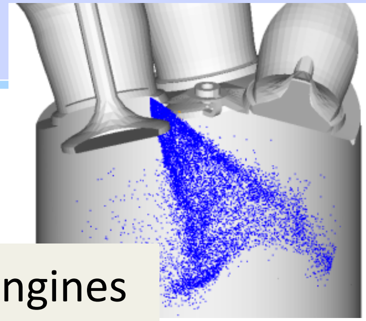
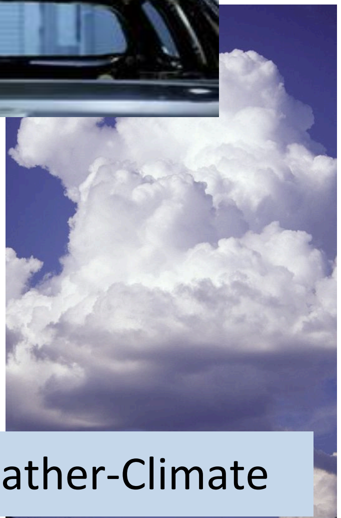
Dynamics of small heavy droplets – results from experiments and simulations





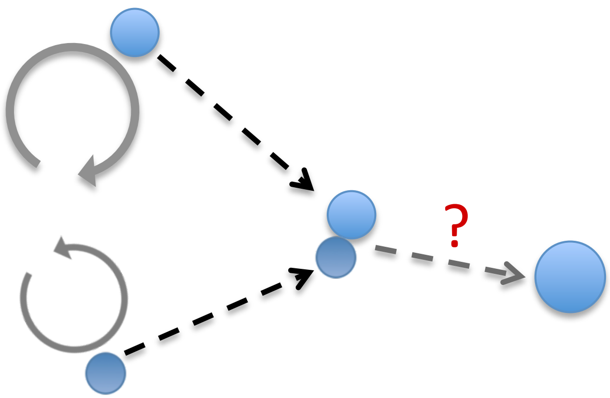
Ewe-Wei Saw [1,2],

Gregory Bewley [1], Samriddhi S. Ray [2], Holger Homann [2], Jeremie Bec [2], Eberhard Bodenschatz [1]

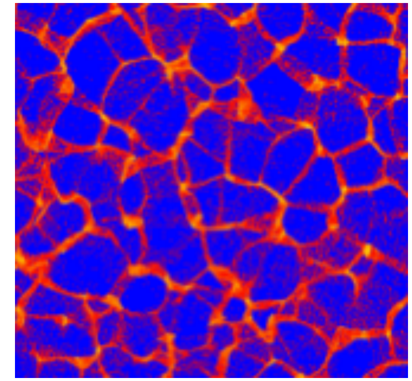
1. Max Planck Inst. for Dynamics and Self Organization, Göttingen, Germany,

2. Lagrange Lab., Observatoire de la Cote d’Azur, Nice, France.

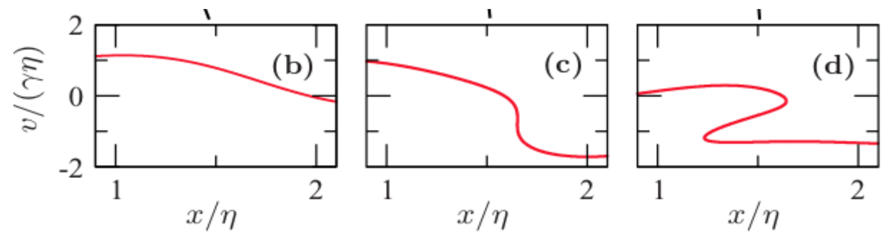
2 Pictures: Sling and Caustics



Sling effect: Falkovich et al. Nature (2002)



Wilkinson, Mehlig, Bezuglyy (2006)



Inertia causes singularity in particles velocity gradient.

- Large relative velocities between neighboring particles.
- Should enhance droplet coalescence in cloud But…
- Extreme velocities → coalescence or breakup ?

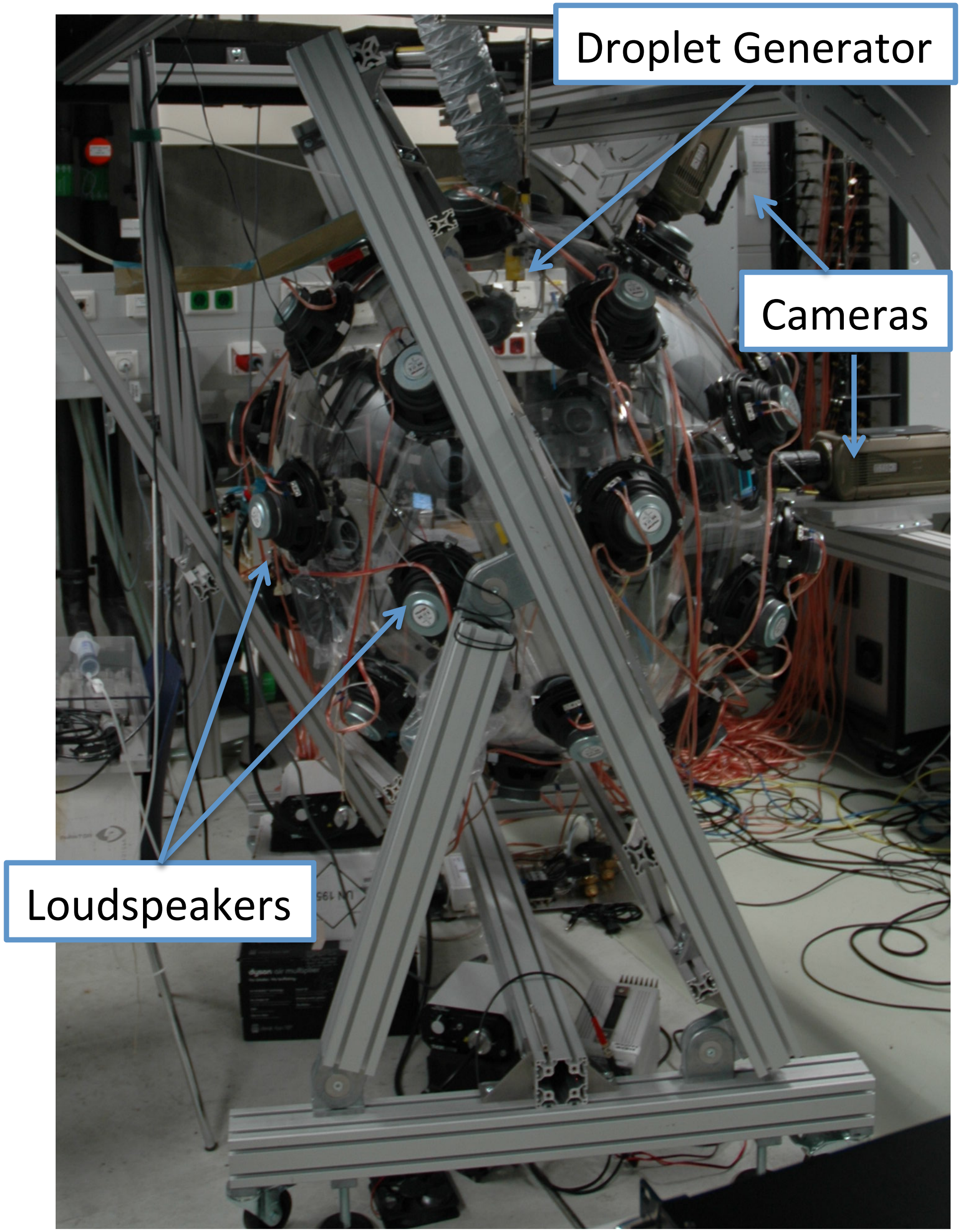
Presentation

1. Describe experiment, simulation.

2. Monodisperse results (same Stokes).

3. Two Stokes (bi-disperse)results.
   ($St_1 \neq St_2$)

Setup : The Acrylic Soccer Ball

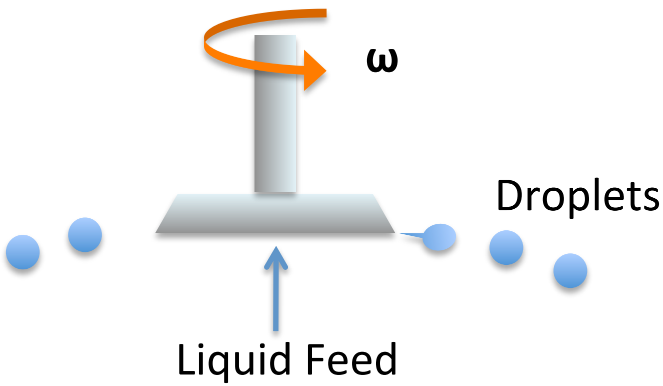


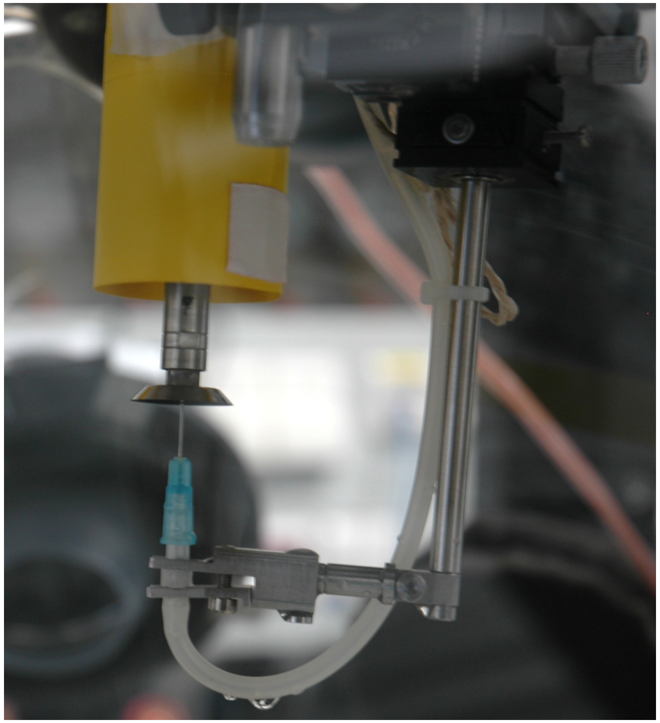
- Acrylic shell , D = 1m.
- 32 Independent-randomly pulsating jets.
- $R_\lambda \approx 100$—400 (200)
- $L \approx$ 10 cm
- $\eta \approx 100$—500 µm (~ 200µm)
- $u_{std,y} / u_{std,z} \approx 1.05$
- $U_{mean} / u_{std} \leq 10\%$

- Droplet generator → next slide
- Size resolved Lagrangian Particle Tracking → next slide

Similar earlier version:
Chang, et al., J. Fluid Mech. (2012)

Setup : Spinning Disk Droplet Generator.





K. May, J. Appl. Phys. **20**, 932 (1949).

- 60,000 rpm spinning disk .
- Liquid wets disk surface.
- Droplets ejected from disk edge .
  (Rayleigh-Taylor Instability)

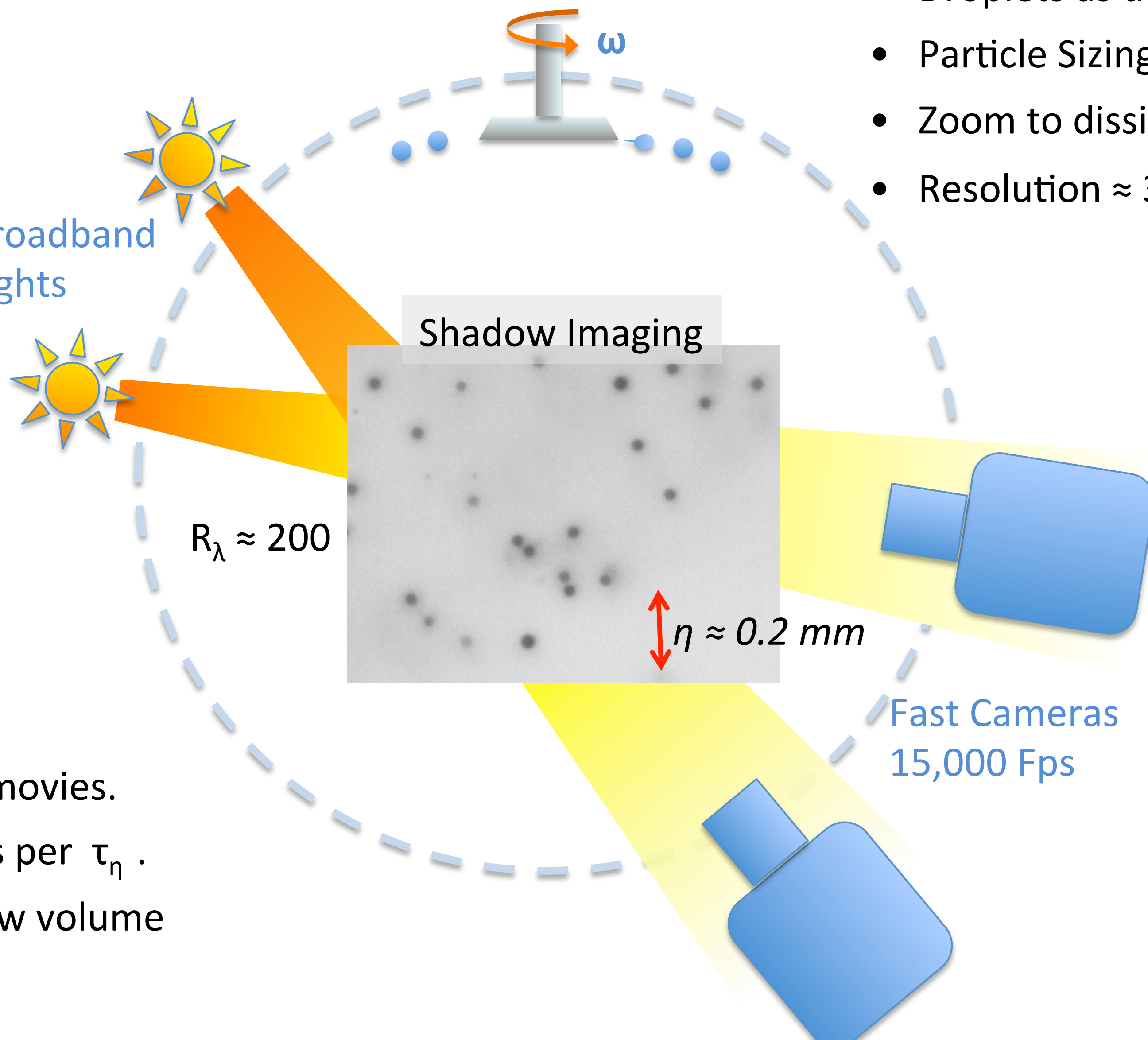
- 40% ethanol - 60% water
- D_{std} = ± 3 μm.

Bi-disperse droplets:

- Big: 19.4 μm mean diameter.
  St = 0.19, 0.31, 0.5
- Small: 6.8 μm mean diameter.
  St = 0.02, 0.05, 0.06

High-Res. Particle Tracking with Shadow Imaging

- Droplets as dark spots.
- Particle Sizing.
- Zoom to dissipative scales.
- Resolution ≈ 3μm < 0.02 η



- High speed movies.
- > 20 frames per τ_η .
- ≈ 2mm^3 view volume

Monodisperse droplets (Same *St*)

How accurate is the Stokes-drag-model for heavy particle?

$$\frac{d\mathbf{v}}{dt} = \frac{1}{\tau_p}(\mathbf{u}_{\mathbf{fluid}} - \mathbf{v}) + \mathbf{g}$$

Sufficient for rare extreme events?

-- Large velocities at very small scales.

Roles of hydrodynamic interactions, history force?

Monodisperse droplets (Same *St*)

How accurate is the Stokes-drag-model for heavy particle?

$$\frac{d\mathbf{v}}{dt} = \frac{1}{\tau_p}(\mathbf{u_{fluid}} - \mathbf{v}) + \mathbf{g}$$

Sufficient for rare extreme events?

-- Large velocities at very small scales.

Compare experiment with DNS using the Stokes drag model.

DNS with Heavy Particles

- Pseudo-spectral Navier–Stokes solver.
- Constant homogenous-isotropic forcing at large scales.
- Point particles with standard yet simplistic Stokes-drag-model:

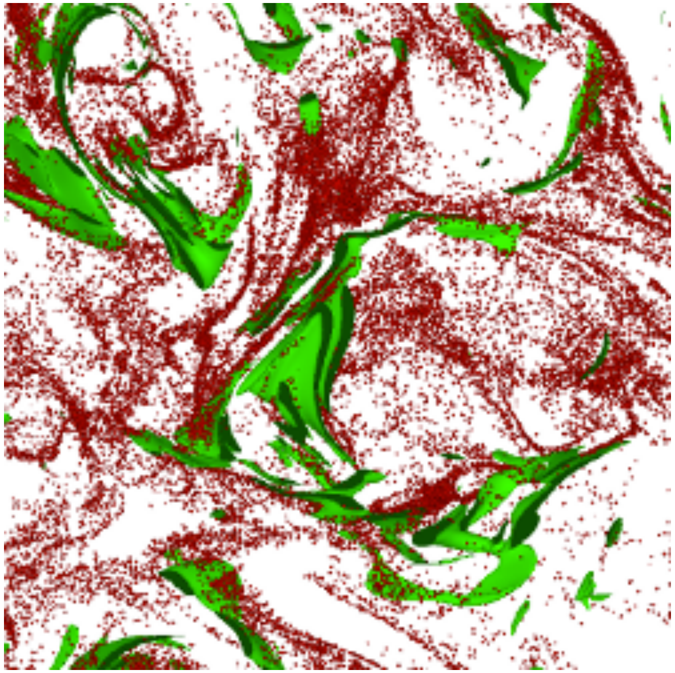
$$\frac{d\mathbf{v}}{dt} = \frac{1}{\tau_p}(\mathbf{u} - \mathbf{v}) + \mathbf{g}$$

- Conditions matched to the experiment:

  $R_\lambda \approx 200$

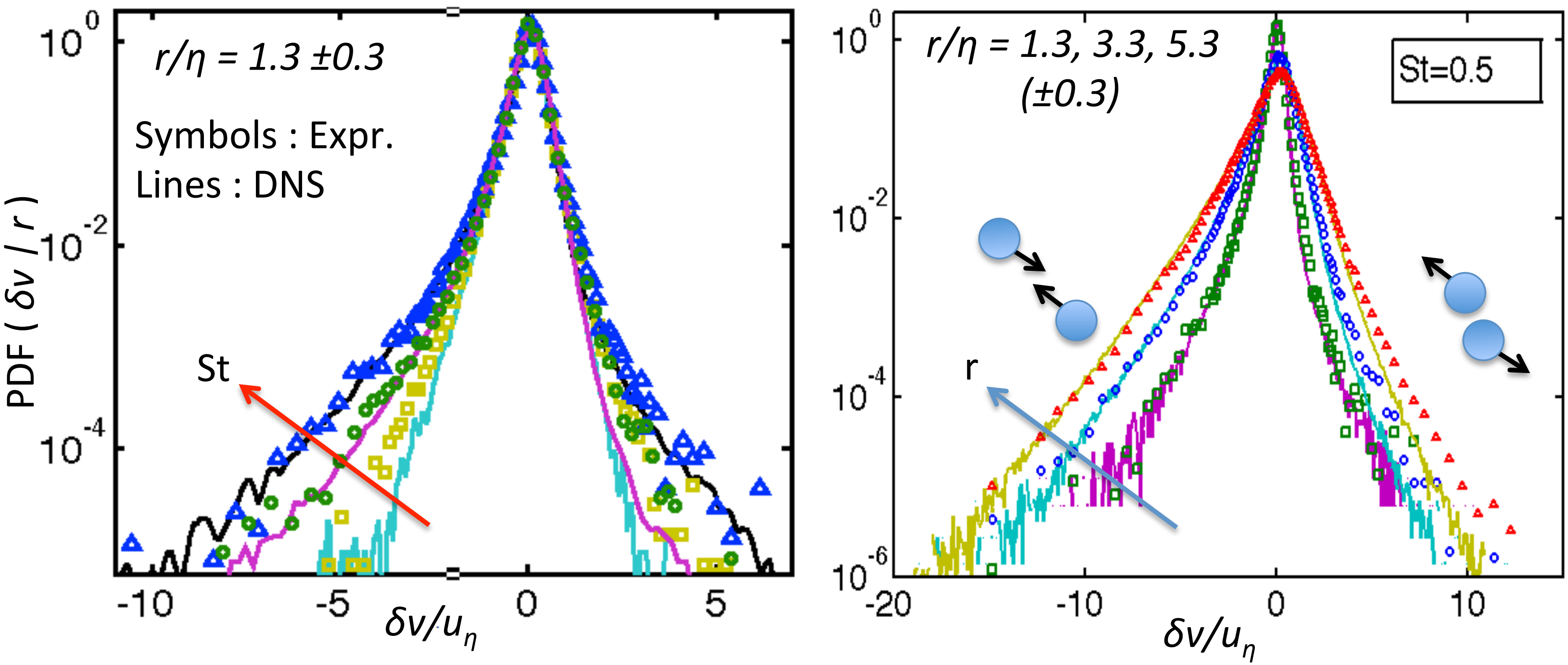
  St = 0.05, 0.31, 0.5

  $S_g < 0.3$ (found negligible influence)

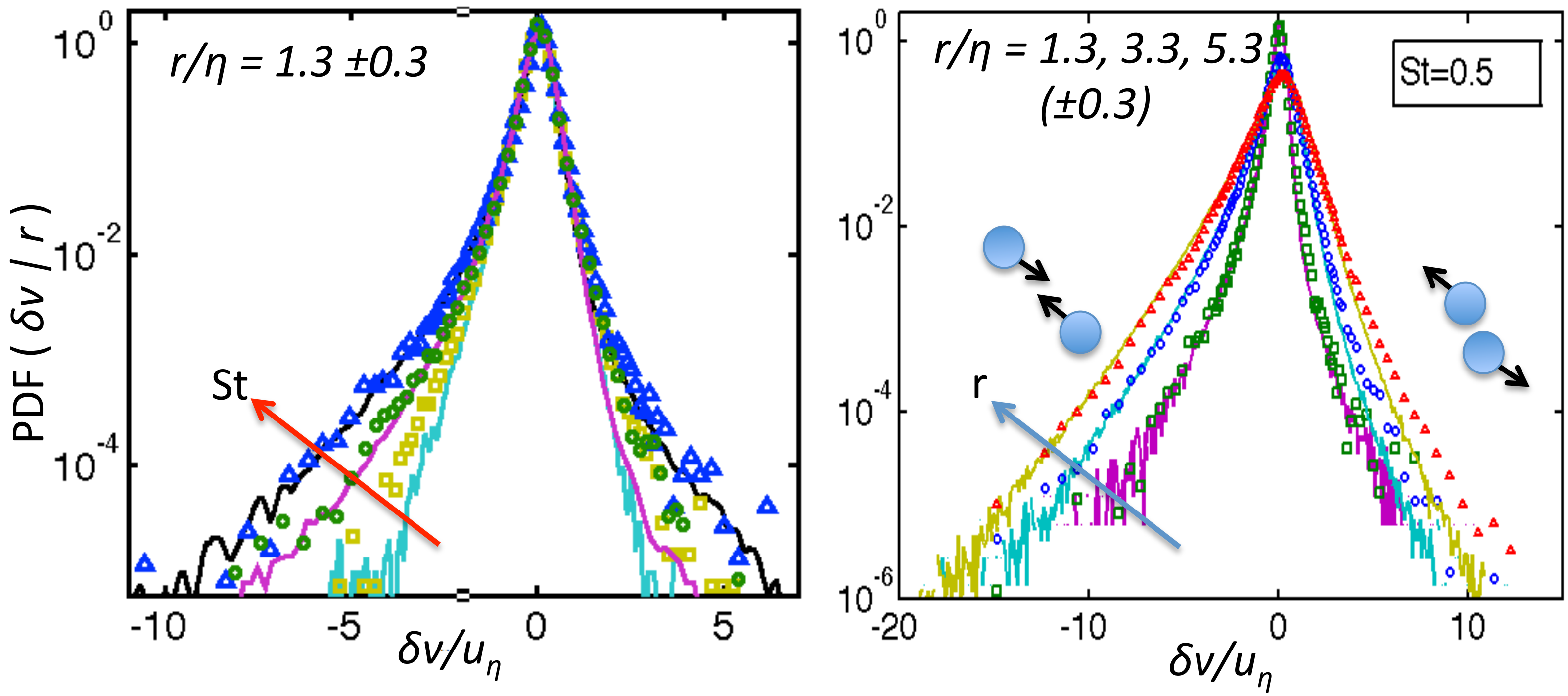


Direct Comparison of Velocity Statistics

PDF (radial relative velocity | separation distance)

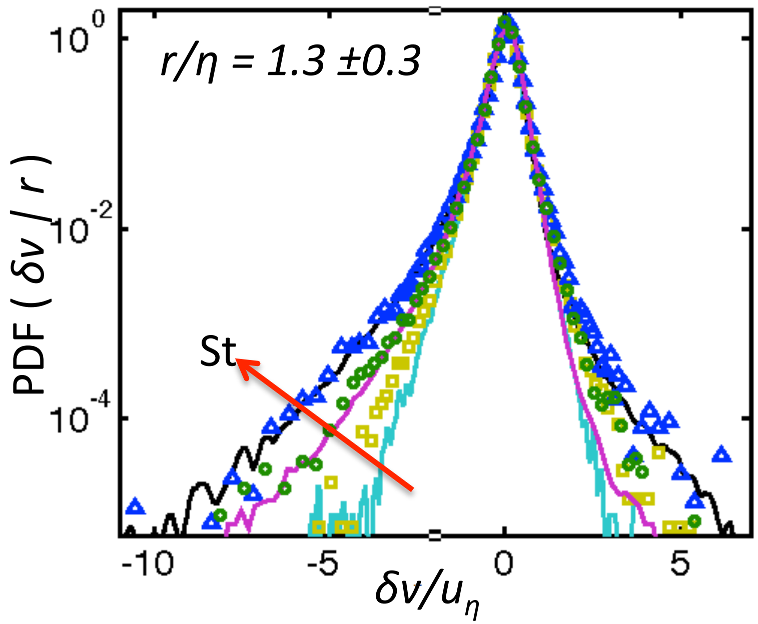


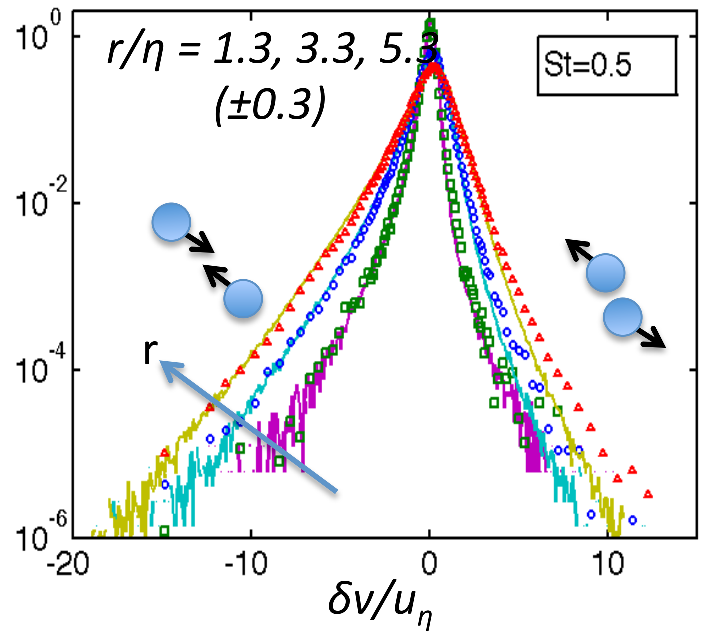
Causes of discrepancy ?



- Measurement noise.
- Miss-match in intermittency, Reynolds number.
- Inaccurate sizing of droplet size.
- Wrong value of viscosity.
- Wrong estimation of energy dissipation rate.

Causes of discrepancy ?

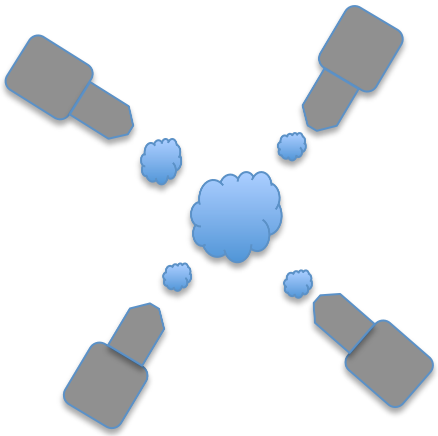




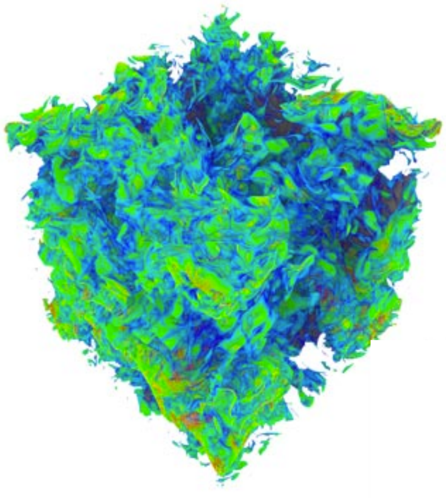
- ~~Measurement noise.~~
- ~~Miss-match in intermittency, Reynolds number.~~
- ~~Inaccurate sizing of droplet size.~~
- ~~Wrong value of viscosity.~~
- ~~Wrong estimation of energy dissipation rate.~~

- Missing physics: History force? ($d/\eta \approx 0.1$)
- Different energy injection mechanisms?

Miss-match of large Scale Energy Injections ?

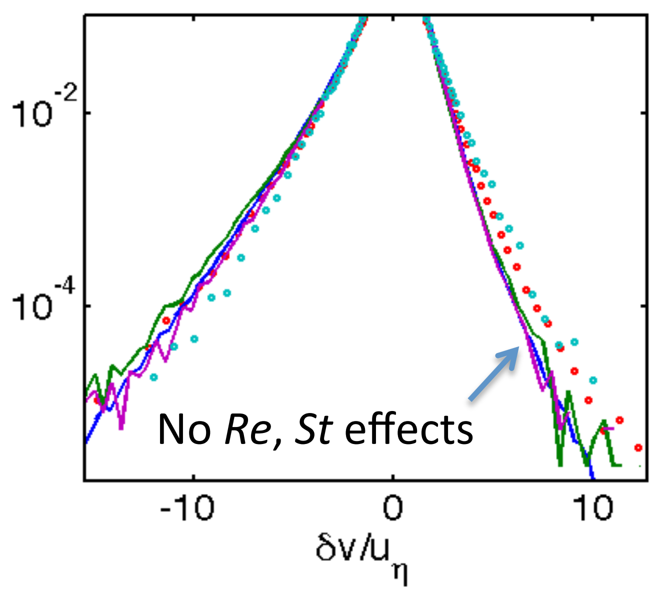


vs.



Forcing localized in space & time

Volumetric constant forcing (DNS)



<u>Experiments</u>

Red : St=0.5, R_λ = 190

Cyan: St=0.06, R_λ = 190

<u>DNS</u>

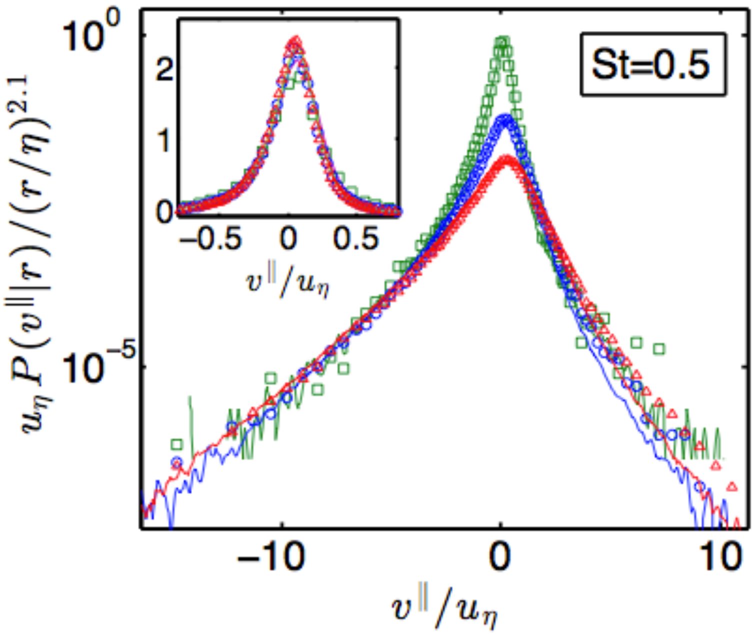
Blue : St=0.5, R_λ = 182

Purple: St=0.4, R_λ = 287

Green: St=0.63, R_λ = 287

r = 5-5.6 η

r- scaling, Frozen Tails and saturated exponents



Tail has similar shape, simple scaling w.r.t. R.

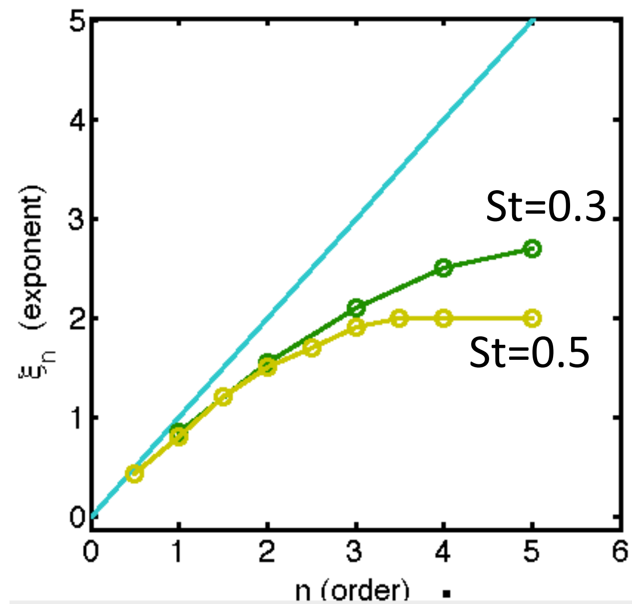
$$P(v|r) \sim r^{\xi_\infty}\phi(v)$$

Core scales like tracers in dissipative scale ((linear):

$$P(v|r) \sim r^{-1}\psi(v/r)$$

Structure function has exponents that plateau at large order.

$$S_{ll}^n(r) = \langle|\delta \mathbf{v}\cdot\hat{\mathbf{r}}|^n\rangle \sim r^{\xi_n}$$



K. Gustavsson & B. Mehlig (2011), (2013)

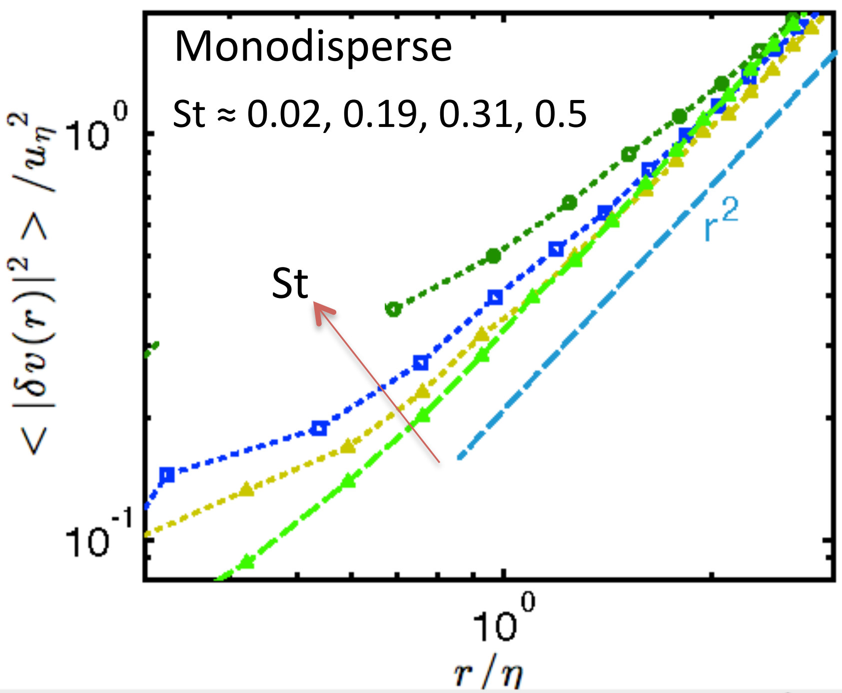
A. Celani, A. Lanotte, A. Mazzino, and M. Vergassola, Physical Review Letters **84**, 2385 (2000).

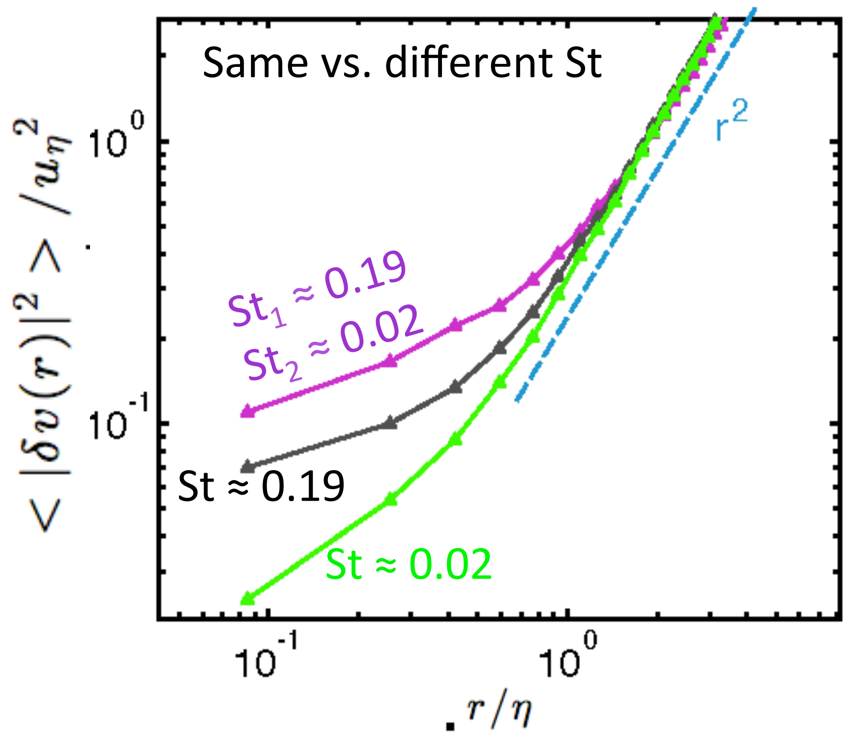
J. Bec, L. Biferale, M. Cencini, A. Lanotte, and F. Toschi, J. Fluid Mech. **646**, 527 (2010).

Relative motion between 2 Stokes Numbers

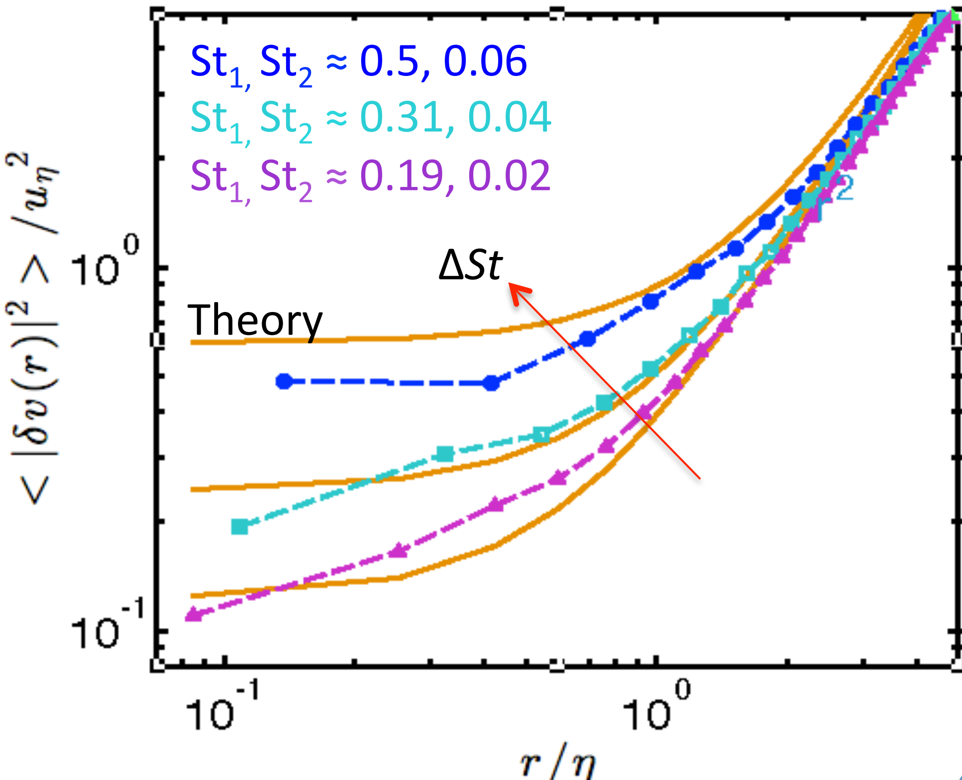
Structure function of modulus of Δv : $S(r) = \left\langle \left| \mathbf{v_1}(\mathbf{x}+\mathbf{r}) - \mathbf{v_2}(\mathbf{x}) \right|^2 \right\rangle_{\mathbf{x}}$

Only experiment data…





Relative motion between 2 Stokes Numbers



- Largest discrepancy ~ 20%.
- Role of *g* significant (~ 20% increase) for lowest *ΔSt* = (0.19 - 0.02) case. ($\varepsilon = 0.45\ \mathrm{m^2/s^3}$)

Theory:

Chang K, Shaw R, Malec B (submitted)

Start with

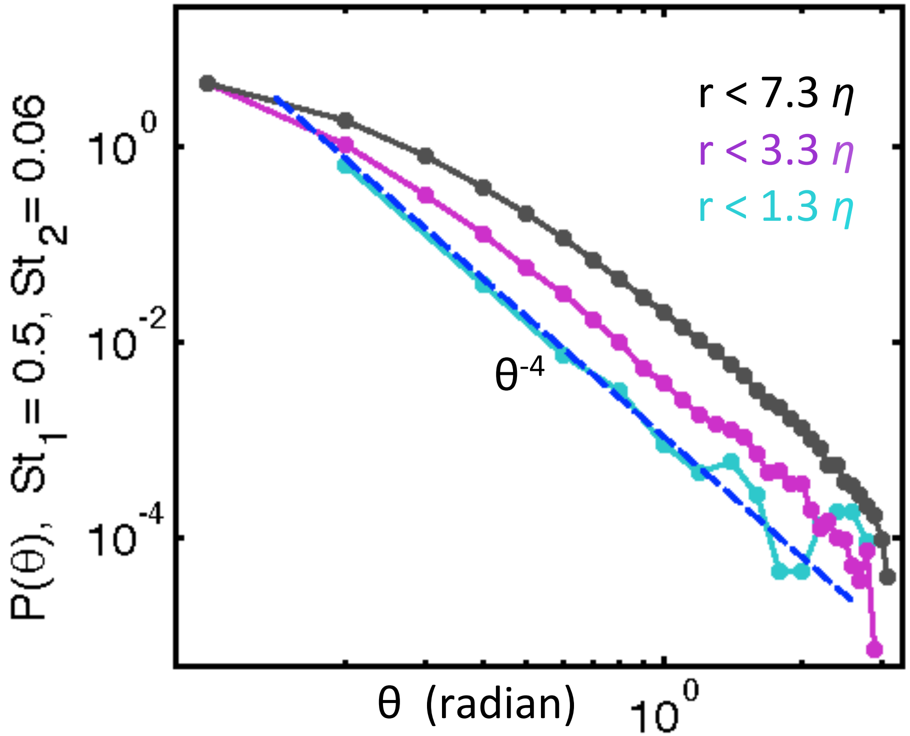
$$\vec{v} = \vec{u}_f + \tau(\vec{g} - \vec{a}_f)$$

$$\boxed{\frac{S_{\Delta St}(r)}{{u_\eta}^2} = \frac{S_f(r)}{{u_\eta}^2} + (\Delta St)^2 (a_0 + \frac{g^2}{{a_\eta}^2})}$$

$$\langle \boldsymbol{a}^2 \rangle = a_0\, a_\eta^2,$$

Values of a_0 from Ishihara et al. (2007), Hills (2002)

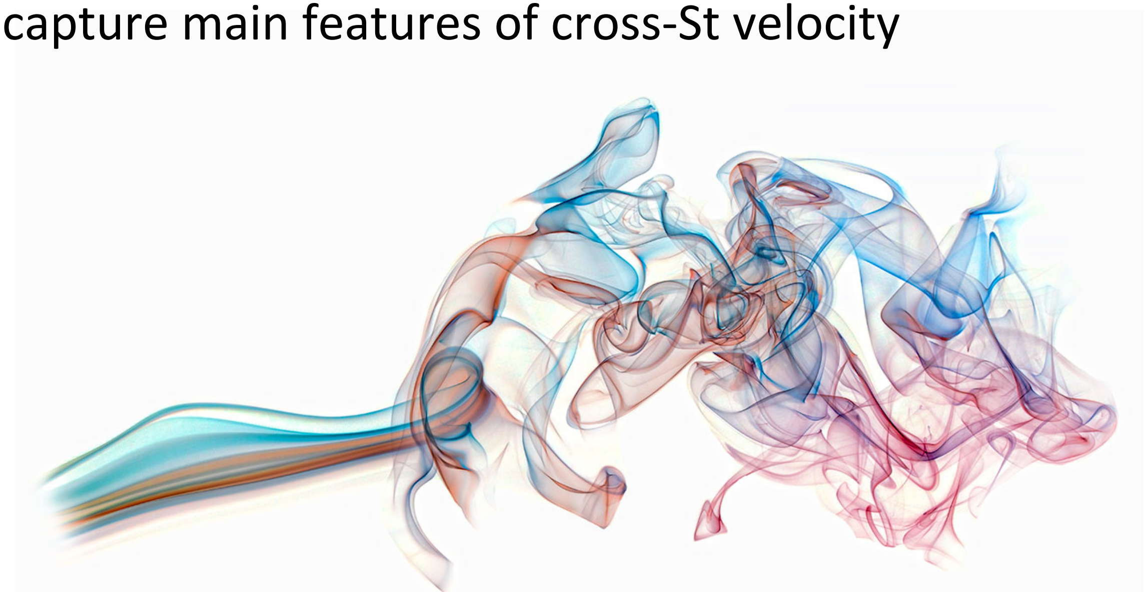
Relative motion between 2 Stokes Numbers



- Seen by simulation: θ^{-4} in intermediate θ, at r=0.
- Check with our Experiment data.

Summary

- Stokes drag (DNS) reproduced qualitative trends of the experiment
- Excellent agreement for inertia dominated regime (large St, v/r).
- Discrepancies for low St, separating particles. Likely due to energy injection peculiarity or missing physics.
- Droplets with different St have much larger relative velocities.
- Simple phenomenology capture main features of cross-St velocity statistics.

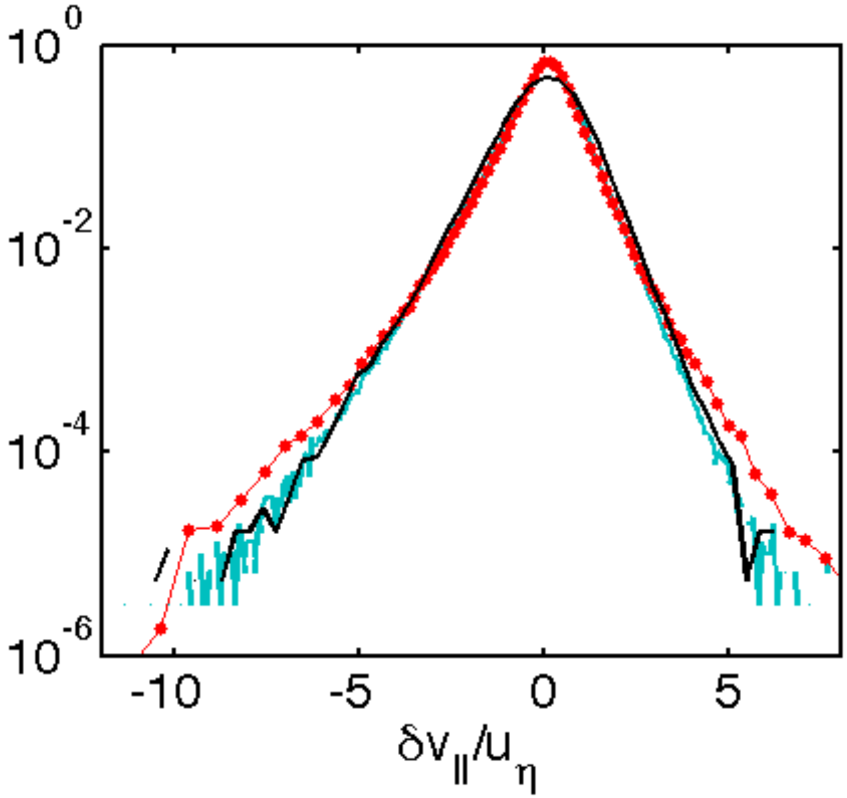


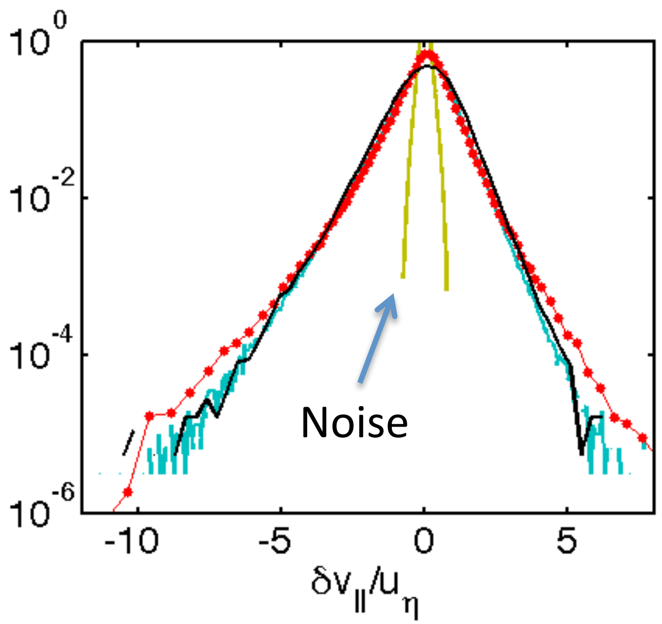
DNS with heavy particles

Effect of Measurement Noise

$St = 0.05$

Red: Experiment
Cyan: DNS
Black: DNS + Experimental Noise



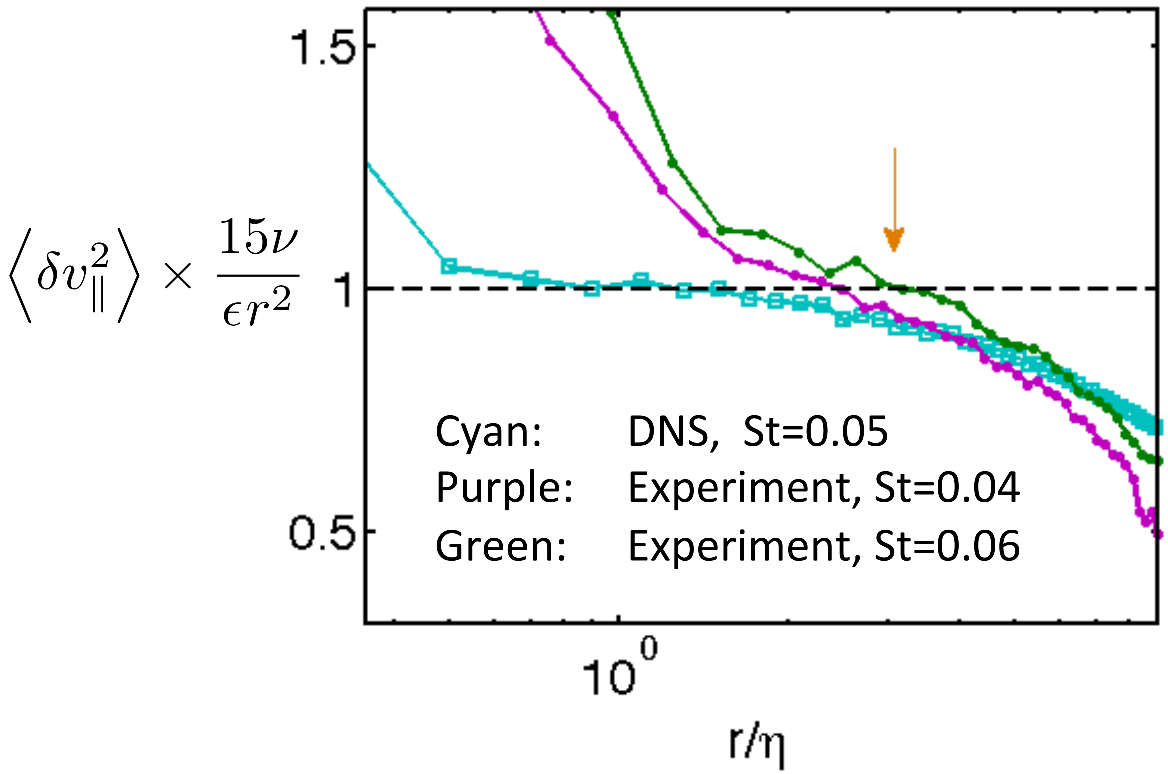


Does not account for the discrepancy.

Accuracy of Measured ε

We used dissipative scaling of structure function: $\left\langle \delta v_{\parallel}^{2} \right\rangle = \frac{1}{15}\frac{\epsilon}{\nu} r^{2}$

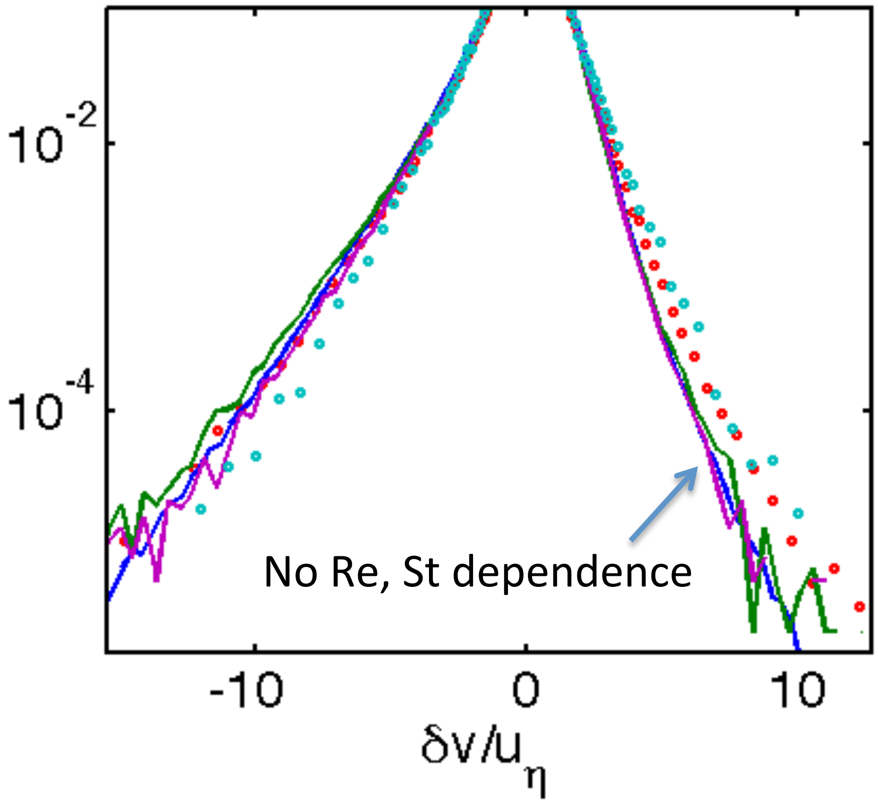
Use DNS to evaluate the accuracy of such method.



Expected Accuracy of ~ 15%

Inaccurate Sizing, Intermittency (Re effect) ?

$$St = \frac{1}{18}\frac{\rho_p}{\rho_f}\frac{d^2}{\eta^2}$$



r = 5-5.6 η

Red: Experiment, St=0.5, R_λ = 190

Cyan: Experiment, St=0.06, R_λ = 190

Blue: DNS, St=0.5, R_λ = 182

Purple: DNS, St=0.4, R_λ = 287

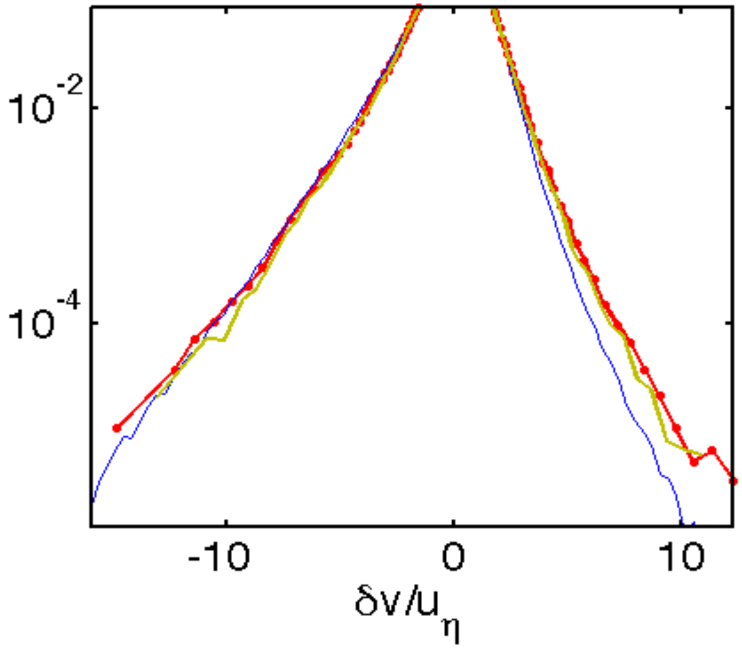
Green: DNS, St=0.63, R_λ = 287

Inaccurate viscosity ?

$$St = \frac{1}{18}\frac{\rho_p}{\rho_f}\frac{d^2}{\eta^2} \qquad \eta = (\nu^{\frac{3}{4}}/\epsilon^{\frac{1}{4}})$$

Physically, changes of *ρ* and *ν* of air with humidity is at most few percents.

Increase *ν* by 30%



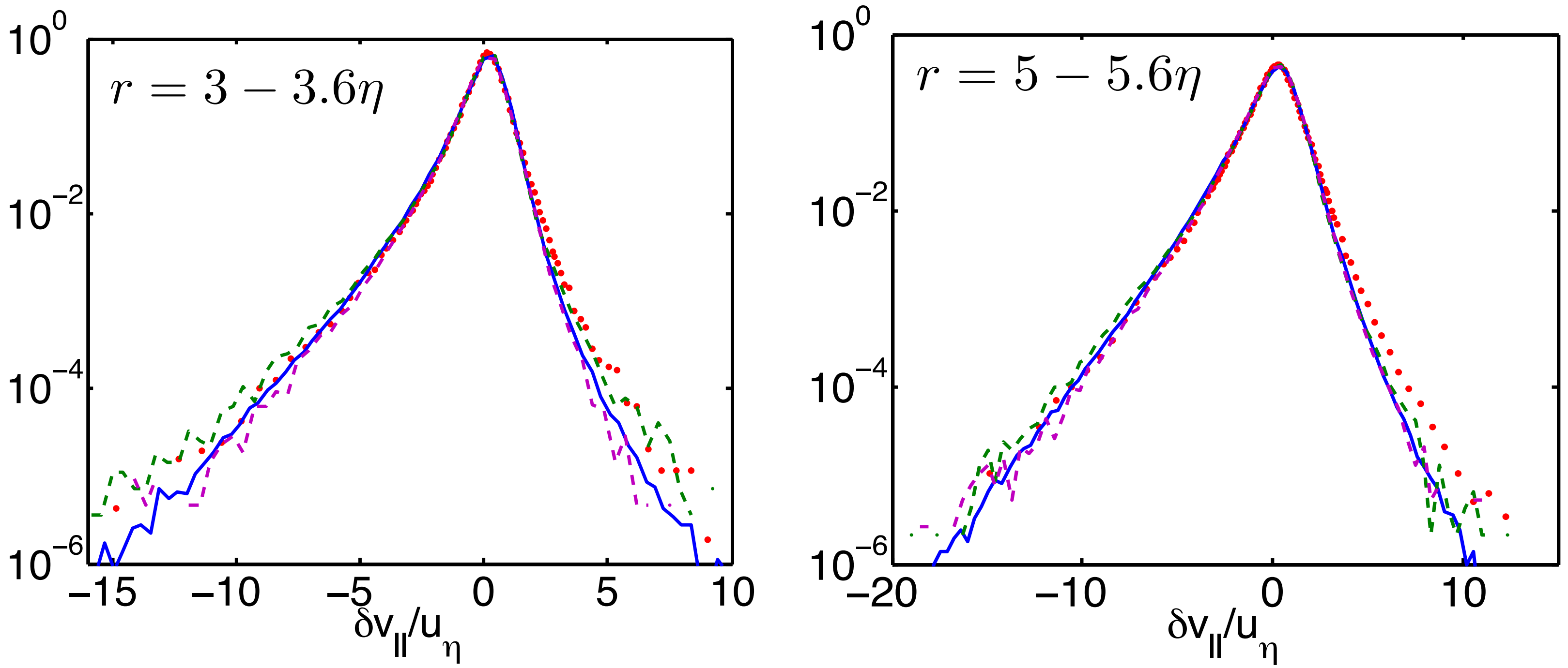
St=0.5, r = 5-5.6 η

Blue: DNS
Red: Experiment
Cyan: Experiment, $\nu = 1.3\ \nu_o$

Effect of Increasing Re

$$St = 0.5$$

Red: Experiment
Blue: N_{Grid}=512, St=0.5, (R_λ = 182)
Purple: N_{grid}=1024, St=0.4 (R_λ =287)
Green: N_{grid}=1024, St=0.6 (R_λ = 287)



Does not account for the discrepancy.

Testing a theory

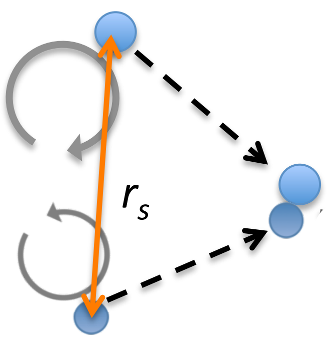
PRL **101,** 174503 (2008) PHYSICAL REVIEW LETTERS week ending 24 OCTOBER 2008

Variable-Range Projection Model for Turbulence-Driven Collisions

K. Gustavsson,[1] B. Mehlig,[1] M. Wilkinson,[2] and V. Uski[2]

A theory for St >> 1.

Predicts: $P(\Delta V) \sim \exp(-C|\Delta V|^{4/3})$



Essential idea: Two particles are thrown together from large distances and follow ballistic motion under Stokes drag.

→ extreme relative velocity

$$\frac{d\mathbf{v}}{dt} = \frac{1}{\tau_p}(\underbrace{\mathbf{u}}_{\approx 0} - \mathbf{v}) + \mathbf{g}$$

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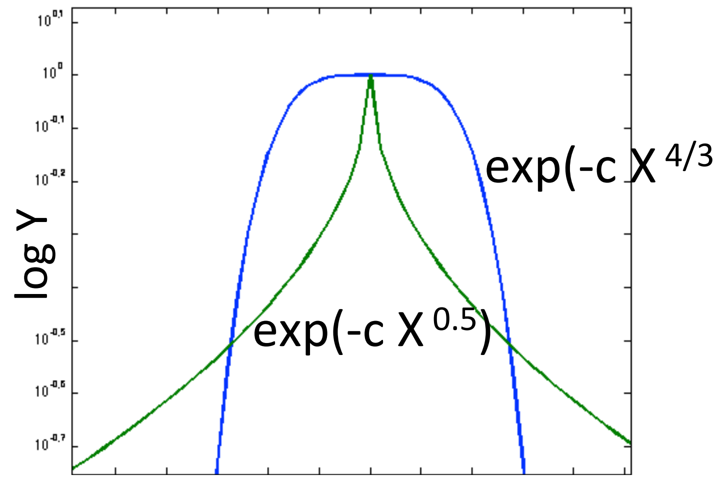
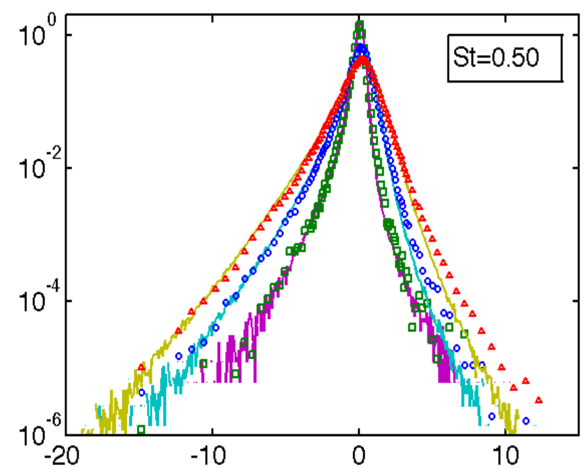
Convex Tail



Our data that have stretched exponential tails (St = 0.2--0.5):

$$exp(-C\,|\Delta v|^{\alpha})\,, \quad \alpha < 1$$

Concave Tail



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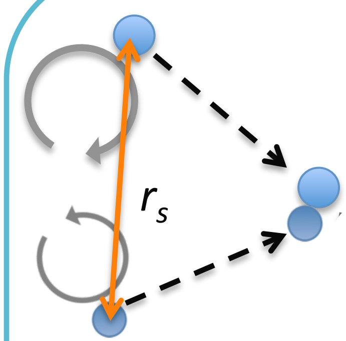
Attempt to test the
essential idea....



Statistics of large velocities are well described by assuming:

Particles approach each other following ballistic motion under Stokes drag.

$$\frac{d}{dt}\Delta\mathbf{v} = \frac{\mathbf{1}}{\tau_{\mathbf{p}}}(\underbrace{\boldsymbol{\Delta}\mathbf{u}}_{\approx 0} - \boldsymbol{\Delta}\mathbf{v}) + \mathbf{g}$$

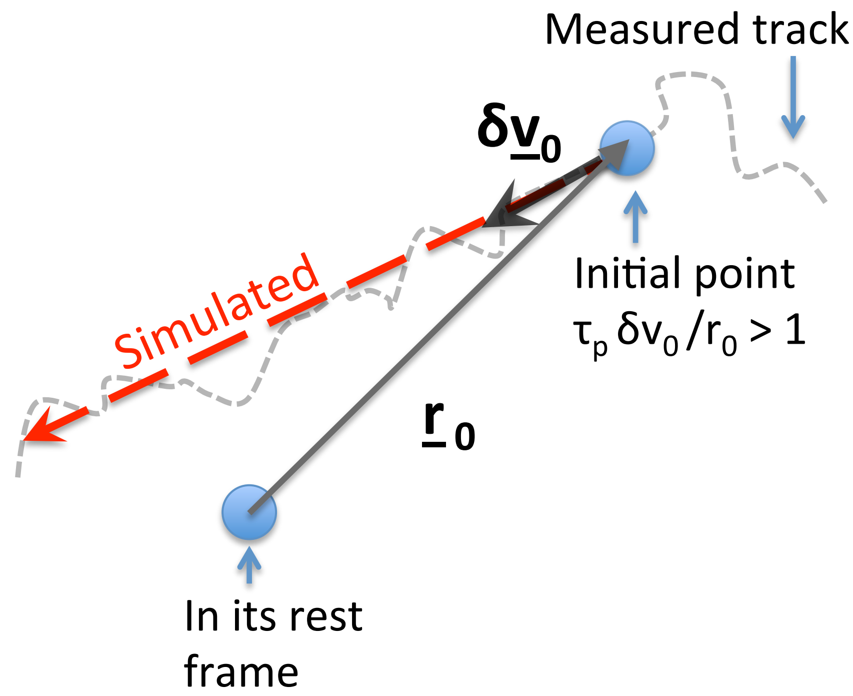
Stokes-Dragged Ballistic Simulation

1) Pick from experimental data, approaching trajectories that are inertia dominated.
(Criteria: $\tau_p \delta v_0 / r_0 > 1$, $\delta \mathbf{v_0} \bullet \mathbf{r} < \mathbf{0}$)

2) Retain only the first point at which the trajectory starts to become inertia dominated.

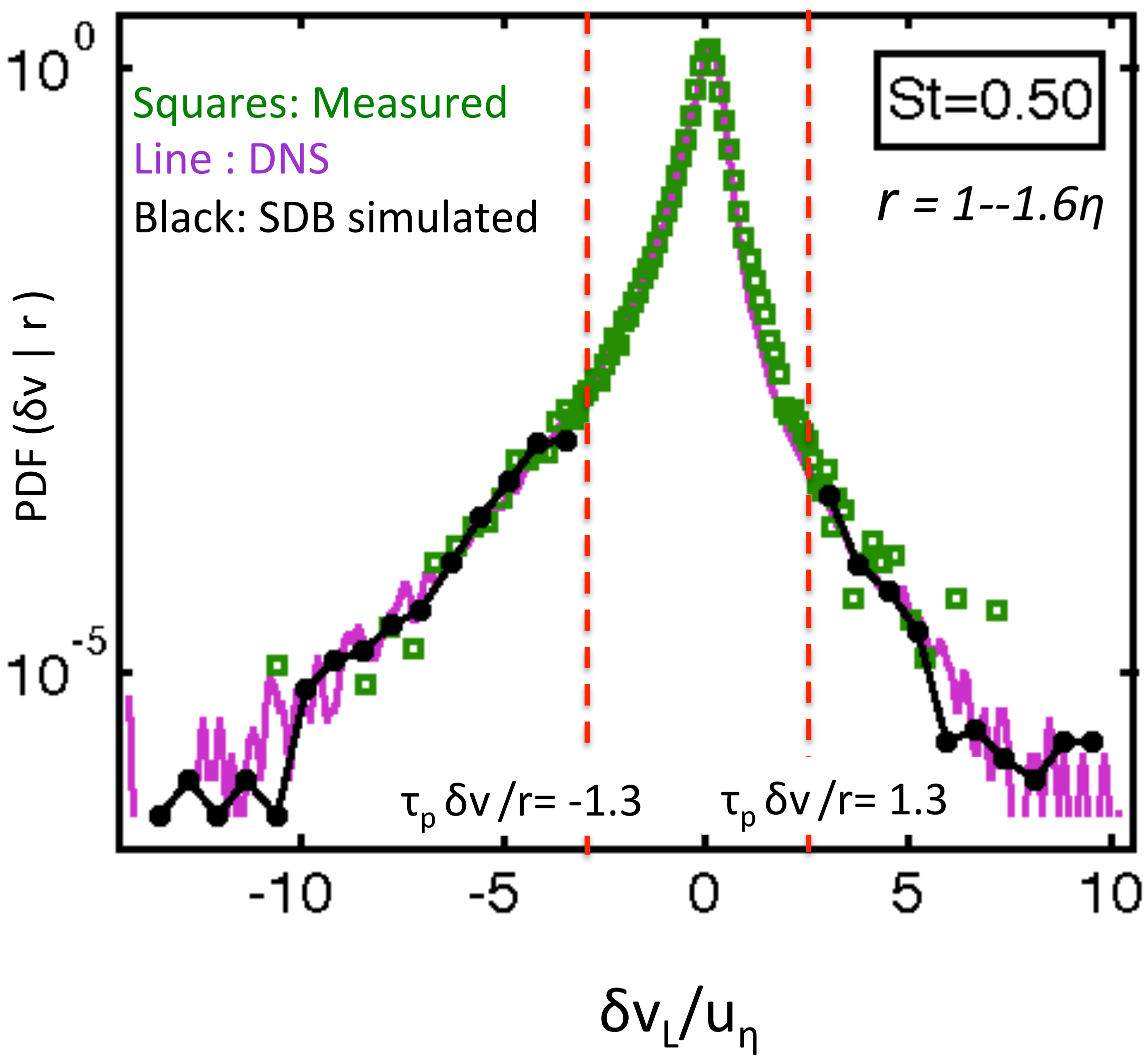
3) Simulated the rest of the trajectory via Stokes-dragged-ballistic motion:

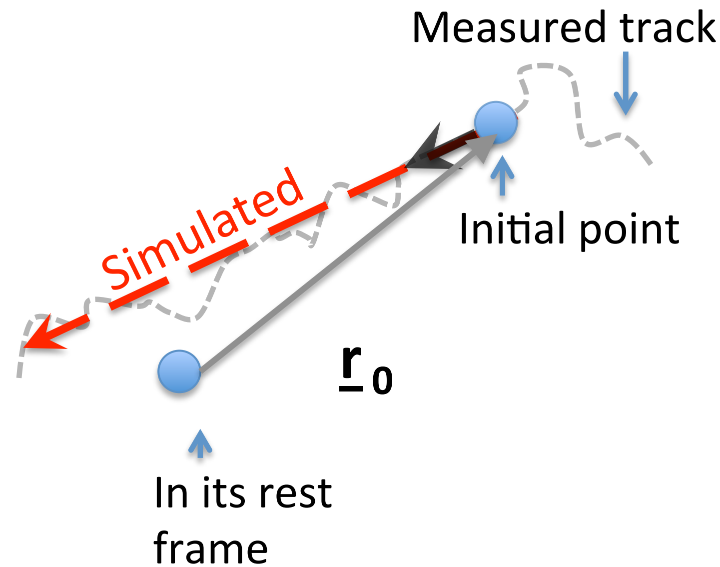
$$\frac{d}{dt}\Delta \mathbf{v} = \frac{\mathbf{1}}{\tau_{\mathbf{p}}}(\underbrace{\mathbf{\Delta u}}_{\approx 0} - \mathbf{\Delta v}) + \mathbf{g}$$



Compare simulated velocity statistics to real (measured) data….

Real data VS. Stokes-Dragged-Balistic Simulation





$$\frac{d}{dt}\Delta\mathbf{v} = \frac{1}{\tau_\mathbf{p}}(\underbrace{\cancel{\Delta\mathbf{u}}}_{\approx 0} - \Delta\mathbf{v}) + \mathbf{g}$$

Summary

- Experimental study of extreme relative velocities between heavy particles.
- Compared experiment and DNS to test the accuracy of the Stokes drag model for the particle advection.
- DNS reproduced all trends of the experiment AND excellent quantitative agreement for inertia dominated regime (large St).
- Small discrepancies for lower Stokes numbers AND for separating particles at larger scale. Likely due to missing physics.
- For large velocities, Data support particle trajectories following ballistic motion under Stokes drag, (for tails of PDF(dv)).

Acknowledgements

- We thank Poh Yee Lim for help in operation in the lab.

- We thank Kelken Chang, Mathieu Gibert, Andreas Kopp, Artur Kubitzek, Ottwin Kurre, Andreas Renner, Udo Schminke, Hengdong Xi for technical support and help with setting up this experiment.

- We thank these individuals for delightful discussions:

  A. Pumir, B. Mehlig, S. Musacchio, S. Lanotte,
  M. Wilkinson, Haitao Xu .

- This work is partially funded by the Max Planck Society and the European Research Council.