Turbulence and droplets in cloud simulator

Conference on Dynamics of Particles in Flows

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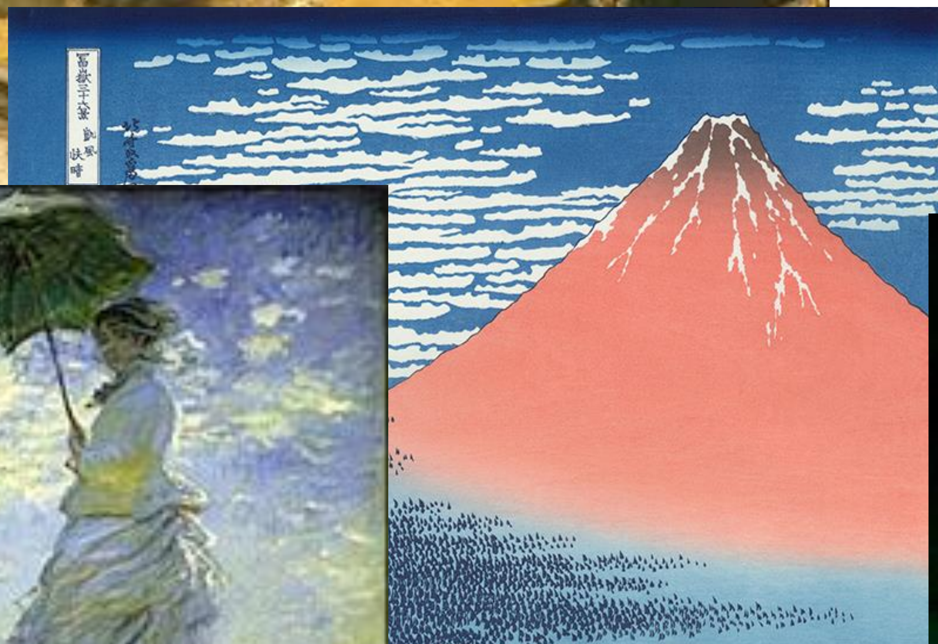
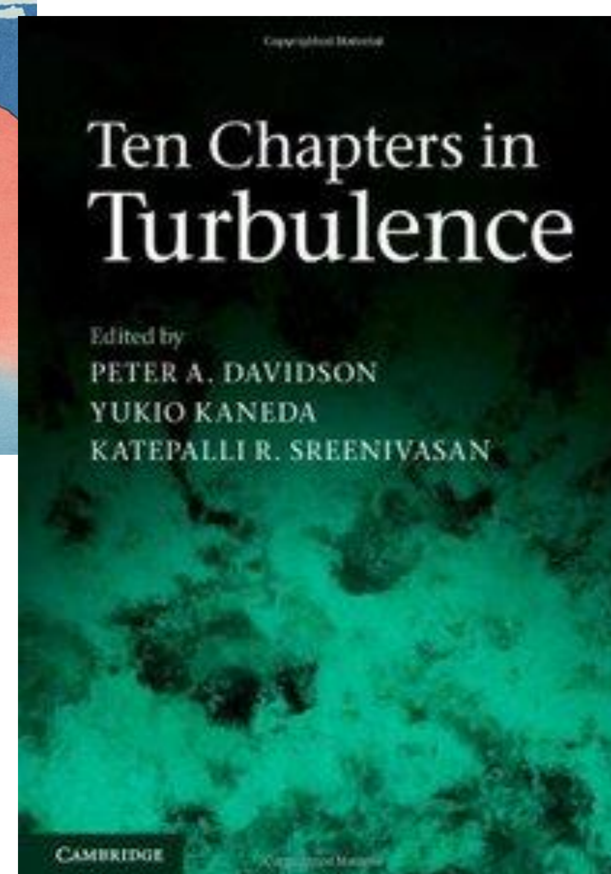
Cloud , Cloud , Cloud, and Cloud

Tail of Genji



Aka Fuji (red Mt.Fuji)
うろこ雲(cirrocumulus clouds)
Hokusai

La promenade, La femme à l'ombrelle
Monet



Vortex cloud

Cloud：dry air, water vapor, water droplets, ice, aerosol,…..

Cloud simulator

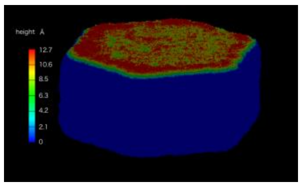
Direct Numerical Sim.　　Turbulence + Cloud droplets　　*0.5 mm < r < 1m*

MD of water molecules

Ogata, Kobayashi and Kajima

Symmetric and time reversal FT method

Potential for water molecule　TIP4P



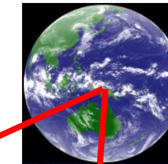
Turbulence, mixing and droplets

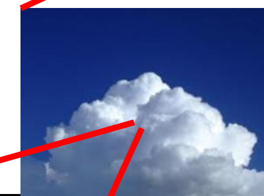
Gotoh and Watanabe

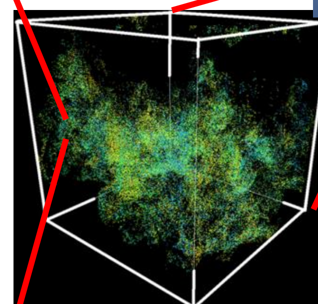
Turbulence　　Spectral method

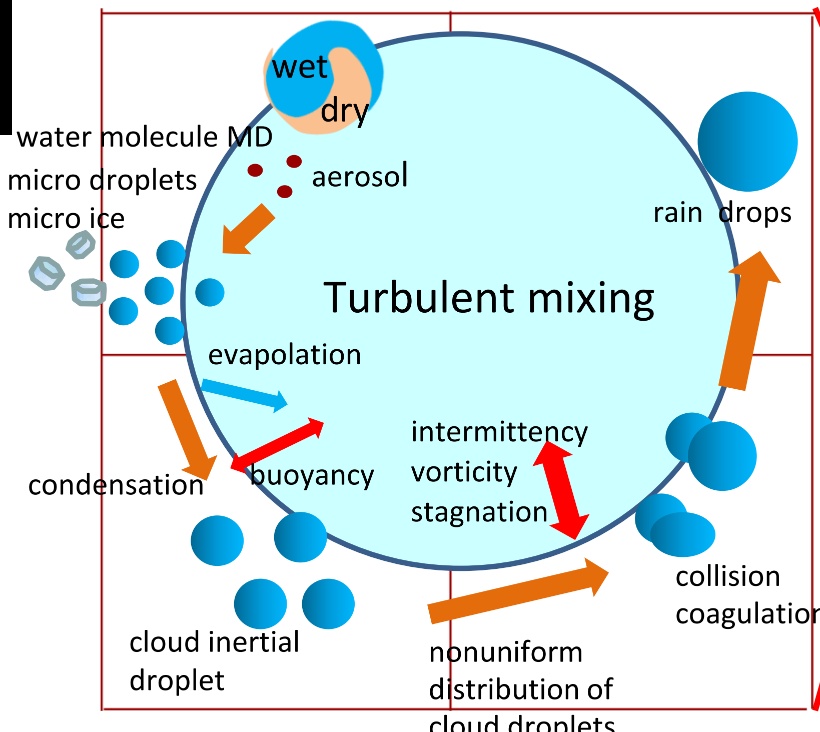
　　　　　　　Hybrid method (spectral +combined compact FD)

Cloud droplets　Lagrangian tracking ＋PIC





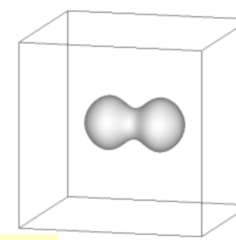




$2\Delta x \approx \eta \approx 1$mm

η :　Kolmogorov length

1m

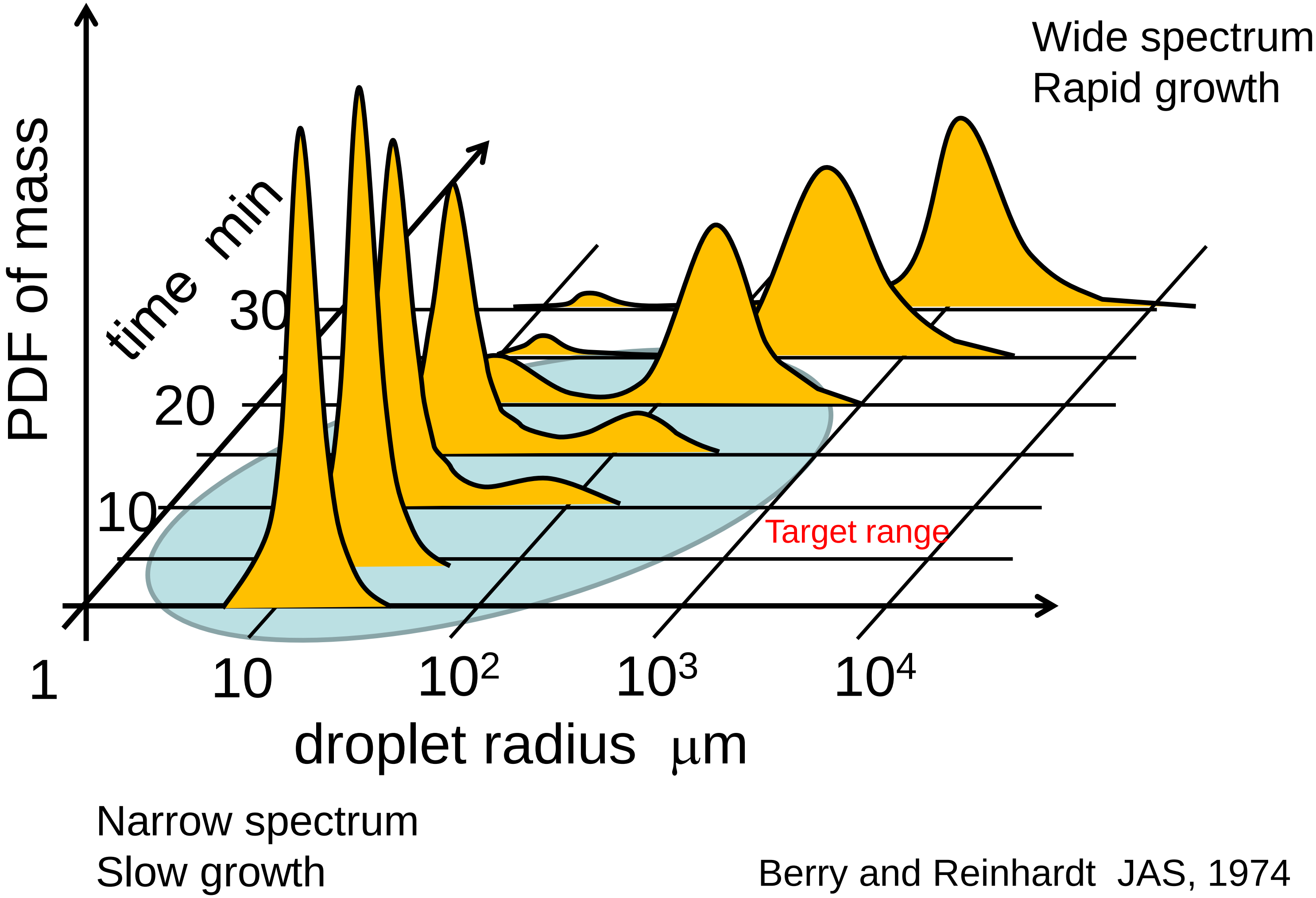


Droplet collision

Yoshino

Two phase flow : LBM (3D19Q)

Growth of cloud droplets



Berry and Reinhardt JAS, 1974

Goal

Cloud droplets

- How fast do the droplets grow in mean and distribution , and become rain drops?
- What are key processes ?
- What is the spatial distribution of cloud droplets?
- To compute the collision efficiency and the collision kernel

Turbulence

- What is role of turbulence in cloud evolution?
- To what extent is turbulence modified or driven? (energy transfer, scalar variance transfer, intermittency)
- How droplets and scalar are transferred and mixed?

Direct Numerical Simulation of Turbulence and Cloud Droplets

Vaillancourt, P. A., Yau, M. K. & Grabowski, W. W. JAS. 2001
Andfrejczuk et al JAP. 2004
Kumar, Schumacher, Shaw, TCFD 2012

Included

- turbulence + buoyancy
- temperature
- water vapor mixing ratio
- water droplets of radius 10μm～20μm
- condensation, evaporation
- Stokes drag + radius dependent relaxation time + gravity

Not included

- Collision of droplets, coagulation
- Nucleation of water droplets
- Rain, ice

Basic Equations

Turbulence (Eulerian) **Boussinesq approximation**

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\nabla p + \nu \nabla^2 u + e_z B + f, \qquad \nabla \cdot u = 0$$

buoyancy external force

$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \kappa \nabla^2 T + \frac{L}{c_p} C_d$$

$$\frac{\partial q_v}{\partial t} + u \cdot \nabla q_v = \kappa \nabla^2 q_v - C_d$$

condensation, evaporation

$$B = g \left(\frac{T - T_0}{T_0} + \epsilon (q_v - q_{v0}) - q_l \right)$$

High Reynolds number turbulence : Spectral method

Scalar transport : Spectral (or hybrid method)

Cloud droplets (Lagrangian)

Collisionless

Vaillancourt *et al.* (2001)
Kumar *et al.* (2012, 2013)

$$\frac{dX_j}{dt} = V_j(t)$$

$$\frac{dV_j}{dt} = \frac{1}{\tau_j(t)}\left(u(X_j(t),t) - V_j(t)\right) - g e_3$$

Stokes approximation

$$R_j(t)\frac{dR_j(t)}{dt} = KS(X_j(t),t)), \qquad R_j = \text{droplet radius}$$

Diffusion process

$$C_d(x,t) \equiv \frac{1}{m_{air}}\frac{dm_l(x,t)}{dt} = \frac{4\pi r_l K}{\rho_0(\Delta x)^3}\sum_{k=1}^{N_\Delta} R_j(t)S(X_j(t),t)$$

Condensation rate

$$S = \frac{q_v}{q_{vs}(T)} - 1, \qquad \text{supersaturation rate}$$

$$K^{-1} = \frac{\rho_l R_v T}{D_v e_{sat}(T)} + \frac{\rho_l L}{\kappa_a T}\left(\frac{L}{R_v T} - 1\right)$$

PIC

Interpolation of velocity and scalar fields at particle position

Redistribution of cloud properties onto grid points

Simulation of cloud evolution

Domain : $(25.6\ \text{cm})^3$ $\Rightarrow$ $(102.4\ \text{cm})^3$

Water droplets : sphere (initially 10～20μm)

Turbulence : homogeneous isotropic steady

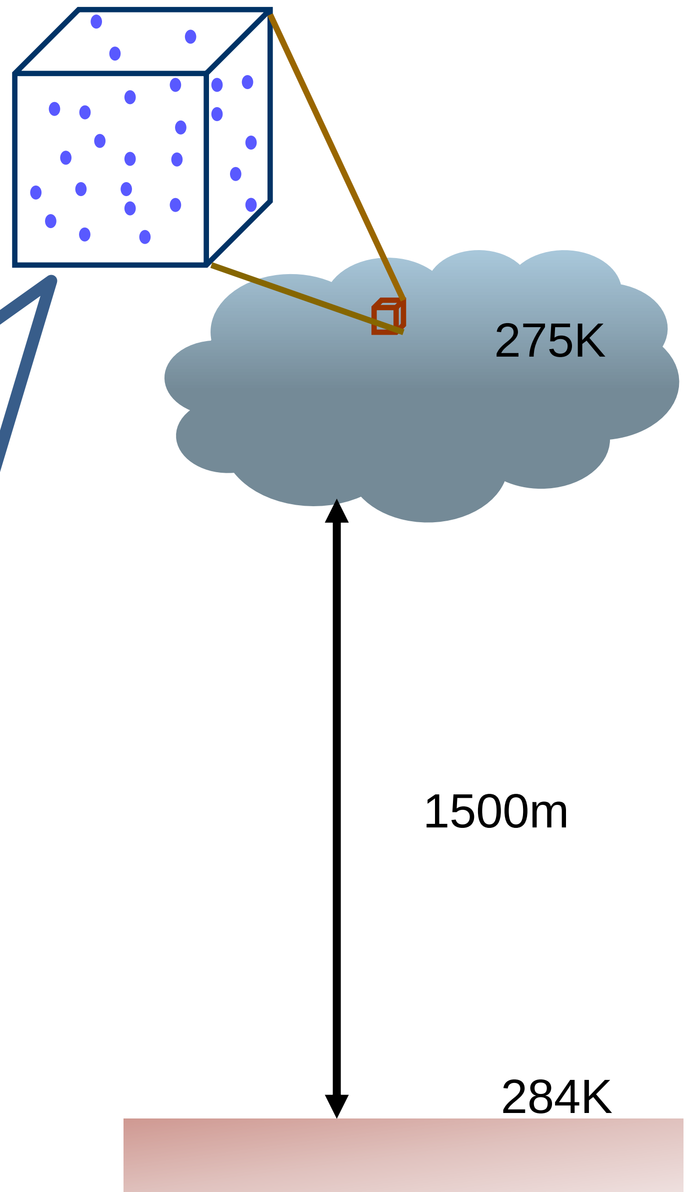
DNS : turbulence + scalar

pseudo spectral method
3DFFT + MPI + Open MP
2D domain decomposition
cell no. relabelling

interpolation for droplets

PIC + TS13 (or linear)

4th order Runge-Kutta-Gill



Initial Condition

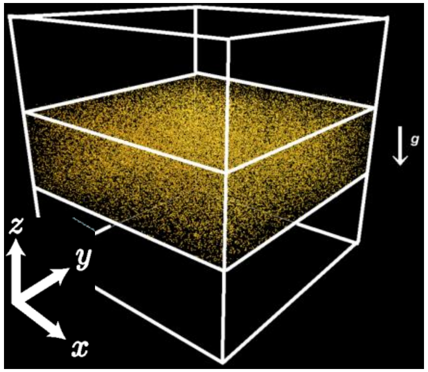
Turbulence

Isotropic Steady turbulence

Temperature fluctuation

θ : 275 K + zero fluctuation

Droplets



- Random in space in the range $-L_B/6 \leqq z \leqq L_B/6$
- No. of droplets : $2^{21} \fallingdotseq 2 \times 10^{6}$, $2^{27} \fallingdotseq 1.3 \times 10^{8}$
- No. density of droplets 31～125/cm^3
- Initial radius ： 10 ～20 μm

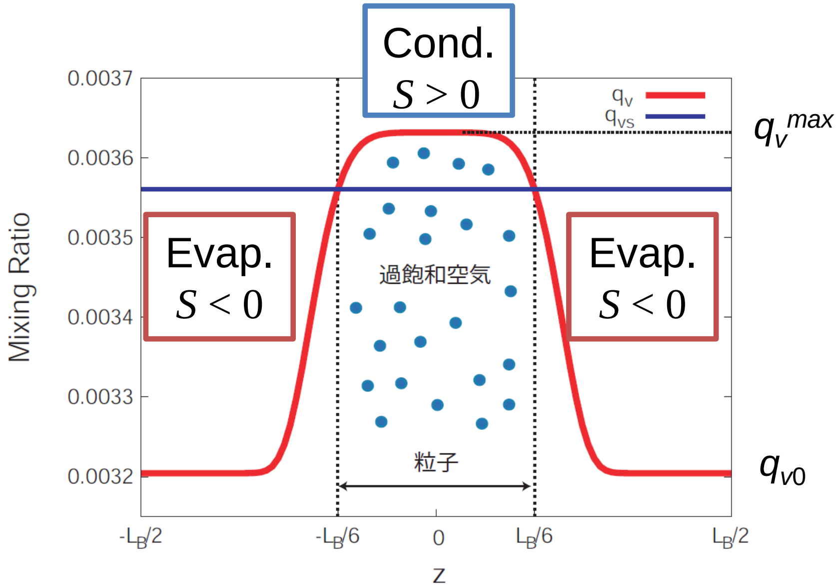
Water vapor mixing ratio q_v

$$q_v(x, t=0) = (q_v^{\max} - q_{v0}) \exp(-Az^6) + q_{v0}$$

$$q_v^{\max} = 1.02 q_{vs} \qquad q_{v0} = 0.90 q_{vs}$$

$$q_v = q_{vs} \quad \text{at} \quad z = \pm L_B / 6$$

Kumar *et al.* (2012, 2013)



Run	A	B	C'	C''	C	D(A)	E(B)	Obs.
N^3	128^3	128^3	128^3	128^3	1024^3	1024^3	1024^3	
N_p	2^{21}	2^{21}	2^{22}	2^{19}	2^{25}	2^{27}	2^{27}	
n_p [cm^{-3}]	125	125	250	31	31	125	125	100~600
q_v	Decay	Grow	Grow	Grow	Grow	Decay	Grow	
θ'	0	0	0	0	0.05K	0	0	
ε[cm^{2}s^{-3}]	68.3	68.3	68.3	68.3	138	138	138	1~100
R_λ	92	92	92	92	252	252	252	3~4x10^4
$K_{max}\eta$	1.2	1.2	1.2	1.2	2.1	2.1	2.1	
r(0) [μm]	20	20	15	15	15	20	20	
St(0)	0.068	0.068	0.068	0.068	0.10	0.18	0.18	

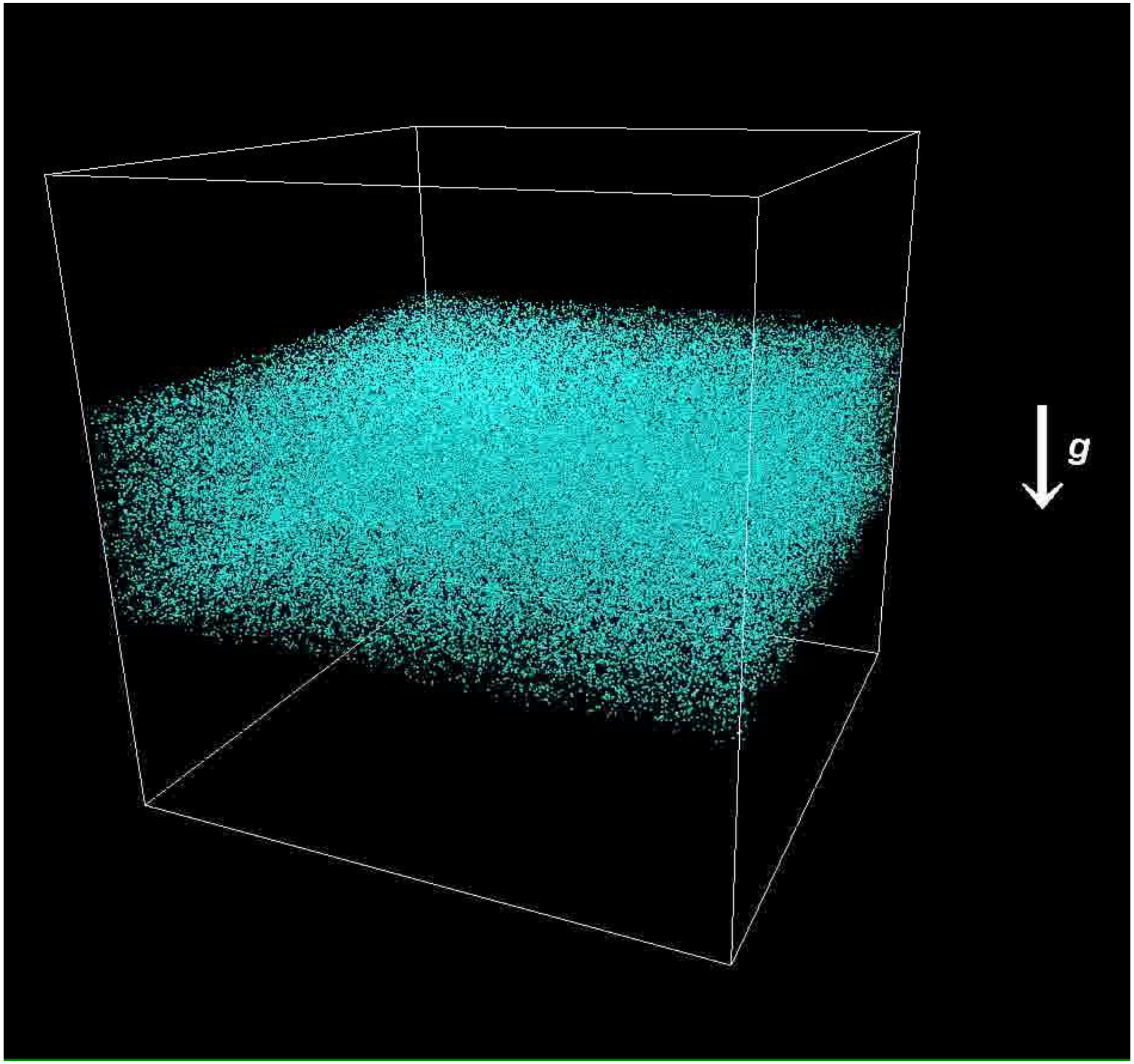
Siebert et al
JAS (2006)

Grow

$$M_{\mathrm{vs}} \le M_{\mathrm{v}} + M_{\mathrm{l}} \Longleftrightarrow \rho_{\mathrm{a}} \int_V q_{\mathrm{vs}} \mathrm{d}V \le \rho_{\mathrm{a}} \int_V q_{\mathrm{v}}(x,t) \mathrm{d}V + \frac{4}{3}\pi\rho_{\mathrm{l}} \sum_{j=1}^{N_{\mathrm{p}}} r_j^3$$

Decay

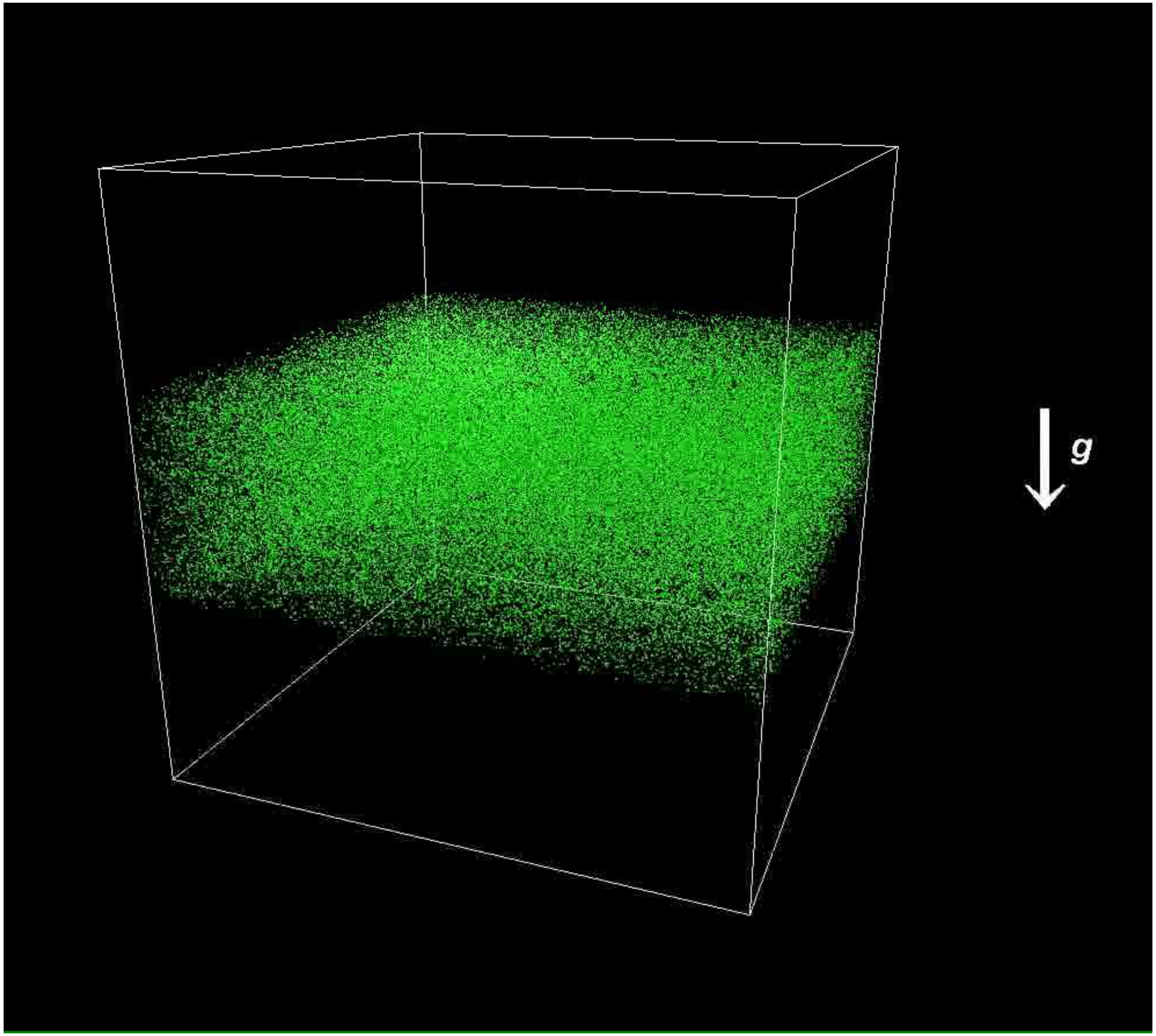
$$M_{\mathrm{vs}} \ge M_{\mathrm{v}} + M_{\mathrm{l}}$$



$R_\lambda = 92$ (Run C')

- Periodic boundary condition
- e(t=0): 68.3 cm^2/s^3
- No. droplets : 4×10^6
- No. density droplets : $250/cm^3$
- No. visualized droplets : 10^5
- r(0) : 15 μm
- color : droplet radius

  red ⇒grow, blue⇒decay
- time : 0s～4s

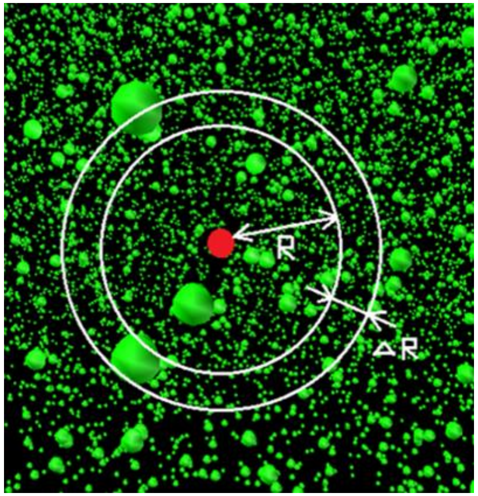


$R_\lambda = 252$ (Run E)

- Periodic boundary condition
- $\varepsilon(t=0)=138\ cm^2/s^3$
- No. droplets: 1.3×10^8
- No. density droplets: $125/cm^3$
- No. visualized droplets: 10^5
- $r(t=0)$: 20 μm
- color : droplet radius
  red ⇒grow, blue⇒decay
- time : 0s〜6s

Radial distribution function of cloud droplets

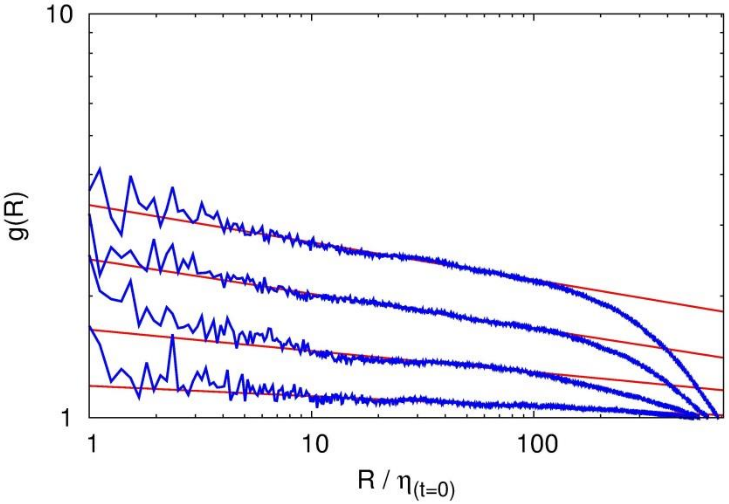
$$g(R) = \frac{G(R)}{4\pi R^2 n_d}$$



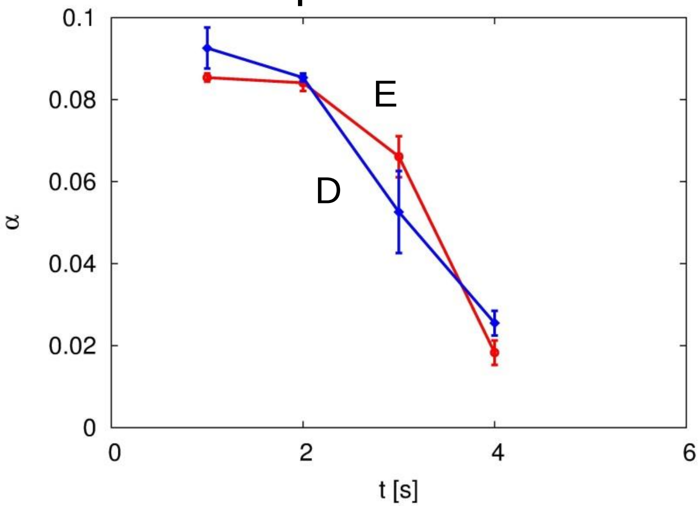
$G(R)\Delta R$: No. of particles within spherical shell between R and ΔR
n_d : Average number density

20 μm: $St(0)=0.18$

Run E

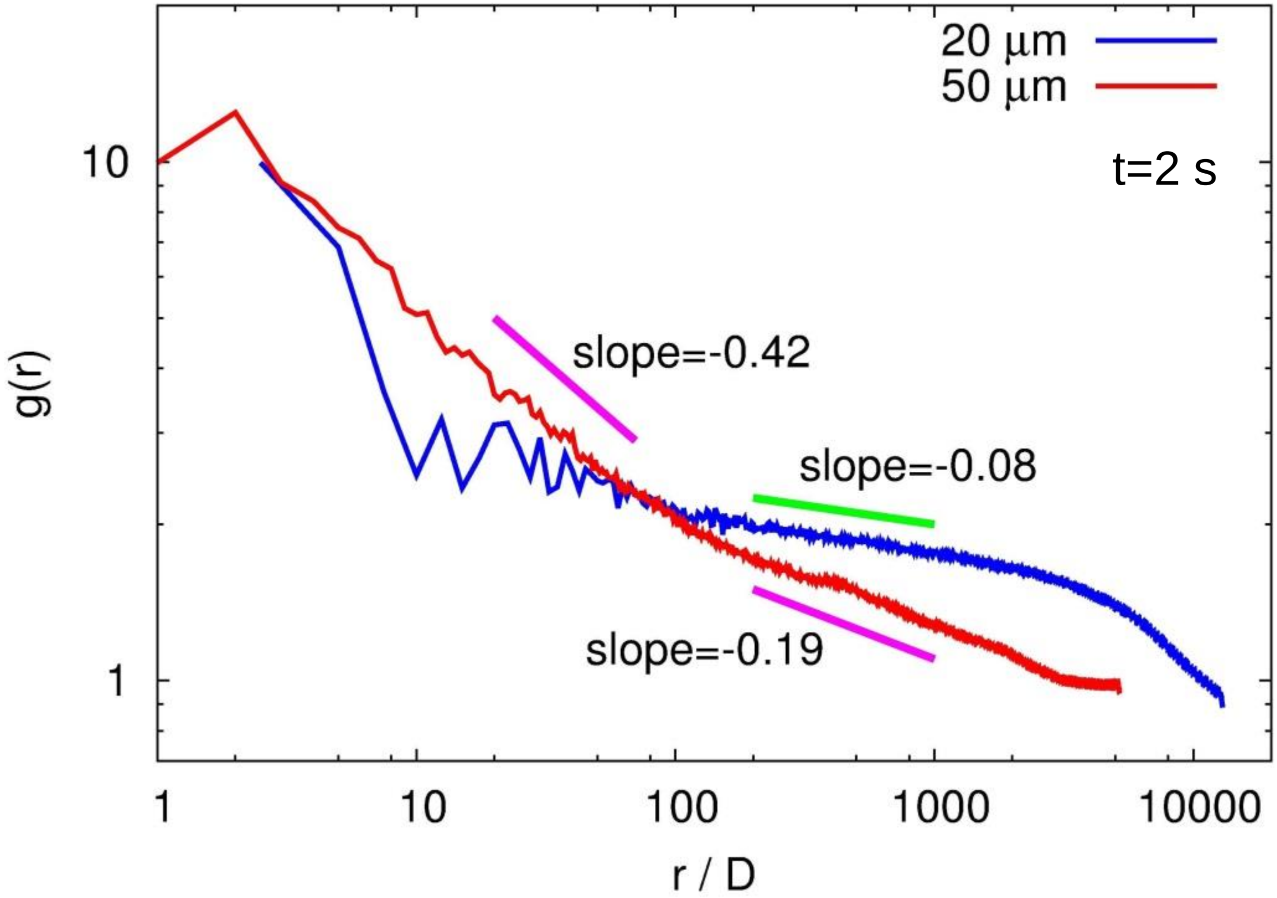


Exponents



Power low $g(R) \propto R^{-\alpha}$

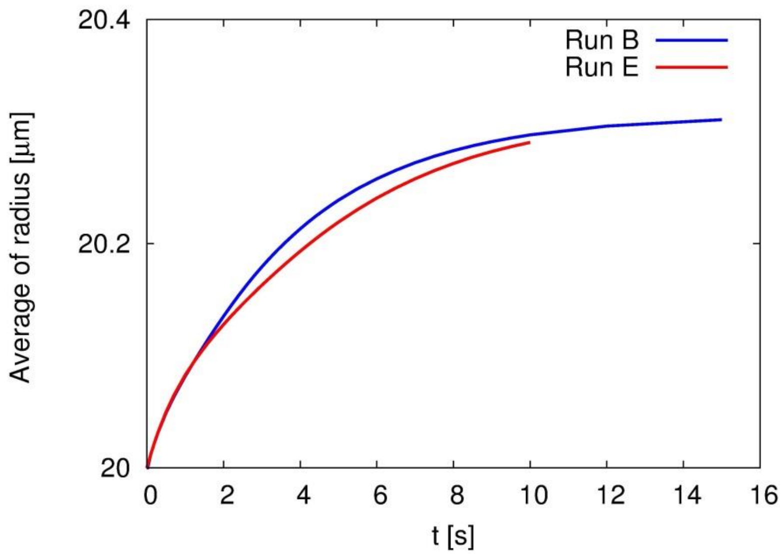
20 μm: *St*(0)=0.18
50 μm: *St*(0)=1.02



Experiments St=0.01~0.21 slope=-0.07
St=0.40~1.00 slope=-0.48

Effects of Reynolds number

Run B (R_λ=92) and E (R_λ=252)

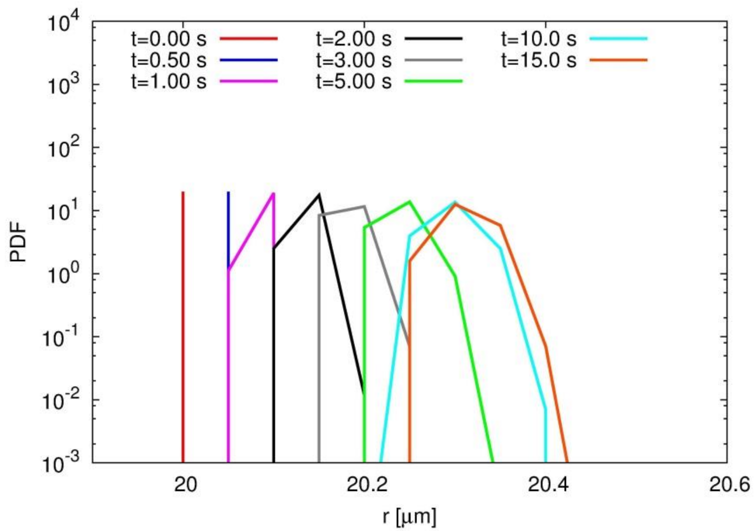


When R_λ increases

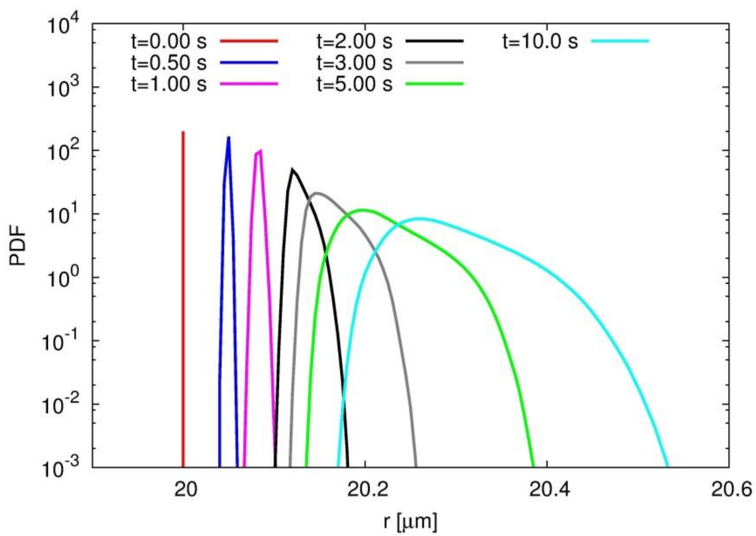
<r> → comparable

PDF → broad

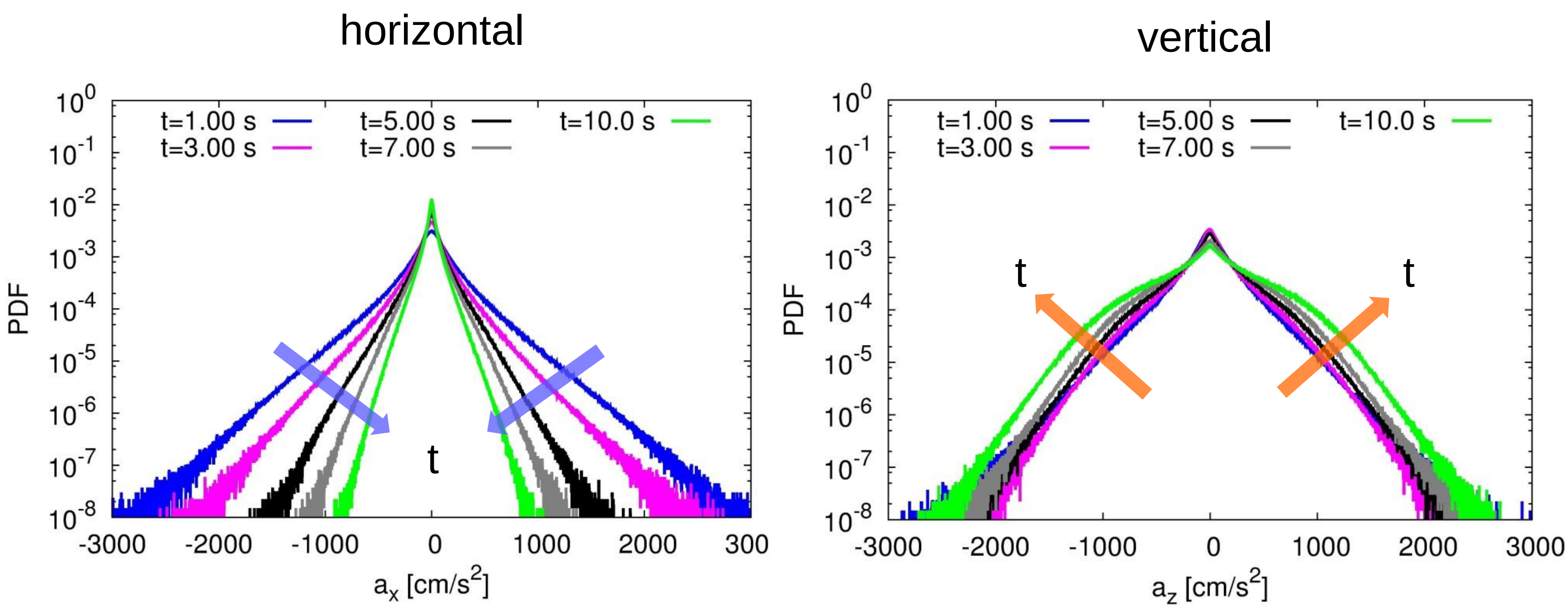
Run B



Run E



PDF of acceleration of cloud droplets $R_\lambda = 252$



Non Gaussianity

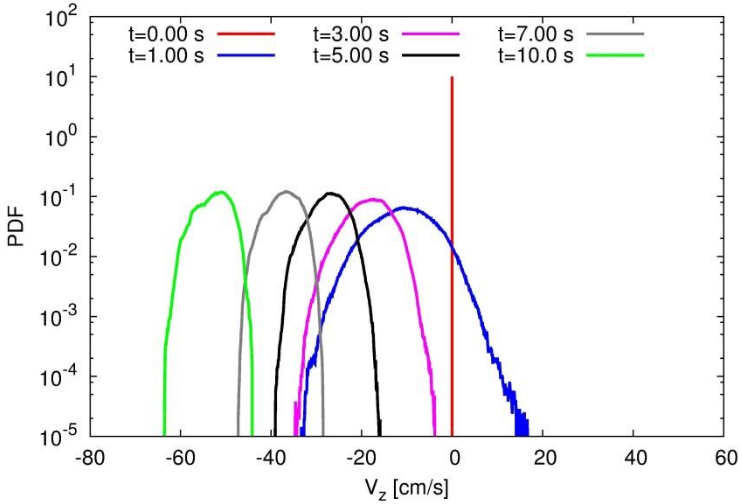
Early time Horizontal > Vertical

Latter time Horizontal < Vertical

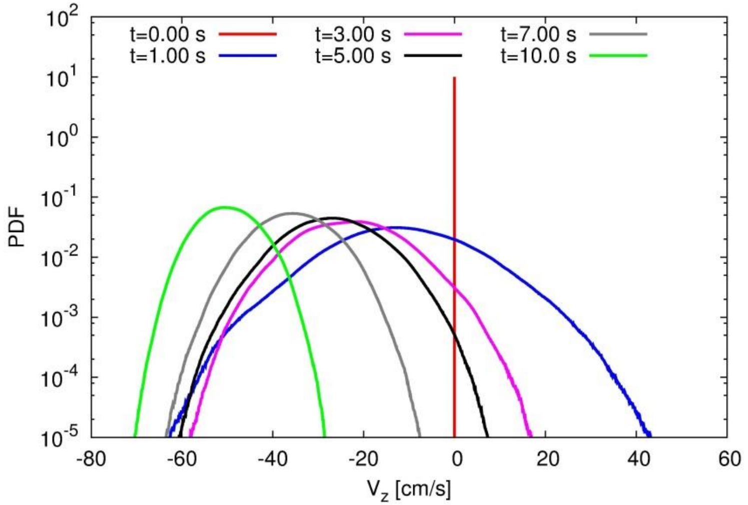
Stronger intermittency in vertical acceleration

PDF of vertical velocity of cloud droplets

$R_\lambda = 92$



$R_\lambda = 252$



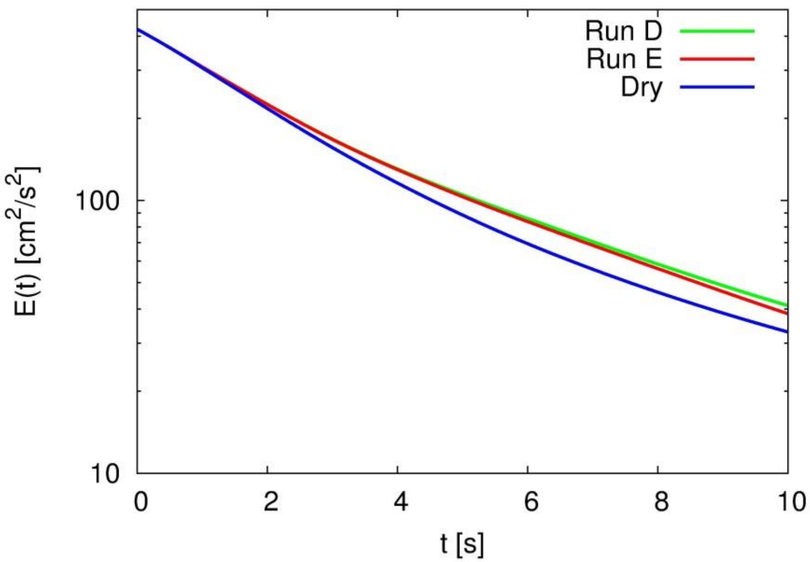
When R_λ increases

Wider PDF of u_3 → More chance to collide

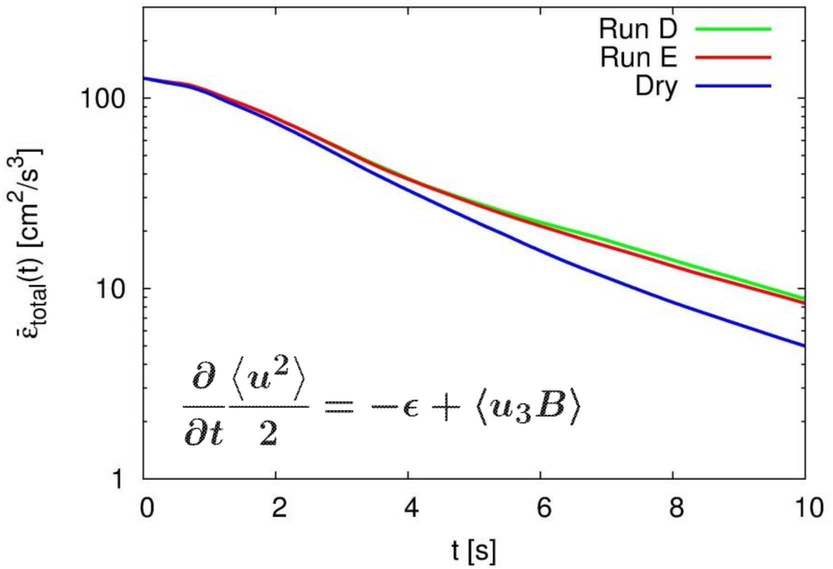
Larger R_λ → Faster growth and wider PDF of r

Turbulence modulation (one point)

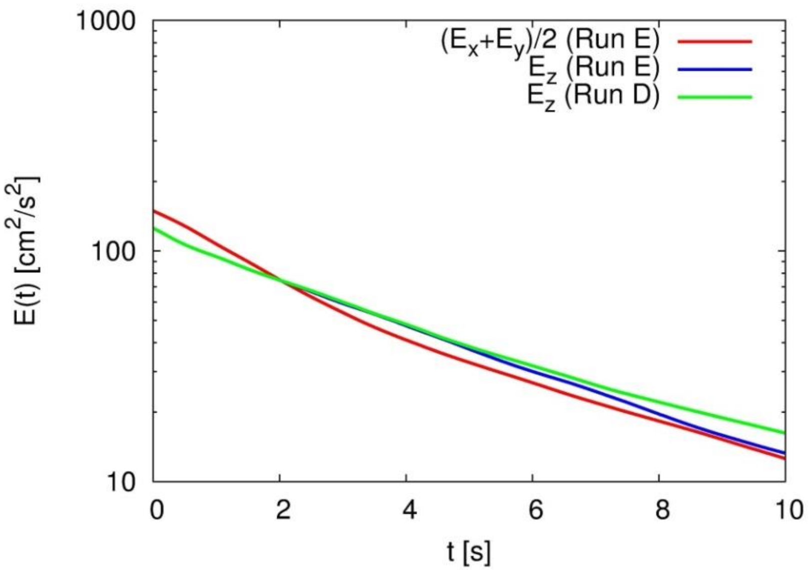
Kinetic energy



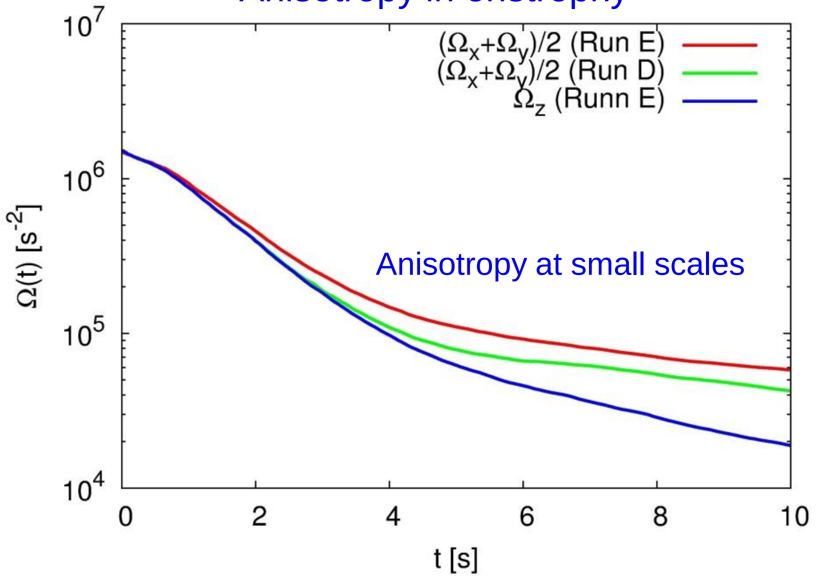
Kinetic energy dissipation



Anisotropy in kinetic energy

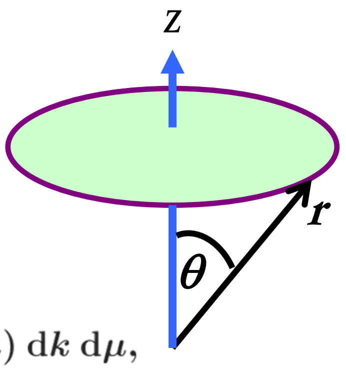


Anisotropy in enstrophy



Turbulence modulation (spectrum)

Turbulence spectrum



Homogeneous, axially symmetric

$$\frac{1}{2}\left\langle u^2 \right\rangle = \int_0^\infty E^v(k)\,\mathrm{d}k = \frac{1}{2}\int \langle u(k)\cdot u(-k)\rangle\,\mathrm{d}k = \frac{1}{2}\int_0^\infty\int_{-1}^{1} 2\pi k^2 Q^v(k,\mu)\,\mathrm{d}k\,\mathrm{d}\mu,$$

$$\left\langle \theta^2 \right\rangle = \int_0^\infty E^\theta(k)\,\mathrm{d}k = \int \langle \theta(k)\cdot \theta(-k)\rangle\,\mathrm{d}k = \int_0^\infty\int_{-1}^{1} 2\pi k^2 Q^\theta(k,\mu)\,\mathrm{d}k\,\mathrm{d}\mu,$$

$$\left\langle q^2 \right\rangle = \int_0^\infty E^q(k)\,\mathrm{d}k = \int \langle q(k)\cdot q(-k)\rangle\,\mathrm{d}k = \int_0^\infty\int_{-1}^{1} 2\pi k^2 Q^q(k,\mu)\,\mathrm{d}k\,\mathrm{d}\mu,$$

Expansion in terms of Legendre polynomial

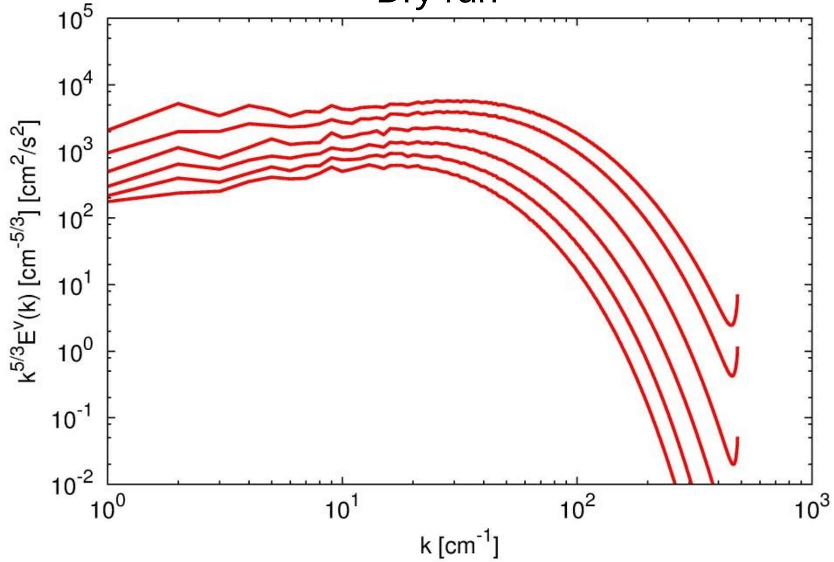
$$Q^\alpha(k,\mu) = \sum_{l=0}^{\infty} Q^\alpha_{2l}(k) P_{2l}(\mu), \qquad \mu = \cos\theta$$

$$E^v_{2l}(k,t) = 2\pi k^2 Q^v_{2l}(k,t), \quad E^\theta_{2l}(k,t) = 2\pi k^2 Q^\theta_{2l}(k,t), \quad E^q_{2l}(k,t) = 2\pi k^2 Q^q_{2l}(k,t).$$

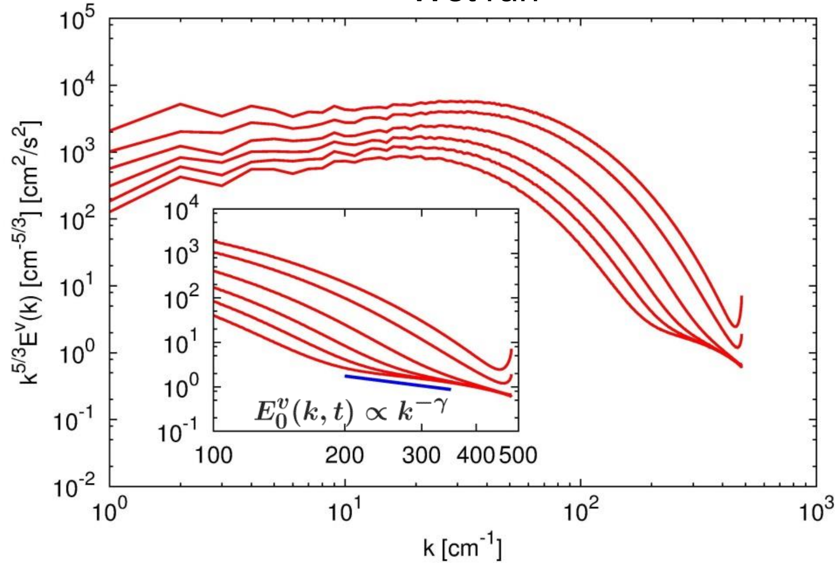
Turbulence modulation (spectrum)

Spectrum modulation begins at high wavenumbers
and propagates toward low wavenumbers

Dry run



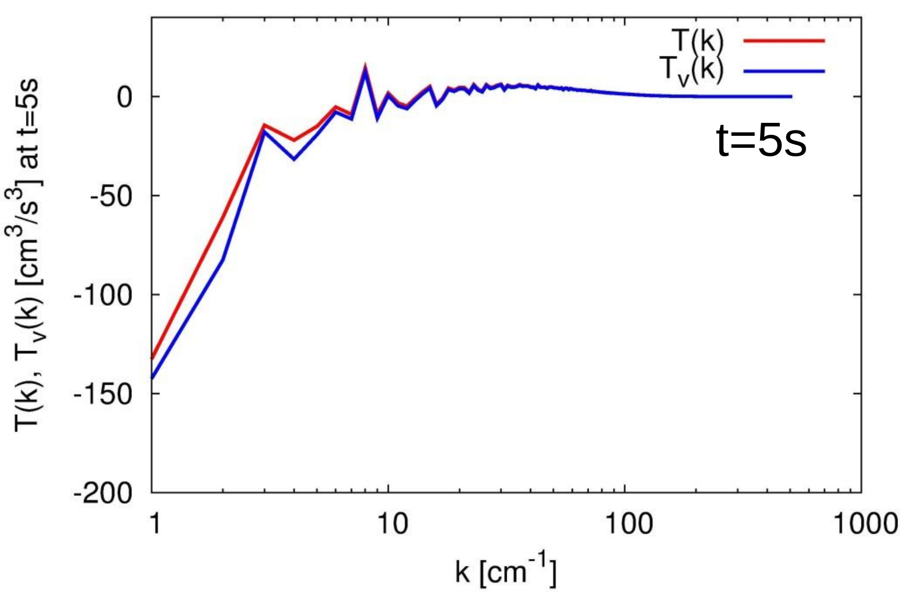
Wet run

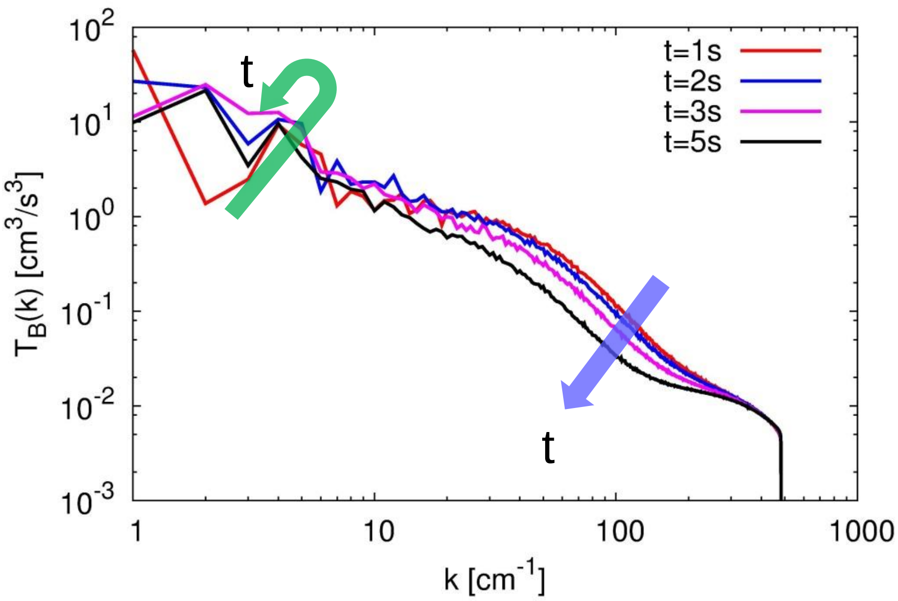


$$E_0^v(k,t) \propto k^{-\gamma}, \quad \gamma = 2.92 \pm 0.05 \quad \text{for} \quad 200 < k < 350$$

Kinetic energy transfer function

$R_\lambda = 252$





$$\frac{\partial E(k,t)}{\partial t} + 2\nu k^2 E(k,t) = T_v(k,t) + T_B(k,t) \equiv T(k,t)$$

$$\begin{aligned} T_B(k,t) &= 2\sum_{shell} \mathrm{Real}\langle u_z(-k,t)B(k,t)\rangle \\ &= 2g\sum_{shell} \mathrm{Real}\Big[\langle u_z(-k,t)\theta(k,t)\rangle + \epsilon\Big\langle u_z(k,t)(q_v(k,t) - q_v^{ref})\Big\rangle - \langle u_z(-k,t)q_l(k,t)\rangle\Big] \\ &= T_{B\theta}(k,t) + T_{Bq_v}(k,t) + T_{Bq_l}(k,t). \end{aligned}$$

Contribution of buoyancy force

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\nabla p + \nu \nabla^2 u + e_z B + f, \qquad \nabla \cdot u = 0$$

$$B = g\left(\frac{T - T_0}{T_0} + \epsilon(q_v - q_{v0}) - q_l\right) = B_\theta + B_q + B_l$$

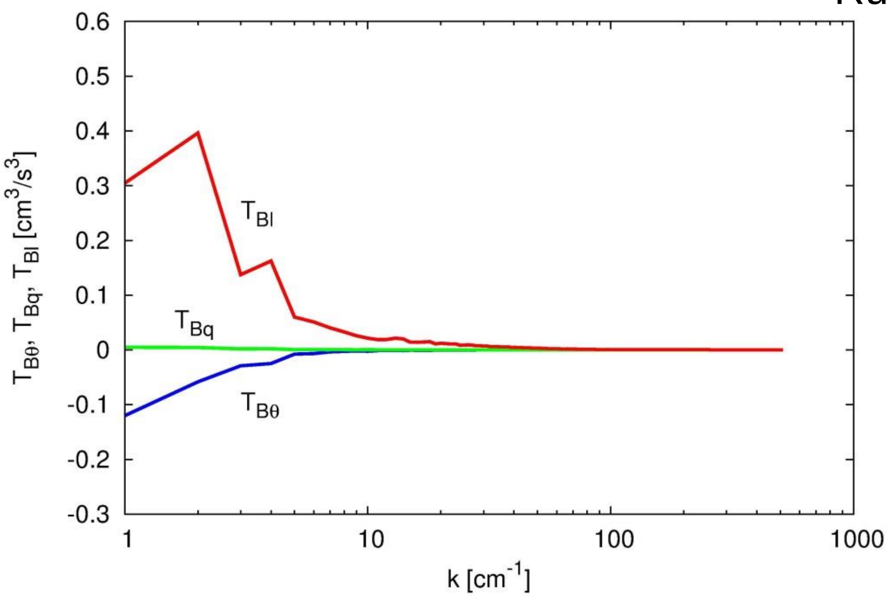
$$\frac{\partial}{\partial t}\frac{\langle u^2 \rangle}{2} = -\epsilon + P_B, \qquad P_B = \langle B u_3 \rangle = \int T_B(k,t)dk$$

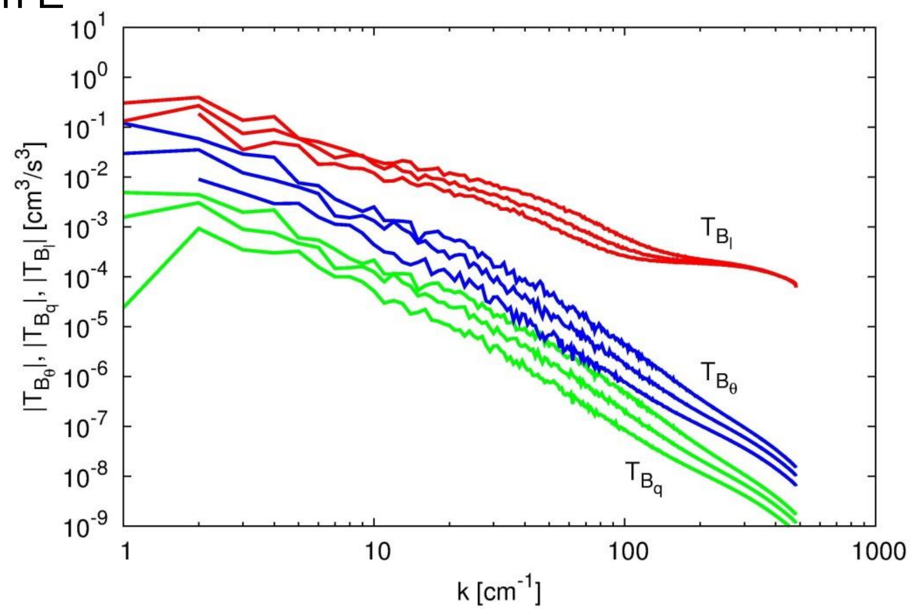
$$T_B(k,t) = 2\sum_{\text{shell}} \text{Real}\left(\langle B_\theta(k,t)u_3(-k,t)\rangle + \langle B_q(k,t)u_3(-k,t)\rangle + \langle B_l(k,t)u_3(-k,t)\rangle\right)$$

$$= T_{B\theta}(k,t) + T_{Bq}(k,t) \underline{+ T_{Bl}(k,t)}$$

dominant

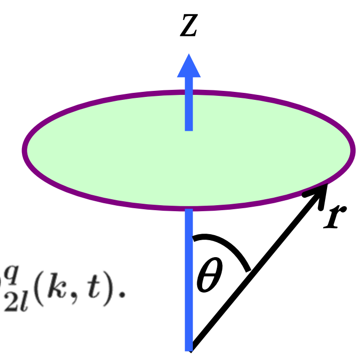
Run E





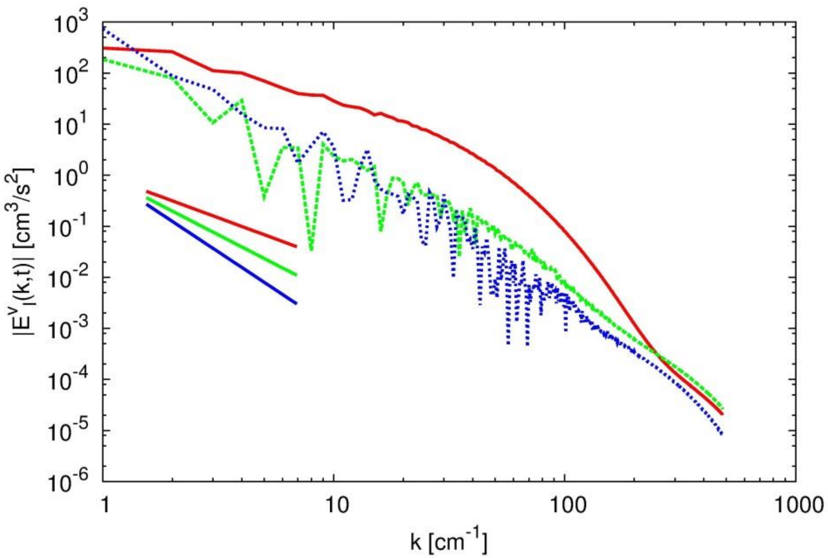
Spectrum of anisotropic sector

$$Q^{\alpha}(k,\mu) \;=\; \sum_{l=0}^{\infty} Q^{\alpha}_{2l}(k)P_{2l}(\mu), \qquad \alpha = v,\ \theta,\ q \qquad \mu = \cos\theta$$

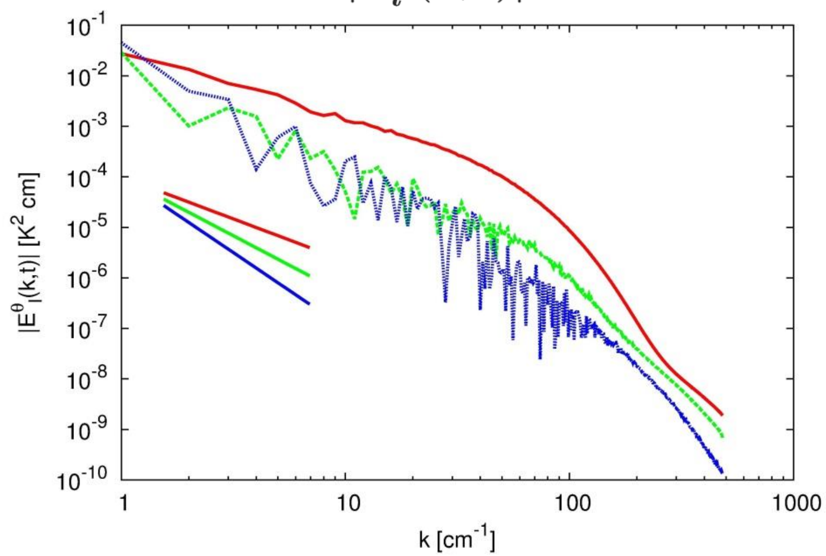


$$E^{v}_{2l}(k,t) = 2\pi k^2 Q^{v}_{2l}(k,t), \quad E^{\theta}_{2l}(k,t) = 2\pi k^2 Q^{\theta}_{2l}(k,t), \quad E^{q}_{2l}(k,t) = 2\pi k^2 Q^{q}_{2l}(k,t).$$

$$|E^{v}_{l}(k,t)|$$

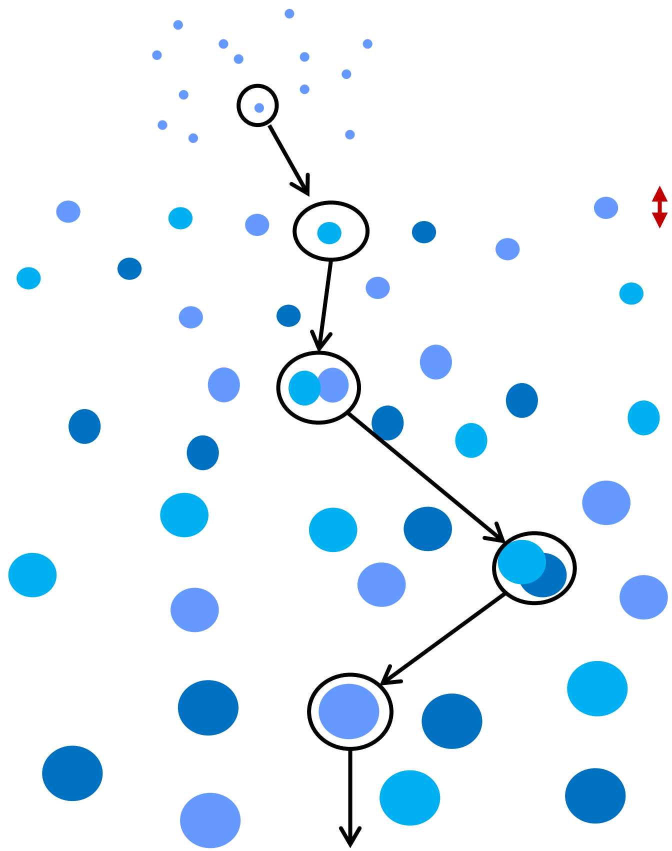


$$|E^{\theta}_{l}(k,t)|$$



$$|E^{\alpha}_{l}(k,t)| = 2\pi k^2 |Q^{\alpha}_{l}(k,t)| \propto k^{-5/3-l/3}, \qquad \text{for} \quad l = 0, 2, 4$$

Droplet Collision



condensation

~30μm

collision / coalescence

two or many body collision

collision scheme
perfect or imperfect coalescence

conservation
mass, momentum, energy

DNS

- **Navier Stokes**

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f}$$

- **External force**

$$\hat{\boldsymbol{f}}\ (\boldsymbol{k}, t) = \begin{cases} a(t)\hat{\boldsymbol{u}}(\boldsymbol{k}, t) & \text{for } 1 \le |k| \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$a(t) = \frac{\epsilon}{\sum_{1 \le |\boldsymbol{k}| \le 2} < |\boldsymbol{u}(\boldsymbol{k})|^2 >}$$

- **Particles**

$$\frac{d\boldsymbol{x}_j}{dt} = \boldsymbol{v}_j$$

$$\frac{d\boldsymbol{v}_j}{dt} = -\frac{1}{\tau_p}\left(\boldsymbol{v}_j - \boldsymbol{u}\left(\boldsymbol{x}_j, t\right)\right)$$

DNS parameters

N_p	:No. of particles	3×2^{18}	Φ	:volume fraction	4×10^{-3}
σ	:particle diameter	1.35×10^{-2}	ε_{in}	:kinetic energy input	0.5

For St number dependence

R_λ	50	80	120
N	64^3	128^3	128^3
Δt	2.0×10^{-3}	8.0×10^{-4}	5.0×10^{-4}
ν	1.0×10^{-2}	4.0×10^{-3}	2.0×10^{-3}
η	0.0376	0.0189	0.0112
τ_η	0.141	0.089	0.063
St	0.04 , 0.1 , 0.4 , 1.0 , 2.0 , 4.0 , 8.0		

$$S_t = \frac{\tau_p}{\tau_\eta}$$

$$N_c = \frac{C}{{L_{box}}^3 \Delta t}$$

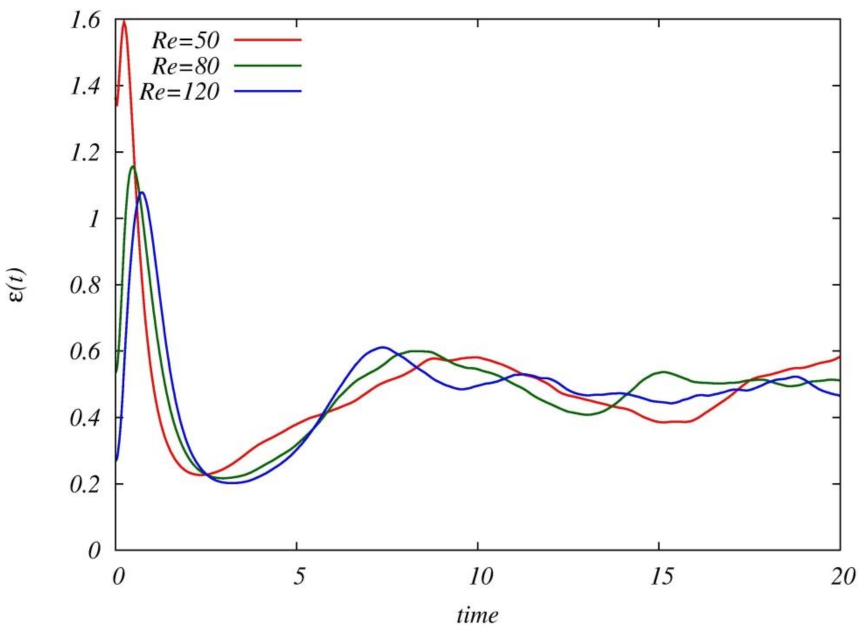
Turbulence statistics

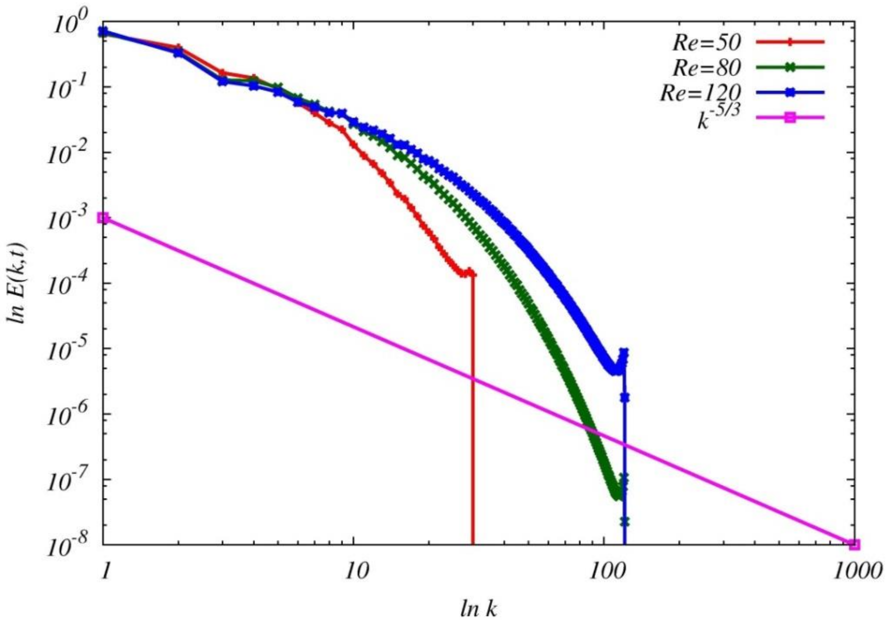
Energy dissipation rate

$$\varepsilon(t) = 2\nu_f \langle (\boldsymbol{\nabla u})^2 \rangle$$

Kinetic energy spectrum

$$E(k,t) = \frac{1}{2} \sum_{k-shell} \left| \boldsymbol{u}(\widehat{\boldsymbol{k}}, t) \right|^2$$

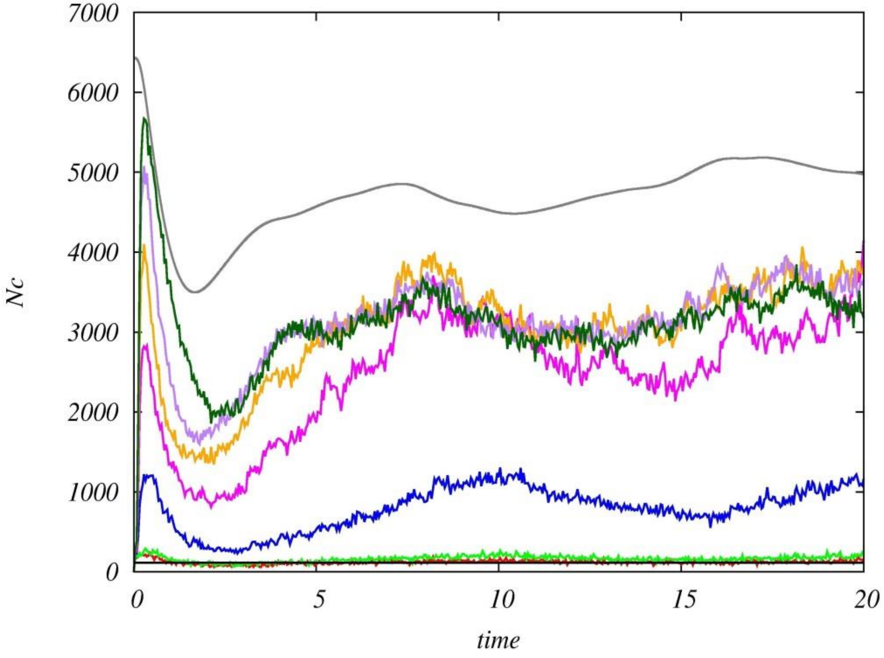




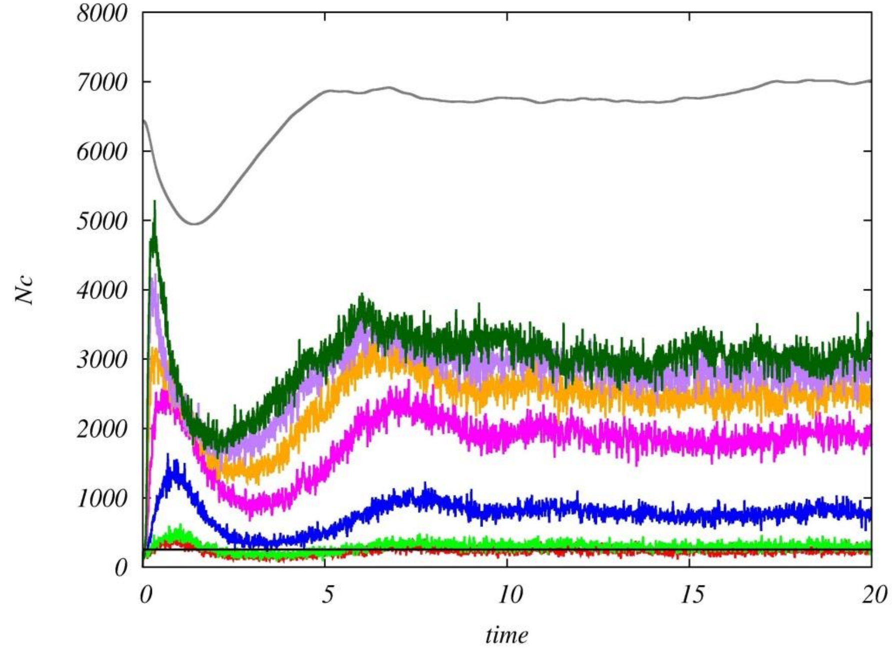
Shiotsu and Watanabe (2014)

Collision frequency

$R_\lambda=50$



$R_\lambda=120$



St=0.04
St=0.1
St=0.4
St=1.0
St=2.0
St=4.0
St=8.0
Saffman-Turner(St→0)
Saffman-Turner(St→∞)

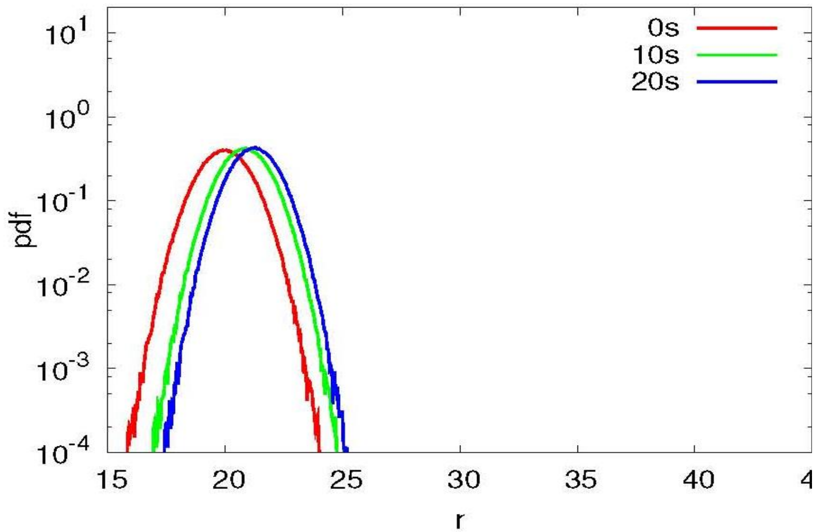
Droplet collision

Condition

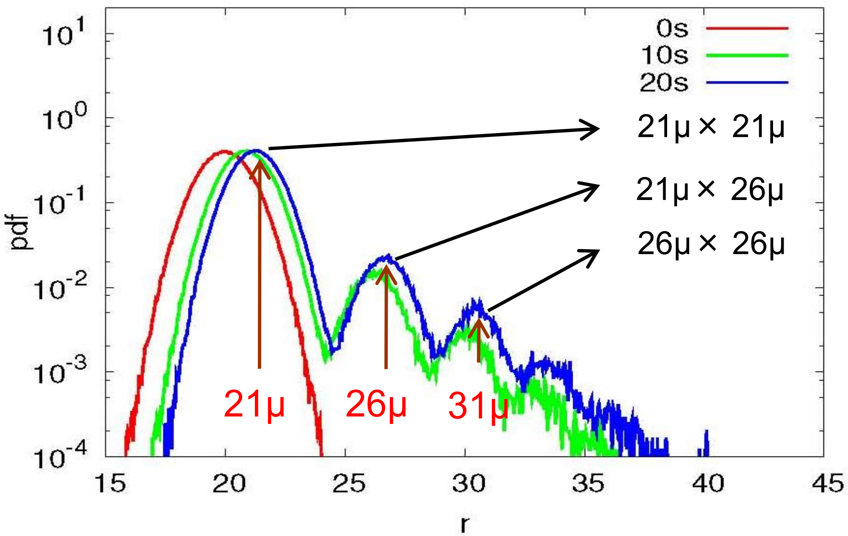
$$r = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \le r_i + r_j$$

- Mass cons. $M_p = M_i + M_j \quad \longrightarrow \quad {r_p}^3 = {r_i}^3 + {r_j}^3$
- Momentum cons. $M_p \boldsymbol{V}_p = M_i \boldsymbol{V}_i + M_j \boldsymbol{V}_j \quad \longrightarrow \quad \boldsymbol{V}_p = \dfrac{M_i \boldsymbol{V}_i + M_j \boldsymbol{V}_j}{M_p}$

No collision

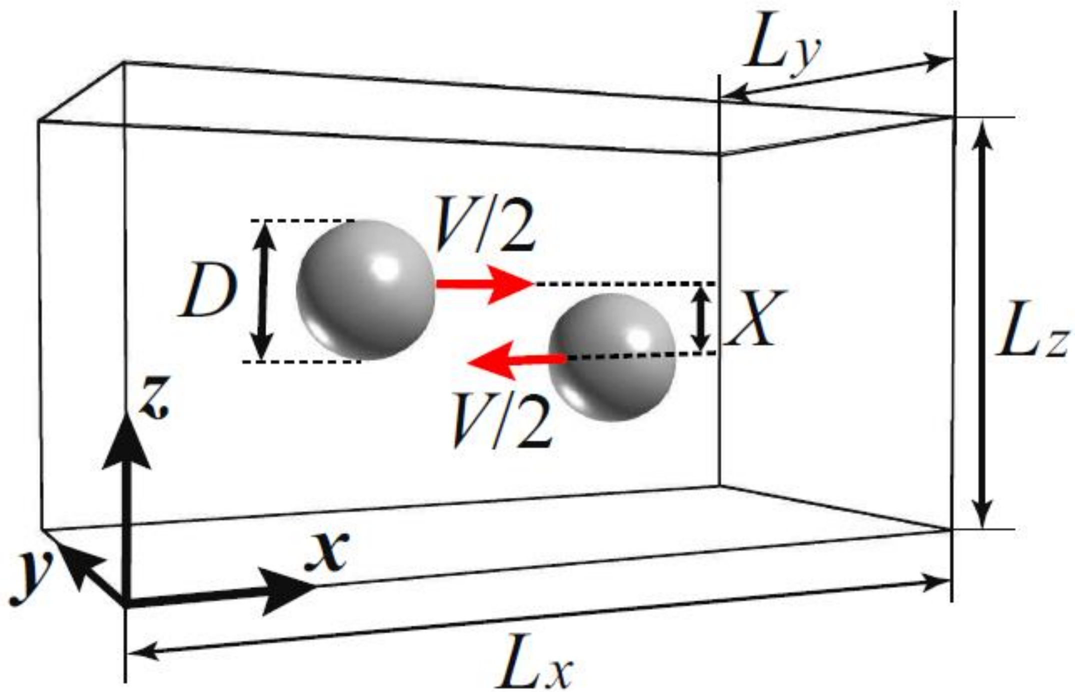


Collision



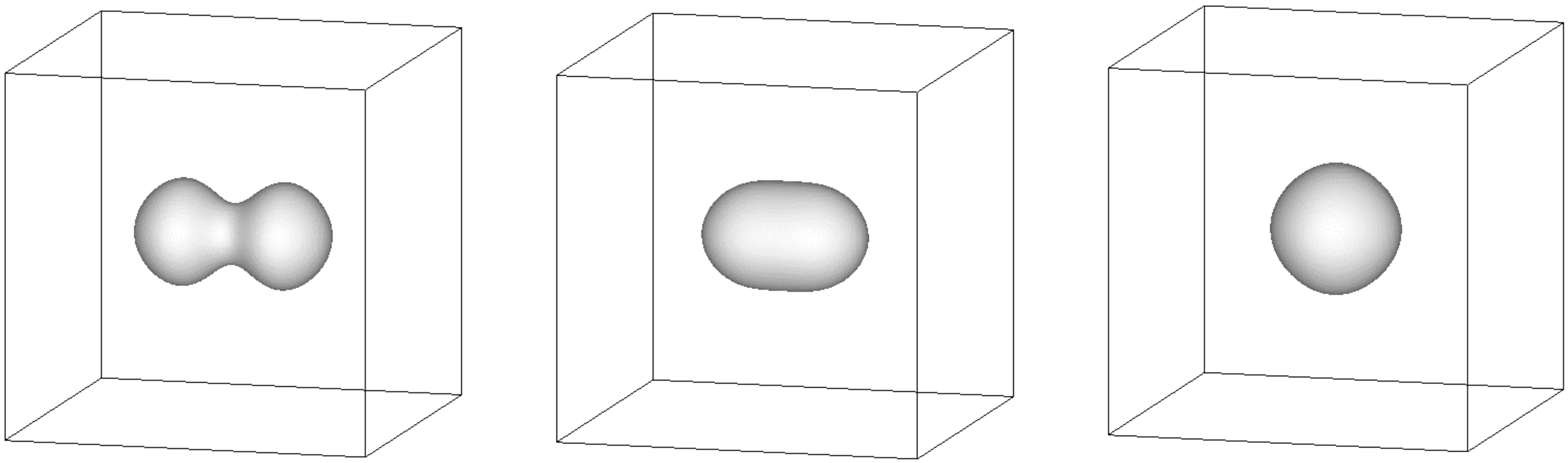
Efficiency : coalescence or separation

Collision of droplets in air by LBM (D3Q15)



Yoshino et al 2014

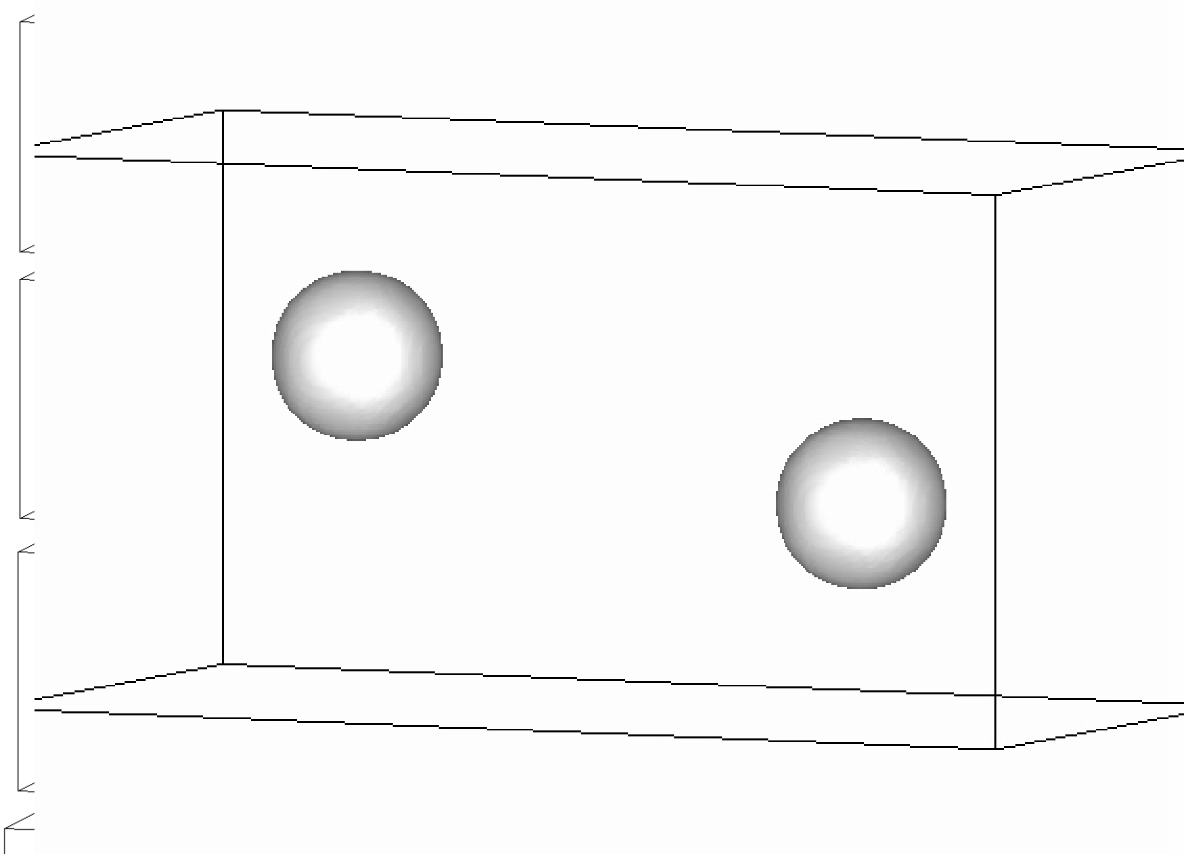
Collision of droplets in air by LBM (D3Q15)



0.34 [m sec]　　0.66 [m sec]　　3.34 [m sec]

d=40 μm　　w= 20 [cm/sec]　　Re=5.26　　ρ_w/ρ_a=50

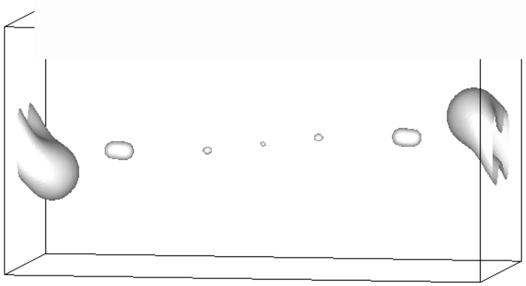
Yoshino et al 2014



r = 250 [μm]
V= 3.42 [m/s]

$B = X / D$

$\mathrm{We} = \rho_{\mathrm{L}} D V^2 / \sigma$



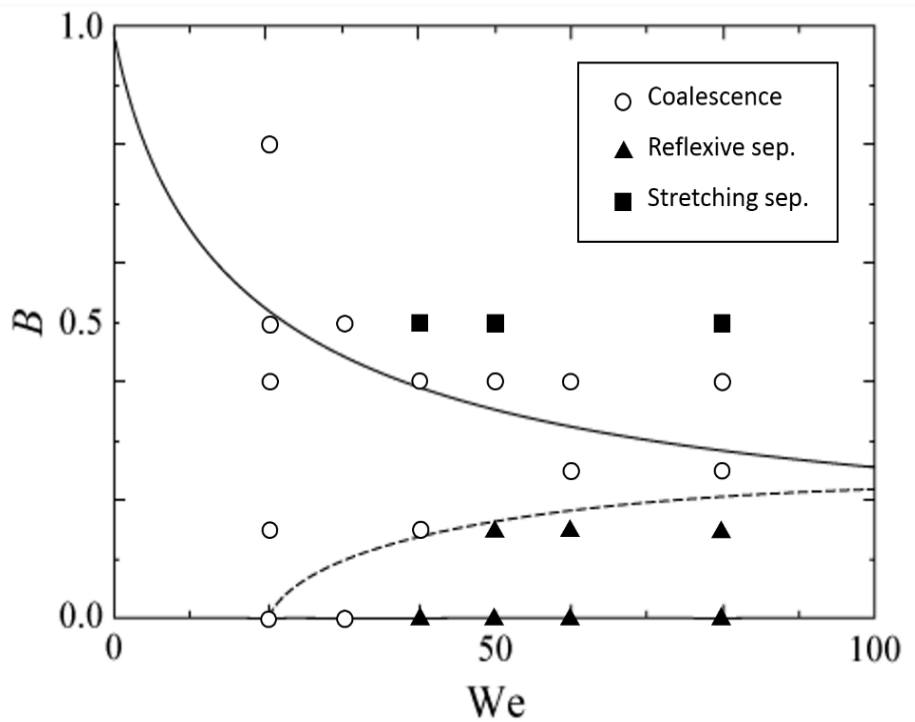
1.92 [m sec]

Yoshino et al 2014

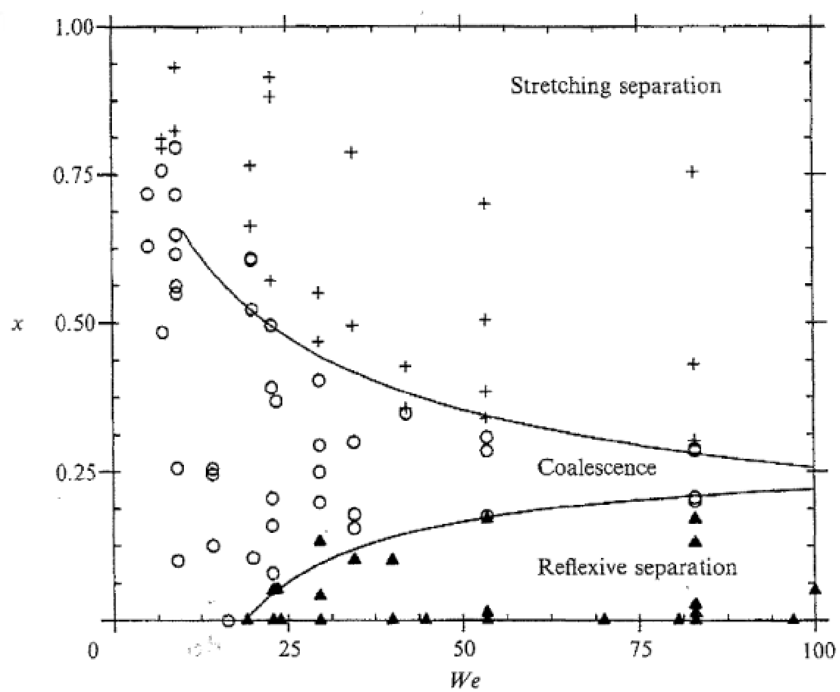
Collision of stretching separation B = 0.5, We = 79.6

We – B diagram

LBM

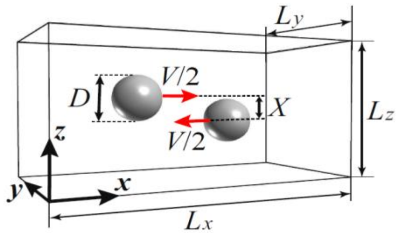


EXP



Yoshino (2014)

Ashgriz and Poo JFM (1990)



$B = X/D$

$\mathrm{We} = \rho_L D V^2/\sigma$

Development of Fast Time-reversible (FT) method

FT method

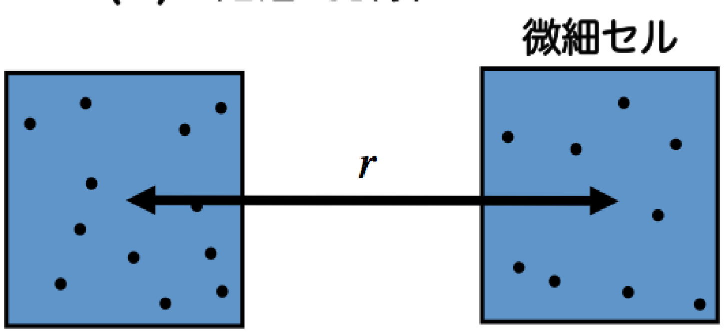
Y.Kajima et al., J. Chem. Phys. 136 (2012) 234105.

- Fastest algorithm for molecular dynamics of water molecules of submicron meter
- Time reversal symmetry in translational and angular momentum evolution
- Use of 4 arithmetic operations (+,−,x,/) alone and 1/3 of the conventional computation
- Total energy is well conserved for larger time step width

O(N) code for large scale computation of the water molecules by combined use of FT method and fast multi-pole expansion

O(N) computation of the Coulomb force

O(N)個の遠方微細セルを中心とした多重極係数セットに関するO(N)の階層的計算:



Quasi-Liquid Layers on Ice Crystal Surfaces

Sazaki et al. (2012)

laser confocal microscopy combined with differential interference contrast microscopy

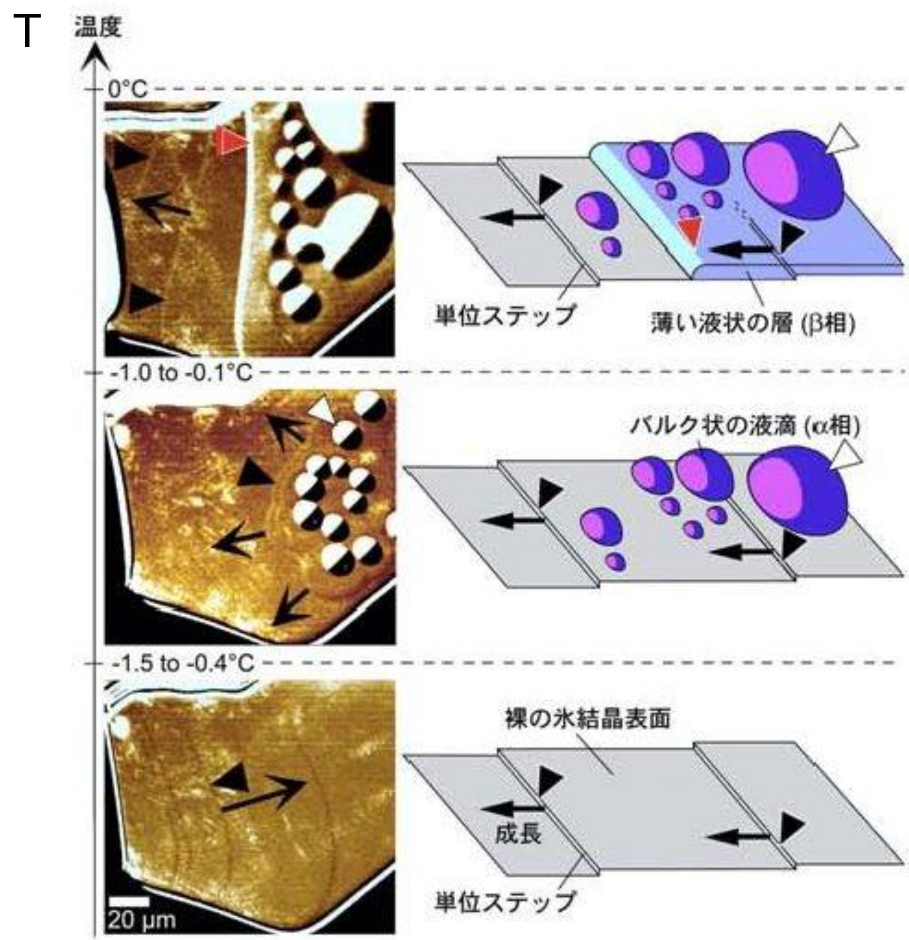
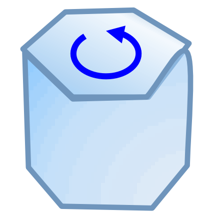


図1 氷結晶の六角底面上で生成する2種類の表面液体相。

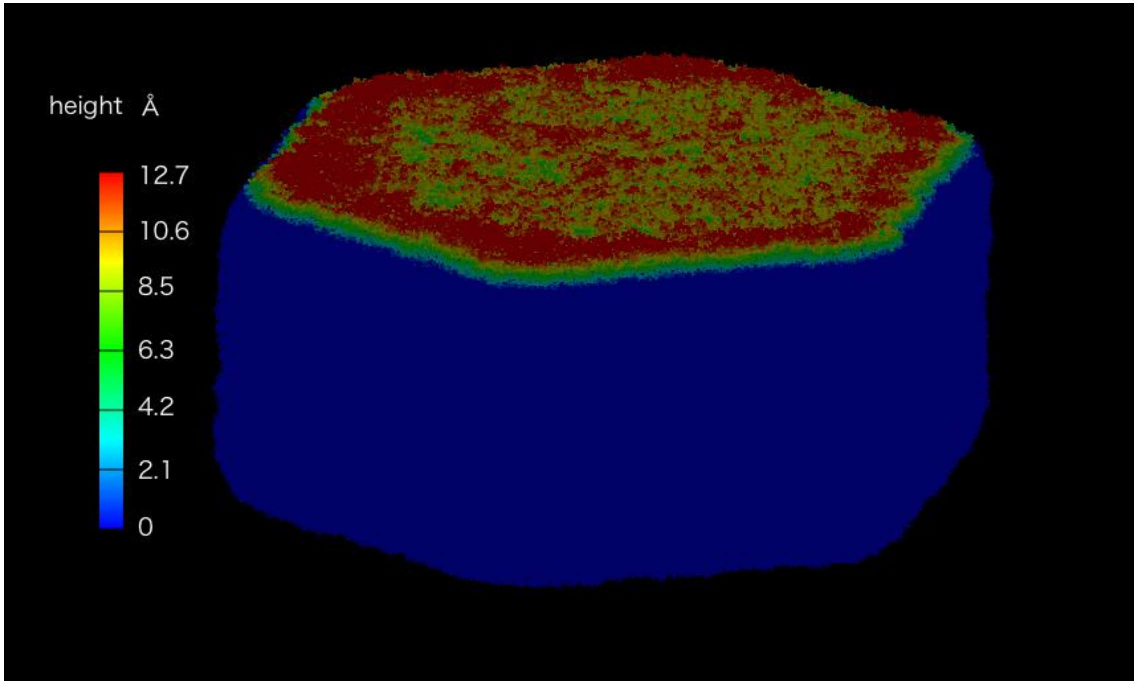


Skating
Thunder
Snow flake
Freezing of food and organ

Large scale MD simulation of submicron ice crystal

Melting of ice crystal at $(T_c\text{-}1)$ K

displacement



~ 0.08μm

Kajima et al. 2014

Summary

- Reynolds number effects (no collision)
  may be small on growth rate of the average particle radius
  broaden PDF of particle radius

- Average growth of cloud droplets governed by diffusion process is very slow
  Needs for particle collision process
  Needs for uniform gradient source of temperature and vapor (updraft)

- Turbulence modulation begins at small scales due to the cloud droplets and the anisotropic modulation is inversely transferred to large scales

- LBM simulation of the cloud droplet collision in air is successful. Detailed data of the collision process will be obtained in the near future.

- MD simulation of the micro ice crystal found that the melting process (two step stage at the temperature near the melting point) is consistent with the experimental observation

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