Gravity-driven enhancement of heavy particle clustering in turbulent flow

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Dynamics of Particles in Flows, NORDITA, Stockholm, Sweden

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Outline

- Introduction
- Settling Velocity
- Two-particle, Small-scale Properties
  - Correlation Dimension
  - Approaching Rates
- Conclusions and Perspective

Introduction

- Many industrial, atmospheric, and astrophysical phenomena involves the interactions between small solid particles suspended in a turbulent carrier flow.
- Two main effects:
  - a viscous drag on the particles (*dominant for small particles*);
  - external forces, such as gravity, on the particles (*dominant for large particles*).
- Standard modelling treats these two limits separately and often fails at the interface.
  - Example: the rate at which rain is triggered in warm clouds.
- An improvement might be to combine the effects of turbulence and gravity.

G. Falkovich, *et al*, Nature **419**, (2002).
W. Grabowski & L.-P. Wang, Annu. Rev. Fluid Mech. **45**, (2013).

Introduction

- In turbulent flows, there is an increase of the terminal velocity of heavy particles.
- This phenomenon is mostly understood on qualitative grounds and has been quantified only in model flows.
- Very little is known on the effect of gravitational settling on two-particle statistics.
- Fundamental theoretical and numerical studies of the clustering of particle pairs and of the enhancement of collisions due to inertia usually neglect gravity.

M. Maxey, J. Fluid Mech. **174**, (1987).
L.-P. Wang & M. Maxey, J. Fluid Mech. **256**, 27 (1993).
E. Balkovsky, *et al*, Phys. Rev. Lett. **86**, (2001).
J. Davila & J. Hunt, J. Fluid Mech. **440**, (2001).
M. Wilkinson, *et al*, Phys. Rev. Lett. **97**, (2006).
O. Ayala, *et al*, New J. Phys. **10**, (2008).
J. Bec, *et al*, Phys. Rev. Lett. **98**, (2007).
J. Bec, *et al*, Fluid Mech. **646**, (2010).

Question

What is the interplay between turbulence, gravity, and particle sizes?

Important for fluid dynamics and non-equilibrium statistical physics.

Principal Results: Summary

- Heavy particles suspended in a turbulent flow settle faster than in a still fluid.
- This effect is due to a preferential sampling of the regions where the fluid flows downward.
- Settling leads to an effective horizontal, two-dimensional dynamics that increases clustering and reduce relative velocities between particles.
- These two competing effects can either increase or decrease the geometrical collision rates between same-size particles and are crucial for realistic modeling of coalescing particles.

Our Approach

- We combine direct numerical simulations with theoretical results based on *standard* asymptotic analysis.
- We make a systematic study of the dynamical and statistical properties of particles as a function of
  - the level of turbulence of the carrier flow (Reynolds number);
  - the inertia of the particles (Stokes number);
  - the ratio between the turbulent accelerations and gravity (Froude number).

The Model

- **The Fluid**
  - The fluid velocity **u** is a solution of the incompressible Navier–Stokes equation and obtained via pseudo-spectral, direct numerical simulations.
  - Statistically steady, homogeneous, isotropic turbulence is maintained by a large-scale forcing.
- **The Particles**
  - Particles are much smaller than the Kolmogorov scale, much heavier than the surrounding fluid, and with a small Reynolds number associated to their slip velocity.
  - Non-dimensionless numbers:
    - *Stokes number*: $St = \tau_{\mathrm{p}}/\tau_\eta$, where $\tau_\eta = \sqrt{\nu/\varepsilon}$.
    - *Froude number*: $Fr = \varepsilon^{3/4}/(g\nu^{1/4})$, where $a_\eta = \varepsilon^{3/4}/\nu^{1/4}$.
  - We use 10 different Stokes numbers and 5 different values of the Froude number

The Model : Equations

- **The Fluid**
  - The incompressible, forced Navier–Stokes equation:

$$\begin{aligned} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} &= -\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}; \\ \nabla \cdot \mathbf{u} &= 0. \end{aligned}$$

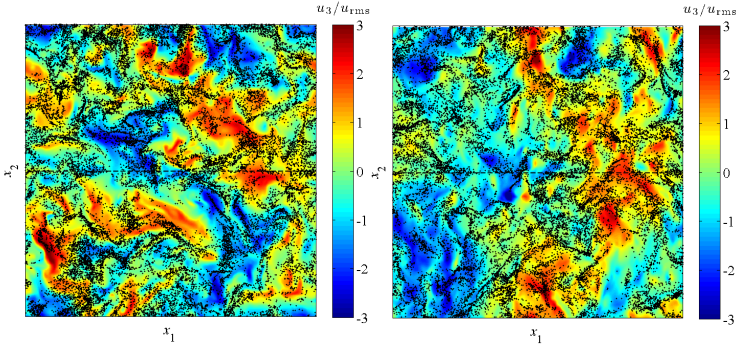
    - ν is the fluid kinematic viscosity and $\mathbf{f}$ a large scale forcing.
- **The Particles**
  - Stokes drag and gravity:

$$\begin{aligned} \frac{d\mathbf{x}_{\mathrm{p}}}{dt} &= \mathbf{v}_{\mathrm{p}}; \\ \frac{d\mathbf{v}_{\mathrm{p}}}{dt} &= -\frac{1}{\tau_{\mathrm{p}}} \left[\mathbf{v}_{\mathrm{p}} - \mathbf{u}(\mathbf{x}_{\mathrm{p}}, t)\right] + \mathbf{g}. \end{aligned}$$

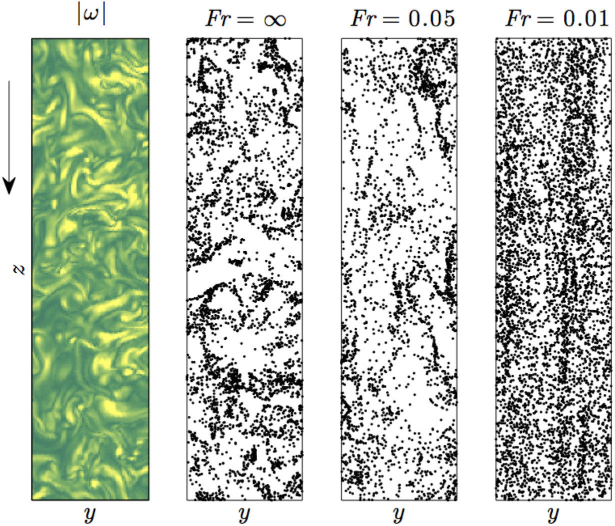
    - $\mathbf{u}(\mathbf{x}_{\mathrm{p}}, t)$ is evaluated by linear interpolation.

Simulation: Details

Re_λ	$u_{\rm rms}$	Δt	η	τ_η	L	T_L	N^3	N_p
460	0.189	0.0012	1.45×10^{-3}	0.083	1.85	9.9	2048^3	10×10^8
290	0.185	0.003	2.81×10^{-3}	0.131	1.85	9.9	1024^3	1.28×10^8
127	0.144	0.02	1.12×10^{-2}	0.45	2.11	14.6	256^3	0.08×10^8



Particle Distribution: Effect of Gravity



Snapshot of the vorticity modulus (Left; yellow = low values, green = high values) and of the particle positions for $R_\lambda = 130$, $\mathrm{St} = 1$ and three different values of the Froude number in a slice of thickness 10η, width 130η, and height 520η. The vertical arrow indicates gravity.

Bec, Homann, and Ray, Phys. Rev. Lett. **112**, 184501 (2014).

Settling Velocity: Qualitative Understanding

- Define : The average settling velocity $V_g = -\langle \mathbf{V}_\mathrm{p} \cdot \hat{\mathbf{e}}_z \rangle$.
- Statistical stationarity $\implies$ $V_g = \tau_\mathrm{p} g - \langle u_z(\mathbf{X}_\mathrm{p}, t) \rangle$.
- Define : The relative increase in settling velocity:

$$\Delta_V = (V_g - \tau_\mathrm{p} g)/(\tau_\mathrm{p} g) = -\langle u_z(\mathbf{X}_\mathrm{p}, t) \rangle / (\tau_\mathrm{p} g)$$

- *If* settling particles in a turbulent flow sample regions where the vertical fluid velocity is aligned with gravity, we expect an enhancement of the average settling velocity.

M. Maxey, J. Fluid Mech. **174**, (1987).
L.-P. Wang & M. Maxey, J. Fluid Mech. **256**, 27 (1993).
K. Gustavsson, *et al.*, Phys. Rev. Lett. **112**, 214501 (2014).

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  - What is its dependence on the particle Stokes number and for different values of Fr and R_λ?

- *If* settling particles in a turbulent flow sample regions where the vertical fluid velocity is aligned with gravity, we expect an enhancement of the average settling velocity.
  - Is there a way to see this preferential sampling from the equations of motion?

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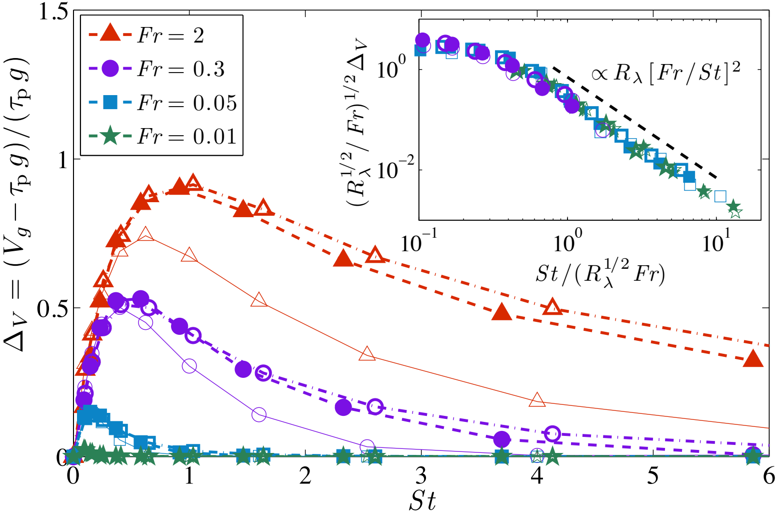
Settling Velocity



Relative increase of the settling velocity Δ_V as a function of the Stokes number St for various Froude numbers, as labeled, and $R_\lambda = 130$ (thin symbols, plain lines), $R_\lambda = 290$ (filled symbols, dashed lines) and $R_\lambda = 460$ (open symbols, broken lines). Inset: $[R_\lambda^{1/2}/Fr]^{1/2}\Delta_V$ as a function of $St/[R_\lambda^{1/2} Fr]$ for the same data.

Bec, Homann, and Ray, Phys. Rev. Lett. **112**, 184501 (2014).

Settling Velocity: Preferential Sampling

Small Stokes Asymptotics

- **Why is there an enhancement?**
  - To leading order, the particles advected by an effective compressible velocity field:

$$\mathbf{v} = \mathbf{u} - \tau_{\mathrm{p}} \left[\partial_t \mathbf{u} + (\mathbf{u} + \tau_{\mathrm{p}}\, \mathbf{g}) \cdot \nabla \mathbf{u}\right].$$

  - Focus on the (x, y) plane.
  - By using isotropy and incompressibility, we obtain:

$$\langle u_z \nabla_\perp \cdot \mathbf{v}_\perp \rangle = \tau_{\mathrm{p}}^2 g \left\langle (\partial_z u_z)^2 \right\rangle > 0.$$

Settling Velocity: Preferential Sampling

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Small Stokes Asymptotics

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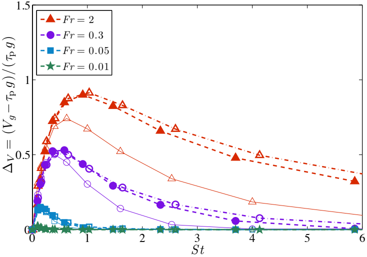
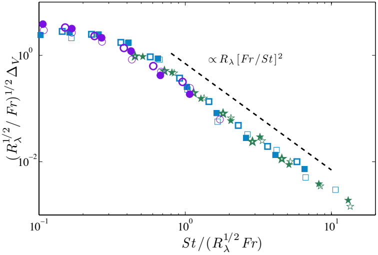
$$\langle u_z \nabla_\perp \cdot \mathbf{v}_\perp \rangle = \tau_{\mathrm{p}}^2 g \left\langle (\partial_z u_z)^2 \right\rangle > 0.$$

  - *Particles preferentially cluster (negative divergence), on average, in the (x, y) plane, at points where the fluid velocity is vertically downwards ($u_z < 0$).*

L.-P. Wang & M. Maxey, J. Fluid Mech. **256**, 27 (1993).
K. Gustavsson, *et al.*, Phys. Rev. Lett. **112**, 214501 (2014).

Settling Velocity: Quantitative Understanding





Small Stokes Asymptotics

$$\Delta_V \propto \tau_\eta \tau_p \left\langle (\partial_z u_z)^2 \right\rangle \propto St$$

Assumptions & Algorithm:

- Relate V_g to $\langle u_z \nabla_\perp \cdot \mathbf{v}_\perp \rangle$.
- Hence $\langle u_z(\mathbf{X}_p, t) \rangle \propto \tau_\eta \langle u_z \nabla_\perp \cdot \mathbf{v}_\perp \rangle$.

G. Falkovich, *et al*, Nature **419**, (2002).

Large Stokes Asymptotics

$$\Delta_V \propto R_\lambda^{3/4} Fr^{5/3} St^{-2}$$

Assumptions & Algorithm:

- Ballistic motion vertically: $L/V_g \ll \tau_L$.
- Effective horizontal dynamics.

Valid:

- $St \gg R_\lambda^{1/2} Fr$ and $Fr \ll R_\lambda^{1/2}$.

I. Fouxon & P. Horvai, Phys. Rev. Lett. **100**, (2008).

Small-scale, Two-particle Statistics

- Describe the evolution of pair separations in terms of $\nabla \mathbf{u}$.
- $V_g \gg u_\eta$: the particles travel η in a time shorter than τ_η.
- Rescale time by $\tau_\eta (V_g/u_\eta)$ and space by η:

$$\frac{\mathrm{d}^2 \mathbf{R}}{\mathrm{d}s^2} \simeq -\frac{1}{\tilde{S}} \left[\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}s} - \mathbf{R} \cdot \boldsymbol{\sigma}(s) \right],$$

  where $\boldsymbol{\sigma}$ is a Gaussian tensorial noise with co-variance $\langle \sigma_{ij}(s) \sigma_{k\ell}(s') \rangle = (\nu/\varepsilon) \langle \partial_i u_j \partial_k u_\ell \rangle \delta(s - s')$.
- The effective Stokes number $\tilde{S} = St\,(u_\eta/V_g)$.
- $V_g \gg u_\eta$: small-scale two-particle statistics depend only on $\tilde{S}$.
- When $\Delta_V \ll 1$, $\tilde{S} \simeq Fr$; the statistics become independent of St when $St \gg Fr$.

Observables

$\mathcal{D}_2$: The Correlation Dimension

- An important observable measuring particle clustering is the correlation dimension $\mathcal{D}_2$ of their spatial distribution.
- It is given by $\mathbb{P}_2(r) \propto r^{\mathcal{D}_2}$ for $r \ll \eta$, where $\mathbb{P}_2(r)$ is the probability that two particles are within a distance r.

The Approaching Rate

- $\kappa(r) = -\langle w\,\theta(-w) \,|\, |\mathbf{R}| = r \rangle\,(\mathrm{d}\mathbb{P}_2/\mathrm{d}r)$, where $w = \mathrm{d}|\mathbf{R}|/\mathrm{d}t$ is the longitudinal velocity difference between particles, θ the Heaviside function, and $\langle\cdot\rangle$ the average over all particle separations $\mathbf{R}$
- This last quantity behaves also as a power of r for $r \ll \eta$ with an exponent ξ_1 given by the first-order structure function of particle velocities.
- This implies that $\kappa(r) \sim r^{\gamma}$ with $\gamma = \xi_1 + \mathcal{D}_2 - 1$.

K. Gustavsson, *et al.*, Phys. Rev. Lett. **112**, 214501 (2014).

$\mathcal{D}_2$: Correlation Dimension

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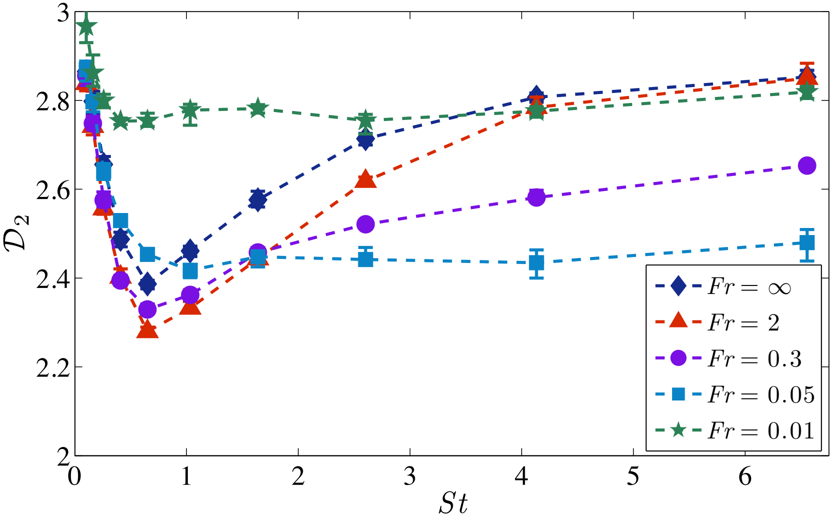
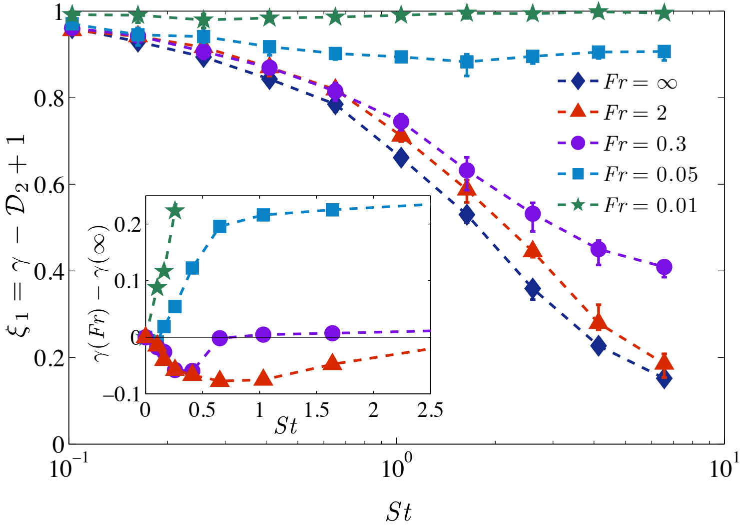


Correlation dimension $\mathcal{D}_2$ of the particle distribution as a function of the Stokes number for $R_\lambda = 460$ and various Froude numbers as labeled. Smaller Reynolds numbers (not shown here) display a similar behavior.

Bec, Homann, and Ray, Phys. Rev. Lett. **112**, 184501 (2014).

Approaching Rates



Exponent of the velocity difference $\xi_1 = \gamma - \mathcal{D}_2 + 1$ as a function of the Stokes number for different *Fr* and $R_\lambda = 460$. Inset: difference between the approaching rate exponent γ associated to the different values of *Fr* and that associated to particles feeling no gravity ($Fr = \infty$).

Bec, Homann, and Ray, Phys. Rev. Lett. **112**, 184501 (2014).

Understanding Approaching Rates

- $\kappa(r) \sim r^{\gamma}$ with $\gamma = \xi_1 + \mathcal{D}_2 - 1$.
- For $Fr = \infty$, $\xi_1 = 1$ at small St (tracers) and $\xi_1 = 0$ for $St \to \infty$ (scale-independent velocity differences).
- When Fr decreases, the effective Stokes number decreases, so that particles get closer to tracers of the effective flow and $\xi_1 \to 1$.

M. Wilkinson, *et al.*, Phys. Rev. Lett. **97**, 048501 (2006).
J. Bec, *et al*, J. Fluid Mech. **646**, 527 (2010).
J. Bec, *et al*, Phys. Fluids **17**, 073301 (2005).

Approaching Rates: Competing Mechanisms

- The two mechanisms determining the rate at which particles collide, namely preferential concentration and large velocity differences, are thus affected in a competing manner by gravity.
- The enhancement of particle clustering takes over the decrease of velocity differences when $St \lesssim Fr$.
- Hence, $\gamma(Fr) < \gamma(\infty)$ for $St \lesssim Fr$, indicating that the collision rates between same-size particles are larger in the presence of gravity.
- These corrections are responsible for an important increase of the geometrical collision rate.
  - Example: In the settings of a highly-turbulent cloud, namely $Fr = 0.3$ and $St = 0.4$, the collision rate doubles when the effect of gravity is included.

Conclusions and Perspectives

- Heavy particles suspended in a turbulent flow settle faster than in a still fluid.
- This effect stems from a preferential sampling of the regions where the fluid flows downward and is quantified as a function of the level of turbulence, of particle inertia, and of the ratio between gravity and turbulent accelerations.
- By using analytical methods and detailed numerical simulations, settling is shown to induce an effective horizontal two-dimensional dynamics that increases clustering and reduce relative velocities between particles.
- These two competing effects can either increase or decrease the geometrical collision rates between same-size particles and are crucial for realistic modeling of coalescing particles.