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Effects of gravity on the spatial clustering of inertial particles in isotropic turbulence

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Organizers of the Program

Contents

1. Background and Motivations

Turbulent clustering has general significances on many physical and chemical processes

eg. Rapid growth of rain droplets at 10-50μm by turbulent coalescence.
Geometrical collision kernel (Sundarim & Collins(1997))

$$\Gamma = 2\pi R^2 \langle |w_r| \rangle g(R)$$

The radial distribution function $g(R)$ denotes the level of clustering.

Mechanisms of clustering of inertial particle, for examples

(1) Centrifugal effects of the vortical structures on particles (Maxey, 1987)

Preferentially cluster at regions of low vorticity and high strain rate

(2) Sling effects, caustics, uncorrelated random motion: multiple-value of particle velocity field (*Falkovich,et al. Nature, 2002; Wilkinson & Mehlig, EPL, 2005; Gustavsson et al. NJP, 2013*) **Cluster at caustic lines or surfaces**

(3) Random multiplicative process in a random flow at large Stokes number and small Kubo number (*Wilkinson, Mehlig, Ostlund &Duncan, PoF, 2007)*

$$Ku = u\tau/\xi \to 0; St = \tau_p/\tau \to \infty; \varepsilon = Ku\sqrt{St} \sim O(1)$$

1. Background and Motivations

Gravity suppresses the level of clustering at small Stokes numbers (Falkovich & Pumir, PoF, 2004)

$$g(r) = C_0 \left(\frac{\eta}{r}\right)^{C_1}, \quad \text{for } r \ll \eta.$$

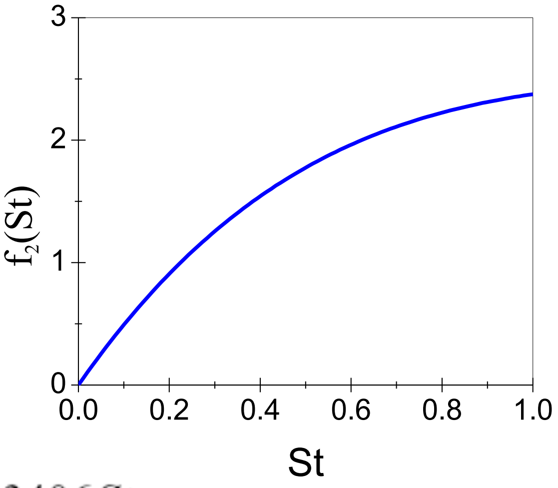
Chun et al, JFM, 2005

$$C_1 = \frac{f_2(St)}{(|\mathbf{g}|/(v_k/\tau_k))^{f_1(R_\lambda)}},$$

where

$$f_1(R_\lambda) = 0.1886 \exp\left(\frac{20.306}{R_\lambda}\right),$$

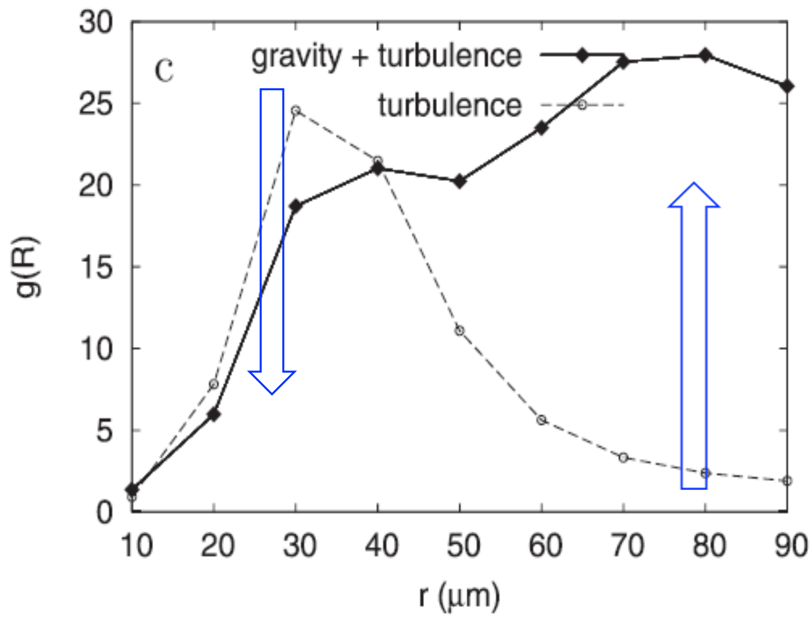
$$f_2(St) = -0.1988St^4 + 1.5275St^3 - 4.2942St^2 + 5.3406St.$$

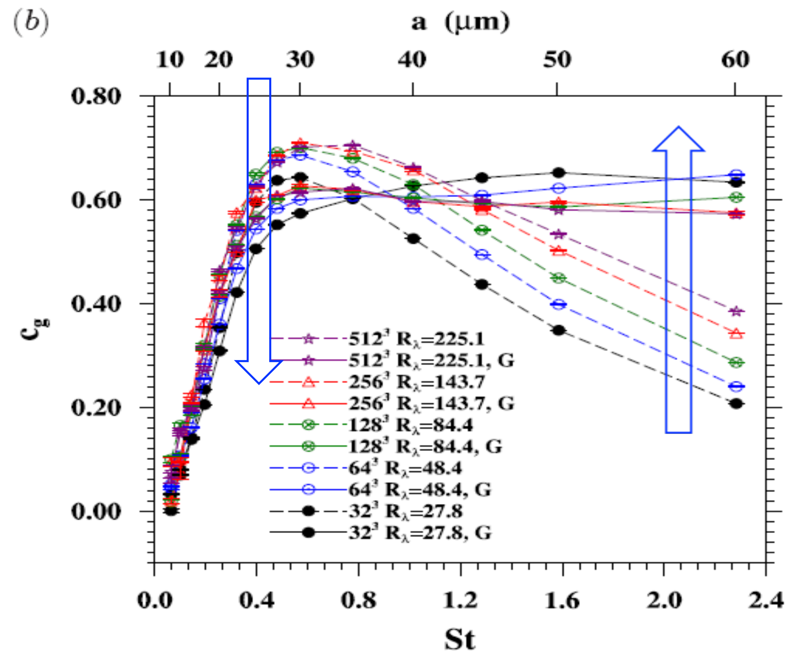


Ayala, Rosa & Wang, NJP, 2008

1. Background and Motivations

How about the effects of gravity on clustering at large Stokes numbers?





r is related to Stokes number

$$g(r) = c_0(\eta/r)^{c_g(St,S_v)},$$

(Woittiez, Jonker & Portela, J. Atmos. Sci., 2008)

Rosa & Wang, NJP, 2013

Motivations:

(1) Effects of gravity on clustering, reduce or enhance, parameter range?

(2) Possible reasons?

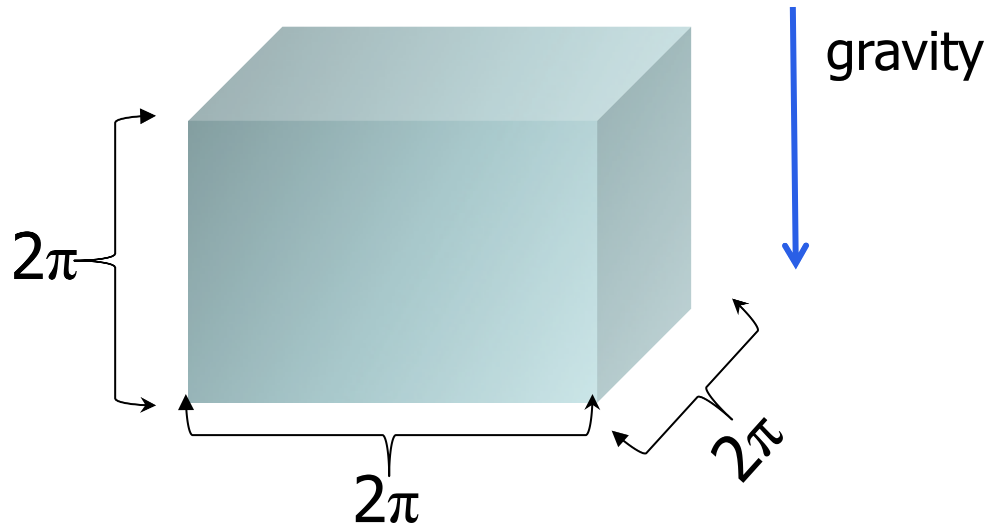
2. Numerical methods for isotropic turbulence

Navier-Stokes equations in spectral space

$$\left(\partial/\partial t + \nu k^2\right)\hat{\mathbf{u}}(\mathbf{k},t) = \mathbf{P}(\mathbf{k})F(\mathbf{u}\times\boldsymbol{\omega}) + \hat{\mathbf{f}}(\mathbf{k},t),$$

$k = |\mathbf{k}|$ is the wavenumber

$\hat{\mathbf{f}}(\mathbf{k},t)$ is the large-scale forcing at low wave numbers $k < \sqrt{8}$



Flow domain: cubic box with each side 2π

Numerical Methods for particle motions

Assumptions: (1) Small, $d_p = 0.5\eta < \eta$

(2) Heavy, $\rho_p >> \rho_f$

(3) Dilute: $\alpha_p < 10^{-6}$

One-way coupling

Particle equation:

$$\frac{dx_p(t)}{dt} = v_p(t),$$

$$\frac{dv_p(t)}{dt} = \frac{(U(x_p(t),t) - v_p(t))f(Re_p) + w}{\tau_p}$$

$U(x_p(t),t)$ fluid velocity seen by inertial particle

τ_p Particle relaxation timescale, $\tau_p = \dfrac{\rho_p d_p^2}{18\mu}$

w Particle settling velocity, $w = \tau_p g$

6-order Lagrangian interpolation for fluid velocity seen by inertial particle

Parameters space dominating the system

Nondimensional parameters

Turbulence Reynolds number $Re_\lambda = \dfrac{u'\lambda}{\nu}$, Turbulence structure, Intermittency

Particle Stokes number $St_K = \dfrac{\tau_p}{\tau_K}$, Particle inertia

Particle Froude number based on velocity $Fr_{p,v} = \dfrac{w}{u'}$ Settling velocity

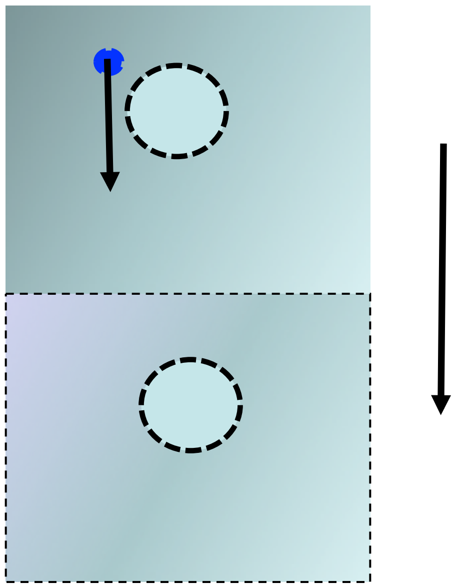
Particle Froude number based on acceleration $Fr_{p,a} = \dfrac{g}{a_K}$ gravity

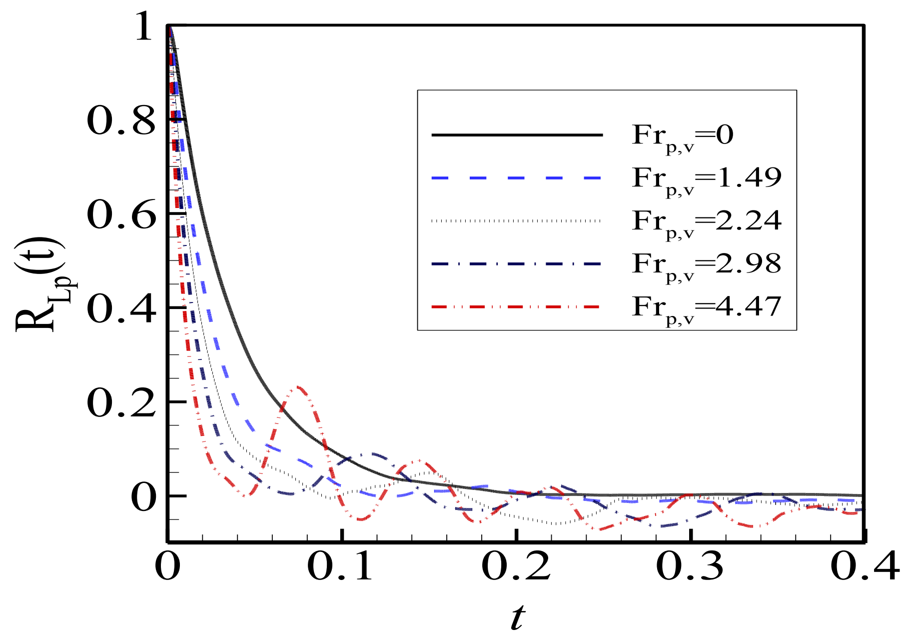
$$Fr_{p,v} = \frac{w}{u'} = \frac{\tau_p g}{\sqrt{\mathrm{Re}_\lambda} v_K} = \frac{\tau_p g}{\sqrt{\mathrm{Re}_\lambda} a_K \tau_K} = \frac{St_K Fr_{p,a}}{\sqrt{\mathrm{Re}_\lambda}}$$

Parameters space $\left\{\mathrm{Re}_\lambda, St_K, Fr_{p,v}\right\}$

Settling velocity limit due to periodic BCs

Particle-eddy interaction





If $Fr_{p,v}$ is too large, a settling particle may encounter the same eddies in next cycle.

$$R_{Lp}(t) = \frac{\left\langle u_y(x_p(t_0),t_0)u_y(x_p(t_0+t),t_0+t)\right\rangle}{\left\langle u_y^2(x_p(t_0),t_0)\right\rangle}$$

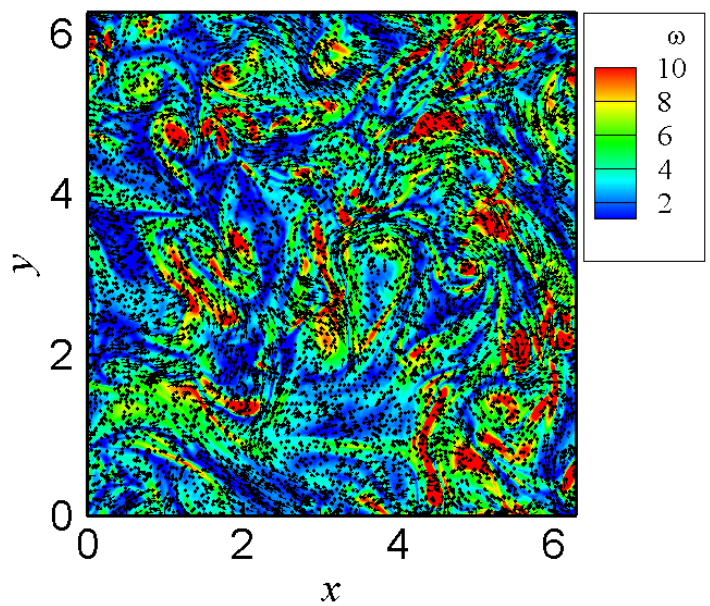
Eddy decays slower than particle going through the box

Oscillations indicate the settling velocity is too large

$$Fr_{p,v} < 3.0$$

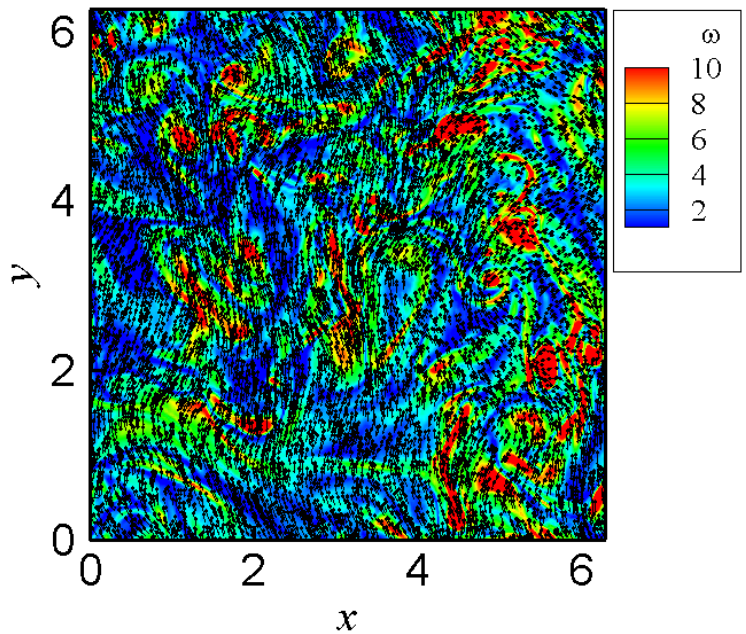
3. Results and Discussions

3.1 Visualization of clustering at St_K =1.0



St_K =1.0 and $Fr_{p,v}$=0.0

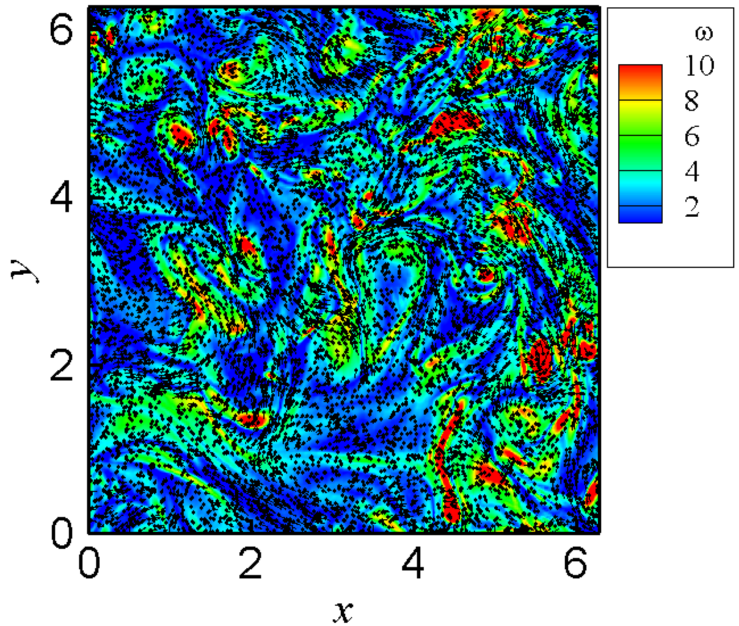
At small Stokes without gravity, particles in blue regions



St_K =1.0 and $Fr_{p,v}$=2.98

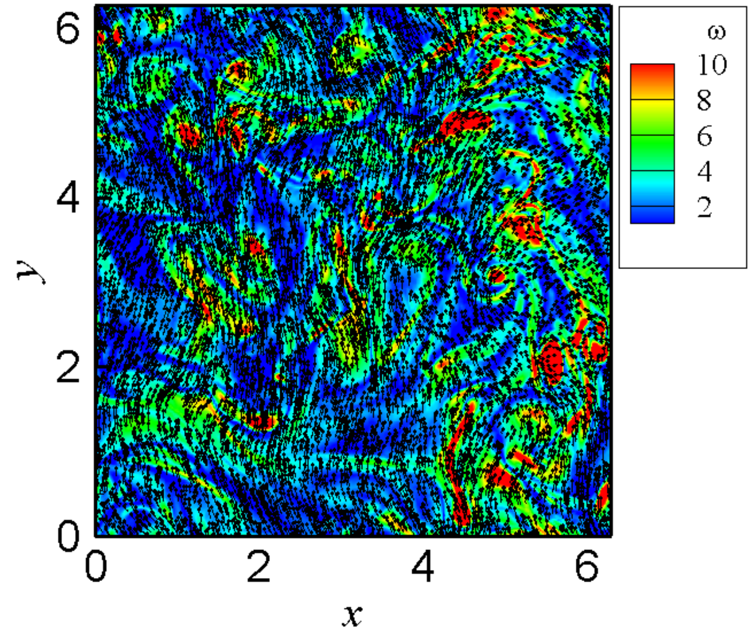
Gravity pulls the particle out of blue region and concentration becomes uniform

3.1 Visualization of clustering at St_K =5.0



St_K =5.0 and $Fr_{p,v}$=0.0

At large Stokes without gravity, level of clustering decreases due to large response timescale compared to Kolmogorov timescale

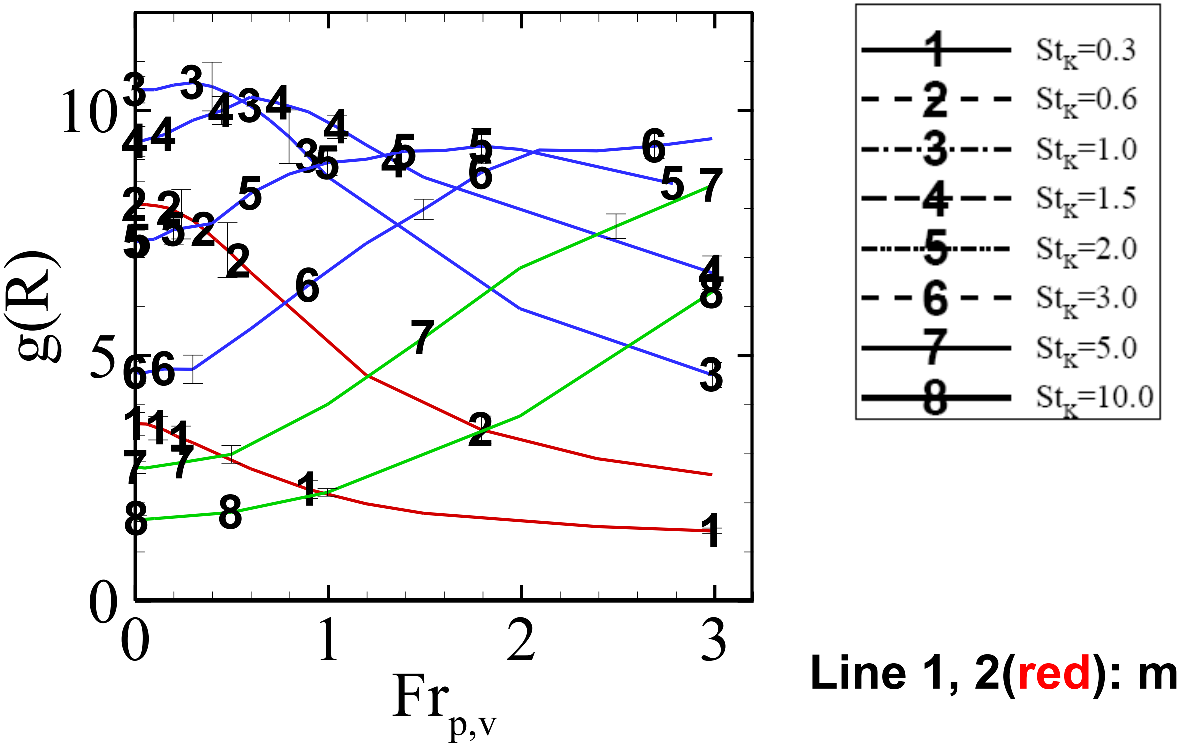


St_K =5.0 and $Fr_{p,v}$=2.98

1) At large Stokes with gravity, level of clustering increases when settling velocity dominate compared to turbulence velocity
2) Vertical structures are observed

3.2 Quantity the level of clustering

3.2.1 Radial distribution function: g(R)



(a) RDF vs. $Fr_{p,v}$

Line 1, 2(red): monotonically decreases

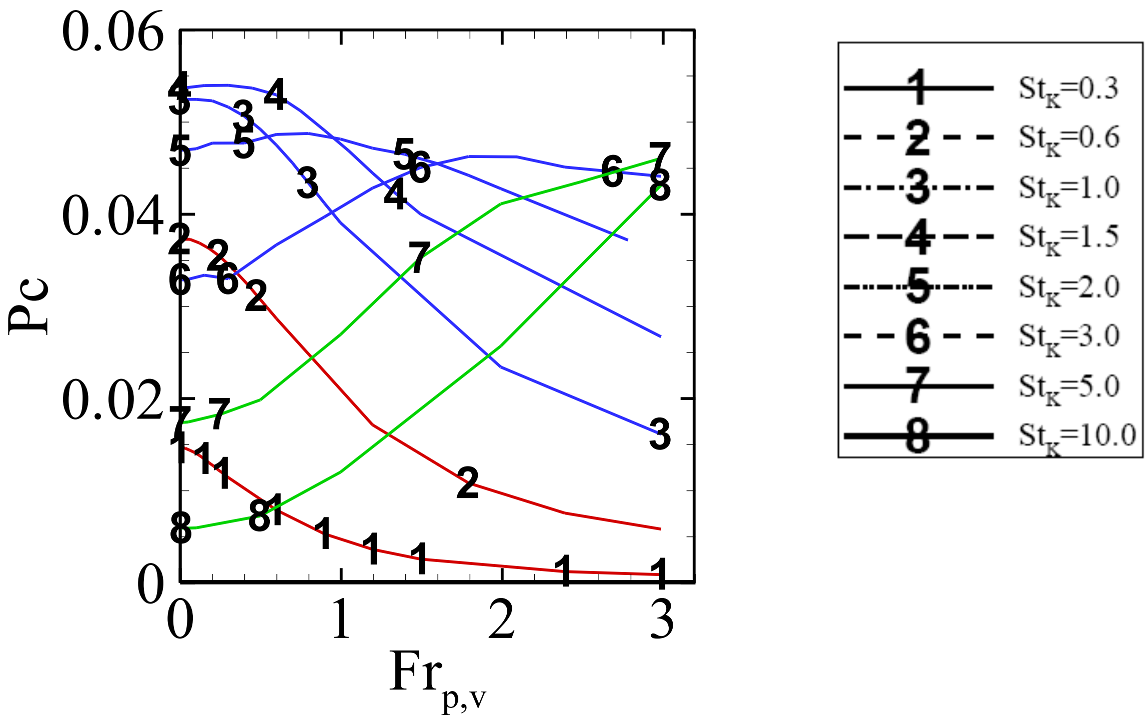
Line 3, 4, 5, 6(blue): Increases, then decreases

Line 7,8(green): monotonically increases

3.2.2 Box counting method

$$Pc = \sum_{i=0}^{10} (p(i) - p_{Poisson}(i))^2$$

The initial PDF of No. of particles in cells is Poisson



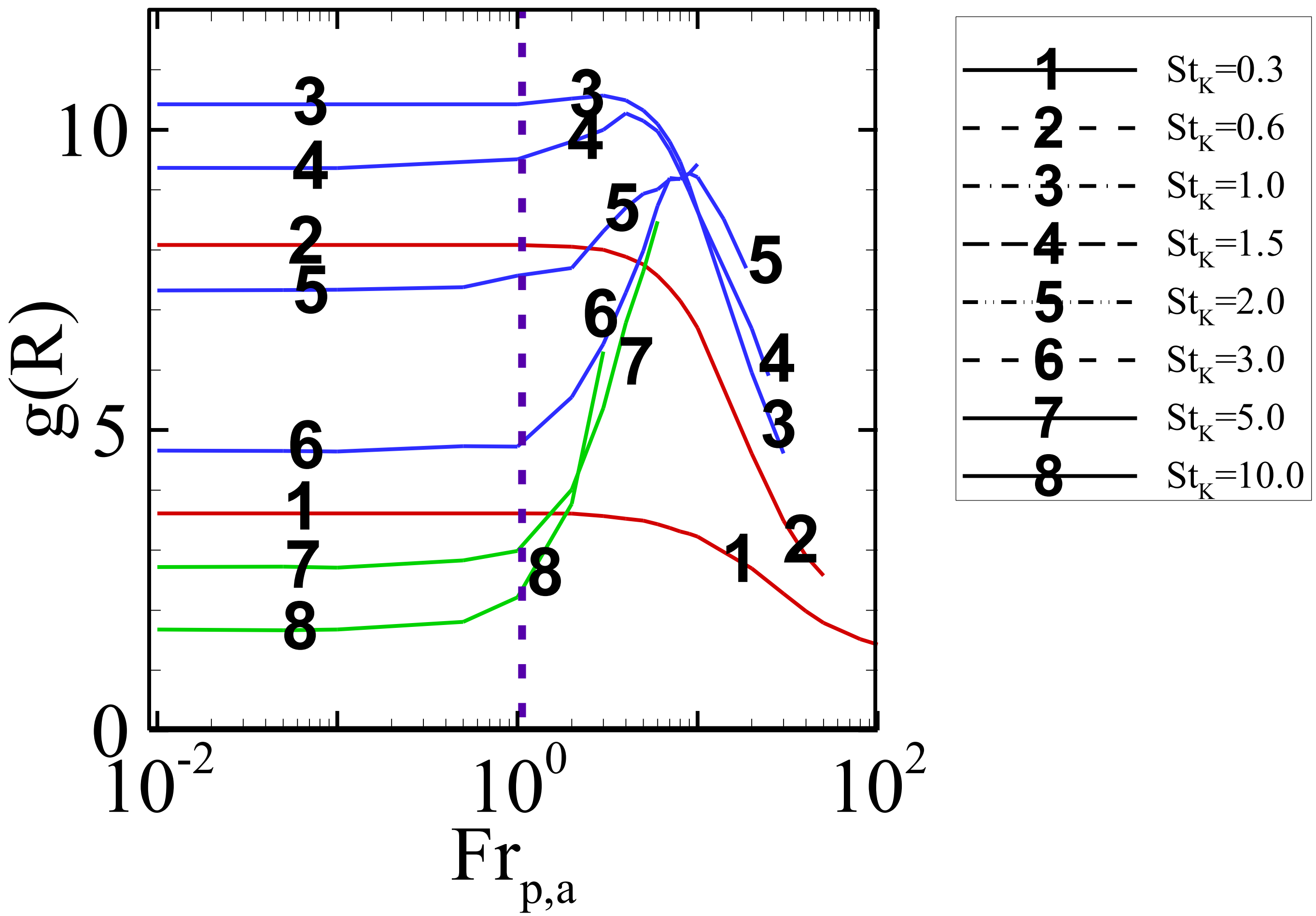
(b)Pc vs. $Fr_{p,v}$

Line 1, 2(red): monotonically decreases

Line 3, 4, 5, 6(blue): Increases, then decreases

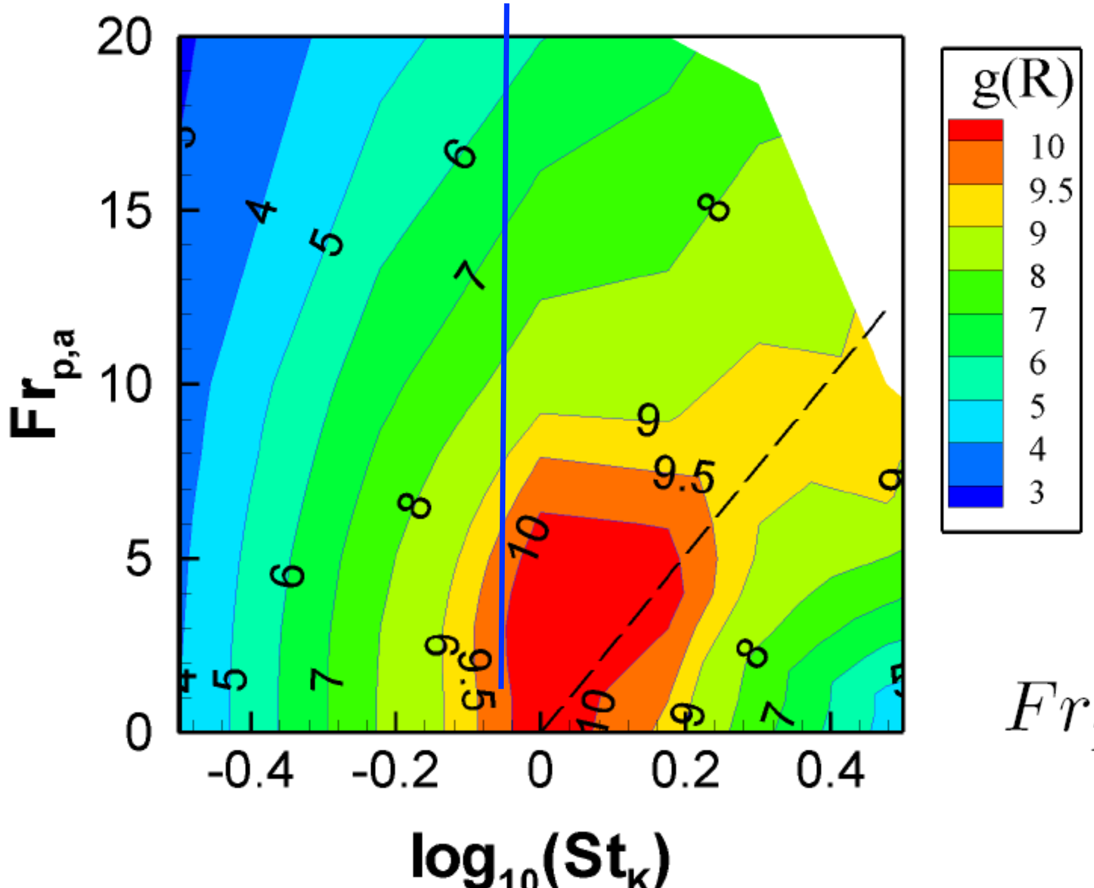
Line 7,8(green): monotonically increases

3.2.3 RDF Vs. $Fr_{p,a}$



Gravity only functions at $Fr_{p,a} = g/a_K > 1$

3.2.4 Map of pattern



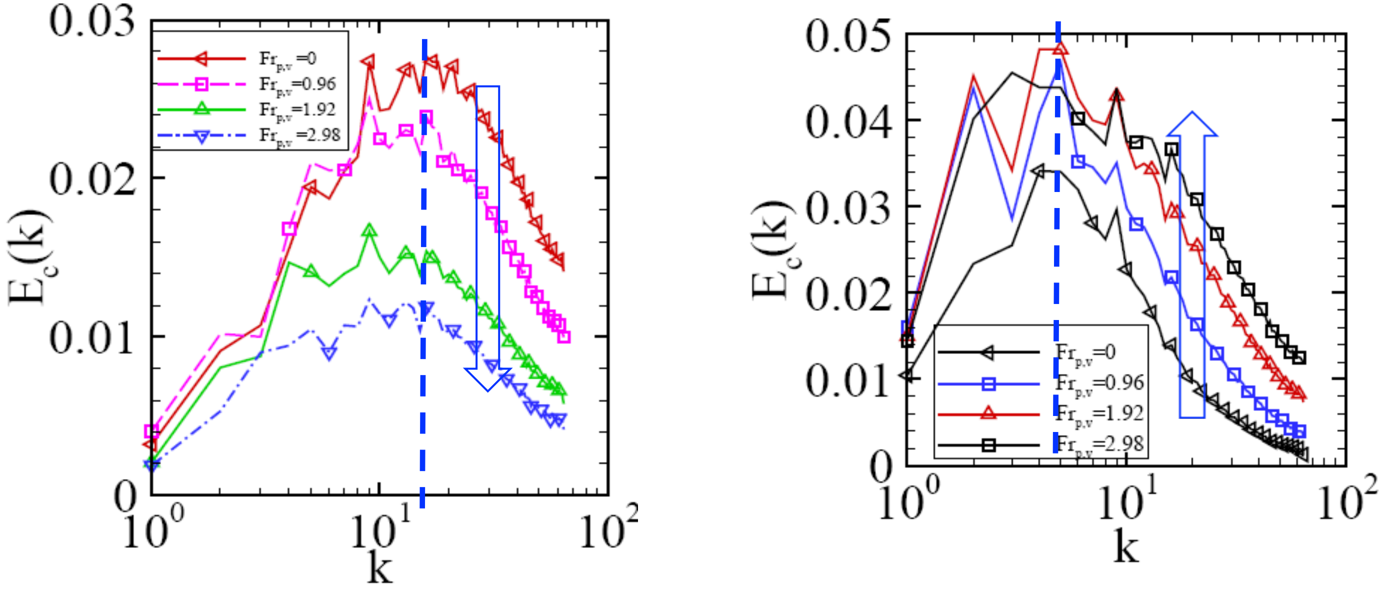
$$Fr_{p,a} = 25.5\log_{10}(St_K)$$

For St_K<1, gravity monotonically reduces clustering

For St_K>1, Clustering has a dividing line, non-monotonical

3.2.5 Spectrum of concentration fluctuations

$E_c(k)$ of $C(\boldsymbol{x},t) - C(\boldsymbol{x},0)$:contribution of spatial cluster at wave number k



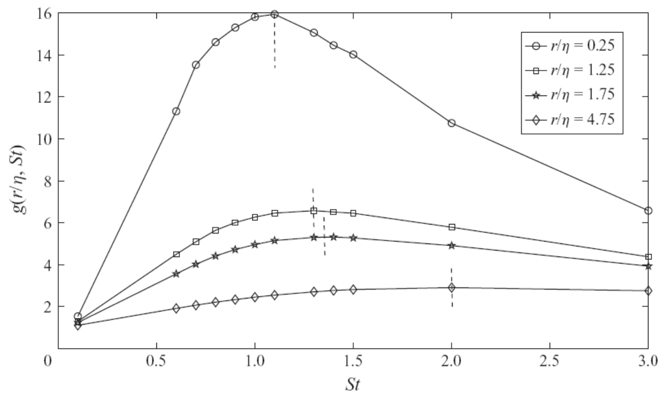
(a) $St_K = 1.0$ (b) $St_K = 5.0$

1) The wave number at peak: characteristic length scale of the clusters

2) The integration area under each line: variance, the level of clustering

$$\langle C^2 \rangle - \langle C^2(0) \rangle = \int_1^\infty E_c(k)dk$$

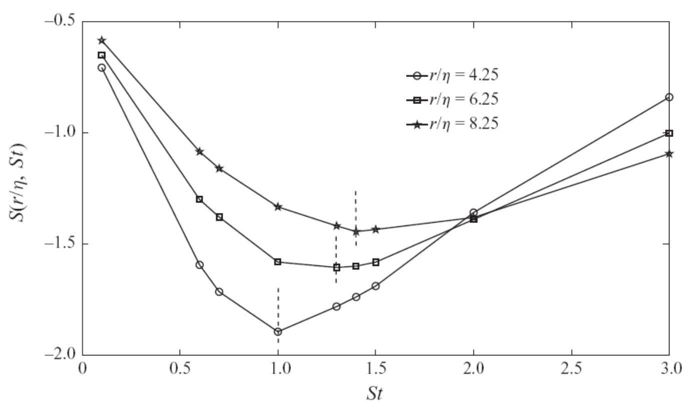
3.3 Level of clustering and skewness of particle relative velocity(1)



Ray & Collins, JFM, 2011

RDF

FIGURE 3. Variation of the RDF with St at the indicated values of r/η. Vertical dashed lines are at the approximate maximum in each curve.



$$S = \langle w_r'(R)^3 \rangle / \langle w_r'(R)^2 \rangle^{3/2}$$

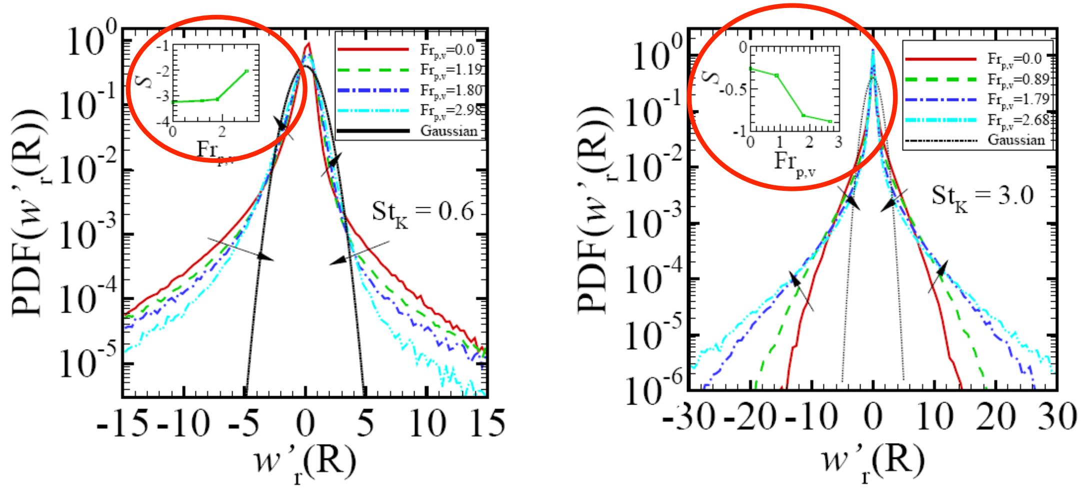
Skewness: the tendency of clustering after a stationary state or compression of particle velocity field

FIGURE 14. Variation of the skewness of the PDF of the radial relative velocity as a function of St at the indicated values of r/η. Dashed vertical lines show approximate peaks.

3.3 Level of clustering and skewness of particle relative velocity(2)

Standardization of particle relative velocity

$$w'_r(R) = (w_r(R) - \langle w_r(R) \rangle)/\sqrt{\langle w_r(R)^2 \rangle}$$



(a) $St_K = 0.6$ (b) $St_K = 3.0$

1) The rms of relative velocity decreases at all Stokes numbers
2) Gravity makes PDF at small St approach to be Gaussian, while makes PDF to be apart from Gaussian distribution
3) Gravity decreases the magnitude of S at small Stokes number
4) Gravity increases the magnitude of S at large Stokes number

3.4 Explanations(1)

Particle 1

$$\frac{dv_{p,1}(t)}{dt} = \frac{(U(x_{p,1}(t),t) - v_{p,1}(t)) + w}{\tau_p}$$

Particle 2

$$\frac{dv_{p,2}(t)}{dt} = \frac{(U(x_{p,2}(t),t) - v_{p,2}(t)) + w}{\tau_p}$$

Relative motion

$$\frac{d\left[\Delta v_p(t)\right]}{dt} = \frac{\Delta U(x_{p,1}(t), x_{p,2}(t), t) - \Delta v_p(t)}{\tau_p} \approx \frac{\frac{\partial U}{\partial x}\Delta x_p(t) - \Delta v_p(t)}{\tau_p}$$

Decorrelation time scale of velocity gradients is on the order of the Kolmogorov turnover time scale (Yu and Meneveau, PRL, 2010)

The timescale of *fluid* velocity increment or velocity derivative seen by the nearby particle pair is on the order of Kolmogorov timescale τ_K

3.4 Explanations(2)

The timescale of particle pair go through the Kolmogorov eddy is η/w_0
where $\mathbf{w_0}$ is the settling velocity, η Kolmogorov length scale

The timescale between the particle pair and fluid under gravity is

$$\tau = [(1/\tau_K)^2 + (w_0/\eta)^2]^{-1/2} \approx \eta/w_0$$

Effective Kubo number seen by inertial particle pairs

$$k_e = v_K\tau/\eta \propto v_K(\eta/w_0)/\eta \quad = v_K/w = a_K\tau_K/g\tau_p \quad = 1/(St_K Fr_{p,a}) < 1$$

Random multiplicative mechanism begins to dominate and contribute to the clustering (Wilkinson, Mehlig, Ostlund &Duncan, PoF, 2007)

4. Summary

(1) Two regimes for the effects of gravity on clustering are identified and related to the dynamics of particle relative motion:

Gravity suppresses clustering when $St_K<1$

Gravity enhances clustering when $St_K>1$ *and settling velocity dominate turbulence*

(2) Gravity suppresses clustering by reducing the centrifugal effects of particle at small Stokes number

(3) Gravity enhances clustering by random multiplicative mechanism for particle at small Stokes number with an effective Kubo number less than 1.

$$k_e \propto 1/(St_K Fr_{p,a}) < 1$$

Thank you for your attention!

Multiplicative random process

Density fluctuations are generated by a multiplicative random process:
volume elements in the particle flow are randomly compressed or expanded, and

the ratio of the final density to the initial density after many multiples of the correlation time can be modeled as a product of a large number of random factors.

The particle density is expected to have a log-normal probability distribution, and the mean value of the logarithm is related to the Lyapunov exponents of the flow.