

# One Model Doesn't Fit All: Recent Results of a Detailed Analysis of Sunspot Demographics

**Andrés Muñoz-Jaramillo**

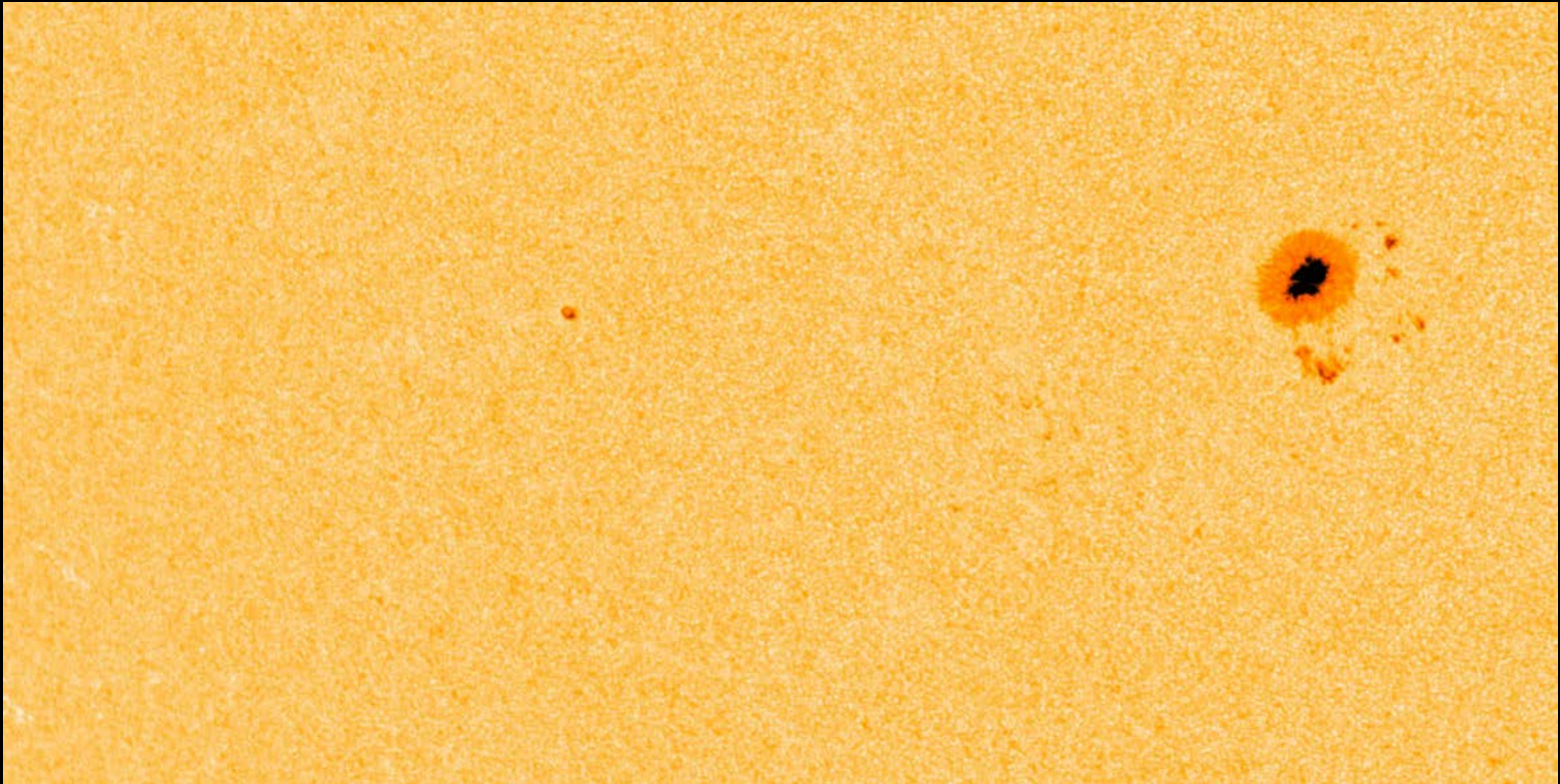
[www.solardynamo.org](http://www.solardynamo.org)

**Funded by:**

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Contract SP02H1701R from Lockheed-Martin to the SAO  
NSF REU program**

# **SUNSPOTS AND THE SOLAR CYCLE**

# Sunspots are the optical signature of the presence of strong magnetic fields

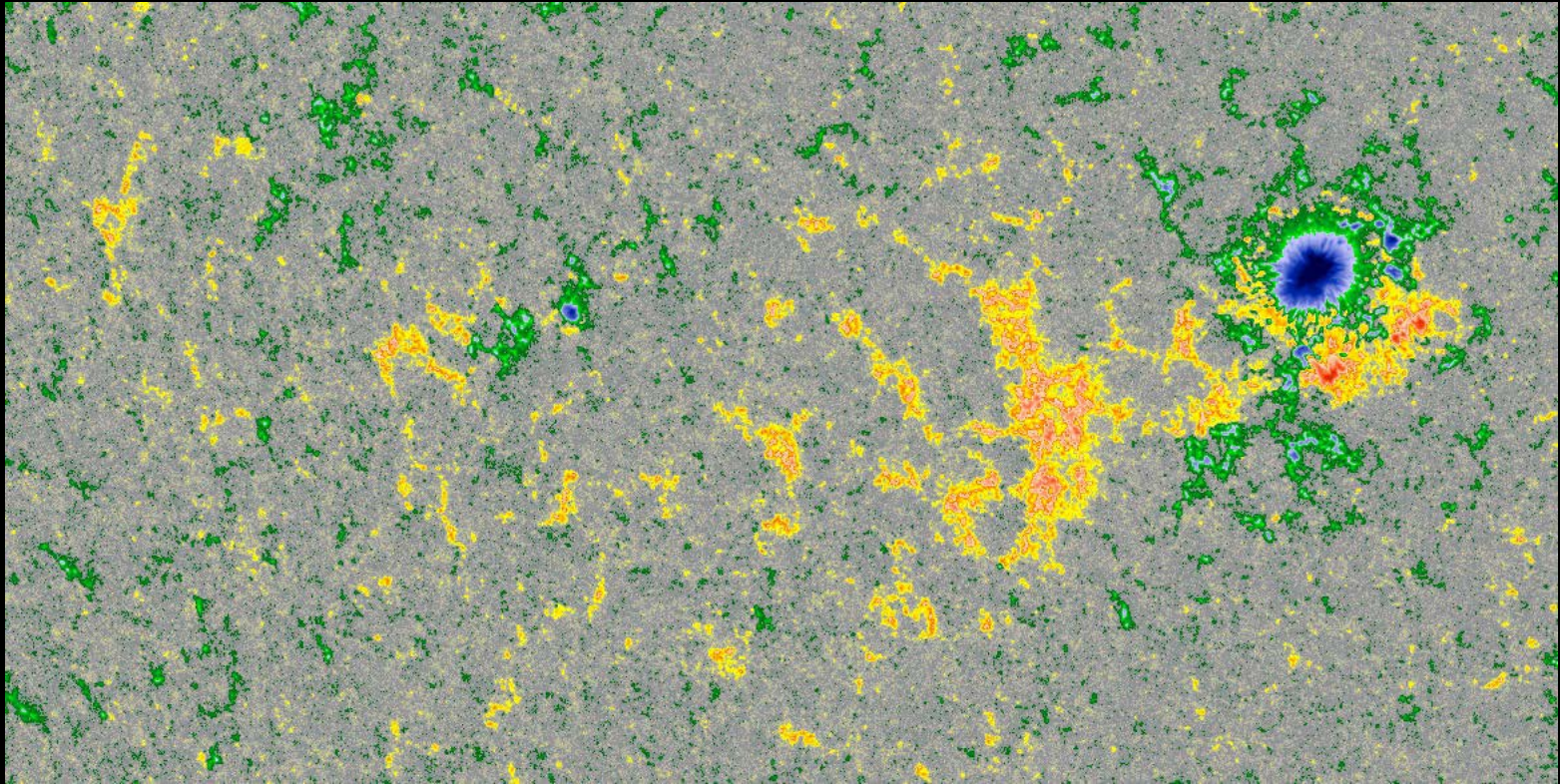


SDO/HMI

They are generally associated with tilted bipolar magnetic regions.



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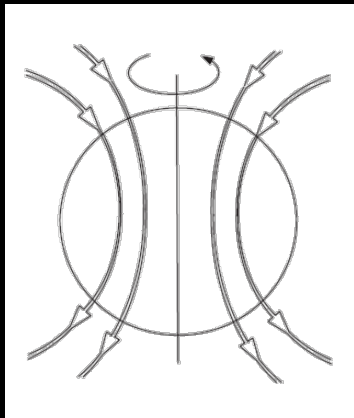
SDO/HMI

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# BMRs are critical for cycle propagation

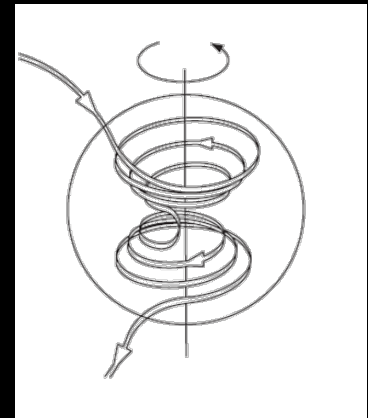
Muñoz-Jaramillo et al. ApJ (2012) & ApJL (2013)

Poloidal  
 $r - \theta$



Polar Fields

Toroidal  
 $\phi$



Credit: J. J. Love

Sunspot  
Numbers/Area

# BMRs are critical for cycle propagation

Muñoz-Jaramillo et al. ApJ (2012) & ApJL (2013)

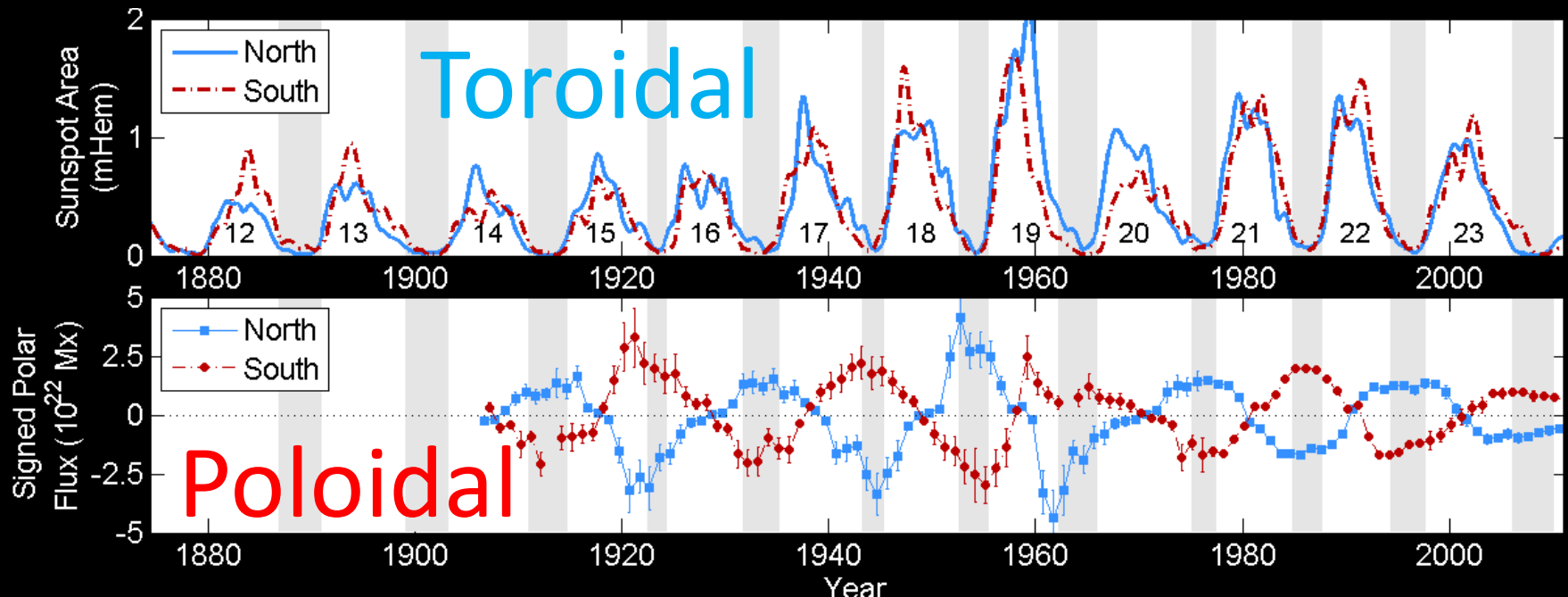
Poloidal

$r - \theta$

Toroidal

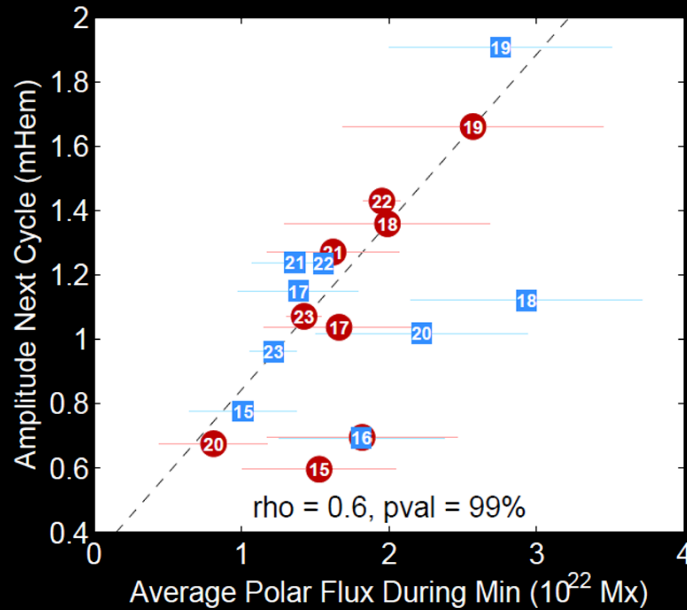
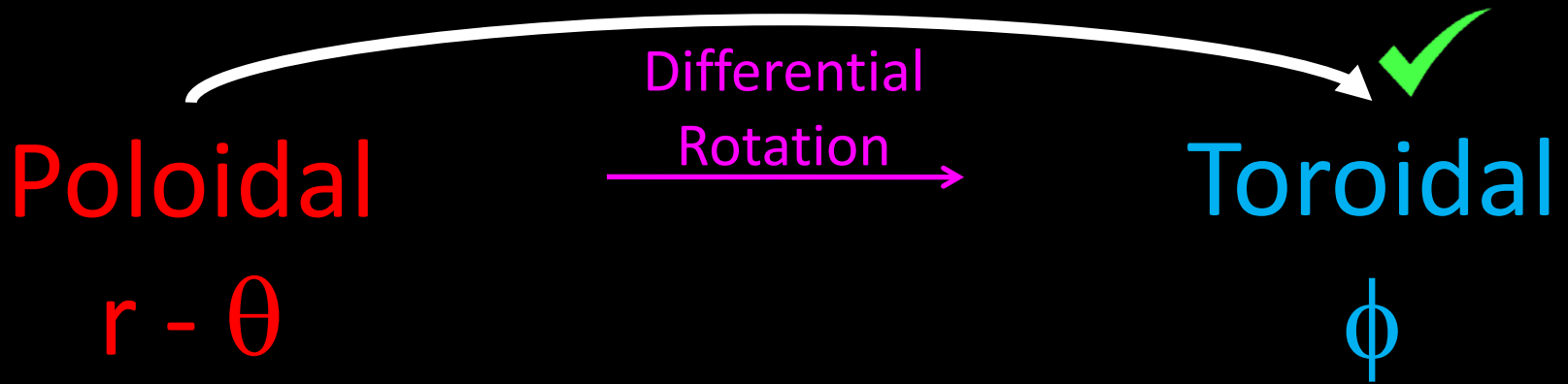
$\phi$

We used optical proxies to study the connection between toroidal and poloidal fields



# BMRs are critical for cycle propagation

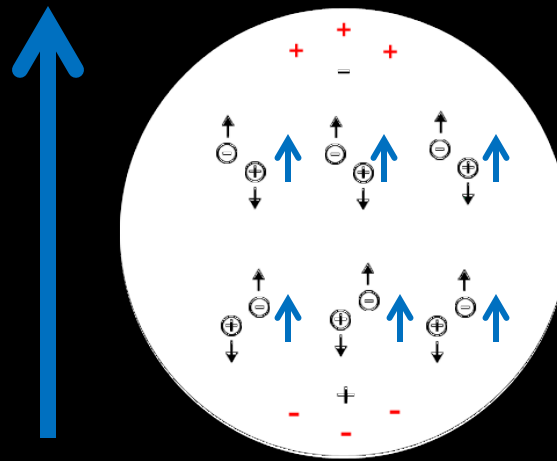
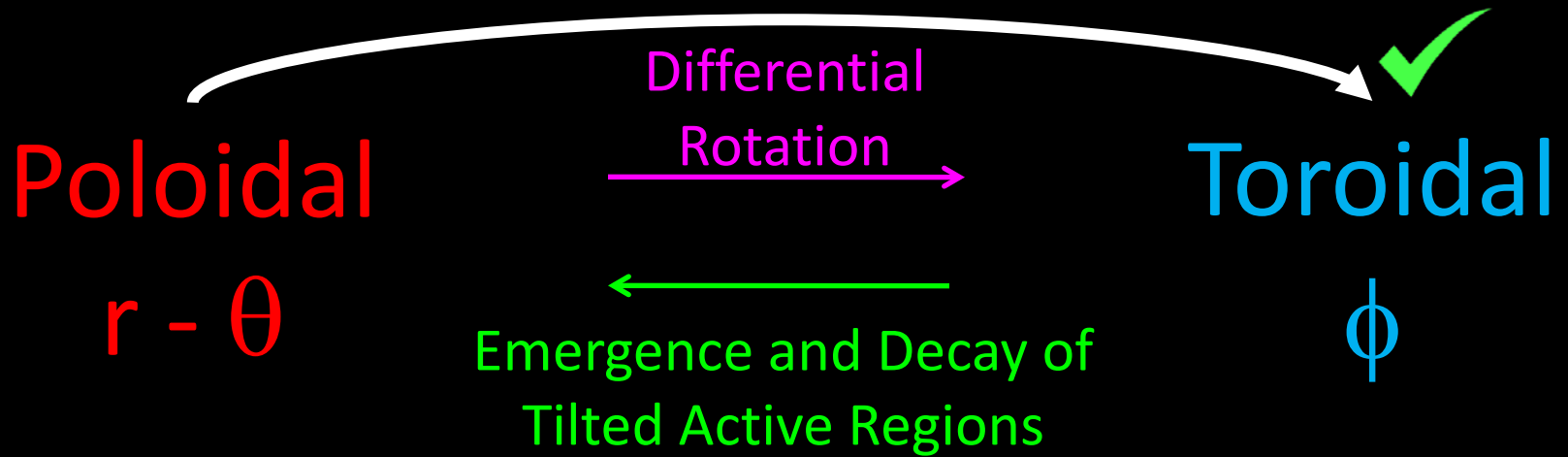
Muñoz-Jaramillo et al. ApJ (2012) & ApJL (2013)



The relationship between polar fields at minimum and cycle amplitude likes at the core of most physics based cycle predictions.

# BMRs are critical for cycle propagation

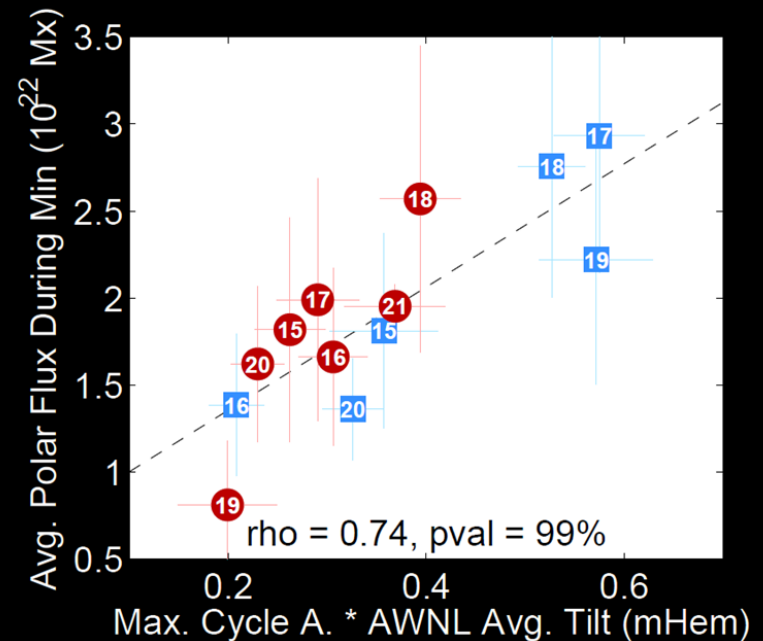
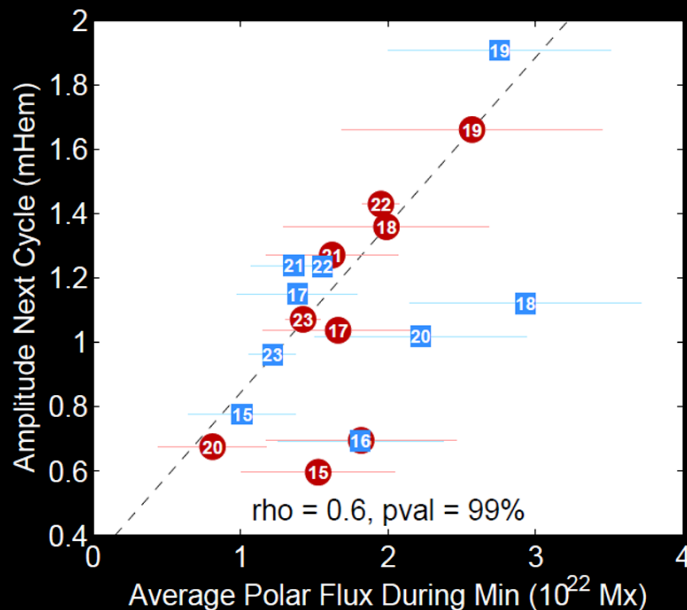
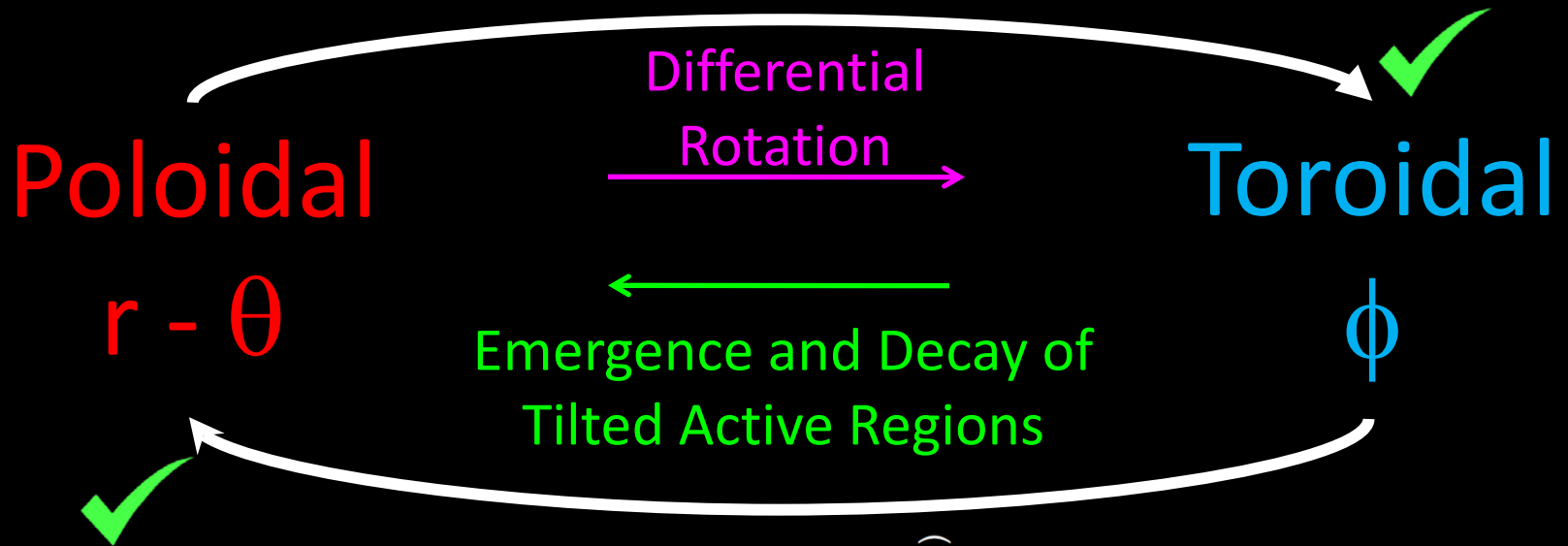
Muñoz-Jaramillo et al. ApJ (2012) & ApJL (2013)





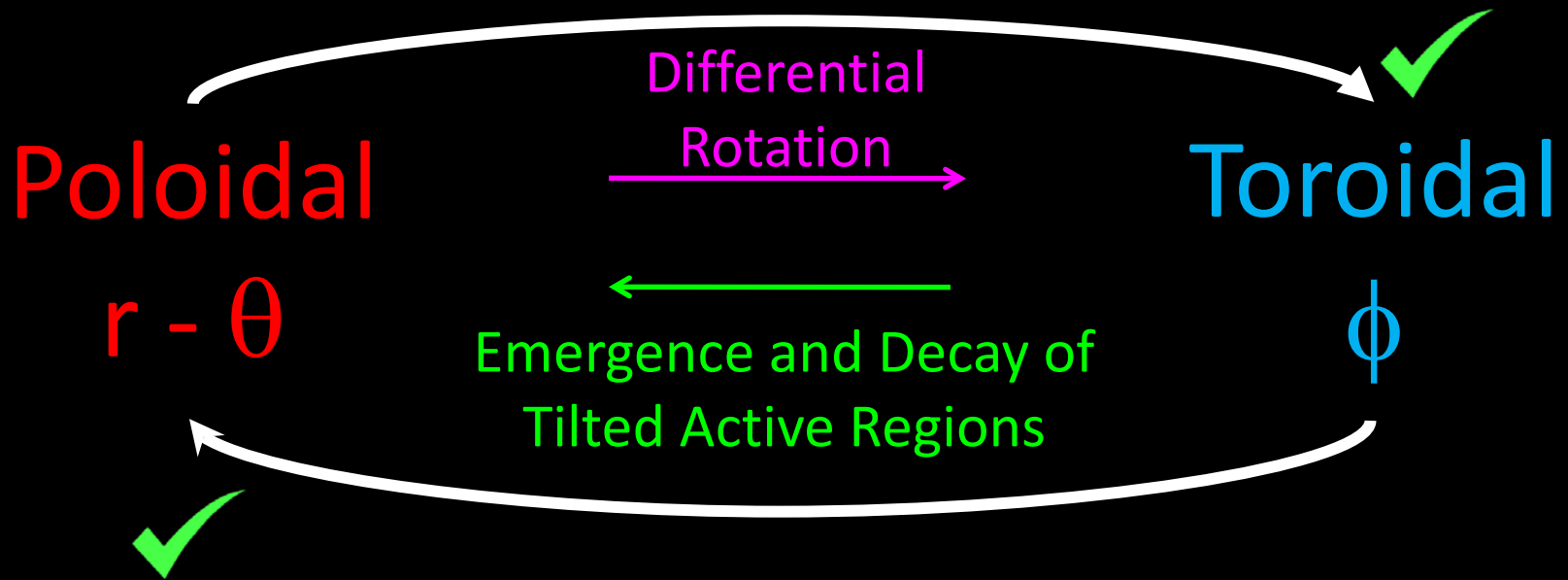
# BMRs are critical for cycle propagation

Muñoz-Jaramillo et al. ApJ (2012) & ApJL (2013)



# BMRs are critical for cycle propagation

Muñoz-Jaramillo et al. ApJ (2012) & ApJL (2013)



- Our results are consistent with this picture of the solar cycle.
- BMR and sunspot properties are highly variable statistical characterizations are necessary.

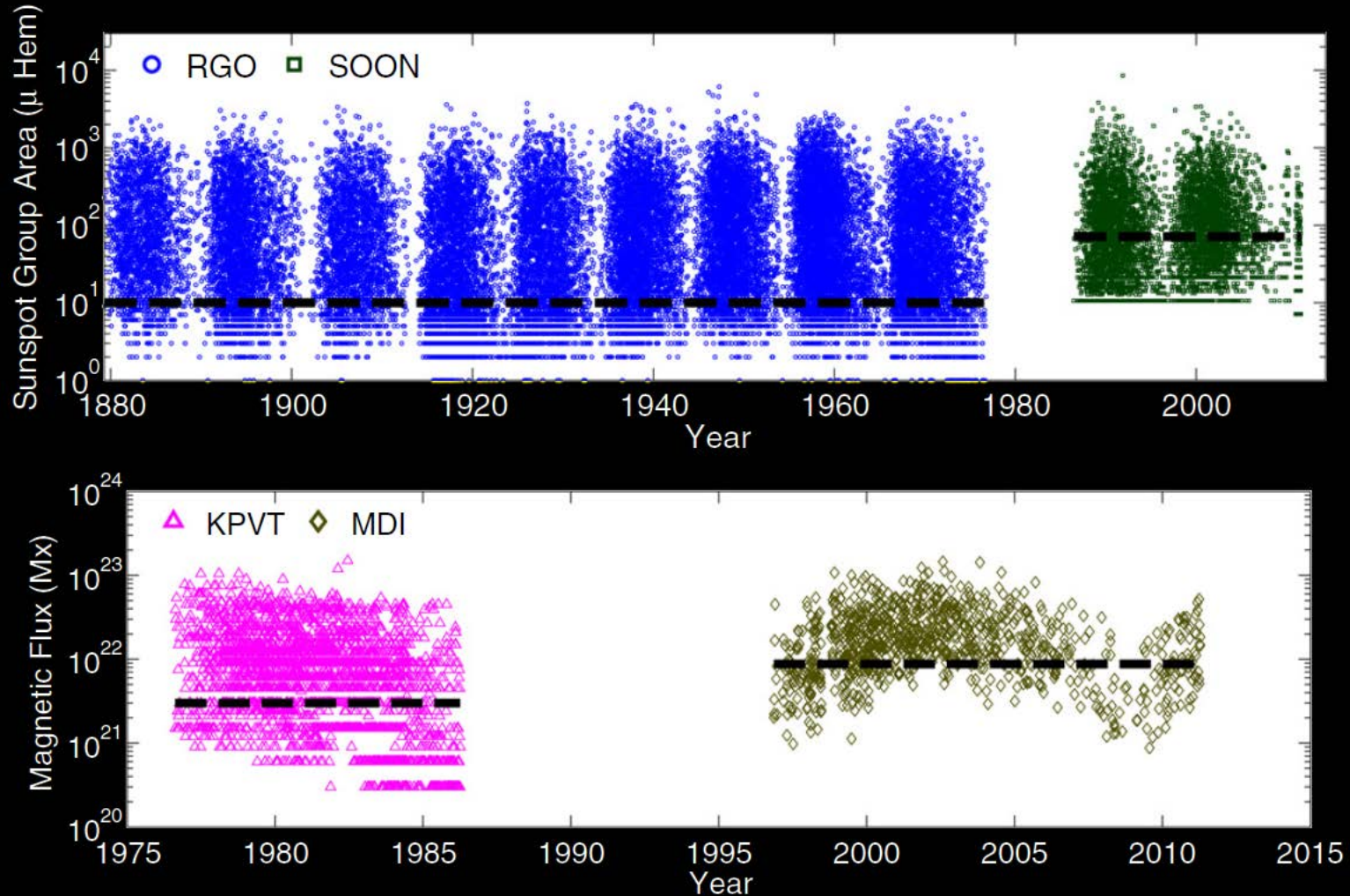
**OUR DATA**

# Our Data

- Sunspot Group Area:
  - Royal Greenwich Observatory (RGO). 1874 - 1976.
  - Solar Observing Optical Network (SOON). 1985 - present.
  - Pulkovo's catalogue of solar activity (PCSA). 1938 - 1991.
  - Kislovodsk Mountain Astronomical Station (KMAS). 1954 - present.
  - SDO/HMI. 2010 - present.
- Sunspot Area:
  - SOHO/MDI Umbral. 1996 - 2010.
  - SDO/HMI Umbral. 2010 - present.
  - San Fernando Observatory (SFO). 1983 - present.
- Bipolar Magnetic Region Flux:
  - KPVT. 1976 - 1986.
  - SOHO/MDI. 1996 - 2010.
  - KPVT/SOLIS. 1996 - present.

# Data Truncation

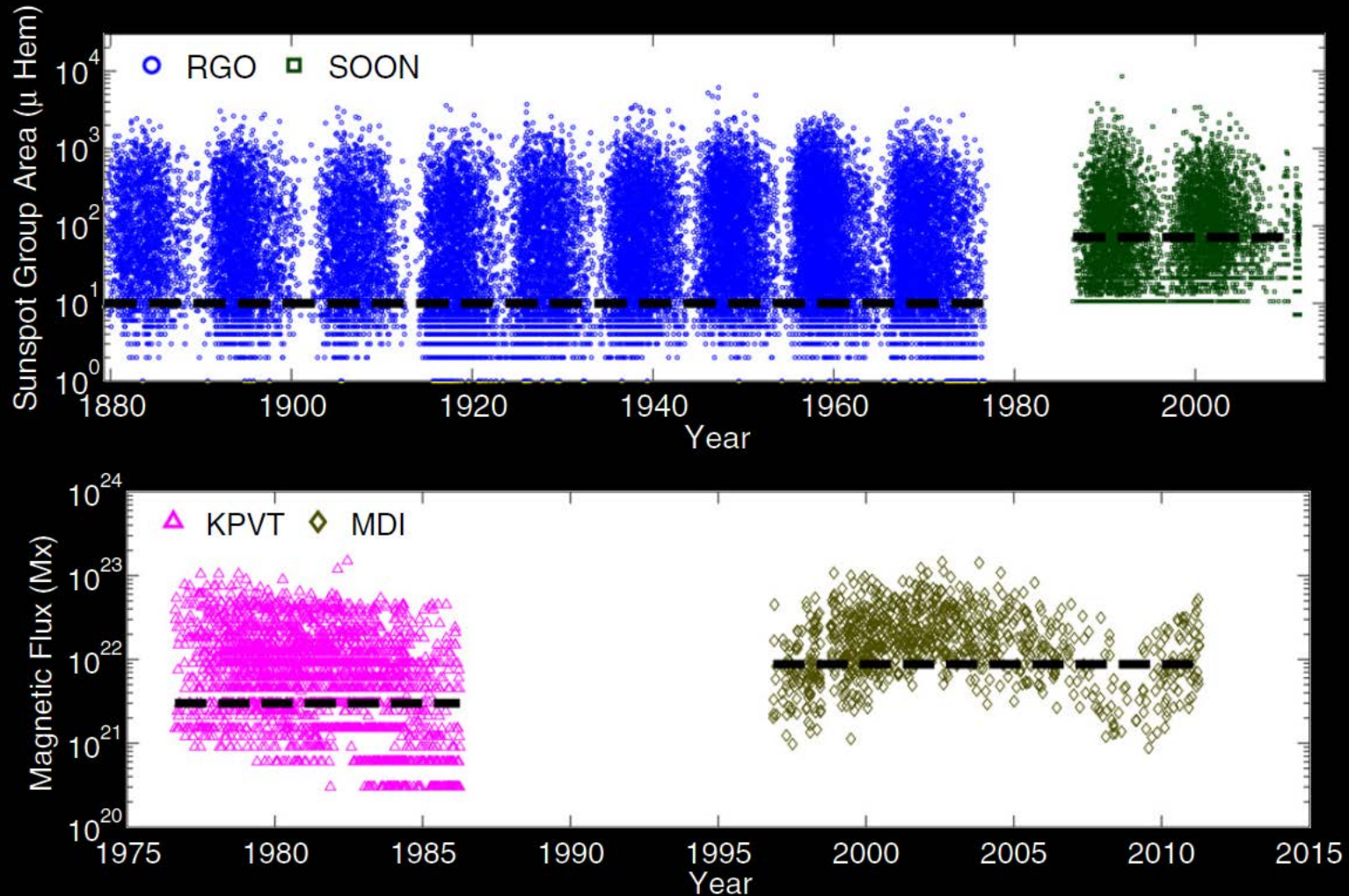
- Structures near the lower detection threshold suffer from a host of issues that can potentially distort our statistical analysis.





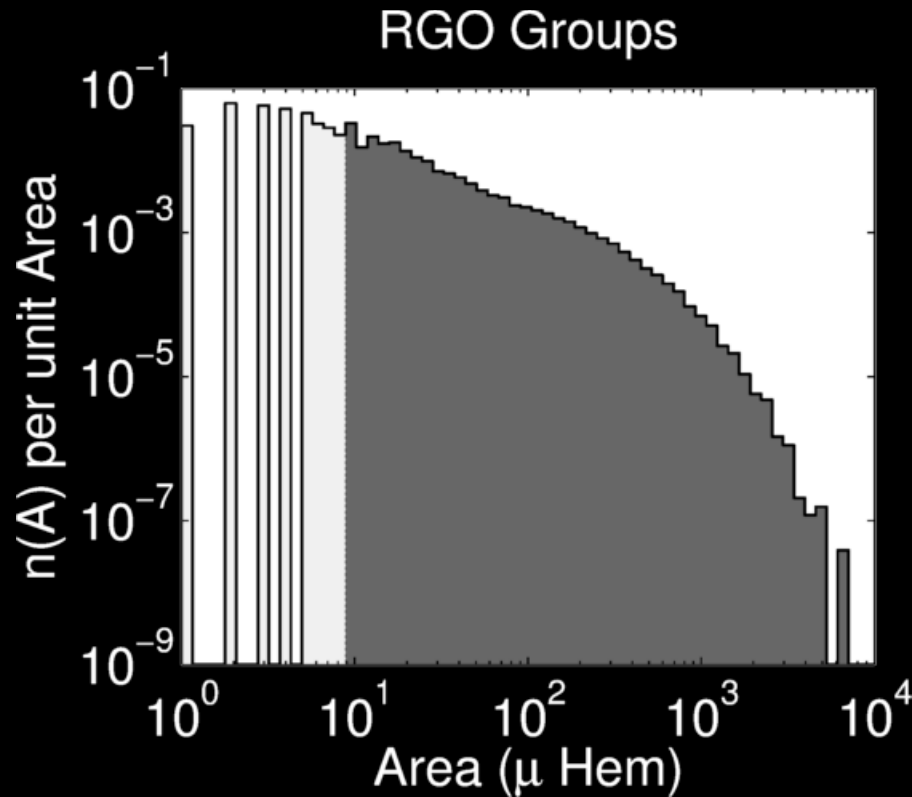
# Data Truncation

- To avoid this issues, we impose a truncation limit one order of magnitude above the minimum size of detection.



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Only data inside dark areas is included in our fits and analysis, light areas are shown for visual reference.

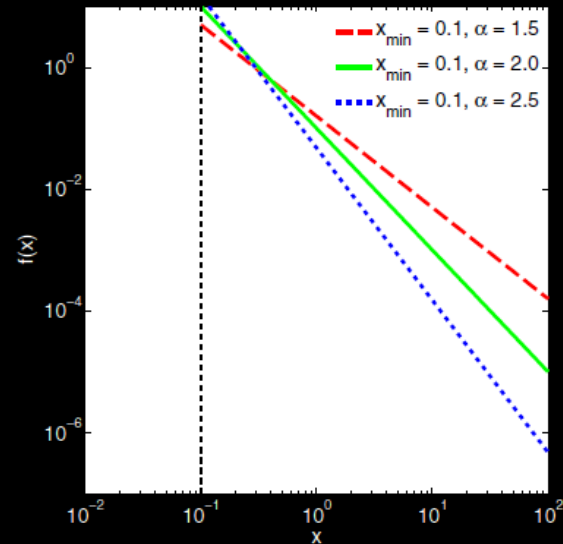
# AREA AND FLUX DISTRIBUTION

Muñoz-Jaramillo et al., *ApJ*, 800:48, 2015

In collaboration with Ryan Senkpeil, John Windmueller, Ernest Amouzou, Dana Longcope, Andrey Tlatov, Yury Nagovitsyn, Alexei Pevtsov, Gary Chapman, Angela Cookson, Anthony Yeates, Fraser Watson, Laura Balmaceda, Piet Martens, & Ed DeLuca

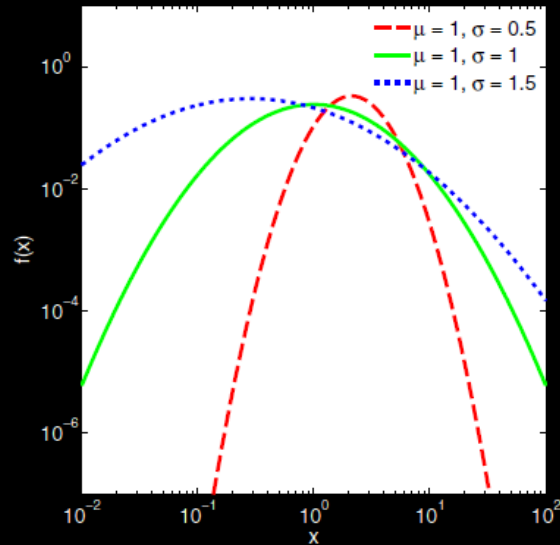
# Which distribution to use?

## Power Law



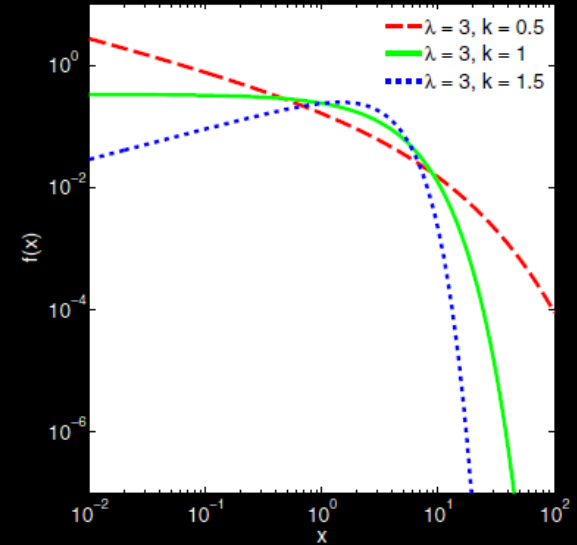
Zharkov et al. (2005)  
Meunier (2003)  
Hagenaar et al. (2003)  
Parnell et al. (2009)

## Log-Normal



Bogdan et al. (1988)  
Baumann & Solanki (2005)  
Zhang et al. (2010)  
Schad & Penn (2010)

## Weibull



Parnell (2002)

## Exponential

Tang et al. (1984)  
Schrijver et al. (1997)

## Composite Distributions

Kuklin (1980)

Harvey & Zwaan (1993)

Jiang et al. (2011)

Nagovitsyn et al. (2012)

# Which distribution to use?

- We fitted our 11 databases with these four distributions (power-law, log-normal, Weibull, exponential), and a log-normal-Weibull composite.
- We applied a quantitative model selection criterion called Akaike's Information Criterion (AIC; Akaike 1983):

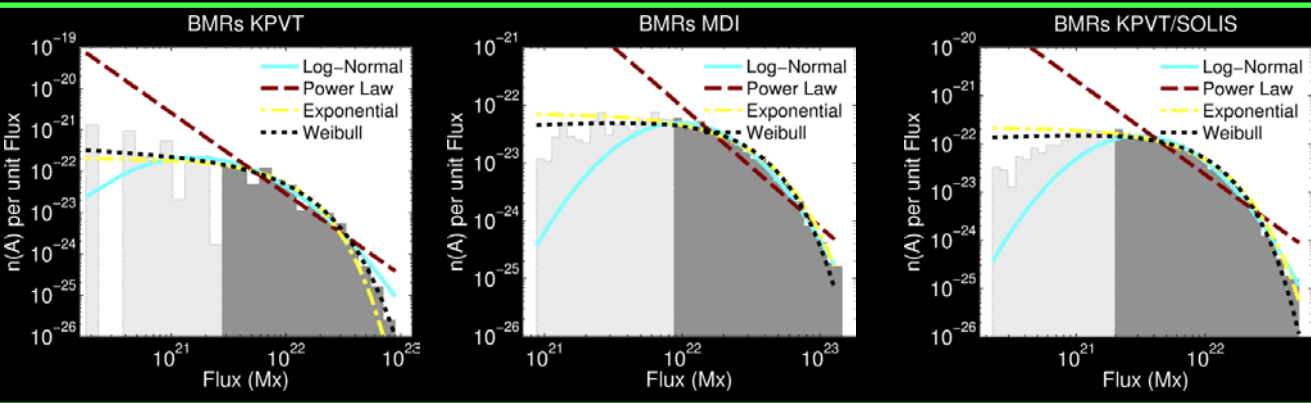
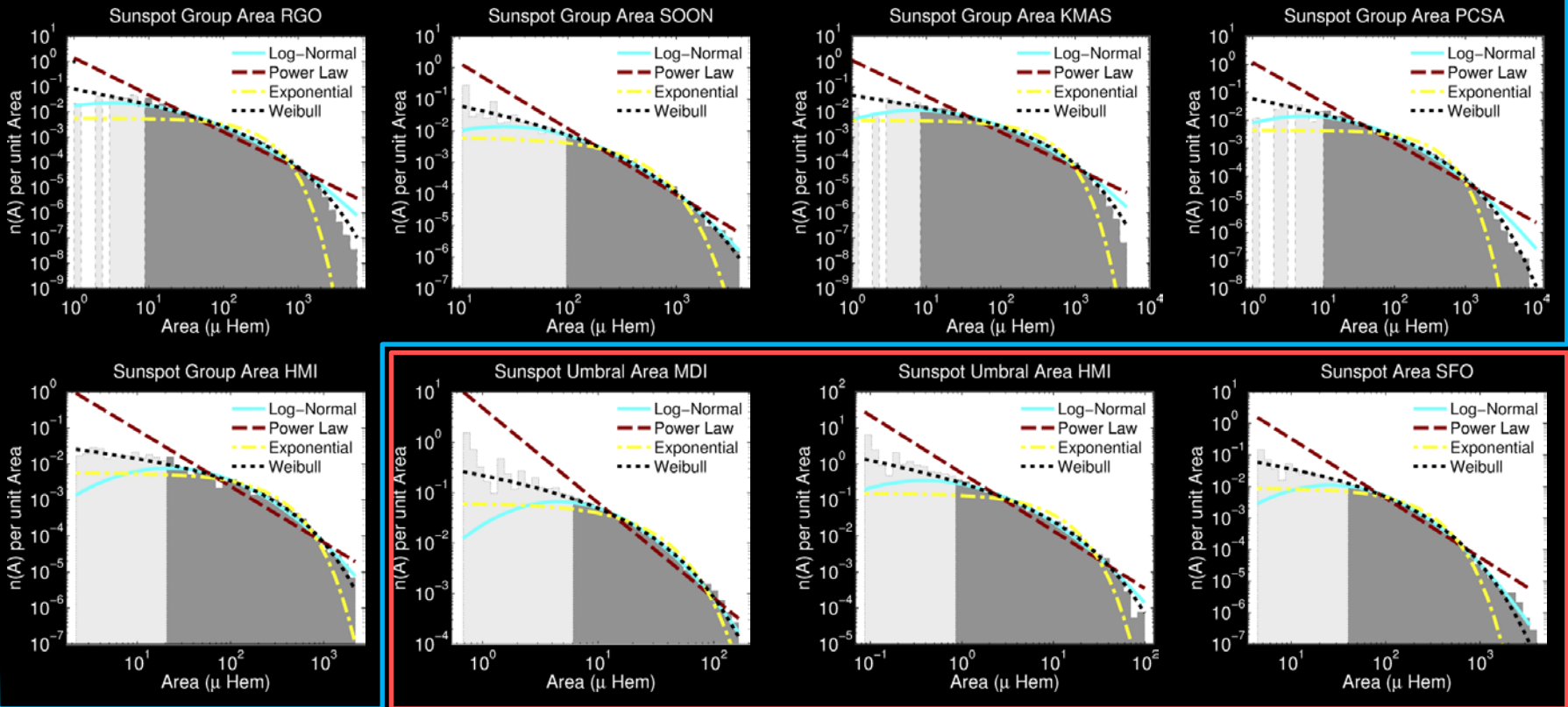
$$\text{AIC} = -2\ln(M) - 2n$$

- In AIC, the model's log-likelihood ( $\ln$ ) and the fitted model's degrees of freedom ( $n$ ) are used to strike a balance between underfitting and overfitting.
- AIC is a relative method of model discrimination, the **BEST** model is not necessarily the "**TRUE**" model.



# Single Fits

## Sunspot Group Area

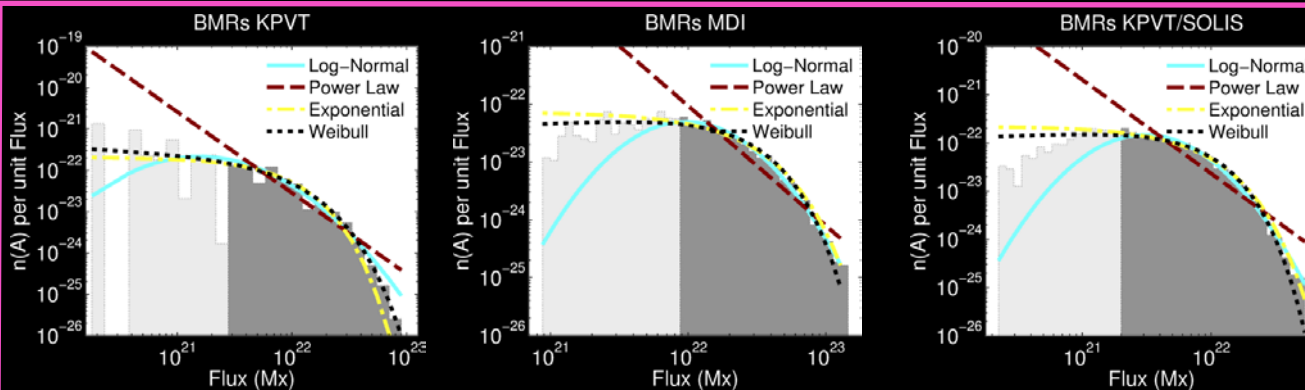
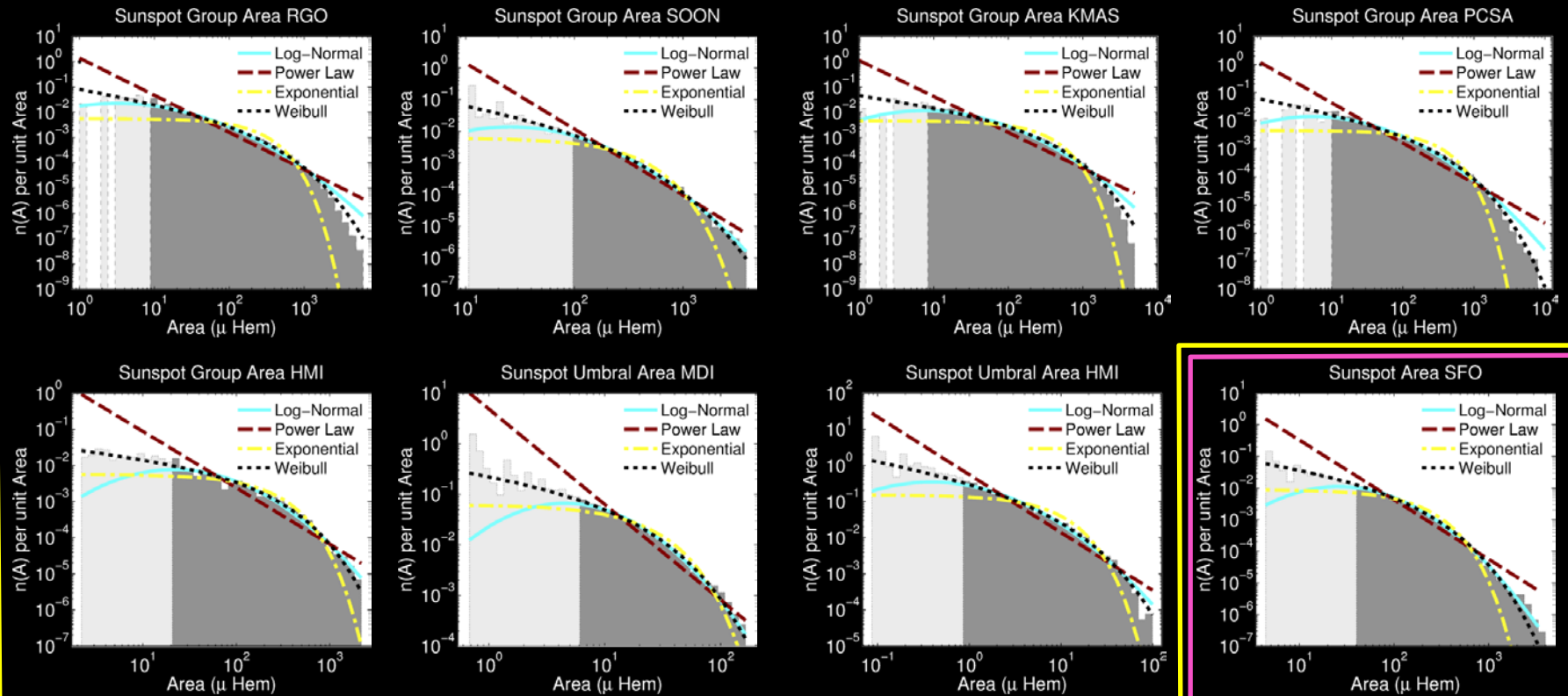


## Sunspot Area

## BMR Flux

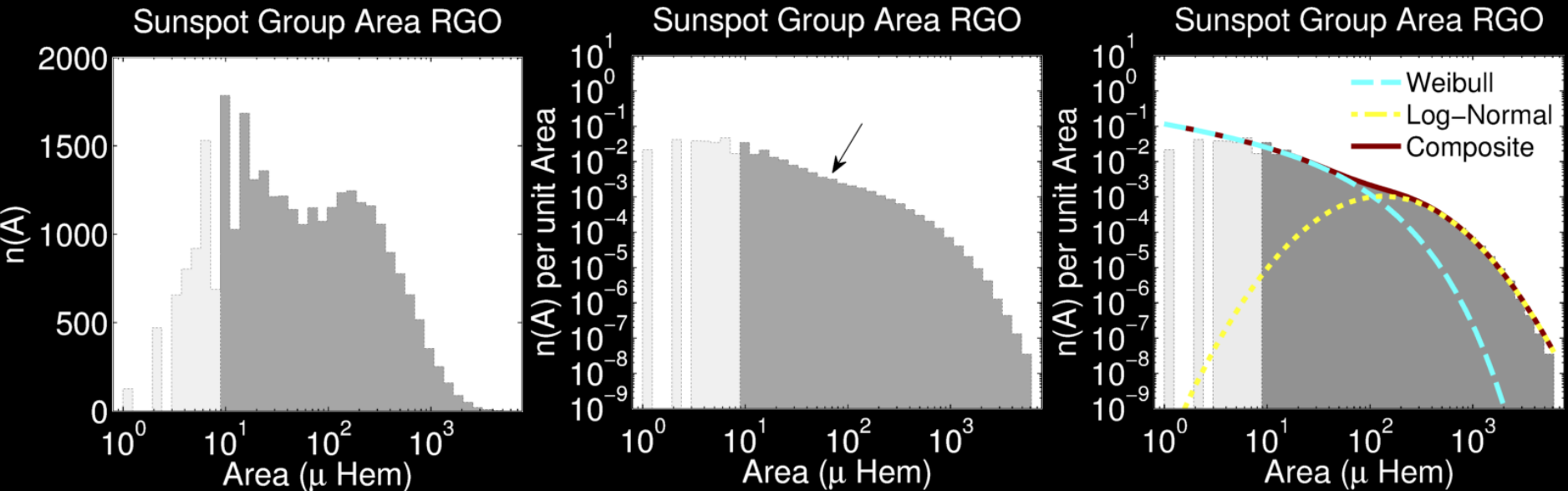
# Single Fits

Better Fitted by Weibull



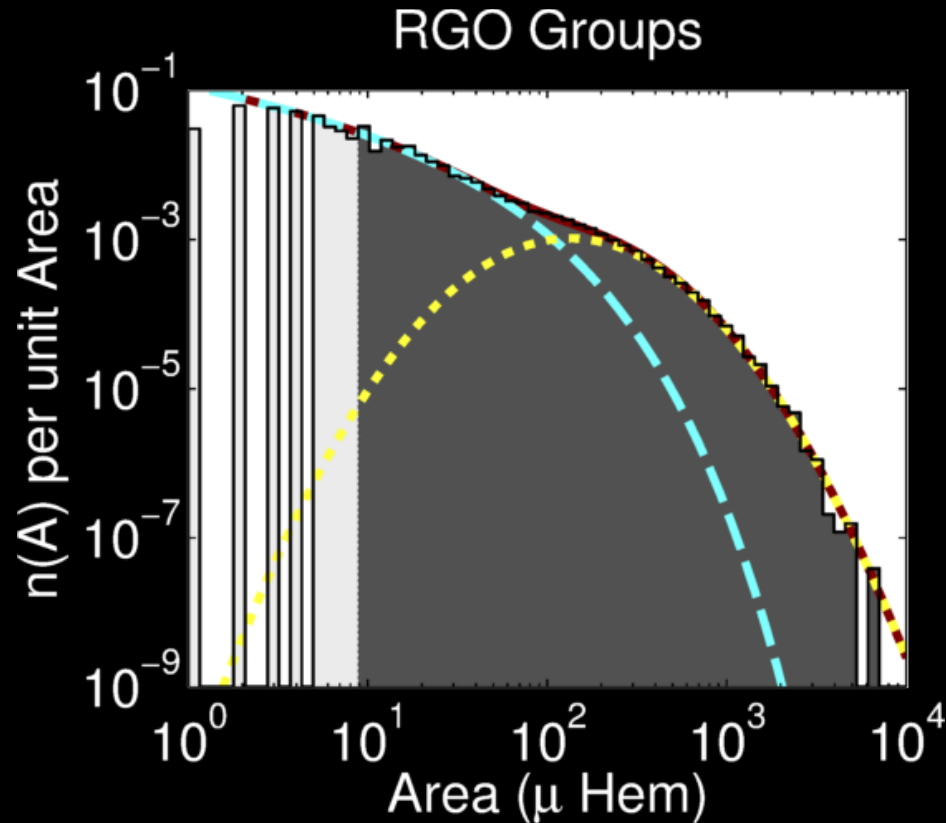
Better Fitted by  
Log-Normal

# A combination of Weibull and Log-normal distributions fits the data best.



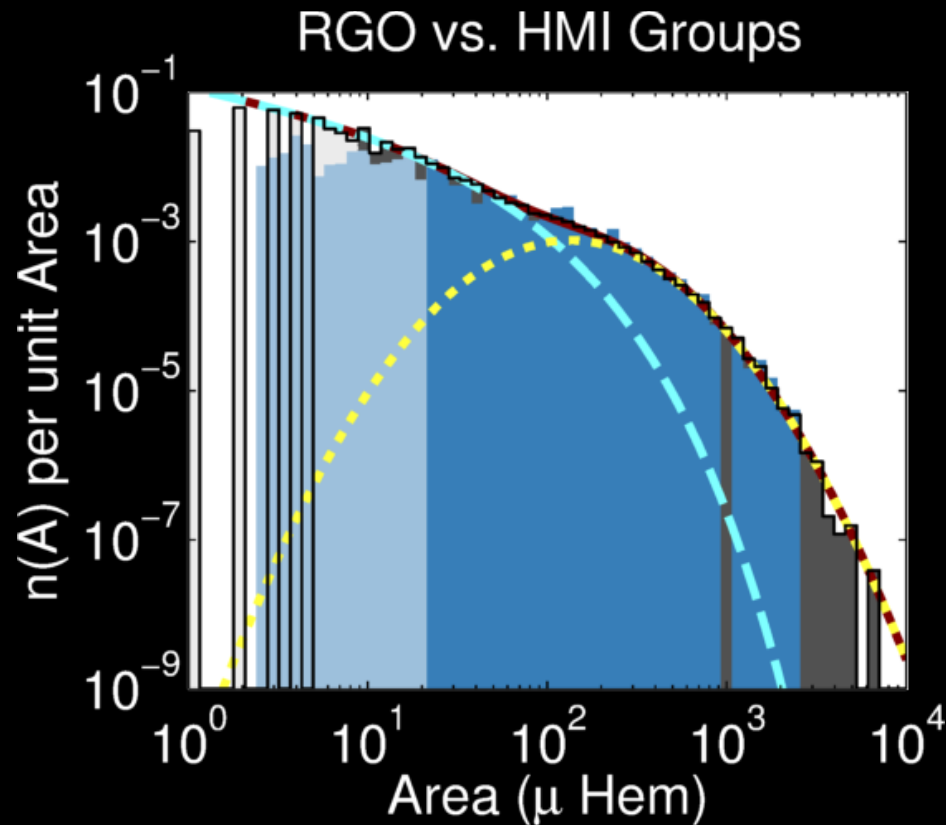
- Different sets are sampling different sections of a single distribution.
- Conflicting results arise from different data types sampling different part of this distribution.

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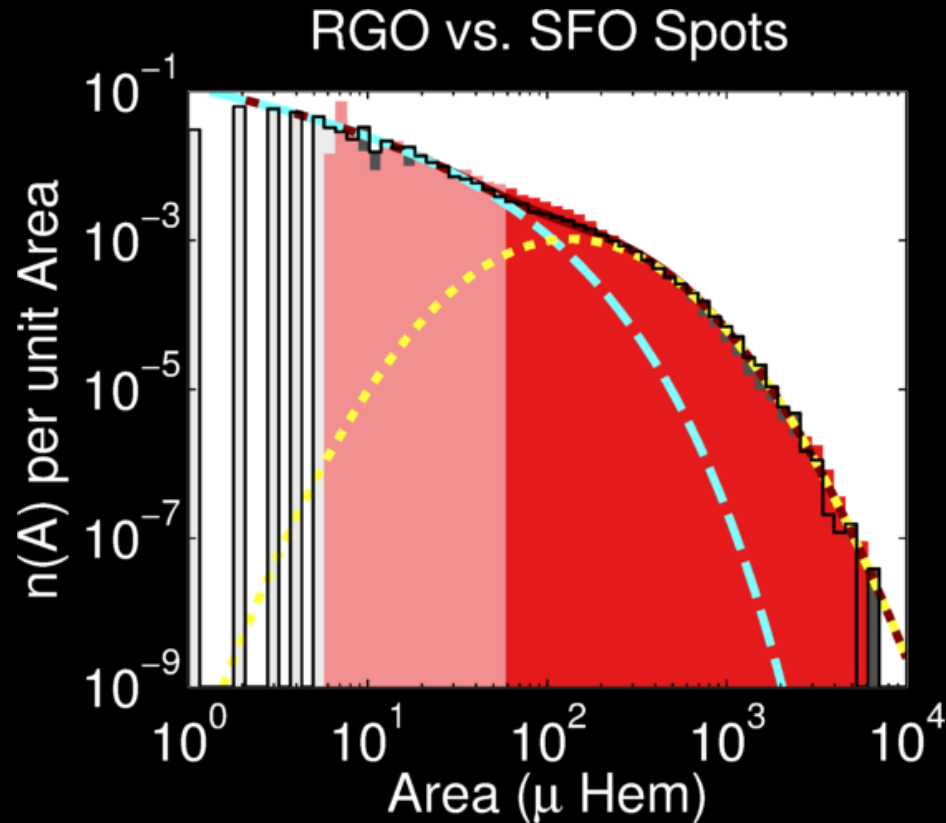
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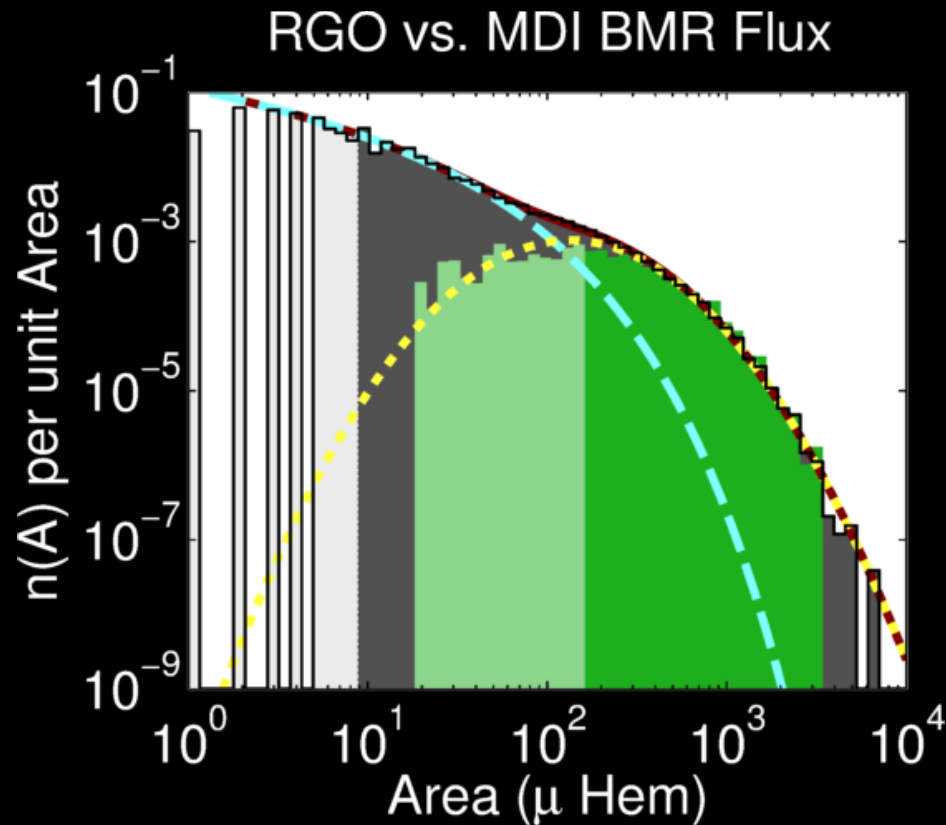


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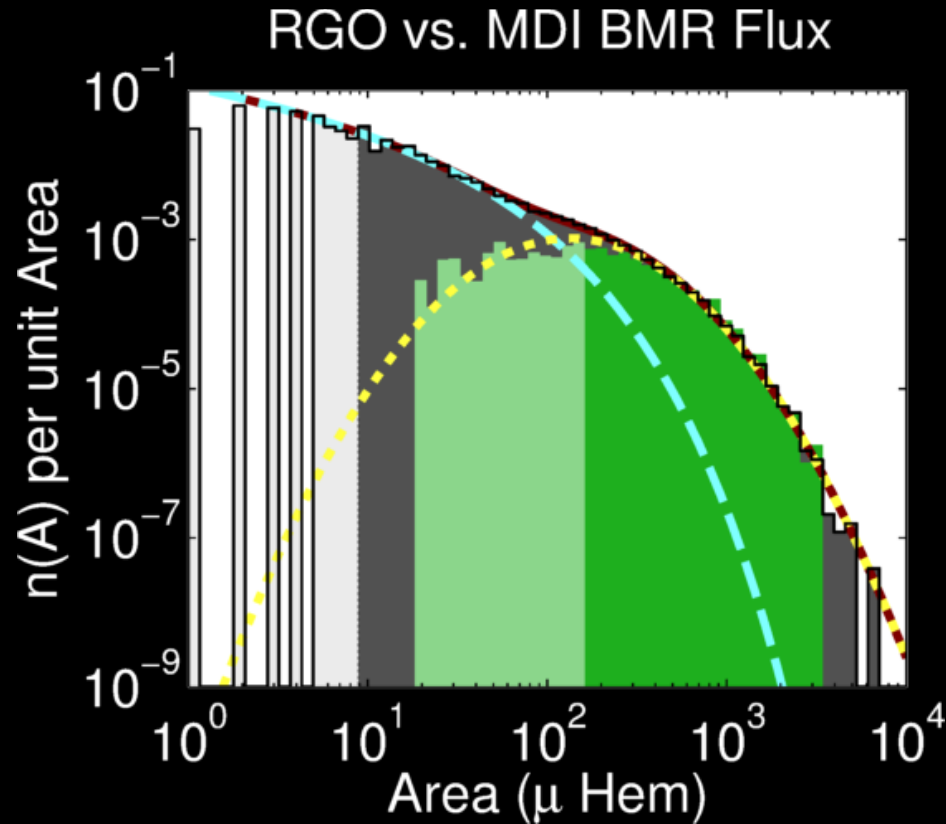
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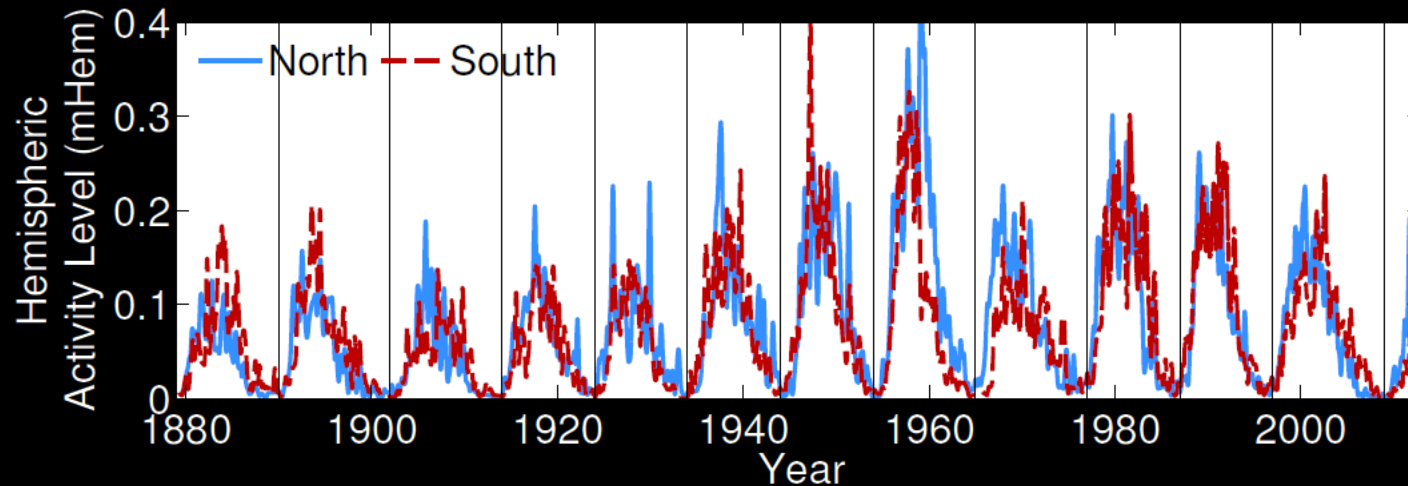
- The log-normal component is populated by the largest sunspot groups/BMRs, the Weibull component by small pores.

# CYCLE DEPENDENCE OF SUNSPOT GROUP PROPERTIES

Muñoz-Jaramillo et al., *ApJ*, in press, 2015

In collaboration with Ryan Senkpeil, Dana Longcope, Andrey Tlatov, Alexei Pevtsov, Piet Martens, & Ed DeLuca

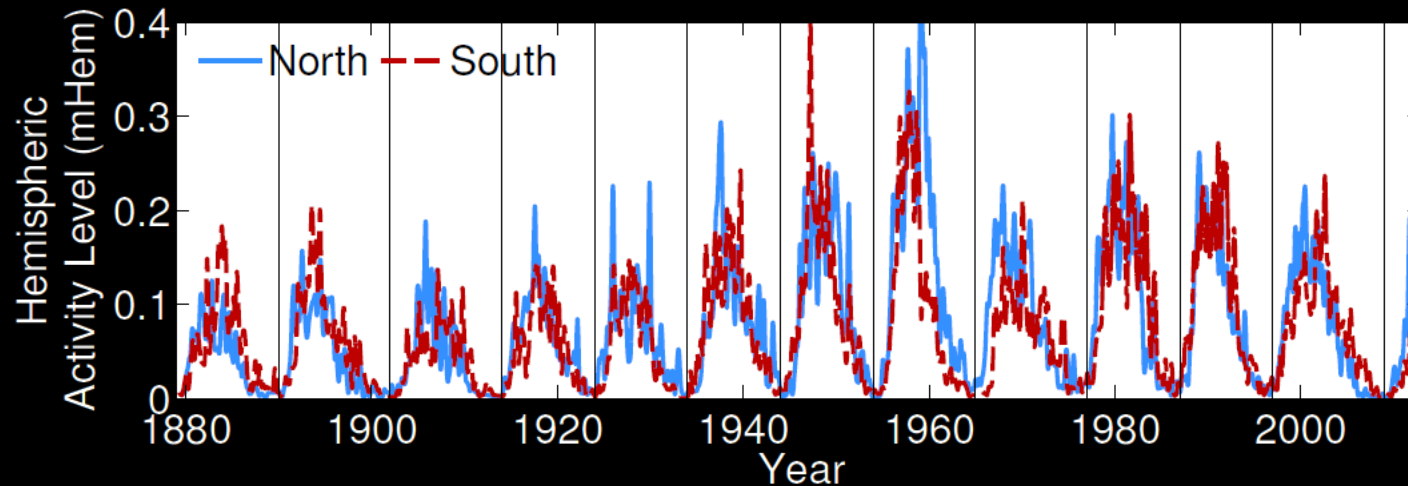
# Activity Level: A New Way of Binning Data



- Cycle evolution of active regions and sunspots is normally studied by comparing separate cycles or phases (minimum vs. maximum).
- This approach is sub-optimal for studying sunspot and BMR properties. Why?

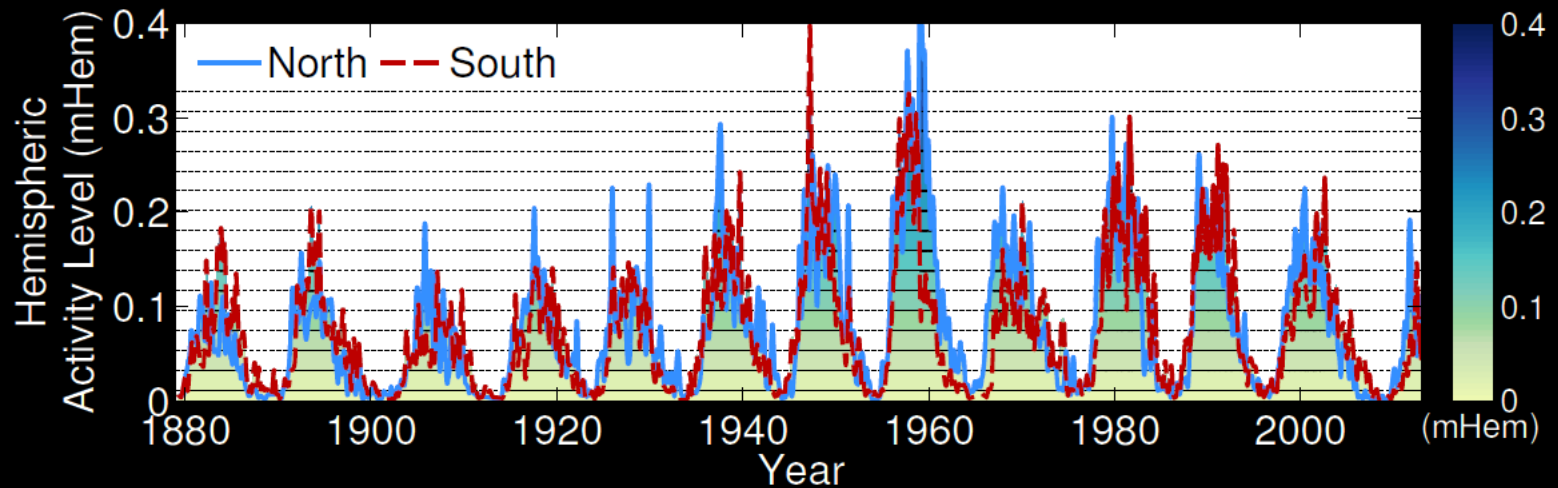


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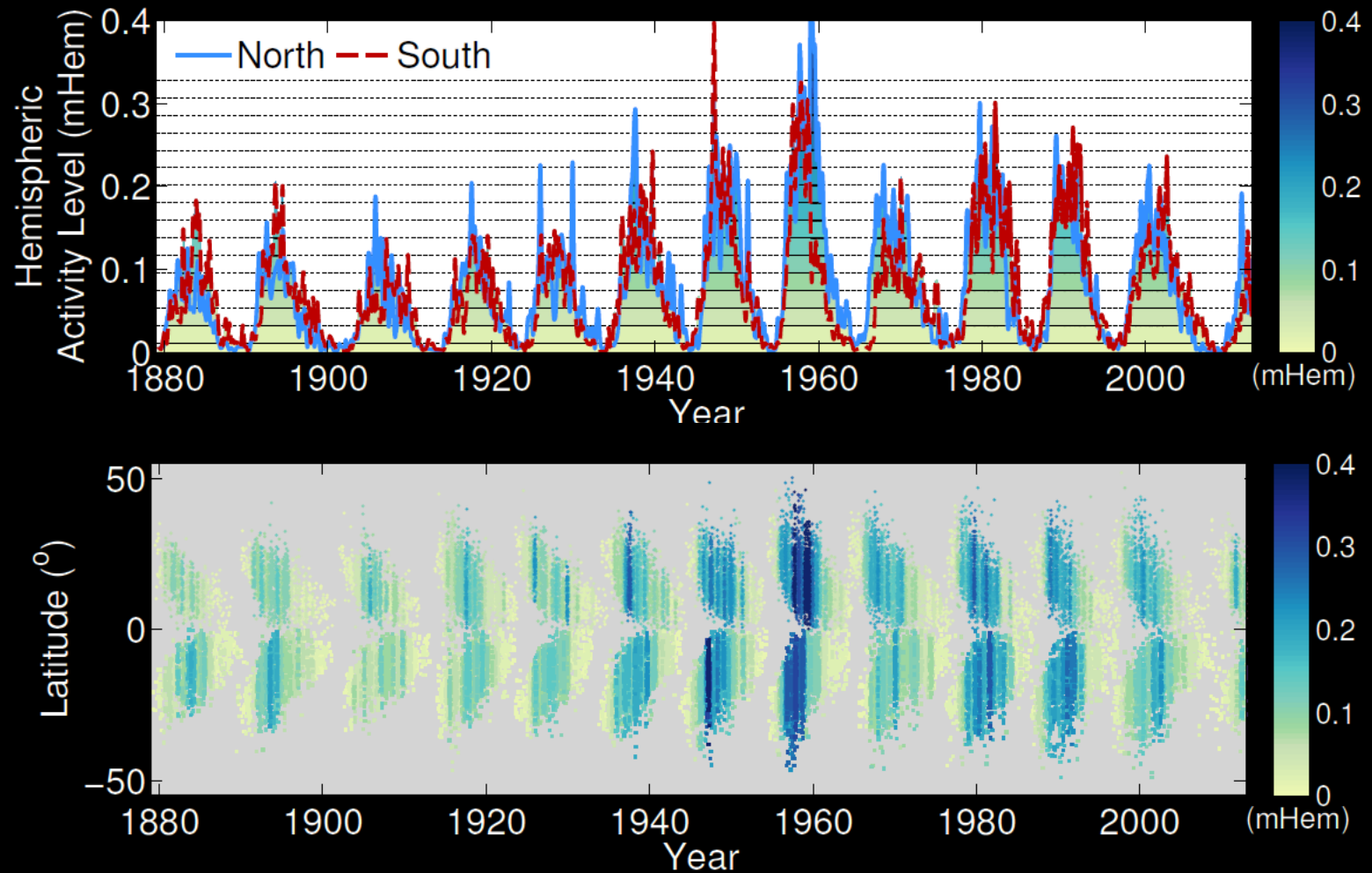
- The lifetime of an active region is but an instant compared with the cycle.
- Assumption: The global properties of a cycle are irrelevant for determining the properties of active regions. Only activity level is important.

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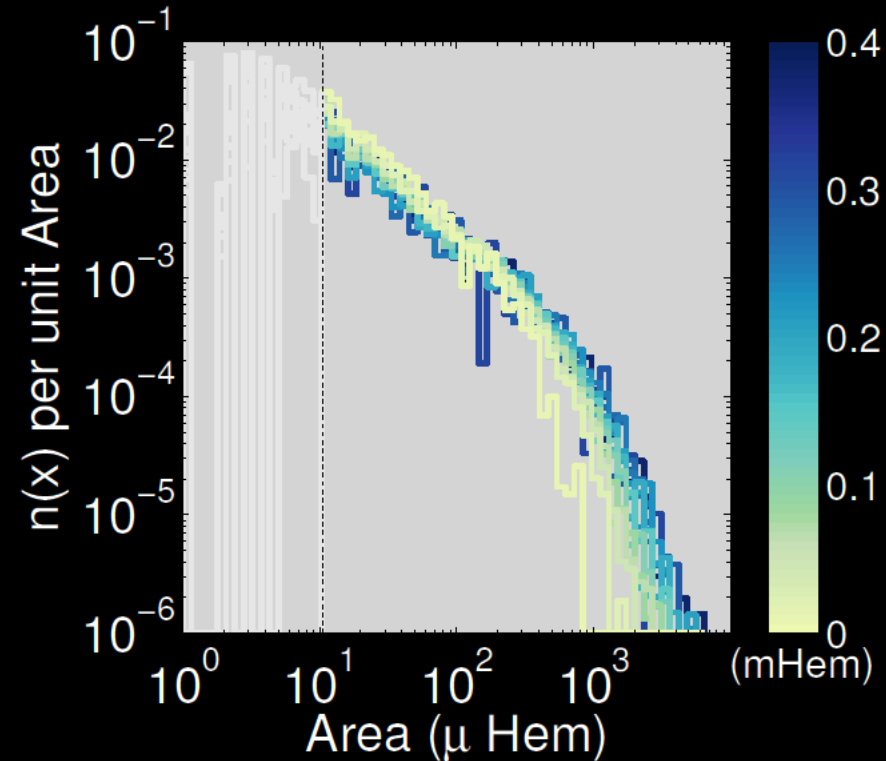
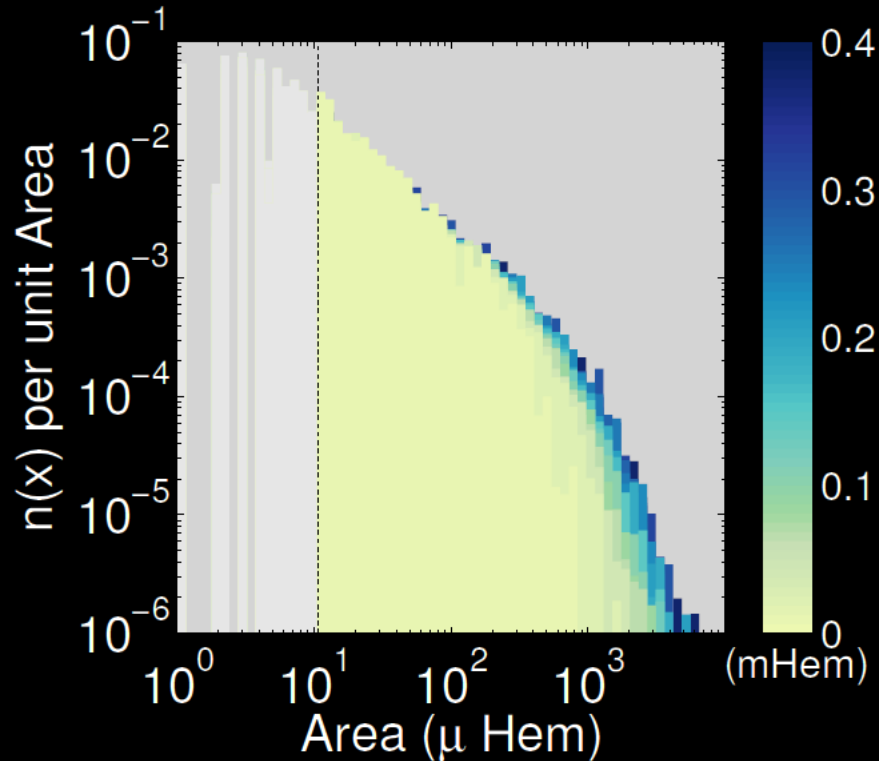
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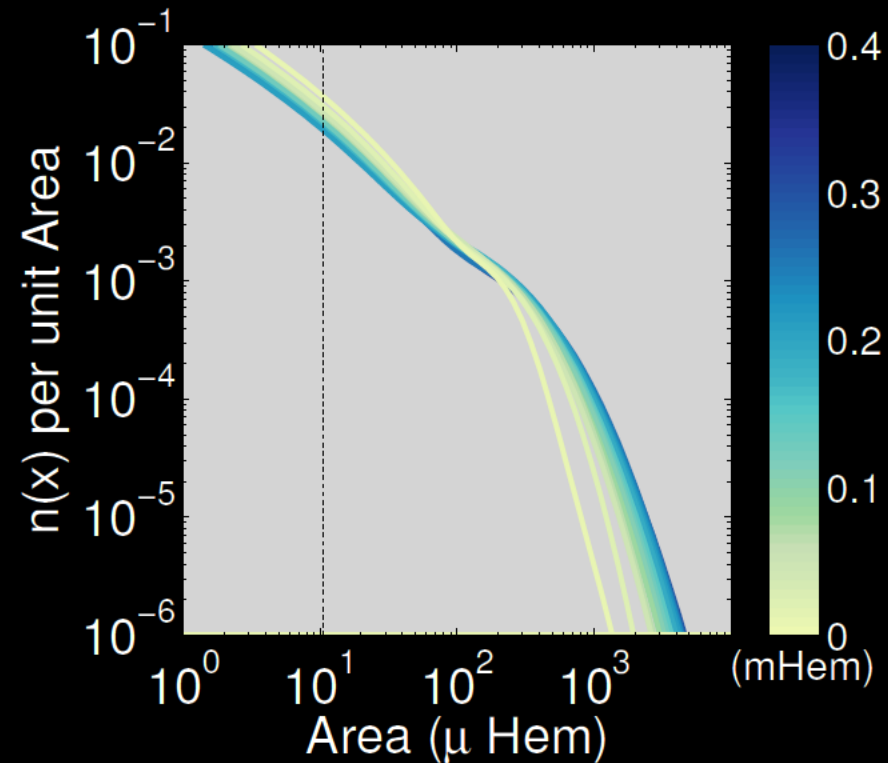
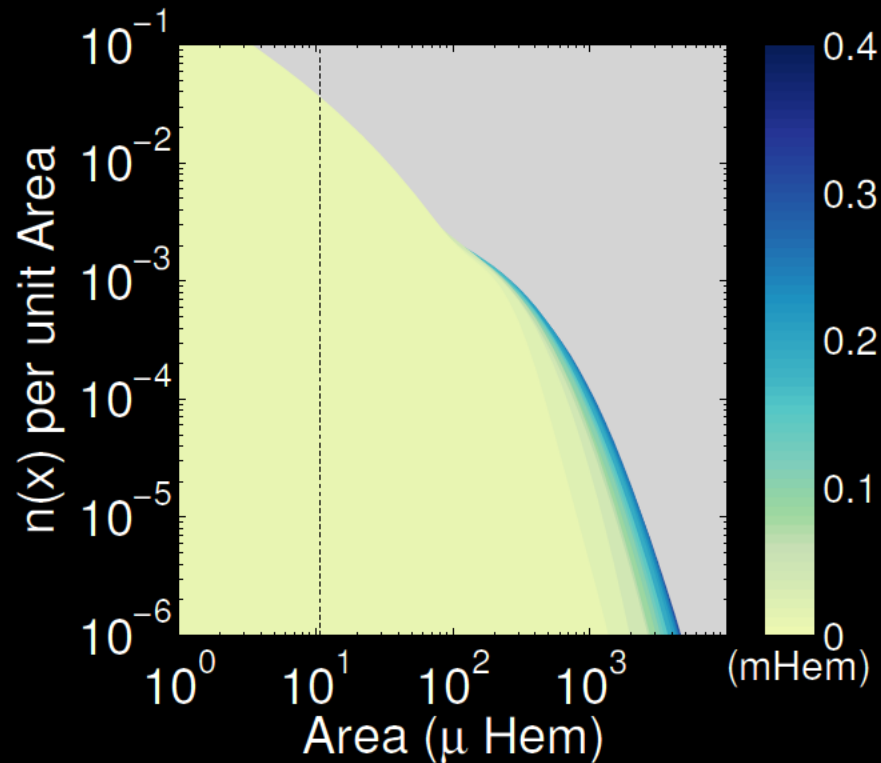
Statistical properties associated with low activity levels are observed in every cycle. Statistical properties can be different in each hemisphere.

# Activity Level and the empirical distribution function



- There is a very clear dependence of the relative amount of large sunspot groups and higher activity levels.
- The Weibull-Log-Normal composite captures successfully this variation

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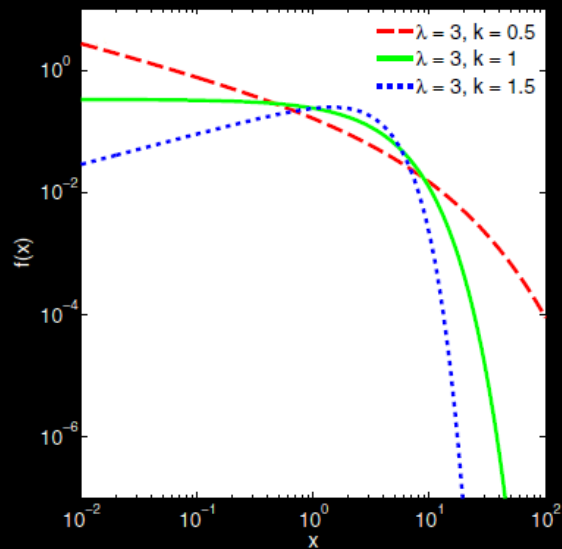


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# Activity level and the analytic probability density function

$$f(x | k, \lambda, \mu, \sigma, c) = (1 - c) \frac{k}{\lambda} \left( \frac{x}{\lambda} \right)^{k-1} e^{-\left( \frac{x}{\lambda} \right)^k} + \frac{c}{x \sigma \sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}$$

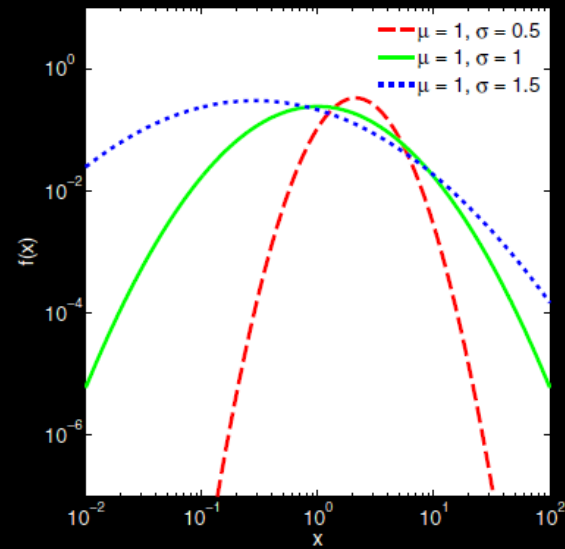
Weibull



Scale Factor ( $\lambda$ )

Shape Parameter ( $k$ )

Log-Normal



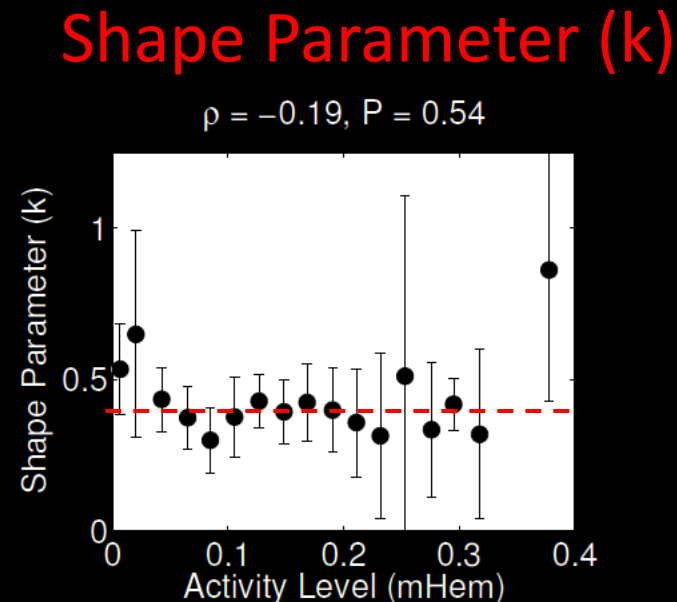
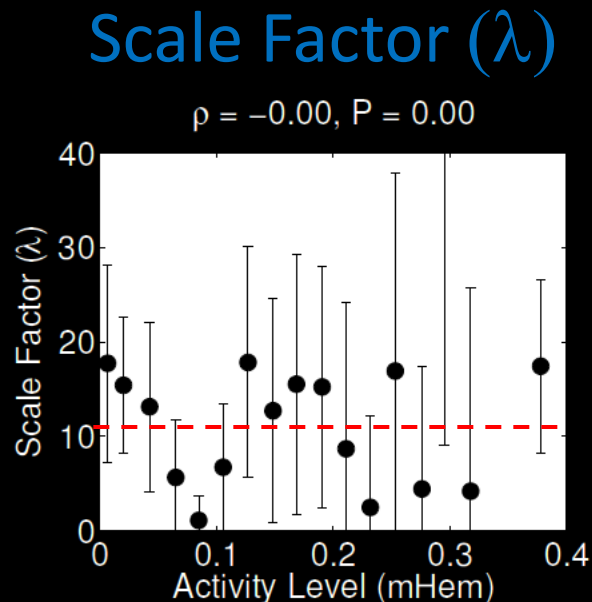
Mean ( $\mu$ )

Variance ( $\sigma$ )

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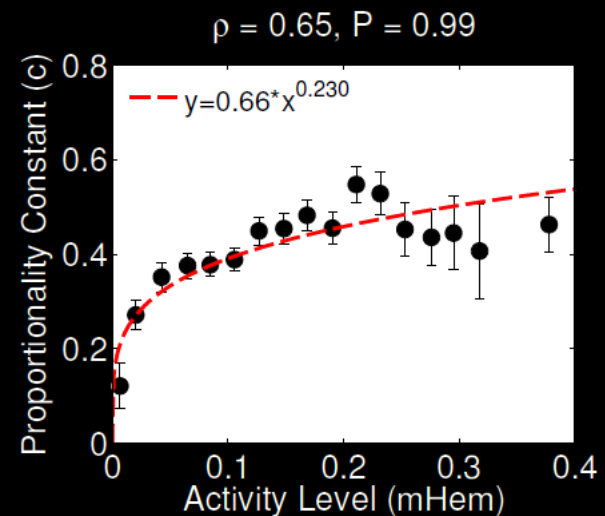
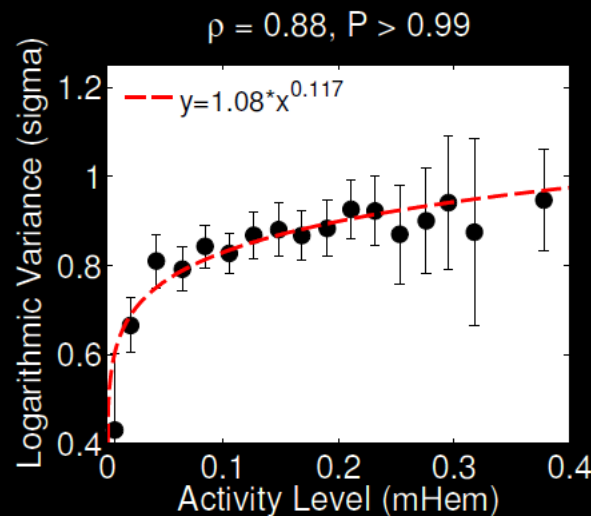
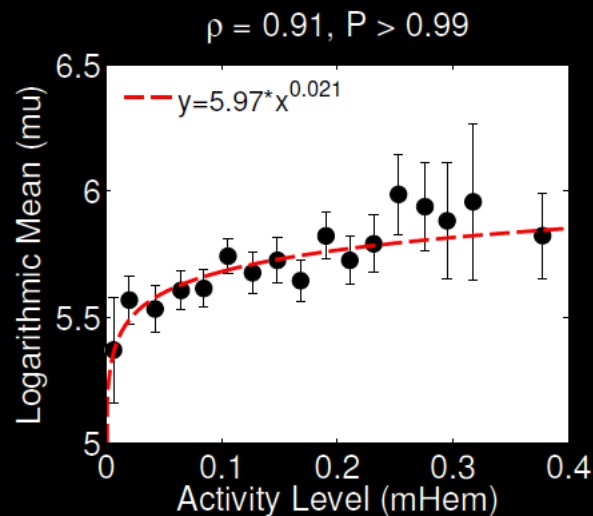
- The Weibull's parameters are not correlated activity level (Hagenaar et al 2003, 2008).

- The Log-Normal parameters correlate strongly with activity level

Mean ( $\mu$ )

Variance ( $\sigma$ )

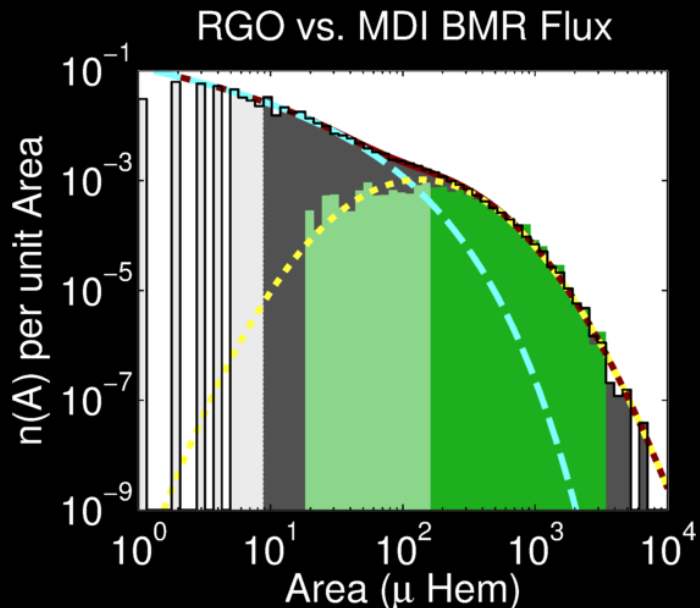
Proportionality ( $c$ )



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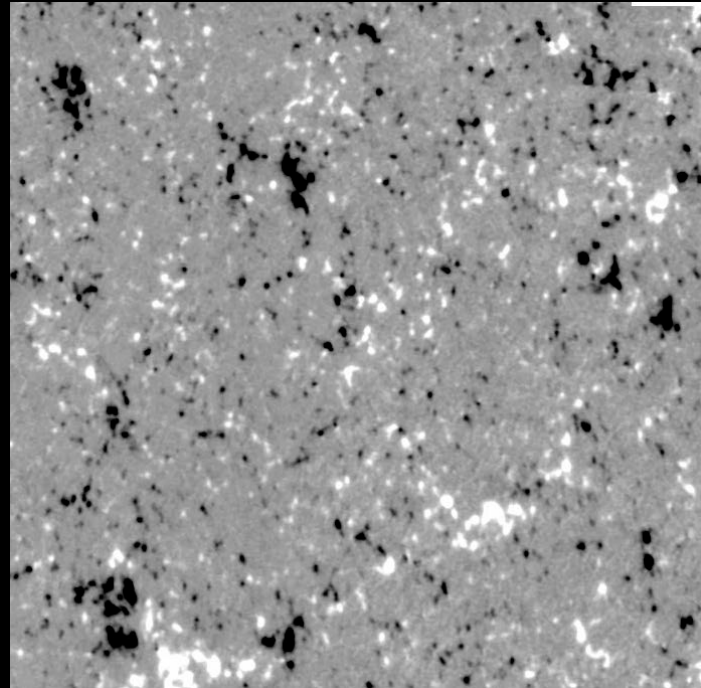
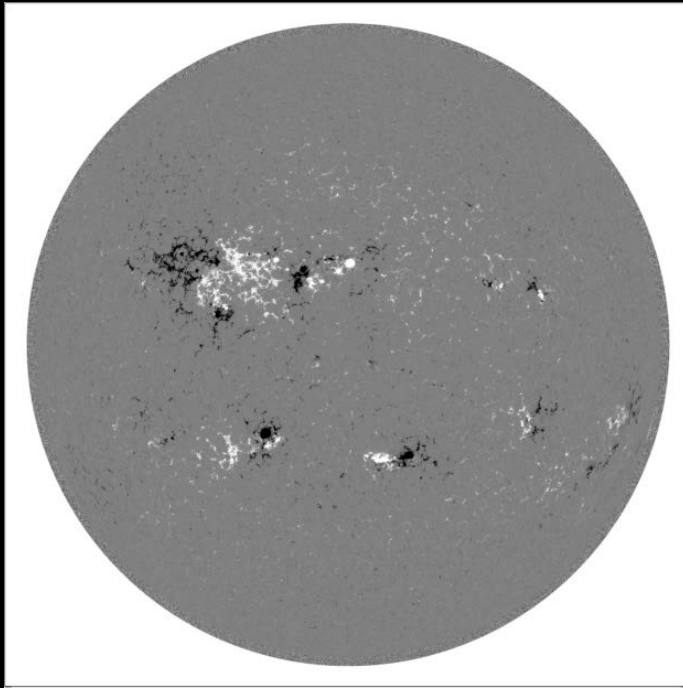
- Cycle-dependent variations of the size-flux distribution are dictated exclusively by the large BMRs!

# IMPLICATIONS

When I submitted my abstract I thought I understood what these results meant.

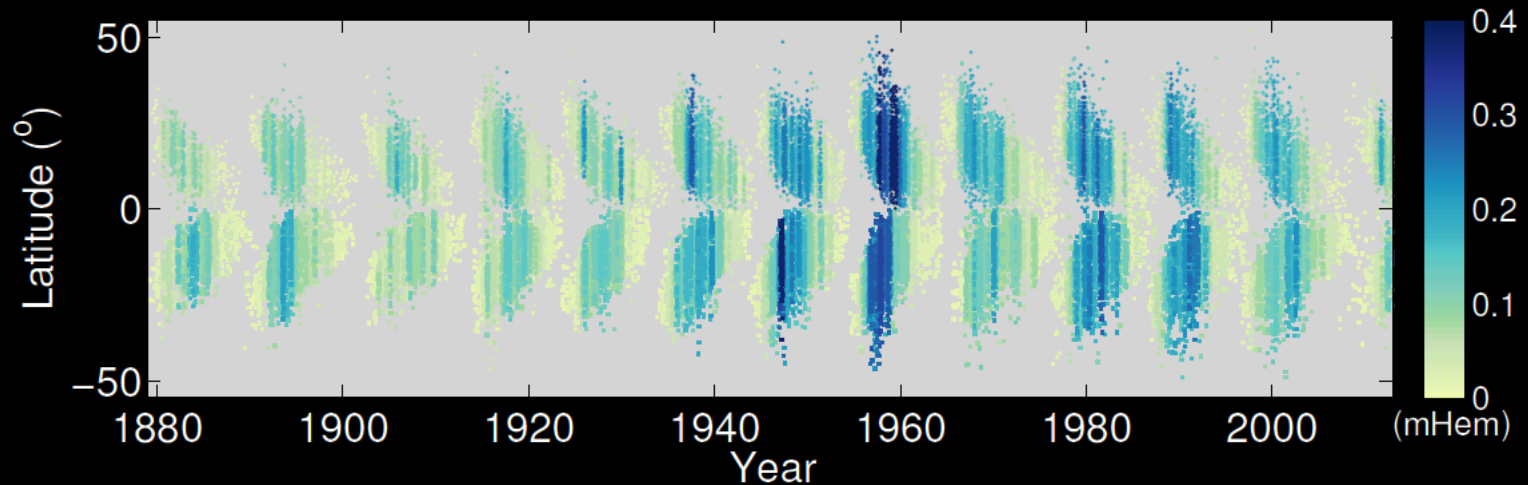
This is not the case anymore

# Small-scale vs. Global Dynamo?



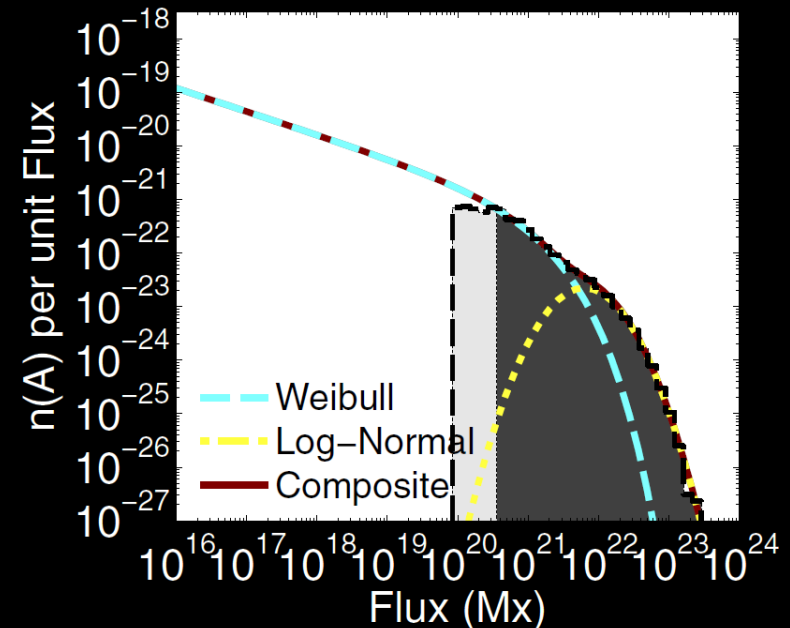
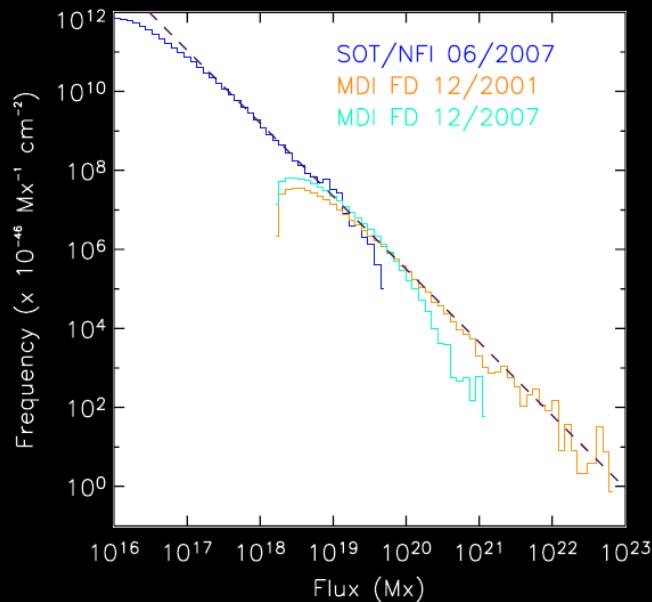
- In our first paper we speculated that the structures associated with each component were associated with different forms of dynamo action.

# ~~Small-scale vs. Global Dynamo?~~



- Sunspot groups and pores are only observed in active latitudes (spatial dependence on cycle evolution).
- Why are the statistical properties of pores fixed?

# ~~Small-scale vs. Global Dynamo?~~



- Sunspot groups and pores are only observed in active latitudes (spatial dependence on cycle evolution).
- Why are the statistical properties of pores fixed?
  - Self-similarity driven by convection?
  - Separate originating mechanisms?

# Concluding Remarks

- The solar size-flux distribution is a composite of Weibull and log-normal distributions. A very simple modulation of its parameters captures cycle dependence.
- Only the parameters that characterize the log-normal distribution change with activity level.
- Our results seem robust and significant, but I have little understanding of the underlying mechanisms.
- Analysis of magnetic data, and of MHD simulations will be critical for furthering our understanding.