

A Capsule in Peristaltic Flows

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PERISTALTIC PUMPING

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M. Y. JAFFRIN AND A. H. SHAPIRO

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1. INTRODUCTION

Peristaltic pumping is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible tube containing a liquid. Physiologically, peristaltic action is

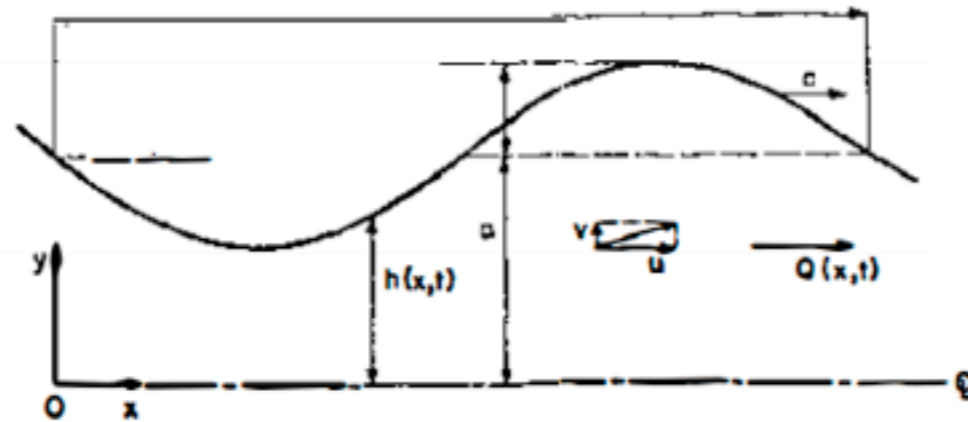


FIGURE 2. Nomenclature for periodic sine wave.

$$\nabla^2 \psi_t + \psi_y \nabla^2 \psi_x - \psi_x \nabla^2 \psi_y = \nu \nabla^2 \nabla^2 \psi \quad \leftarrow \text{streamfunction}$$

After introduction of the dimensionless variables

$$\xi \equiv 2\pi \frac{x}{\lambda}; \quad \eta \equiv \frac{y}{a}; \quad \tau \equiv 2\pi \frac{ct}{\lambda}; \quad \chi \equiv \frac{\psi}{ac}; \quad H \equiv \frac{h(x,t)}{a}$$

wavelength \rightarrow

wave speed \rightarrow

and the two important dimensionless parameters

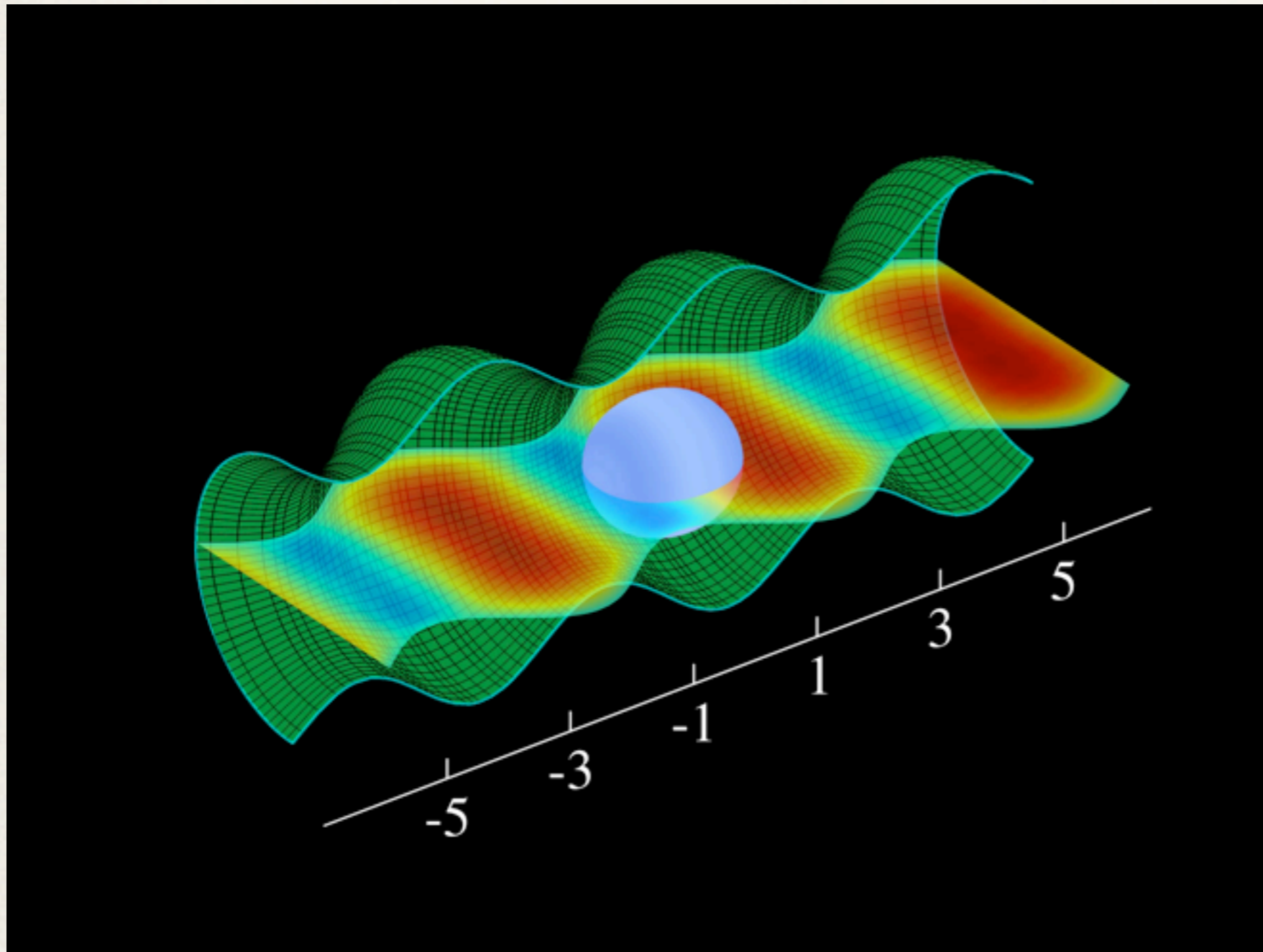
$$\alpha \equiv 2\pi a/\lambda; \quad R \equiv (ac/\nu) \cdot \alpha$$

Reynolds No.

the vorticity equation becomes

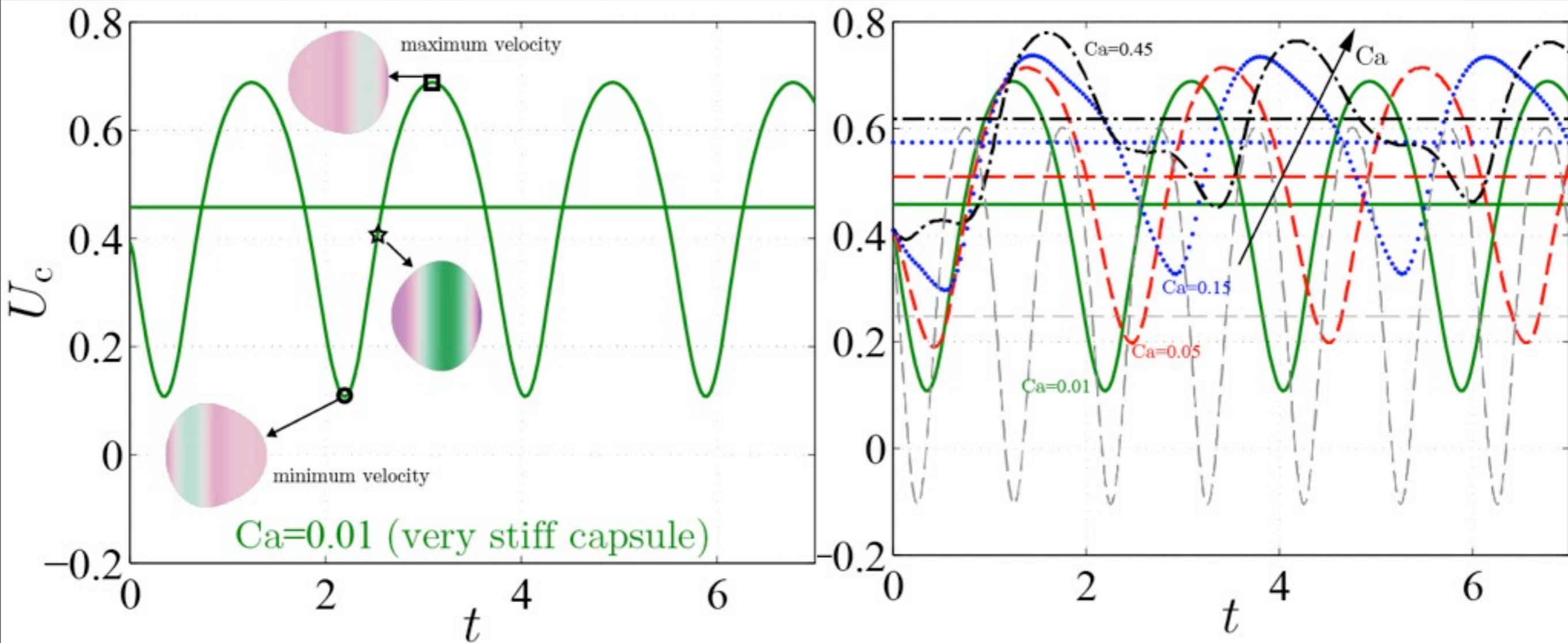
$$R(\bar{\nabla}^2 \chi_\tau + \chi_\eta \bar{\nabla}^2 \chi_\xi - \chi_\xi \bar{\nabla}^2 \chi_\eta) = \bar{\nabla}^2 \bar{\nabla}^2 \chi$$

Stokesian peristalsis with capsule

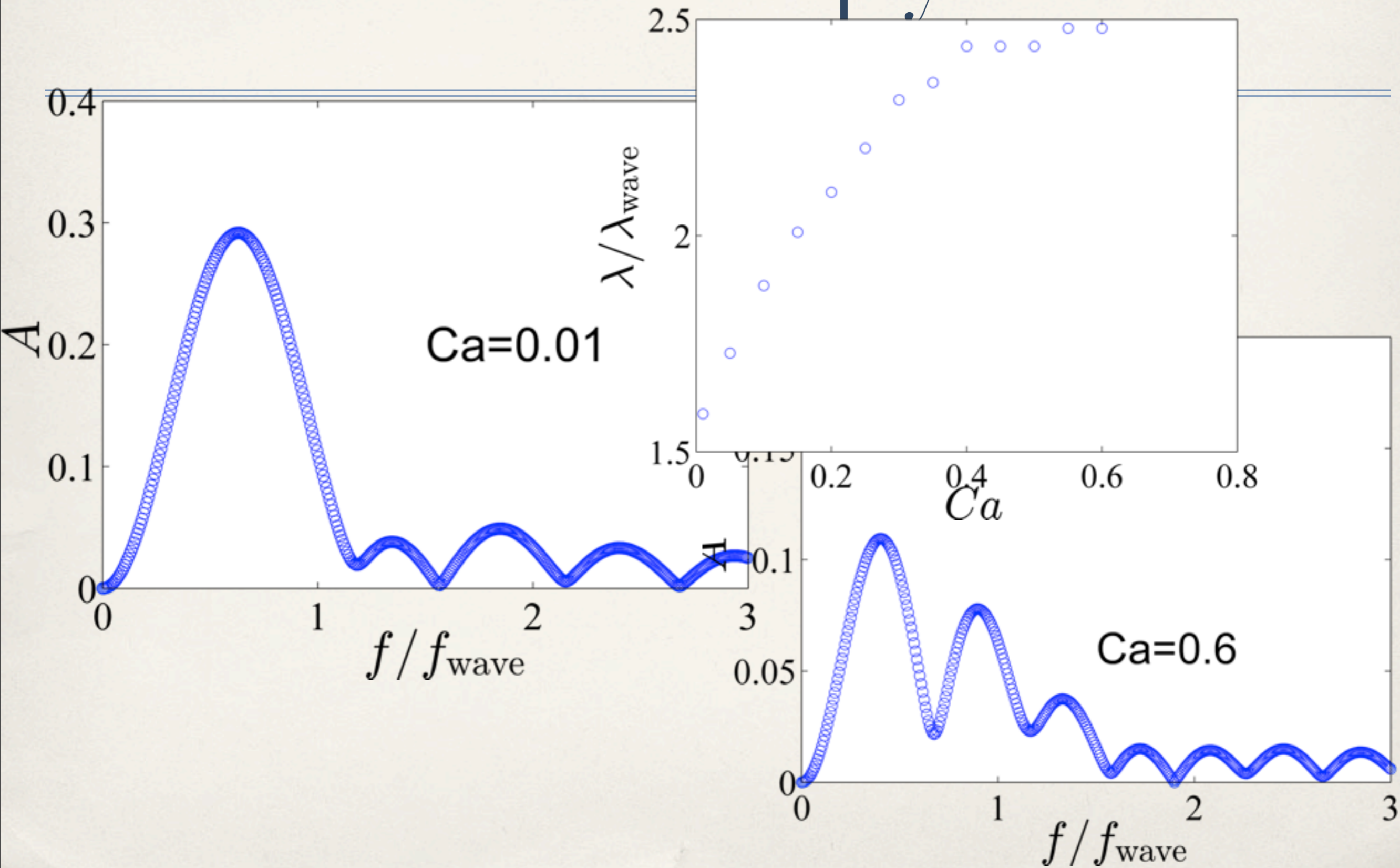


- ❖ Solve Stokes equation with moving boundaries.
- ❖ Calculate elastic forces on the membrane of the capsule
- ❖ Move the capsule by the velocity of the fluid interpolated to boundary points.

Deformability sets speed !



Periodic but not simply harmonic



Conclusion

- * Maximum velocity saturates as a function of capillary number.
- * Can be used to design a deformability based cell-sorting device.
- * Membrane nonlinearity and flow-structure interaction can give rise to a new kind of mixing and maybe even chaotic time-dependent Stokes flows.

