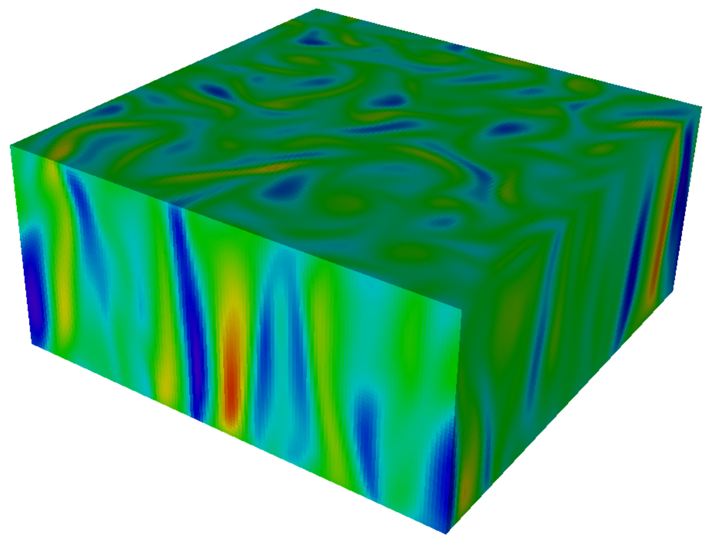
Inertial particles in rotating Rayleigh-Benard convection



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Outline

- Motivation
- Dynamics of inertial particles in a rotating frame
- Rayleigh-Benard Convection
- Rotating Rayleigh Benard Convection
- Inertial particles in rotating Rayleigh Benard Convection
- Conclusion

Motivation

- Transport of Inertial particles on earth

Tropical cyclone and polar deep convection: examples of geophysical flows driven by convection and affected by rotation.

http://www.tue.nl/en/university/departments/applied-physics/research/

T. Sapsis and G. Haller, Inertial particle 's motion in geophysical fluid flows, ENOC 2008

Motivation

- Planetesimals

Tanga et al., Forming planetesimals in vortices, Icarus, **121**, 158 (1996).

In this work we explore the possibility that large-scale, quasi-two-dimensional vortices on the solar nebula act as trapping regions for dust particles. We show that large-scale vortices may be naturally generated in a differentially rotating solar nebula, and we discuss a simple self-similar solution for the vortex pattern. Numerical simulations of the transport of dust in the presence of vortices on the nebula indicate that for a Rossby number less than one the Coriolis force pushes the heavy particles into the cores of anticyclonic (retrograde) vortices. Here the concentration of dust grains becomes much larger than outside in a few periods of revolution of the disk. The cores of anticyclonic vortices are thus identified as candidate regions for rapid planetesimal formation.

Motivation

- Planetesimals

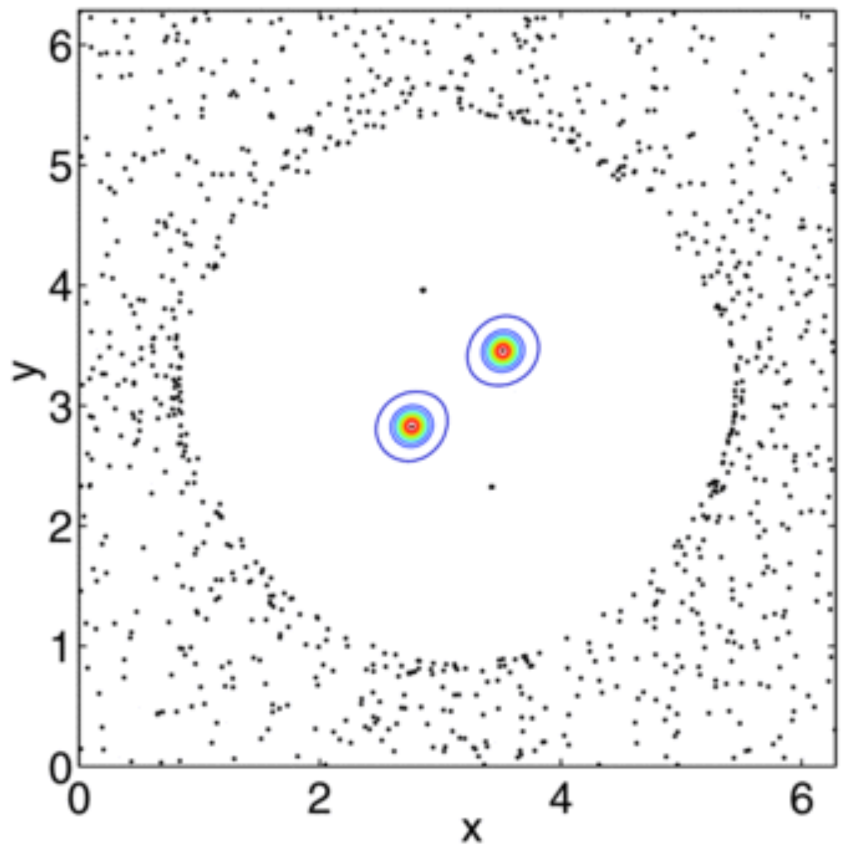
L. Hodgson and A. Brandenburg, Turbulence effects in planetesimal formation, Astron. Astrophysics, **330**, 1169 (1998).

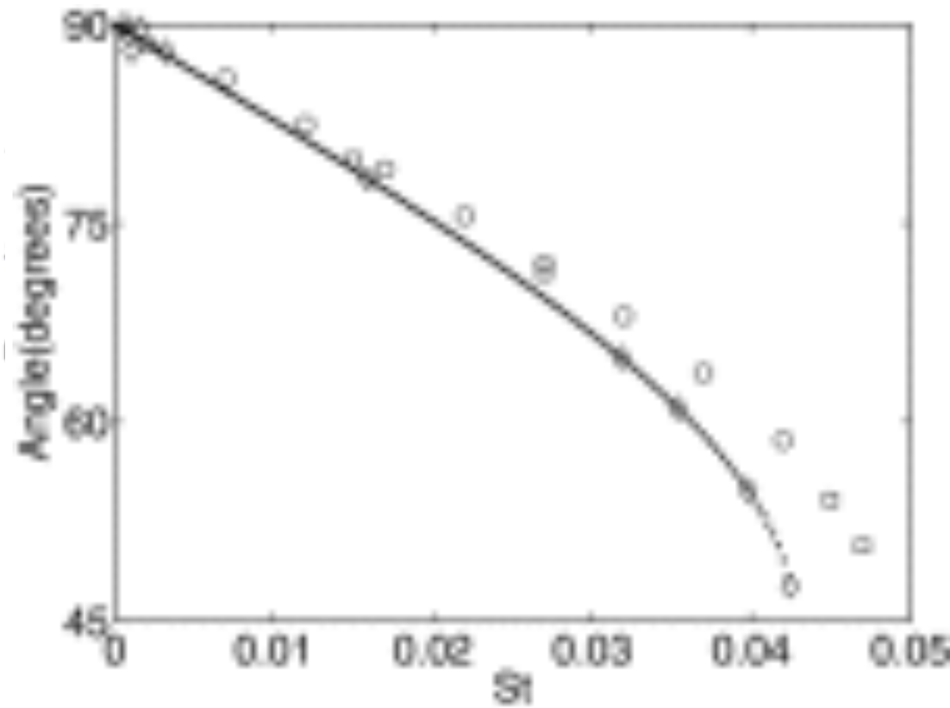
Abstract. The formation of planetesimals is investigated by studying the transport of dust particles in a local three-dimensional simulation of accretion disc turbulence. Heavy particles fall rapidly towards the midplane, whereas lighter particles are strongly advected by the flow. For light particles the turbulence leads to a rapid redistribution of particles such that their density per unit mass is approximately constant with height. There is no pronounced concentration of particles in vortices or anticyclones, as was suggested previously. This is partly because of the adverse effect of keplerian shear and also because in our simulation vortices are only short lived. However, if we assume the gas velocity to be frozen in time, there is a concentration of dust in ring-like structures after a few orbits. This is caused mainly by a convergence of the gas flow in those locations, rather than the presence of vortices or anticyclones.

Motivation

- New fixed points in rotating frames

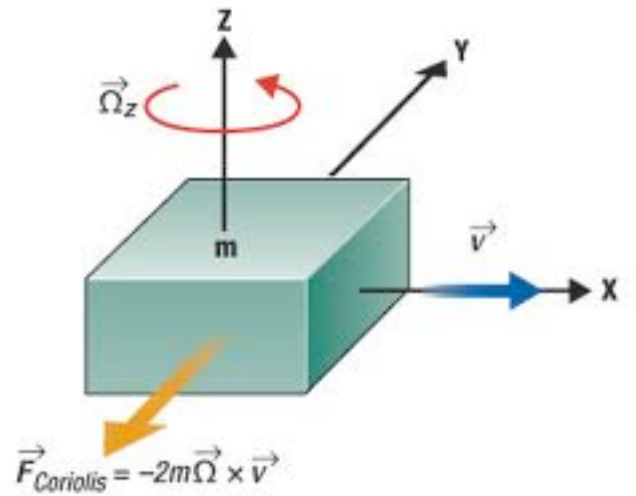
S. Ravichandran et al., Attracting fixed points in the vicinity of a vortex pair , Phys. Fluids, **26**, 013303 (2014).





Clustering of inertial particles with rotation

Inertial particles with rotation



$$\frac{dx}{dt} = v$$
$$\frac{dv}{dt} = -\frac{1}{\tau}(v - u) - 2\Omega \times v$$

Phase space contraction

$$Tr\begin{pmatrix} 0 & I \\ 0 & -d/\tau \end{pmatrix} = -\frac{d}{\tau}$$

Compressibility [small Stokes: Maxey approximation, JFM (1987)]

Coriolis term introduces an additional compressibility mechanism

$$\frac{Du}{Dt} = -\frac{1}{\tau}(v - u) - 2\Omega \times u \quad \Rightarrow \quad \nabla \cdot v = \tau[\Omega \cdot \omega - \nabla \cdot (u \cdot \nabla)u]$$

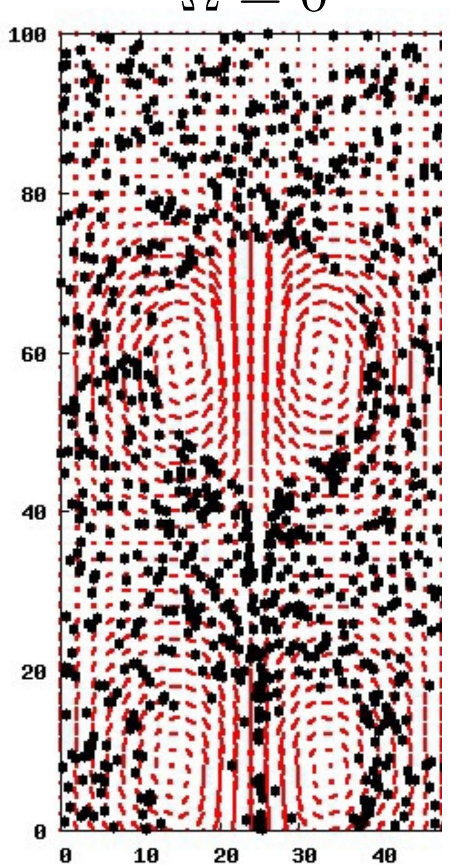
Ignore centrifugal forces => small distance from the centre of rotation.

Inertial particles in a dipole

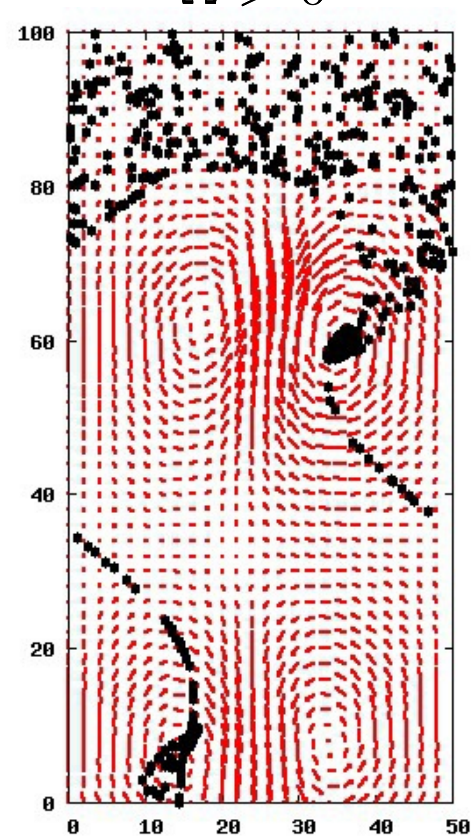
$$\frac{dx}{dt} = v$$
$$\frac{dv}{dt} = -\frac{1}{\tau}(v-u) - 2\Omega \times v$$

Coriolis term: Particles accumulate in anti-cyclonic vortices and are ejected out faster from the cyclonic vortices.

$\Omega = 0$



$\Omega > 0$



Rayleigh-Benard

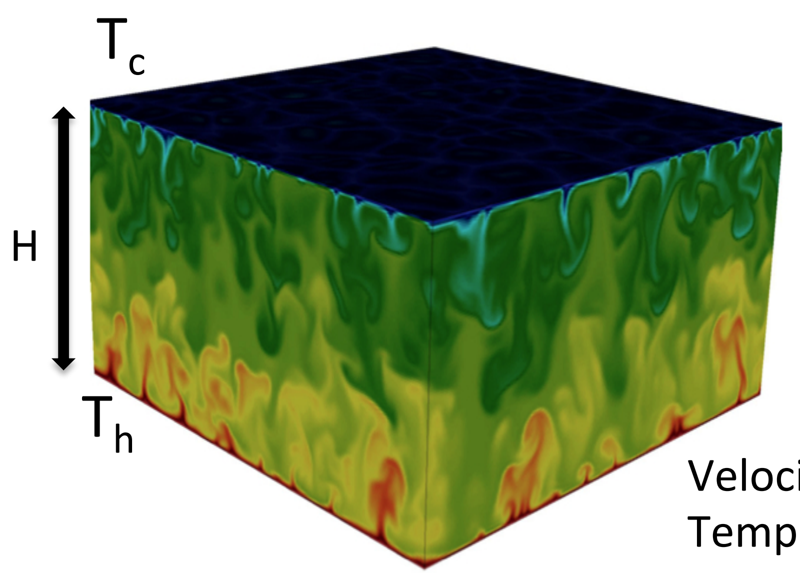
~~Rotating~~ RB convection

$$\frac{Du}{Dt} = -\nabla p + \nu \nabla^2 u - \alpha g \Delta T e_z,$$

$$\frac{DT}{Dt} = \kappa \nabla^2 T$$

$$\nabla \cdot u = 0$$

Aspect Ratio: 2:2:1



$$Ra = \frac{g\alpha \Delta T H^3}{\nu\kappa} \approx 2.5 \times 10^6$$

$$Nu \equiv 1 + \frac{u_z T}{\kappa \Delta T} H \approx 11$$

Velocity: No slip at top and bottom walls

Temperature: T_c at the top wall and T_h at the bottom wall

Rotating RB convection

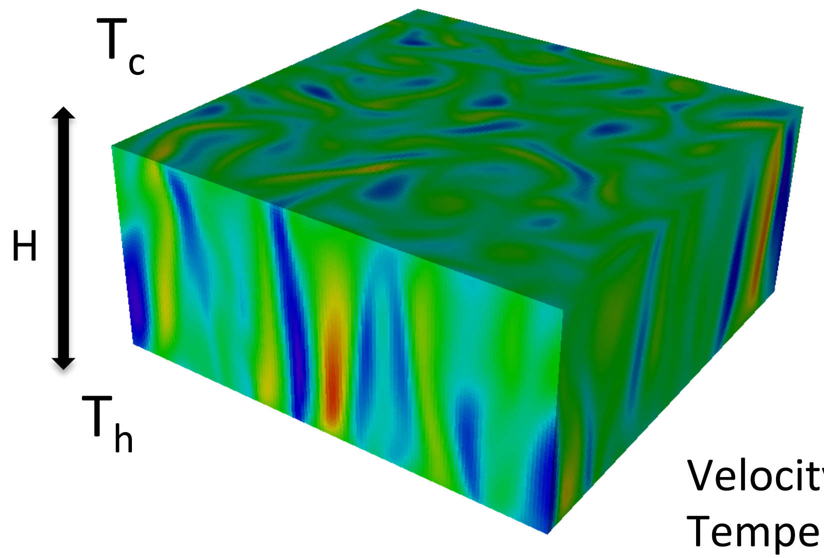
$$\frac{Du}{Dt} = -\nabla p + \nu \nabla^2 u + 2u \times \Omega e_z - \alpha g \Delta T e_z$$

$$\frac{DT}{Dt} = \kappa \nabla^2 T$$

$$\nabla \cdot u = 0$$

Rotation=> Taylor-Proudman
=> Two dimensionalization

Aspect Ratio: 2:2:1



$$Ra = \frac{g\alpha \Delta T H^3}{\nu \kappa} \approx 2.5 \times 10^6$$

$$Ro \equiv \frac{U}{2\Omega H} = 16$$

$$Free\ fall\ velocity\ U = \sqrt{g\alpha \nabla T H}$$

Velocity: No slip at top and bottom walls
Temperature: T_c at the top wall and T_h at the bottom wall

Simulate using Lattice Boltzmann Method (LBM)

(Density & momentum)

$$f_\alpha(x+e_\alpha,t+1)-f_\alpha(x,t)=-\frac{f_\alpha(x,t)-f_\alpha^{(eq)}(\rho,\mathbf{u})}{\tau}$$

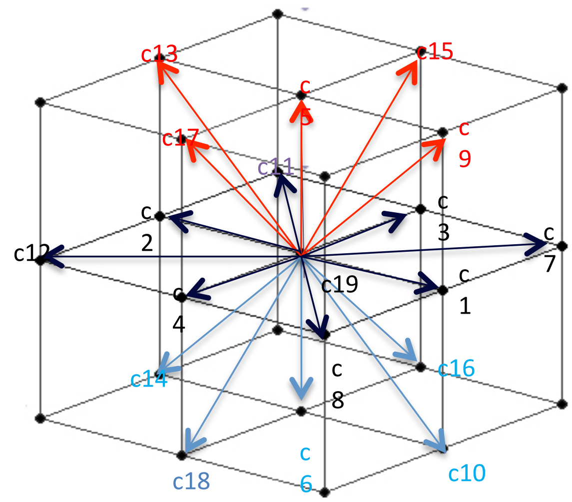
(Temperature)

$$g_\alpha(x+e_\alpha,t+1)-g_\alpha(x,t)=-\frac{g_\alpha(x,t)-g_\alpha^{(eq)}(T,\mathbf{u})}{\tau_g}$$

$$f_\alpha^{eq}=\omega_\alpha\rho\left(1+\frac{3(e_\alpha\cdot\mathbf{u})}{c_s^2}+\frac{9(e_\alpha\cdot\mathbf{u})^2}{2c_s^4}-\frac{3u^2}{2c_s^2}\right)$$

$$g_\alpha^{eq}=\omega_\alpha T\left(1+\frac{3(e_\alpha\cdot\mathbf{u})}{c_s^2}+\frac{9(e_\alpha\cdot\mathbf{u})^2}{2c_s^4}-\frac{3u^2}{2c_s^2}\right)$$

$D_t\rho=0$	(mass balance)
$D_t(\rho\mathbf{u})=\nu\nabla^2\mathbf{u}-\nabla\mathbf{p}$	(momentum balance)
$p=\rho/3$	(Equation of state)
$D_tT=\kappa\nabla^2T$	(Temperature)



D3Q19

$$\rho=\sum_\alpha f_\alpha^{eq};\ \ \rho\mathbf{u}=\sum_\alpha f_\alpha^{eq}\mathbf{e}_\alpha$$

$$T=\sum_\alpha g_\alpha^{eq}$$

$$\nu=c_s^2\left(\tau-\frac{1}{2}\right)$$

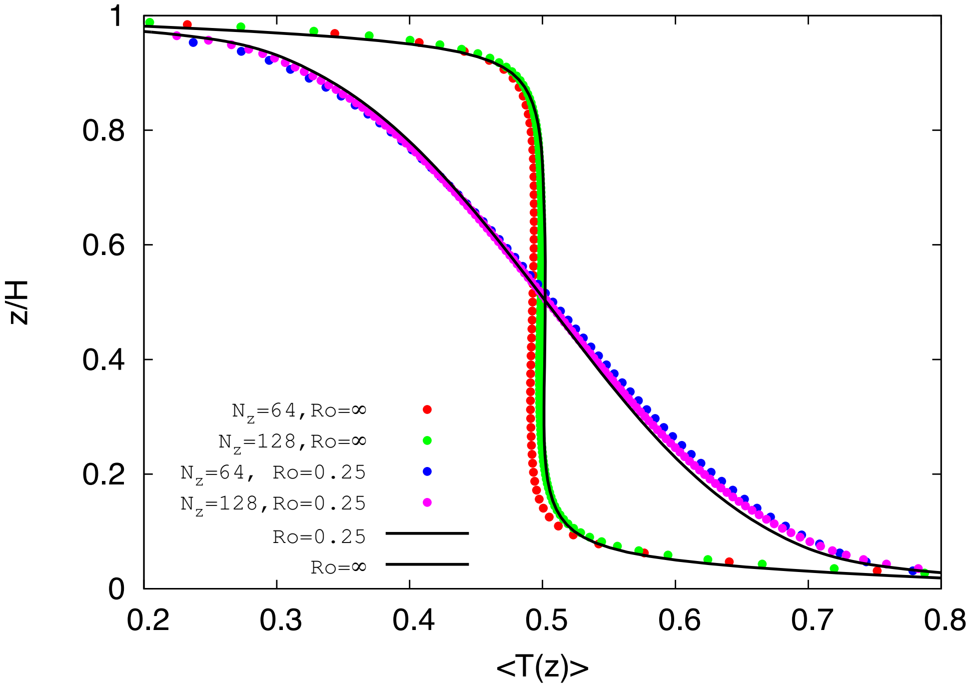
$$\kappa=c_s^2\left(\tau_g-\frac{1}{2}\right)$$

Boussinesq and Corioilis added as external force to the momentum distribution function

Rotating RB convection

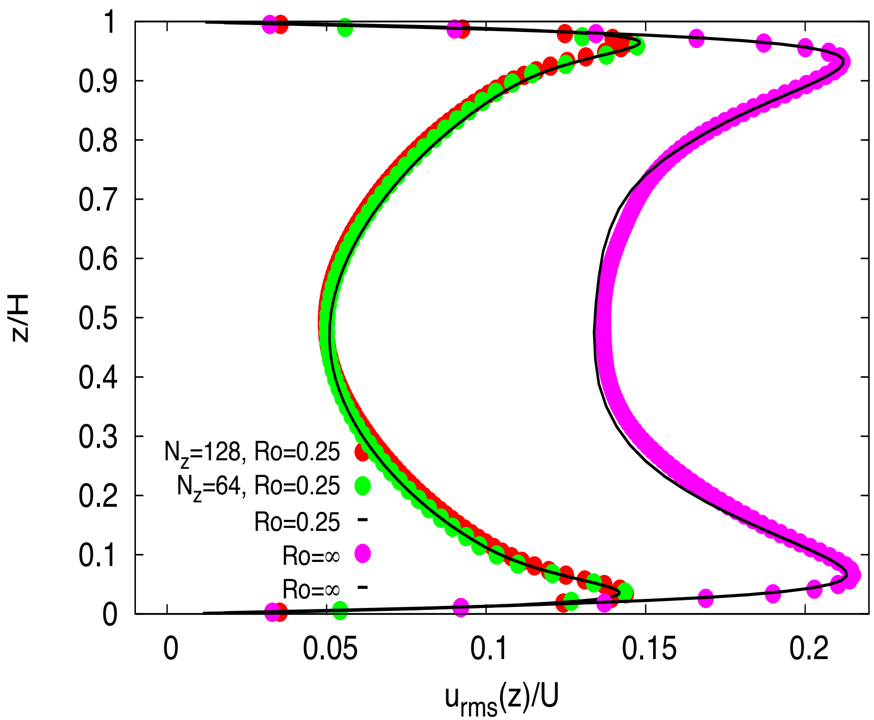
- Rotation modifies heat transfer

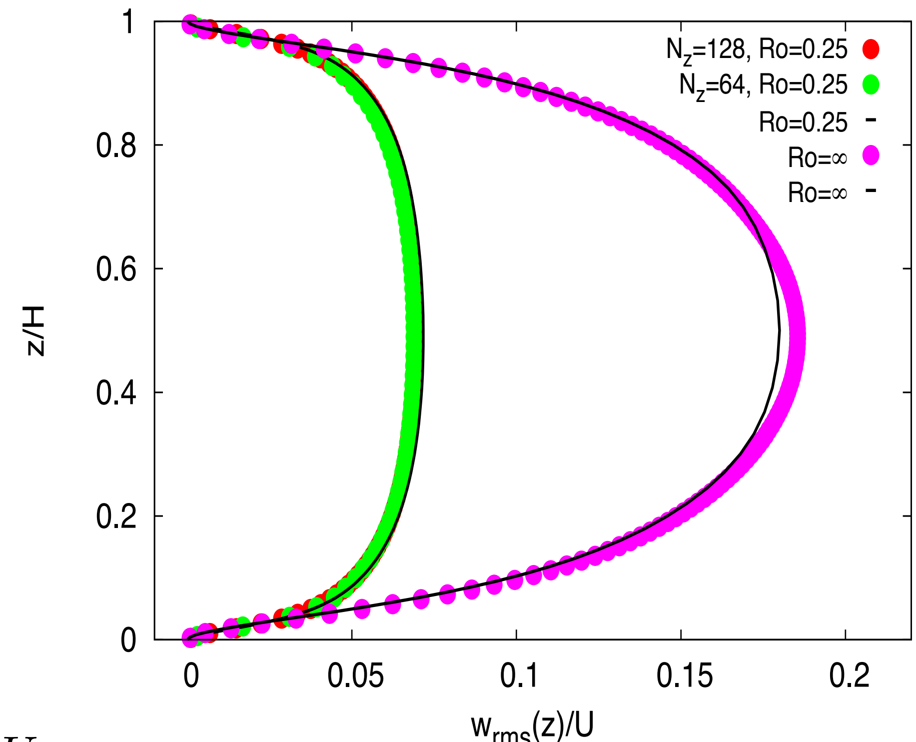
Average Temperature



Rotation modifies momentum transfer

Domain 2x2x1





$$Ra = \frac{g\alpha\Delta T H^3}{\nu\kappa} \approx 2.5 \times 10^6$$

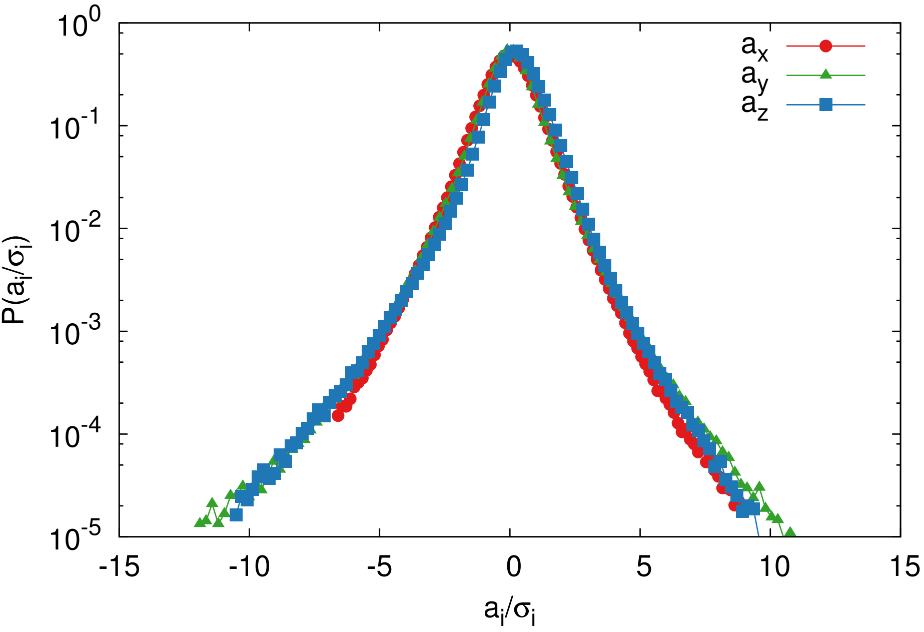
$$Ro = \frac{U}{2\Omega H}$$

$$Free\ fall\ velocity\ U = \sqrt{g\alpha\nabla T H}$$

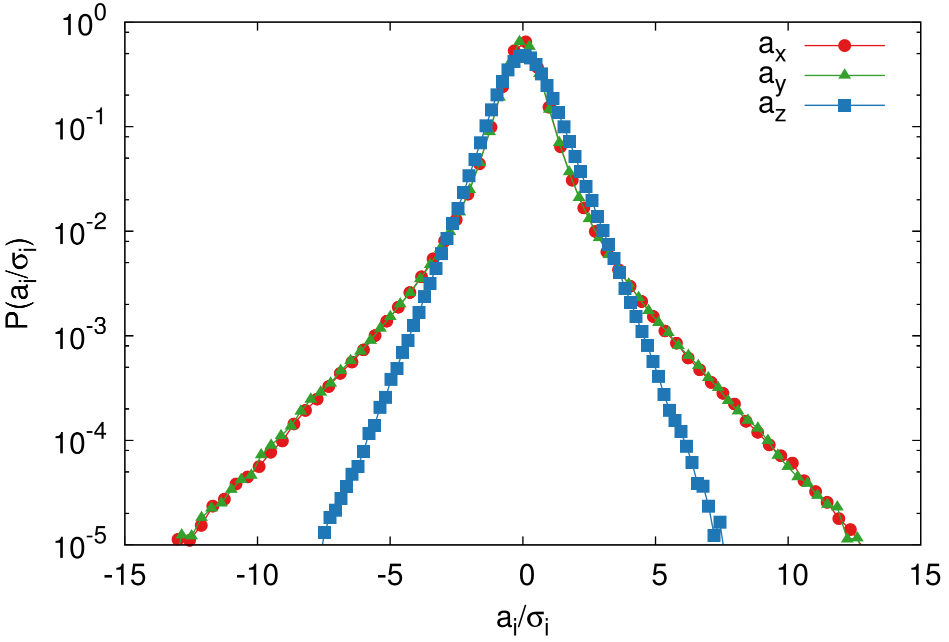
—— R. Kunnen, PhD Thesis (2008).

Acceleration PDF

$Ro = \infty$



$Ro = 0.25$



Acceleration is enhanced in the plane of rotation and is suppressed along the axis of rotation.

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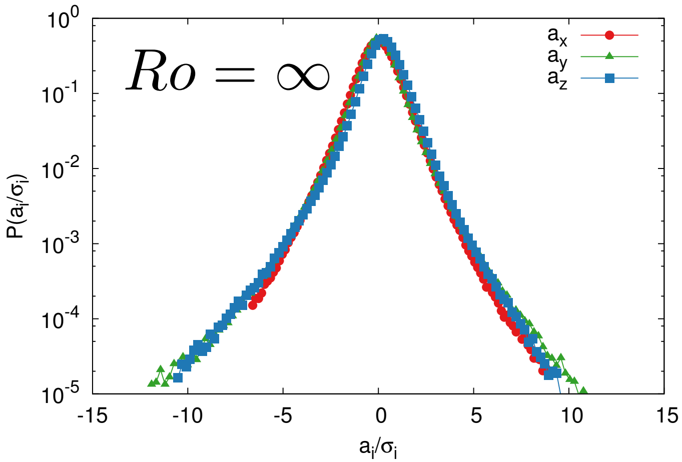
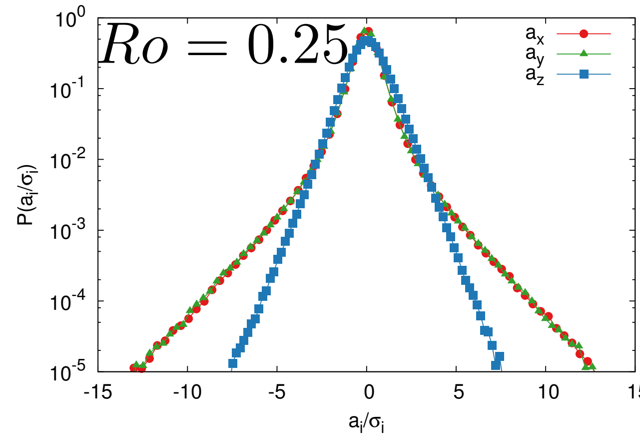
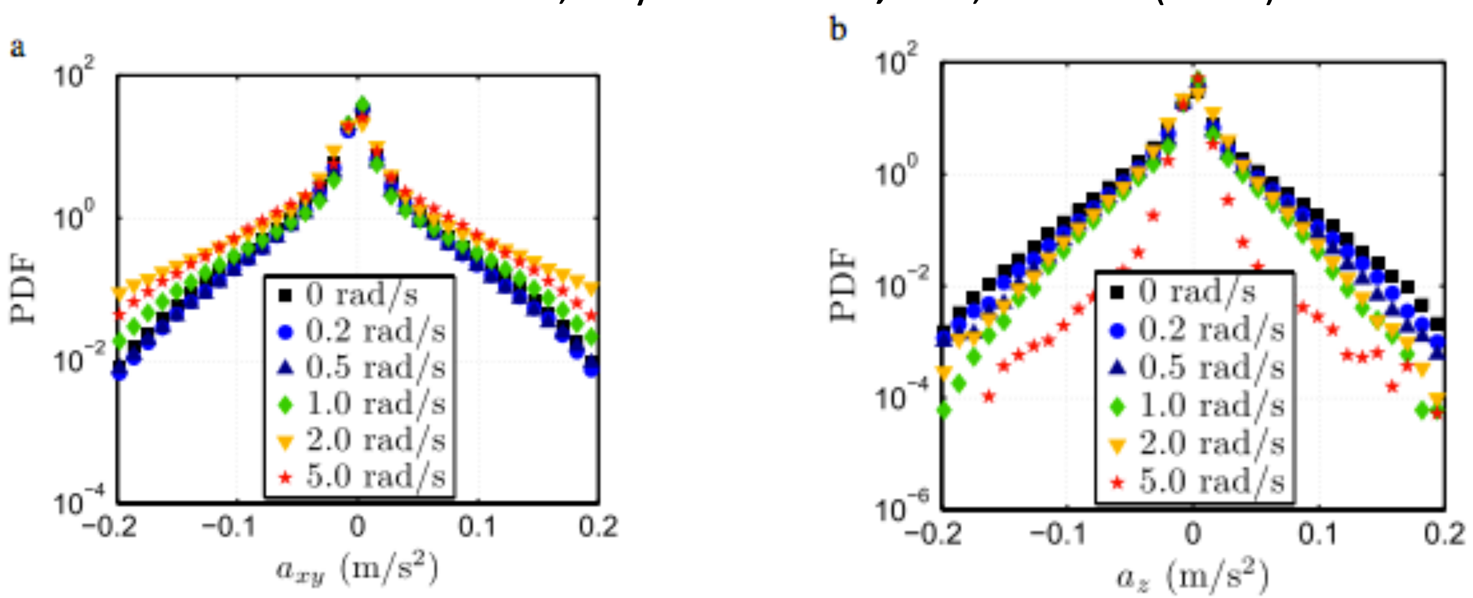




L.D. Castello and H.J.H. Clercx, Phys. Rev. Lett.**, 107**, 214502 (2011).



Inertial particle in rotating RB

$$\frac{Du}{Dt} = -\nabla p + \nu \nabla^2 u + 2u \times \Omega e_z - \alpha g \Delta T e_z$$

$$\frac{DT}{Dt} = \kappa \nabla^2 T$$

$$\nabla \cdot u = 0$$

+

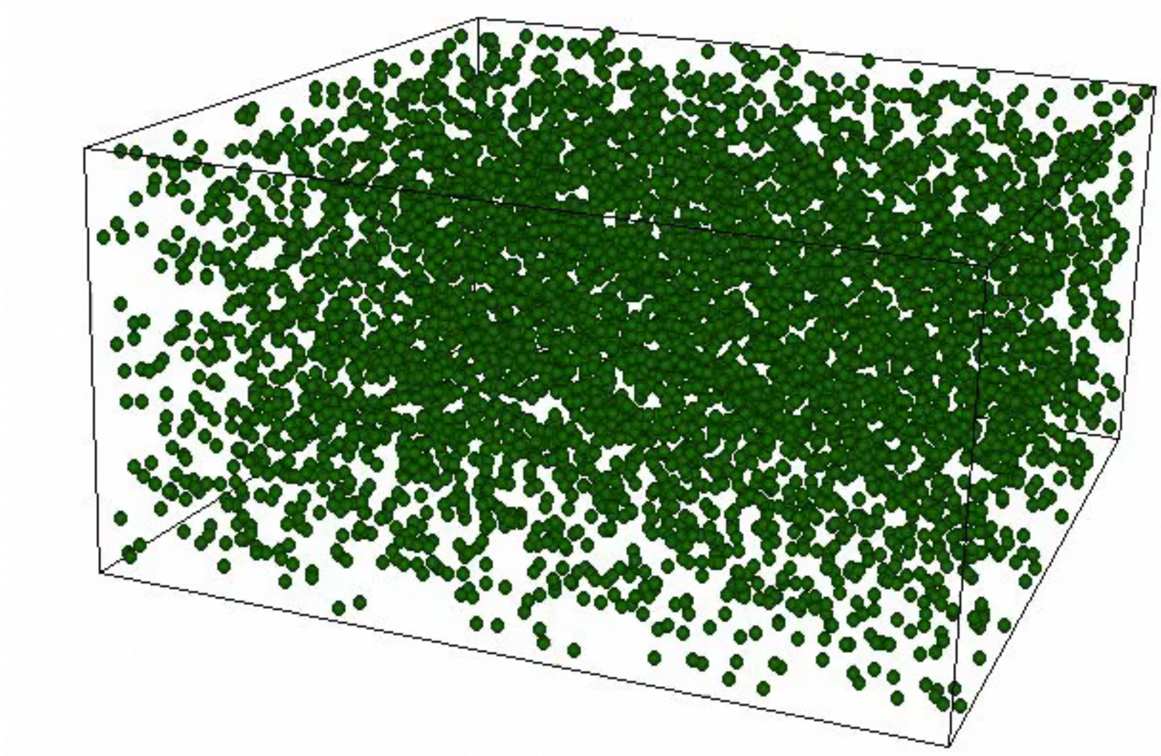
$$\frac{dx}{dt} = v$$

$$\frac{dv}{dt} = -\frac{1}{\tau}(v - u) - 2\Omega \times v$$

Inertial particles in RRB

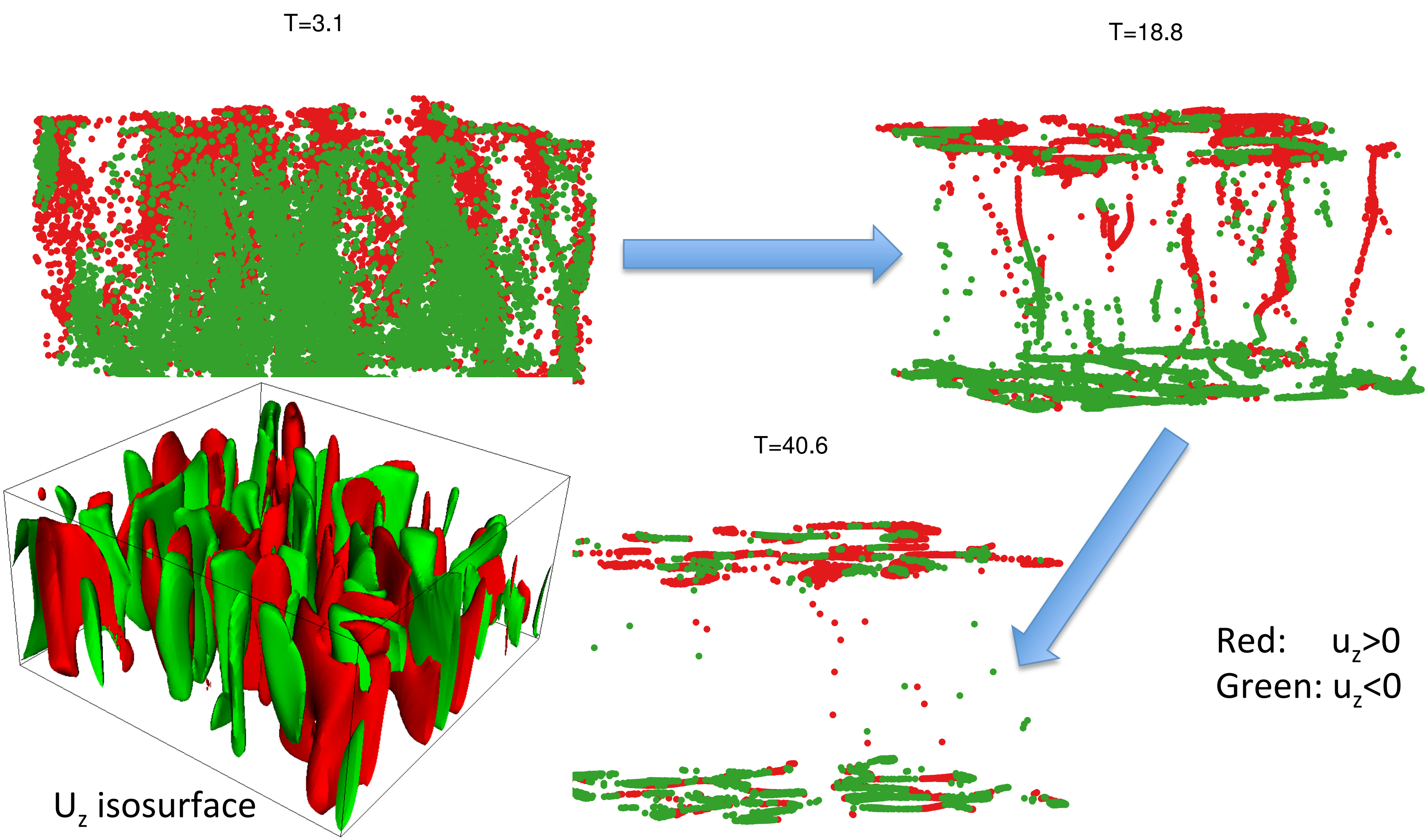
Stokes = 0.22

Particles get trapped in anticyclonic vortices and are then transported to the walls.



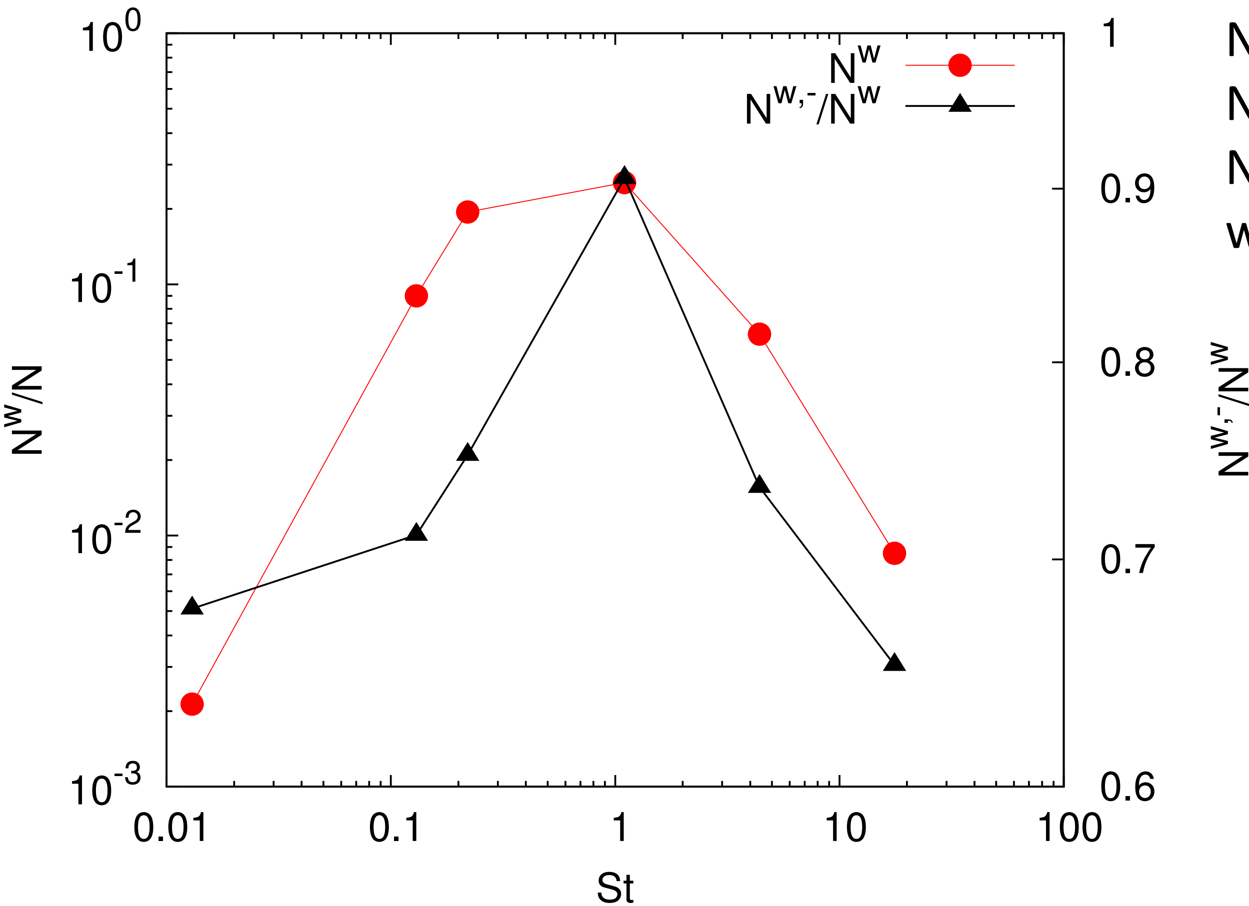
Vertical velocity of the particles

Particles get trapped in anticyclonic vortices and are then transported to the walls.



Inertial particles in RRB

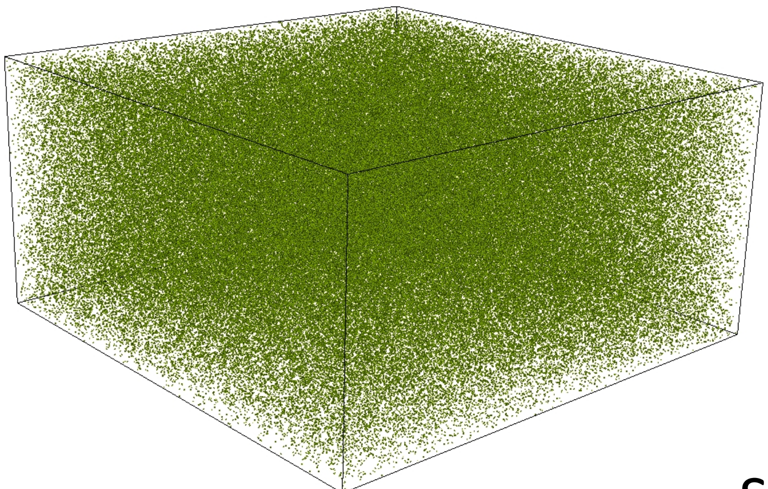
Particles get trapped in anticyclonic vortices and are then transported to the walls.



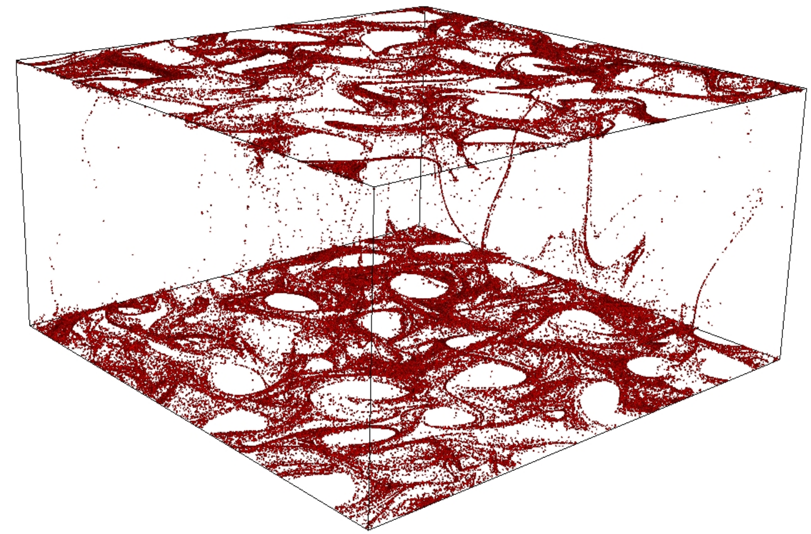
N: # of particles
N^w: # of particles at walls
$N^{w,-}$: # of particles at walls with negative vorticity

Stokes number dependence

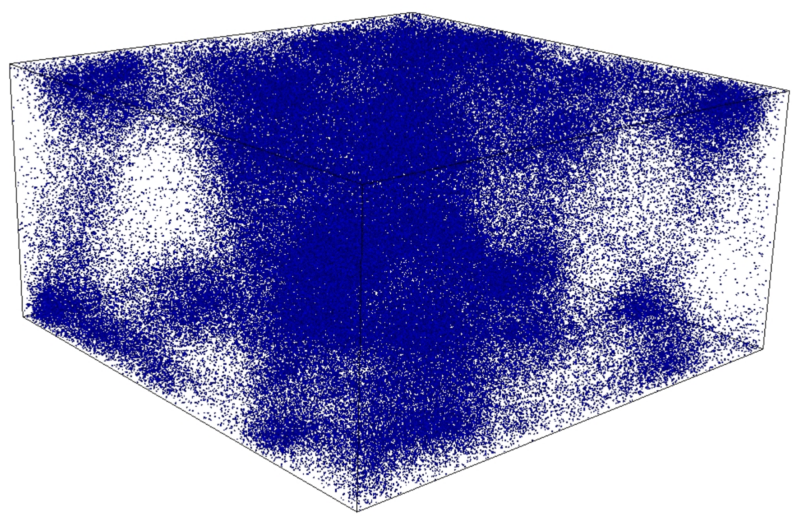
St=0



St=1



St=13



Conclusions

- A Lattice Boltzmann method coupled with Lagrangian particle tracking has been used to solve the turbulent flow laden with inertial particles in a Rayleigh-Benard cell;
- Bulk acceleration PDF in non-rotating RB is HIT whereas, rotation introduces anisotropy similar to experiments in rotating turbulence.
- Inertial particles with St=1 are accumulated in the regions of anti-cyclonic vortices and are transported to the top and bottom walls.