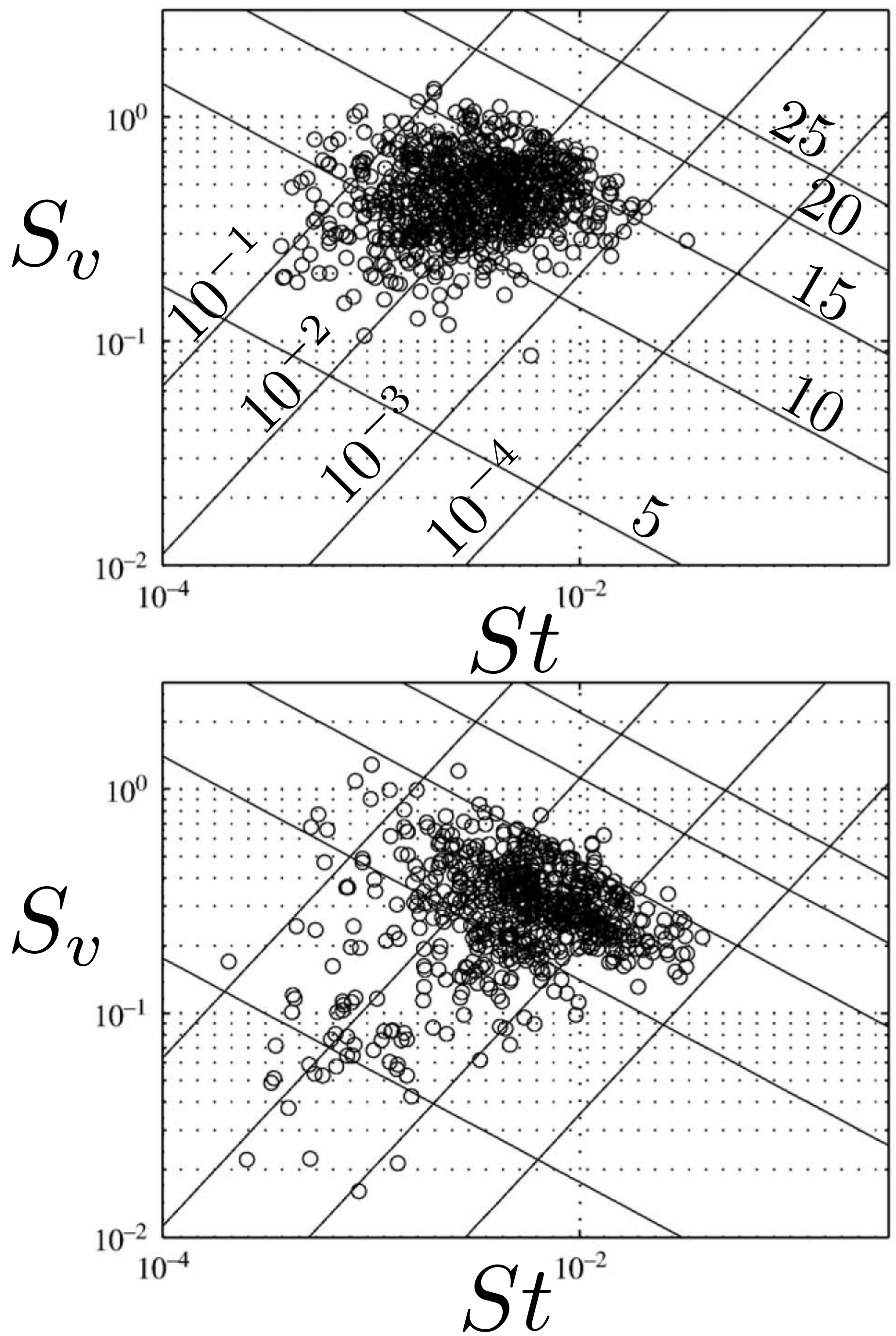
understood → d/η ~~size parameter~~

neglected → Φ ~~volume fraction~~

$$S_v \equiv \frac{v_T}{u_\eta}$$ settling parameter

Cloud parameters

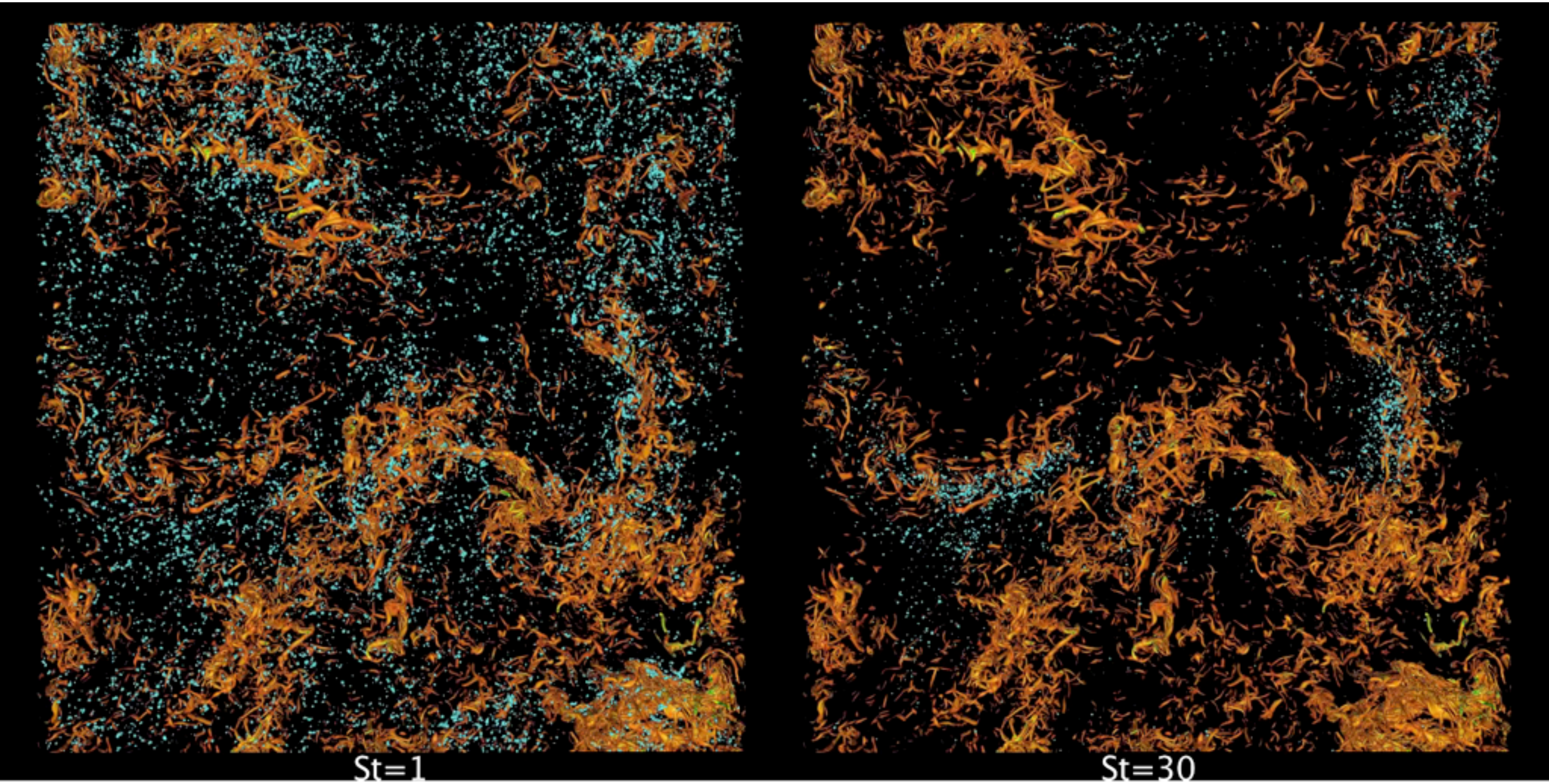


$R_\lambda \sim 10^4$

$\Phi \sim 10^{-6}$

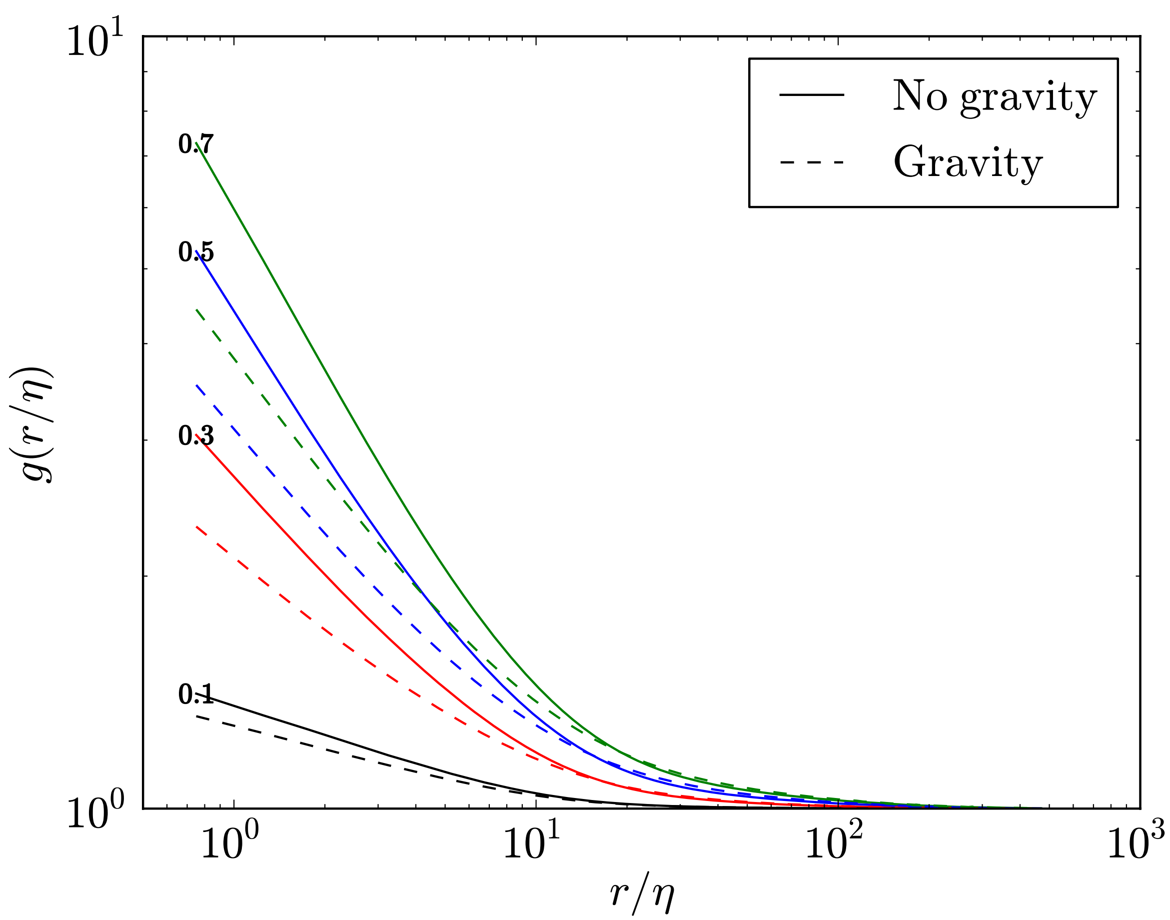
Fig. 9. The distribution of cloud microphysical and turbulence properties in a dimensionless Stokes–settling parameter space. The upper left plot is for a stratocumulus cloud and the remaining three are for small cumulus clouds. Each point represents data in a 1-second (approximately 15 m) average. Diagonal lines with positive slope are contours of constant turbulent energy dissipation rate, ε, at values of 10^{-4}, 10^{-3}, 10^{-2}, and 10^{-1} (lower right to upper left corners). Diagonal lines with negative slope are contours of constant droplet diameter at values of 5, 10, 15, 20 and 25 μm (lower left to upper right corners).

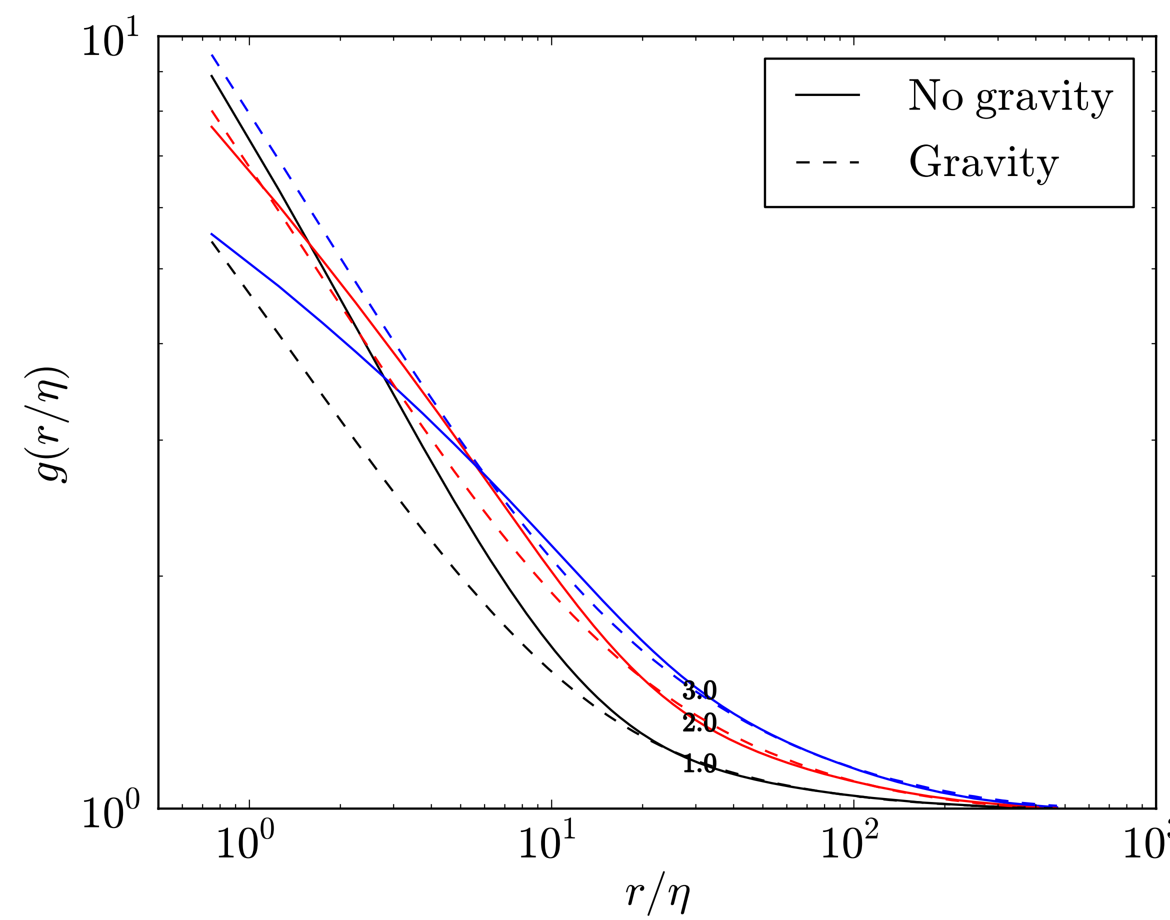
DNS



NCAR Yellowstone 2048^3 $R_\lambda \approx 600$

Effect of gravity on the RDF

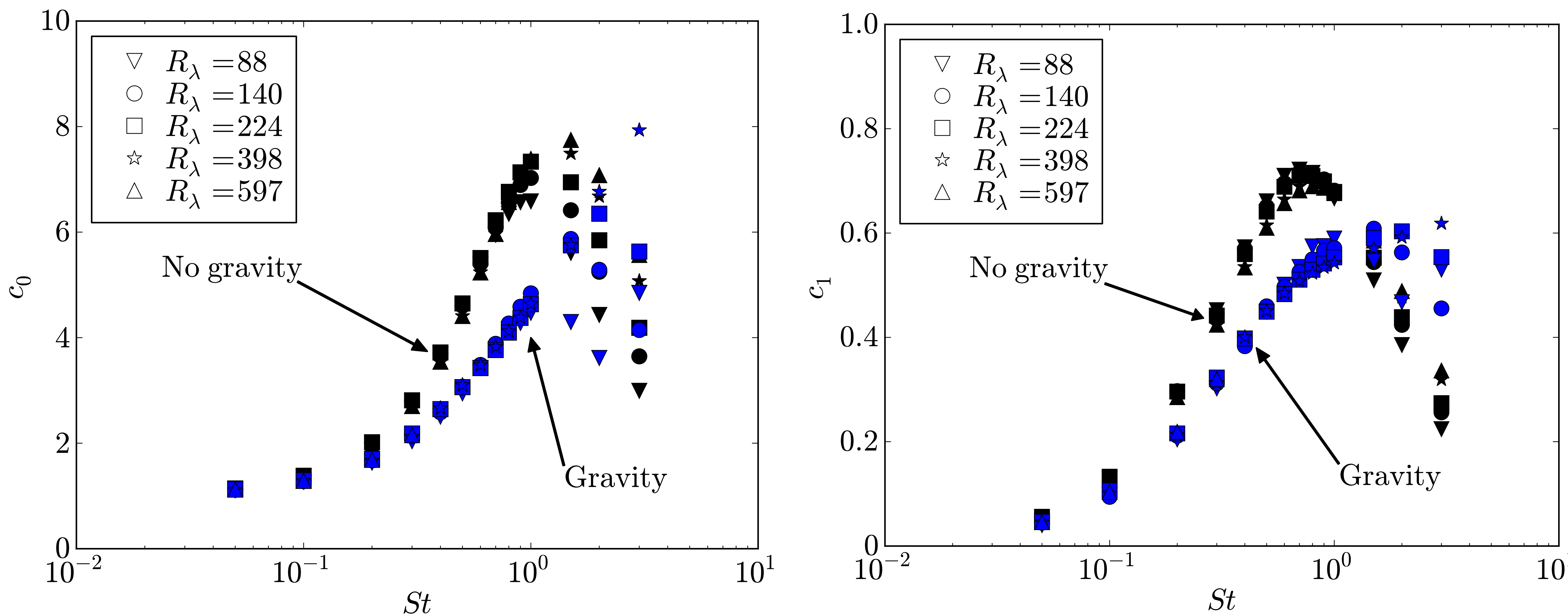




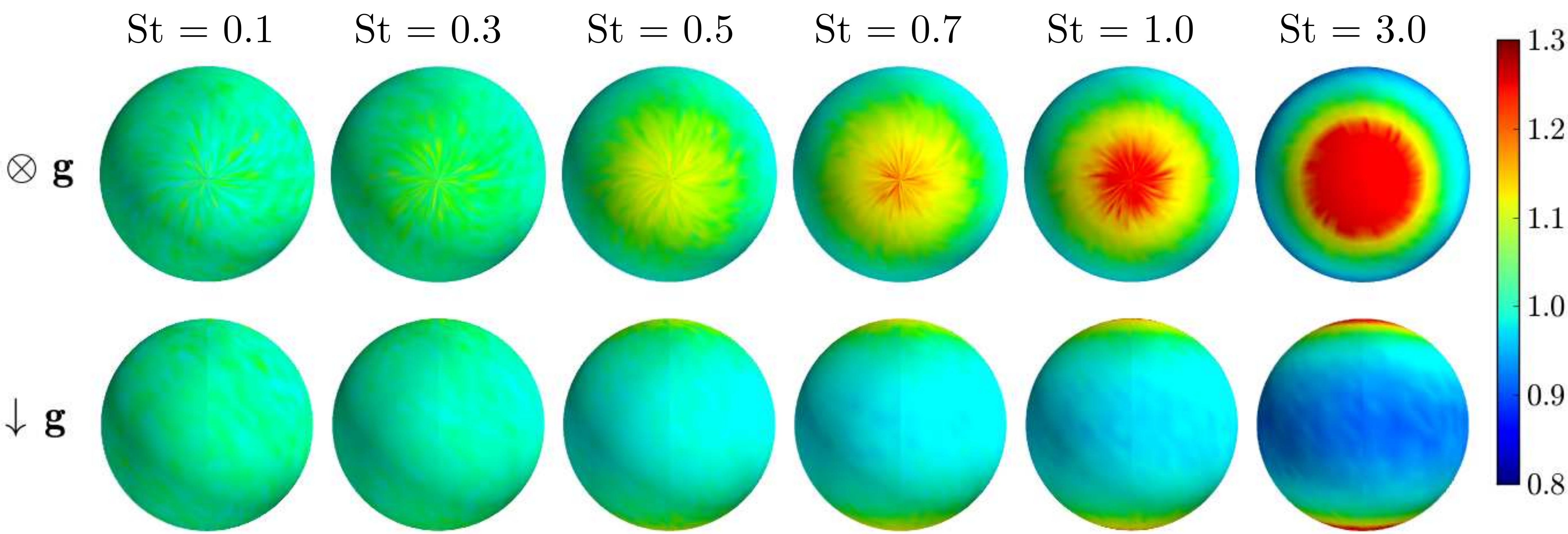
Gravity <u>diminishes</u> clustering at <u>low St</u>, but <u>enhances</u> it at <u>higher St</u>

Power-law Coefficients

$$g(r) = c_0(St)\left(\frac{\eta}{r}\right)^{c_1(St)}$$



Angular distribution of RDF



Accounting for reduction in symmetry

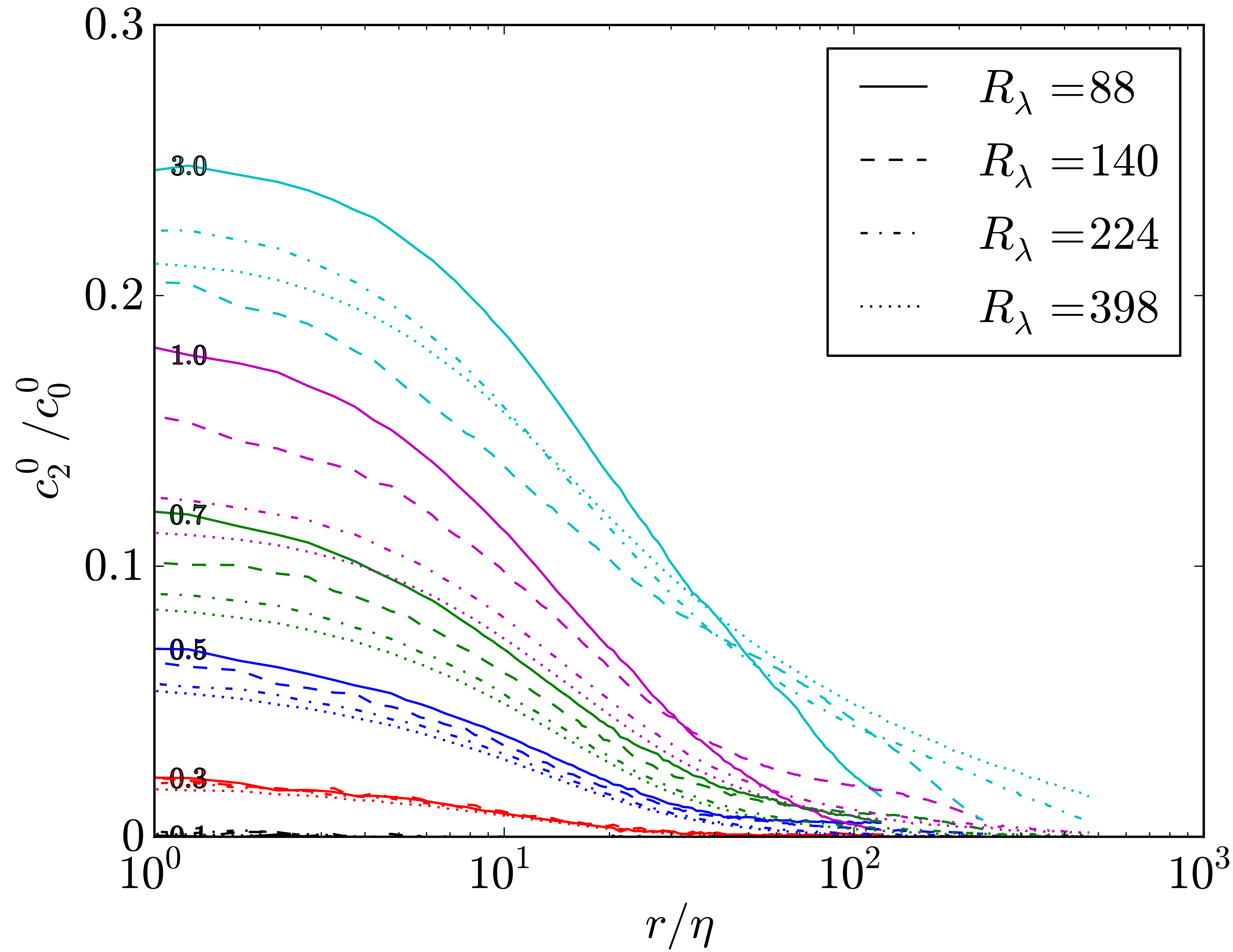
$$g(r,\theta) = c_0 P_0(\mu) + c_2 P_2(\mu) + c_4 P_4(\mu) + \cdots$$

$$\mu \equiv \cos(\theta)$$

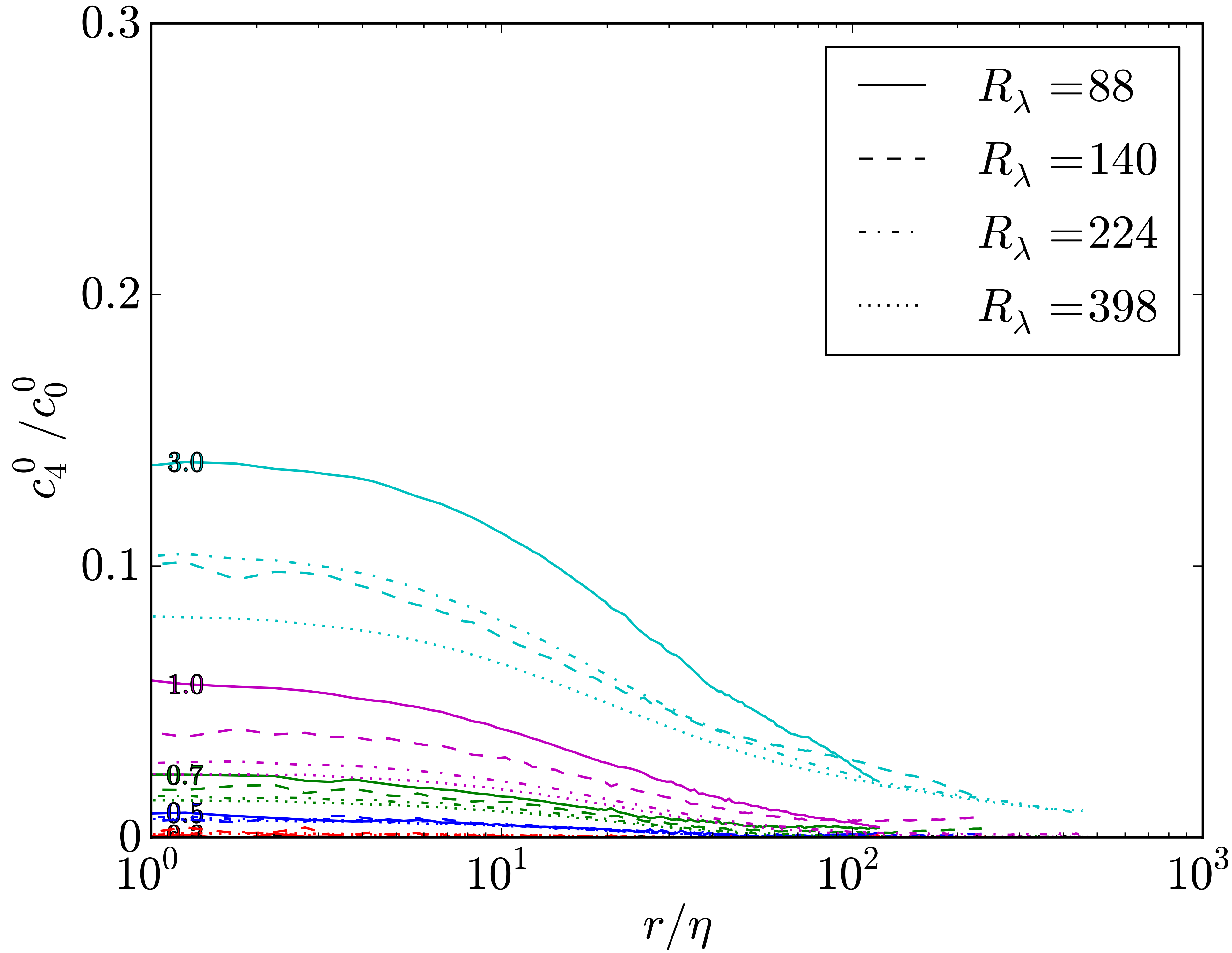
$$P_0 = 1$$
$$P_2 = 1/2(3\mu^2 - 1)$$
$$P_4 = 1/8(35\mu^4 - 30\mu^2 + 3)$$

Legendre Polynomials

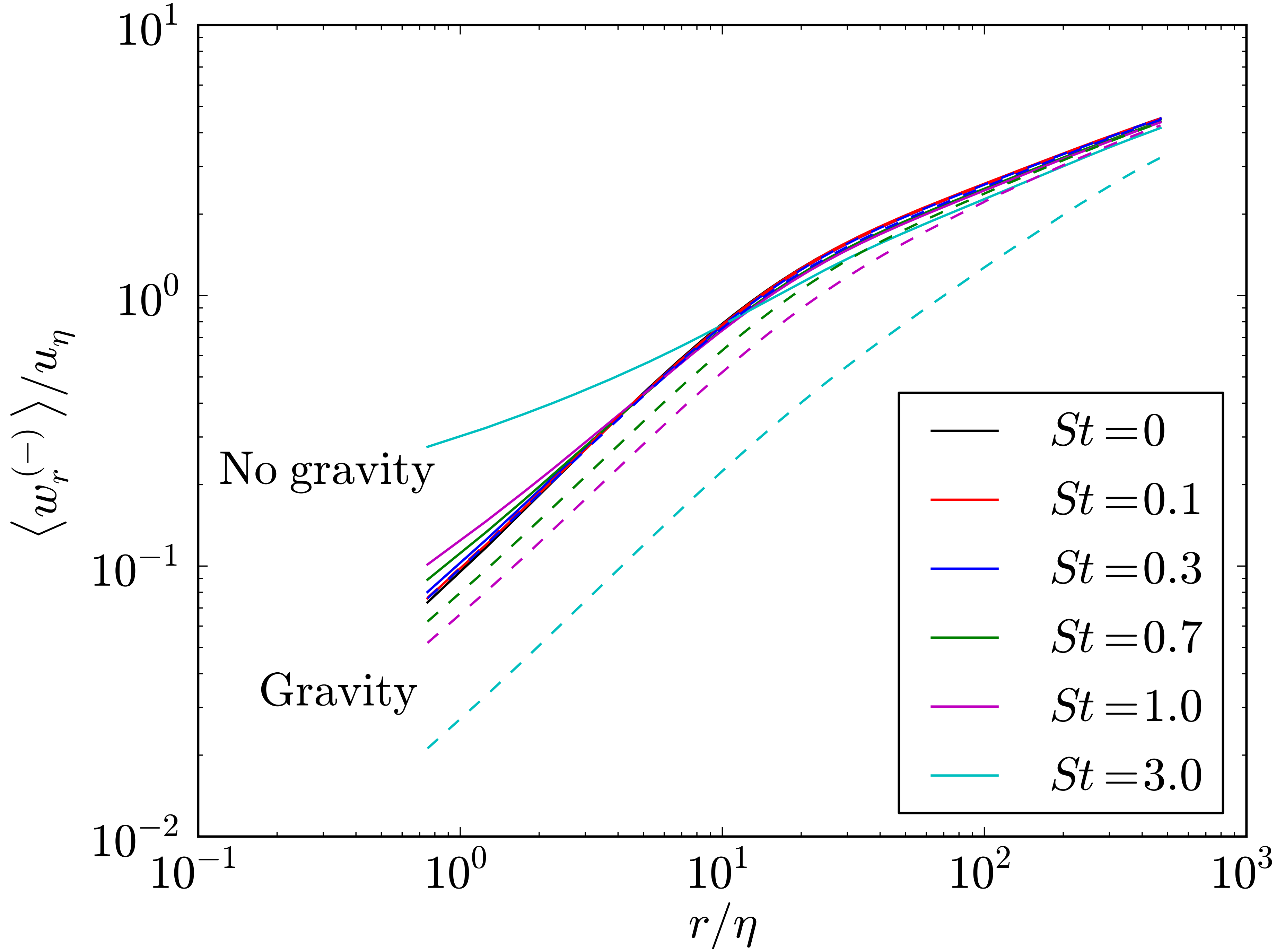
Degree of anisotropy for RDF



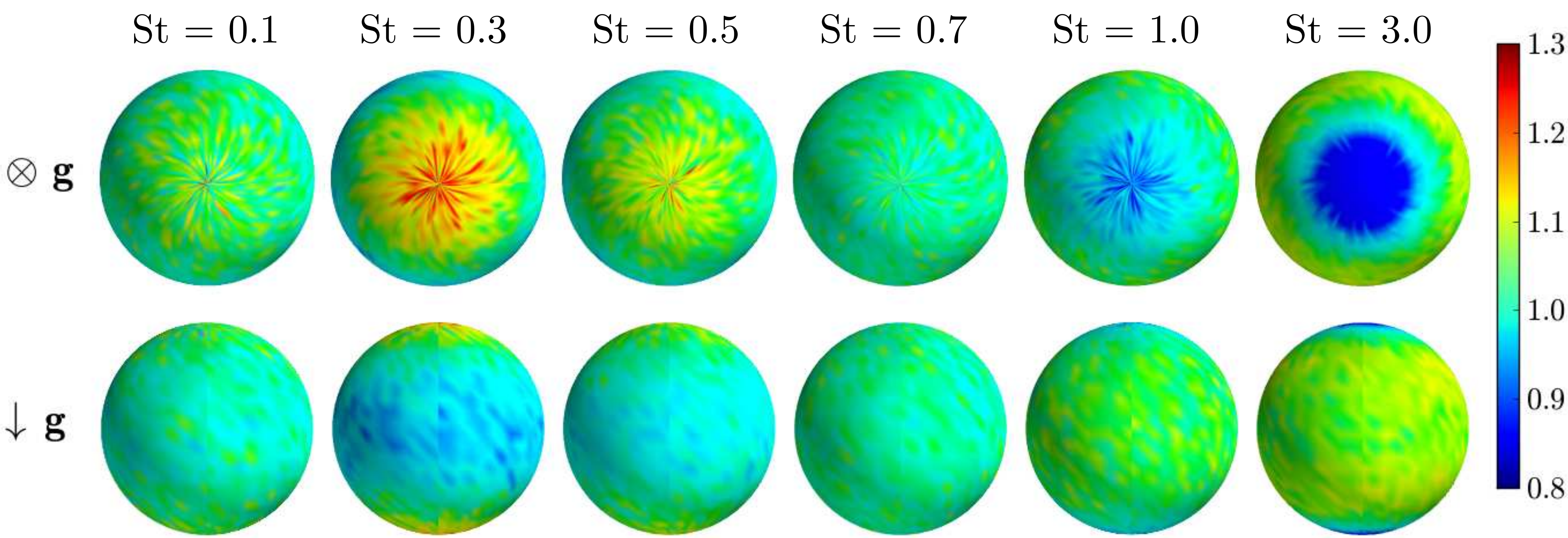
Degree of anisotropy for RDF



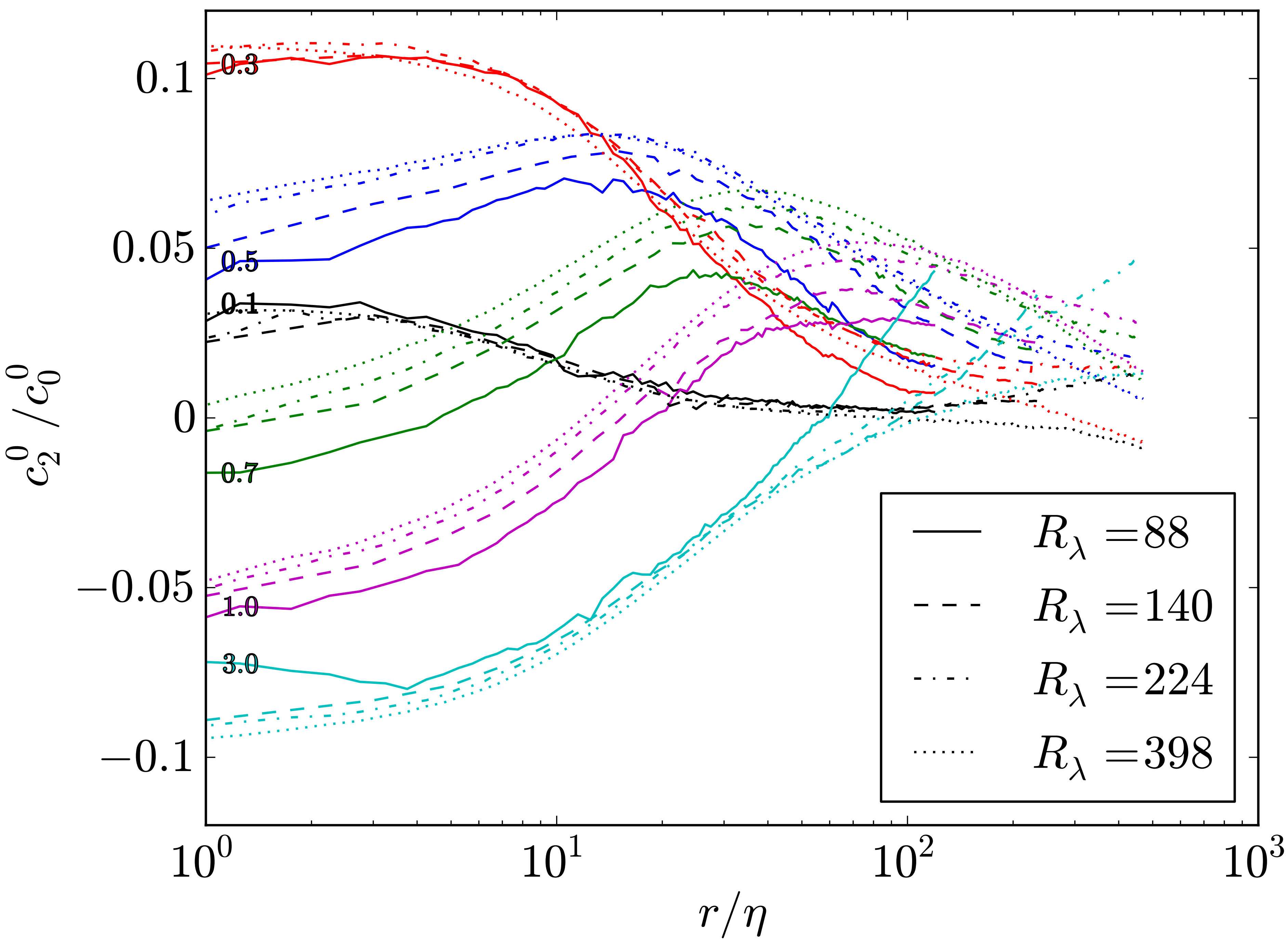
Effect of gravity on the mean inward relative velocity



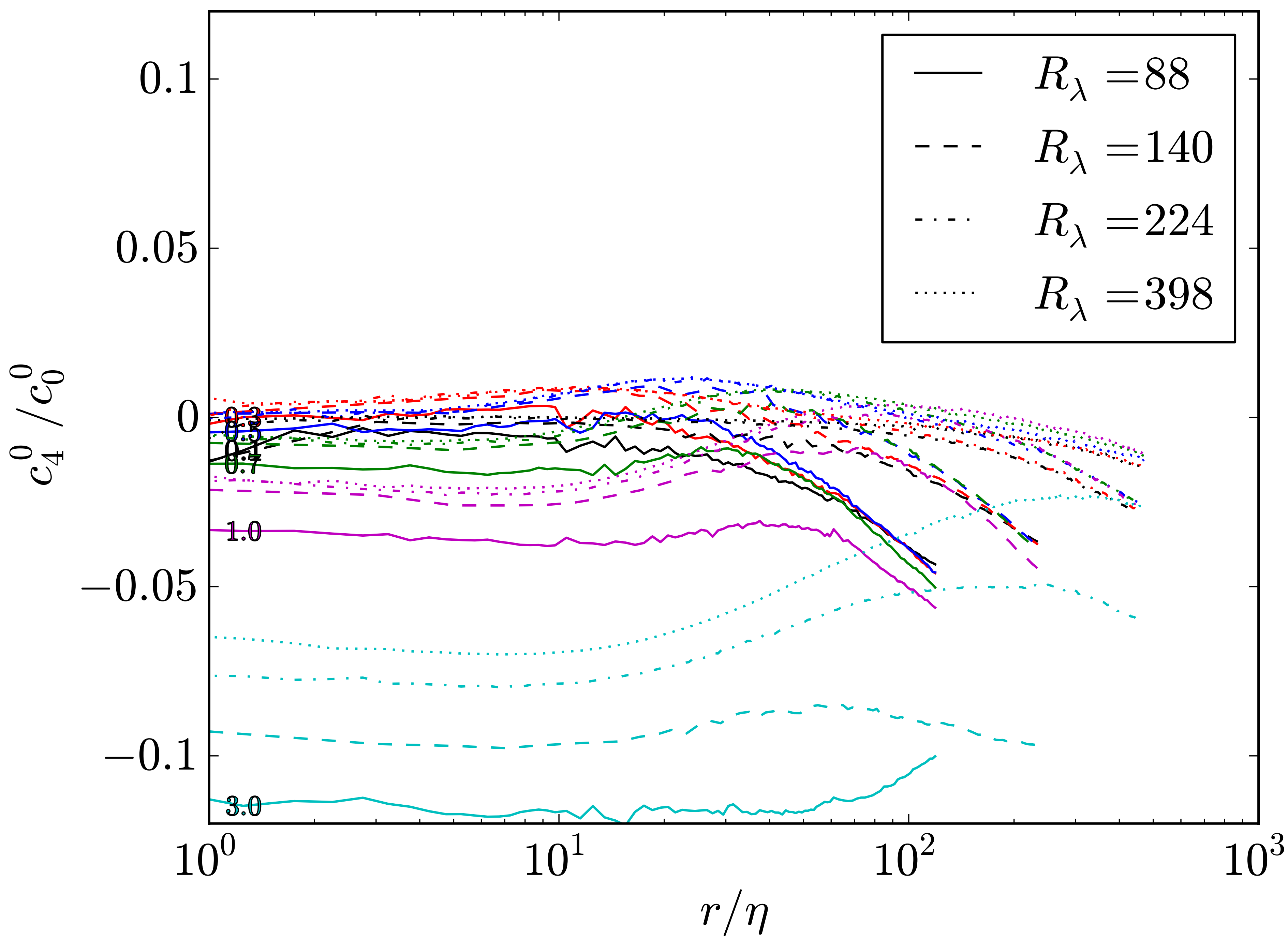
Angular distribution of relative velocity



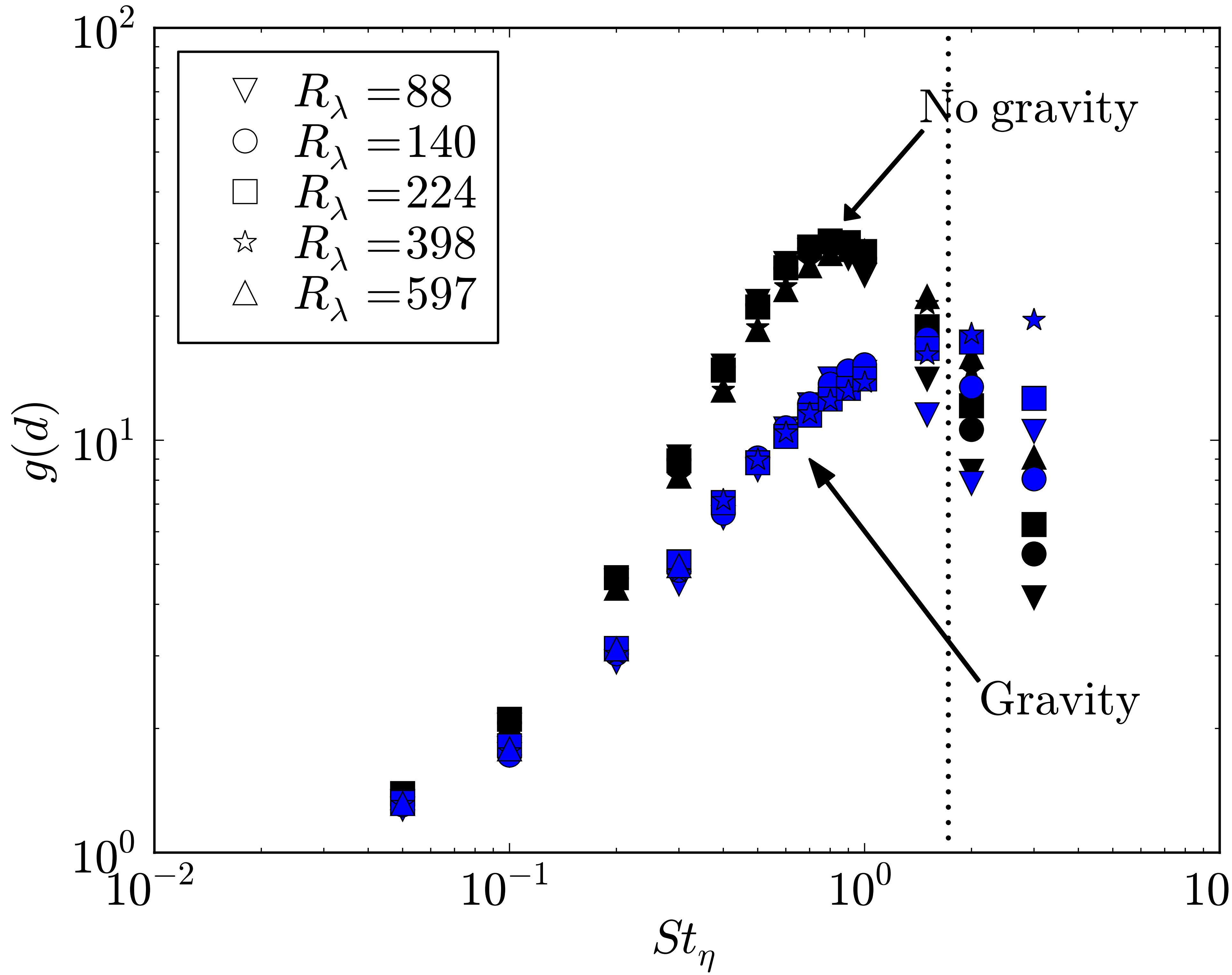
Degree of axisymmetry for the relative velocity



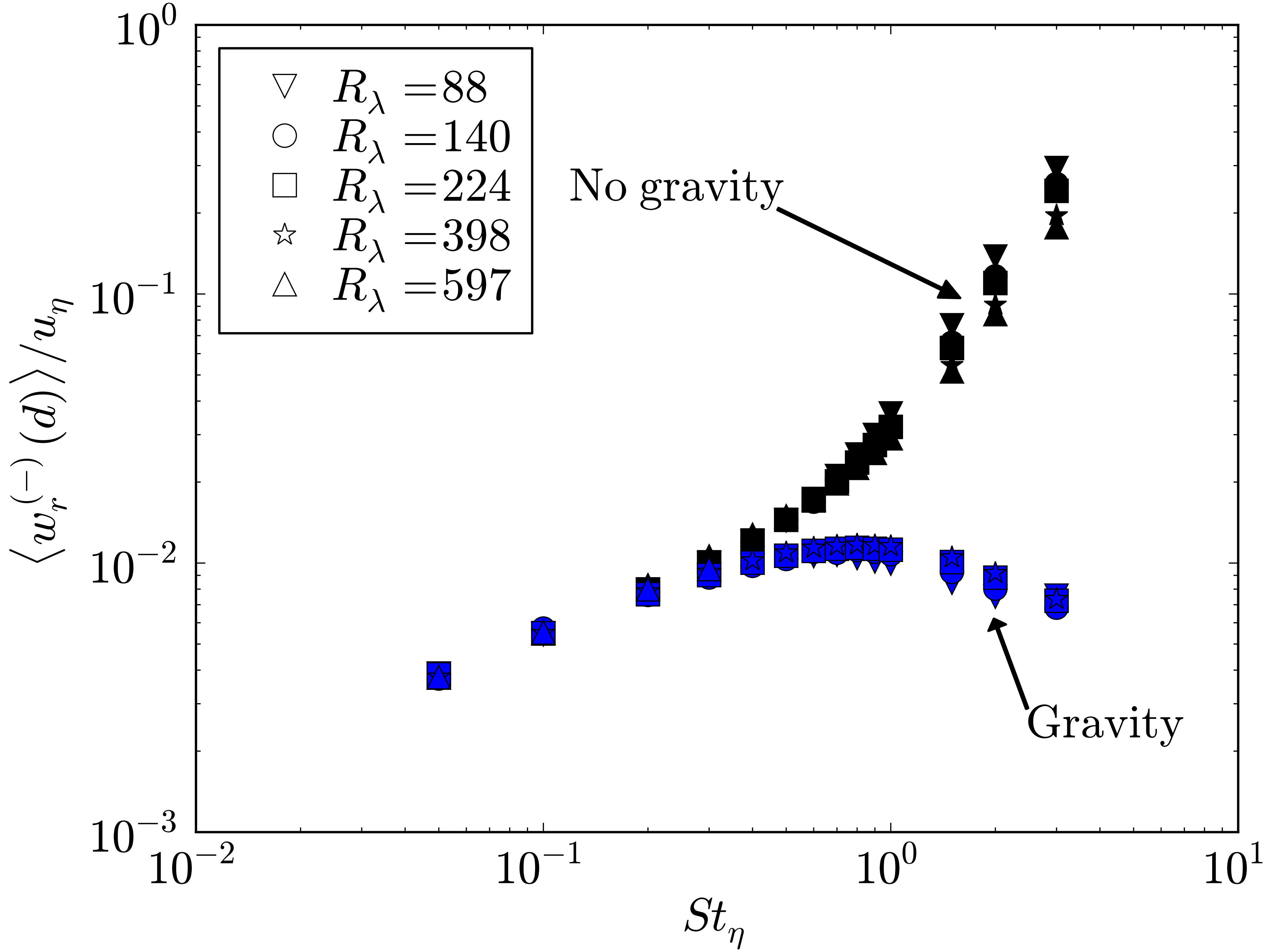
Degree of axisymmetry for the relative velocity



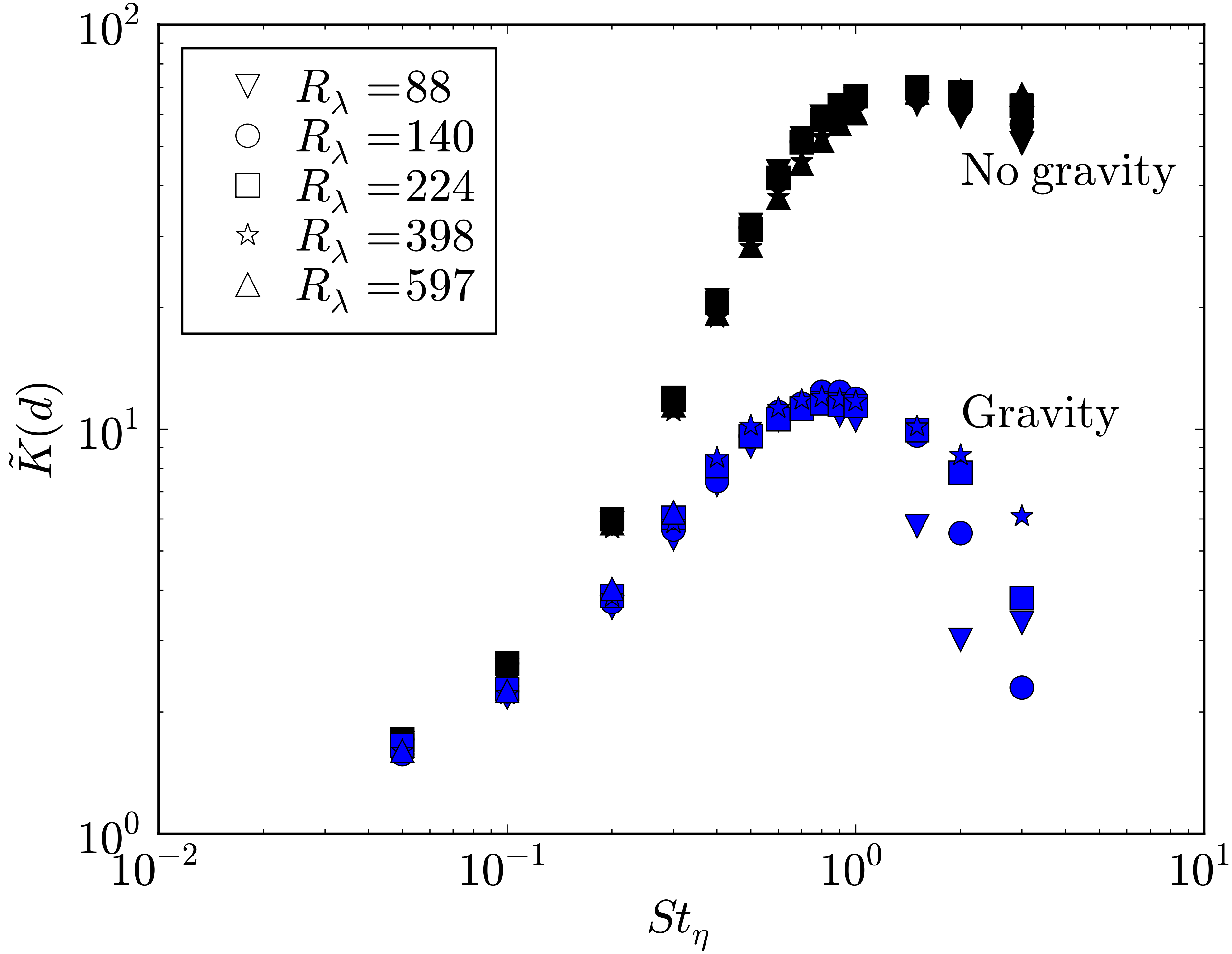
RDF at contact



Relative velocity at contact



Collision kernel



Zaichik Theory

$$-\underbrace{\tau_p\left(\mathbf{S}_2^p+\lambda\right)\cdot\nabla_{\mathbf{r}}g}_{\text{diffusion}}-\underbrace{g\tau_p\nabla_{\mathbf{r}}\cdot\mathbf{S}_2^p}_{\text{drift}}=0$$

$$St\ll 1$$

$$\tau_p\left(\mathbf{S}_2^p+\lambda\right)\approx\tau_p\lambda=B_{\mathrm{NL}}\frac{r^2}{\tau_\eta}\qquad\tau_p\nabla_{\mathbf{r}}\cdot\mathbf{S}_2^p\approx\frac{St\tau_\eta}{3}r\left\langle\mathcal{S}^2-\mathcal{R}^2\right\rangle_p$$

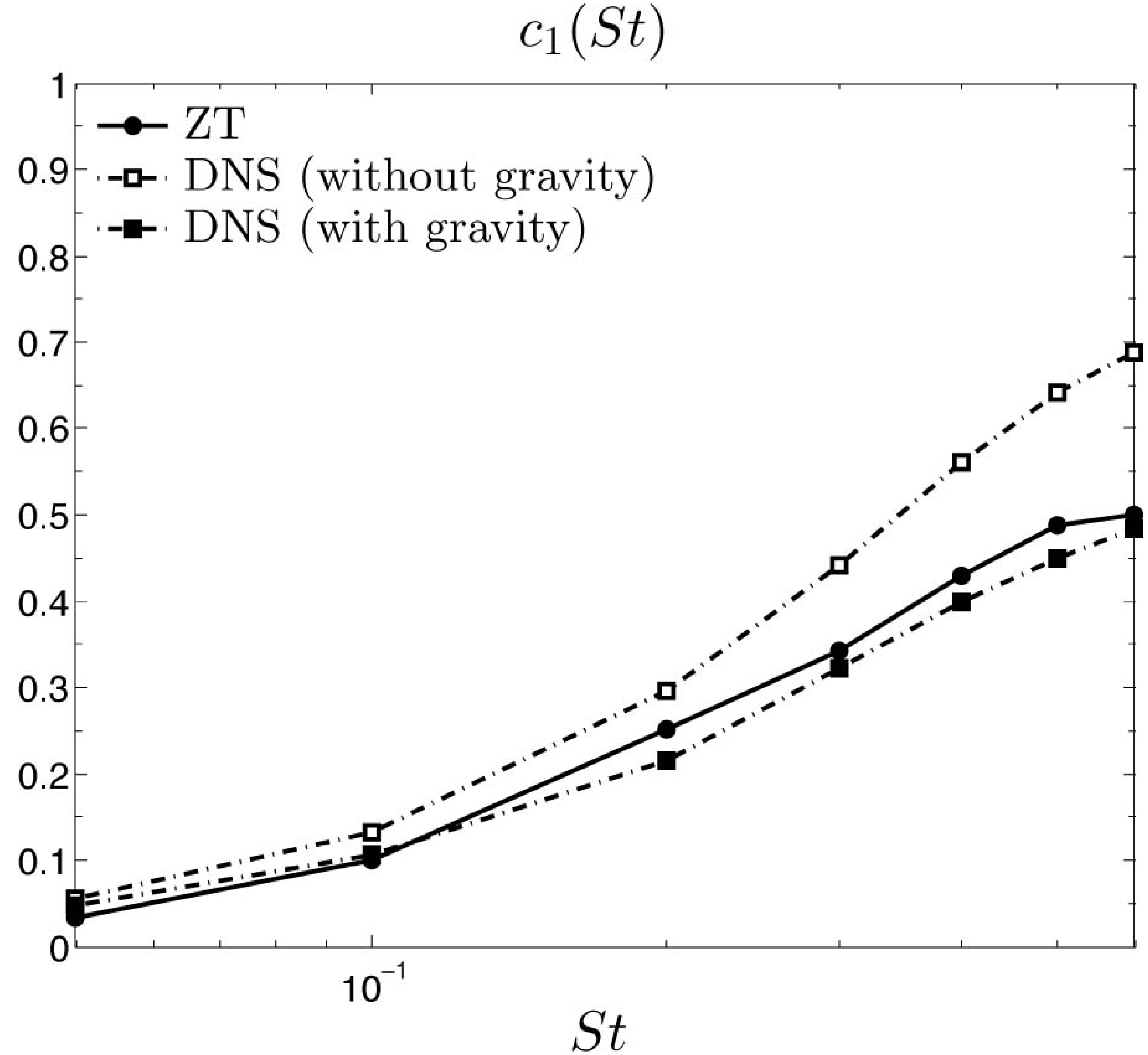
$$St\gtrsim 1$$

$$\tau_p\left(\mathbf{S}_2^p+\lambda\right)\approx\tau_p\mathbf{S}_2^p$$ Gravity diminishes more than drift

Zaichik, Fede, Simonin, Alipchenkov (2009)

Bragg and Collins, NJP (2014)

Zaichik Theory



No clear path to predicting effect of gravity on time scales!
Initially considering the strong gravity limit

Summary

- Considered the effect of gravitational settling on clustering, relative velocity and collision
- Gravity reduces clustering at small St and enhances it at high St
- Gravity causes a significant reduction in symmetry (~30%)
- Gravity increases relative velocity at small St and decreases it at high St; collision kernel is decreased by gravity
- Zaichik theory explains qualitative trends; however, quantitative predictions requires a model for implicit changes in the time scales for strain and rotation along particle paths
- We are first considering the large gravity limit (frozen flow)