Sunspot formation: theory, simulations and observations
Mar. 9-13, 2015 at NORDITA, Stockholm, Sweden
Session 4: Deep seated vs. distributed magnetic field in the Sun

High resolution calculations of solar global convection and dynamo

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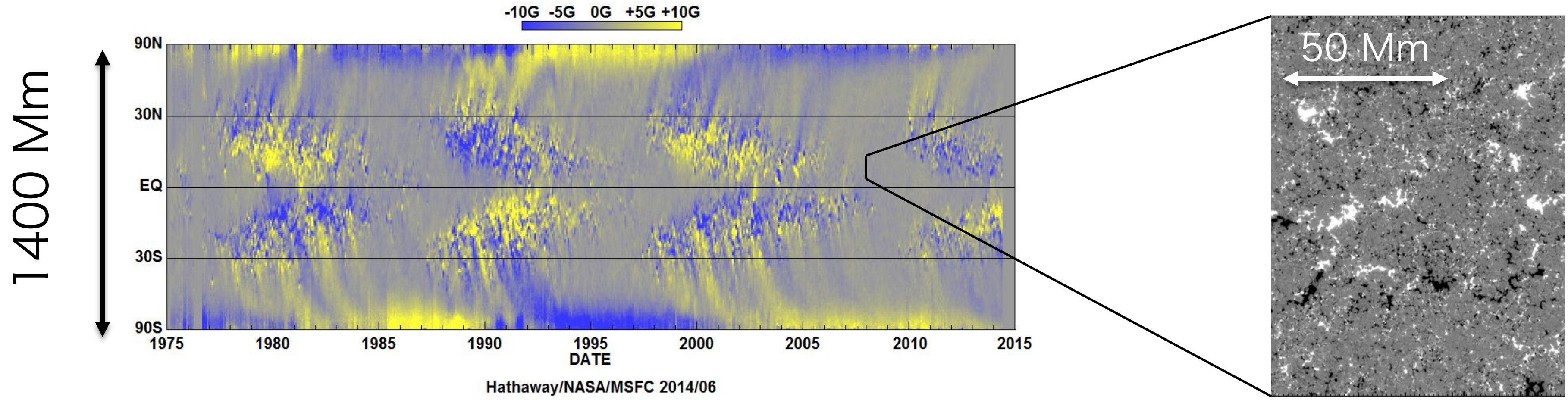
Solar magnetism



(Lites+2008, *Hinode*)

The sun shows large temporal and spatial ranges of magnetic field from the granulation scale (～1Mm) to the global dipole field (～1000 Mm).

Magnetic field is thought to be maintained by the dynamo action in the convection zone.

Small- and large-scale dynamos

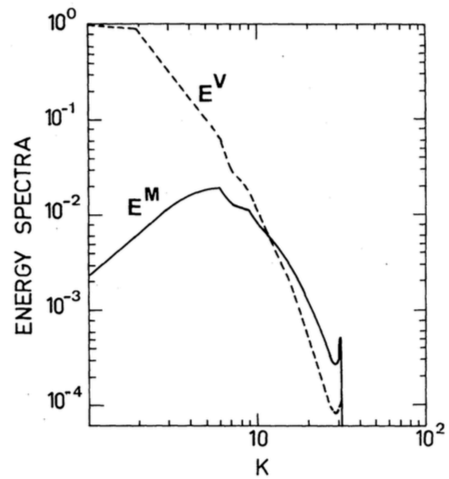
Small-scale dynamo: Dynamo operating with non-helical turbulence in the scale smaller than energy carrying scale generating **no** net magnetic flux (Local dynamo, fluctuation dynamo). Origin of the **photospheric** magnetism (Catteneo 1999). Time scale is less than 1 minute (Rempel, 2014).

Large-scale dynamo: Dynamo with large-scale shear and/or mean turbulent electromotive force by helical turbulence generating net magnetic flux (Global dynamo).
Origin of the features related to solar cycle (e.g. **polar reversal**, **Hale's law**). Time scale is about 10 years.
(See also Brandenburg+2005, Physics Report, Section 5)

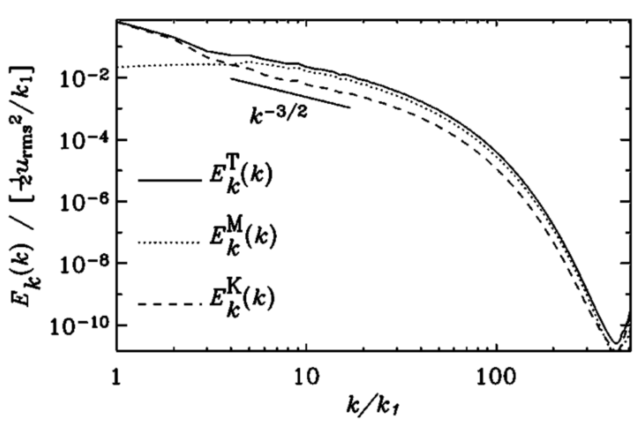
Sometimes there is no clear separation between them.
Combination of these constructs the solar magnetism.

Small-scale dynamo simulations

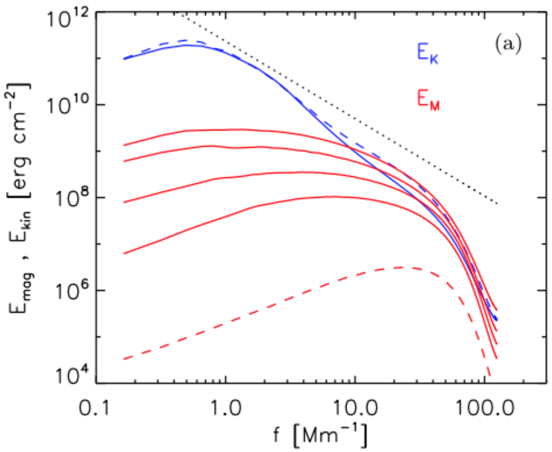
Small-scale dynamo is described with its energy spectra. In the kinematic growth phase, the magnetic energy peaks in the smallest scale. In the saturated regime, if the system can reach it, the magnetic energy exceeds the kinetic energy in the small scale.
The peak shifts to the larger scale.
Interesting discussions about magnetic Prandtl number is seen in Brandenburg 2011, 2014.



Meneguzzi+1981 64^3 grid in forcing turbulence.



Haugen+2004, 1024^3 grid in forcing turbulence



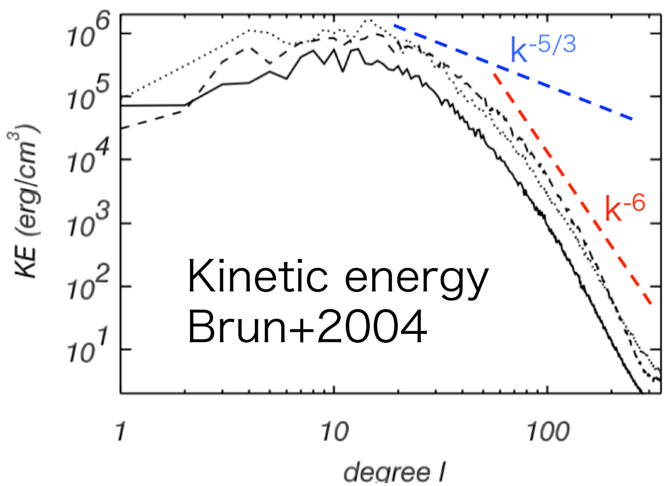
Rempel 2014, 1536^2x768 at the solar photosphere

Large-scale dynamo simulations (1/3)

A lot of large scale dynamo simulations succeed in reproducing the coherent mean field with cycle (ASH, EULAG, FSAM, and pencil...)

In the large-scale dynamo simulations in the convection zone, the small-scale dynamo is **not** efficient, since the small-scale dynamo requires enough resolution for resolving the inertial range of turbulence.

As a result, the turbulent magnetic energy is "small", which is less than 10% of kinetic energy.

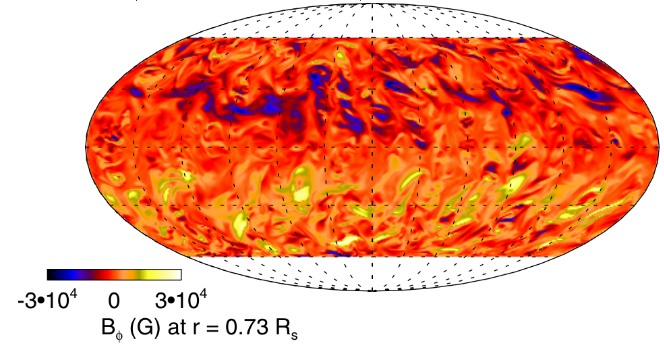


$\kappa = 3\times10^{13}$ cm^2 s^{-1}

$\nu = \eta = 1\times10^{12}$ cm^2 s^{-1}

$\Omega_0 = \Omega_{sun}$, $E_{mag}/E_{kin} \sim 0.1$

(Fan+2014)



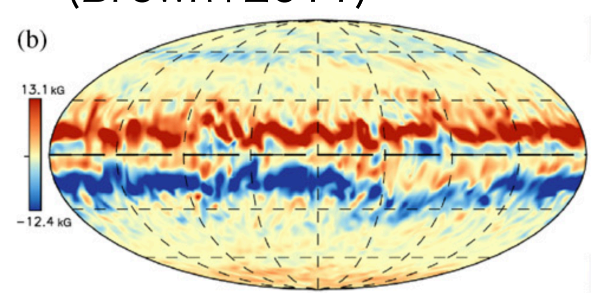
$\kappa = 7.52\times10^{12}$ cm^2 s^{-1}

$\nu = 9.4\times10^{11}$ cm^2 s^{-1}

$\nu = 1.88\times10^{12}$ cm^2 s^{-1}

$\Omega_0 = 5\Omega_{sun}$, $E_{mag}/E_{kin} \sim 0.08$

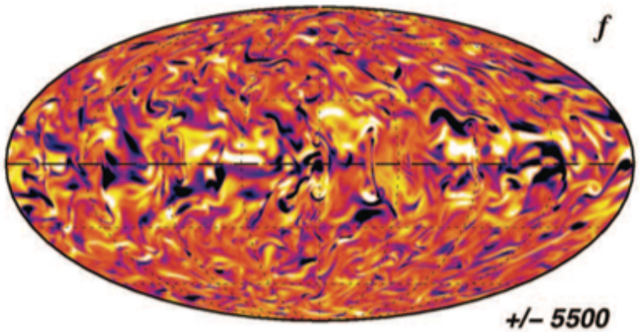
(Brown+2011)



Large-scale dynamo simulations (2/3)

Small magnetic diffusivity tends to cause **small amplitude of the mean field** and short cycle.

$\Omega_0=3\Omega_{sun}$, Pm=0.25

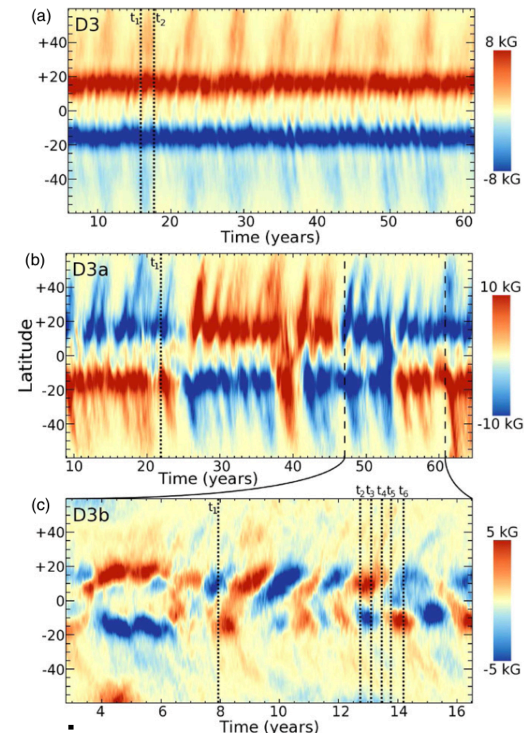


Pm=4, $\Omega=\Omega_{sun}$,
$\eta=3\times10^{11}$ cm^2 s^{-1}
$E_{m(turb)}/E_{m(total)}$=0.98
No clear indication of mean magnetic field.
(Brun+2004)

$\eta=2.6\times10^{12}$ cm^2 s^{-1}
$E_{m(turb)}/E_{m(total)}$=0.53

$\eta=1.8\times10^{12}$ cm^2 s^{-1}
$E_{m(turb)}/E_{m(total)}$=0.59

$\eta=1.2\times10^{12}$ cm^2 s^{-1}
$E_{m(turb)}/E_{m(total)}$=0.85



Mean magnetic energy is decreased by factor of 3. (Nelson+2013)

Large-scale dynamo simulations (3/3)

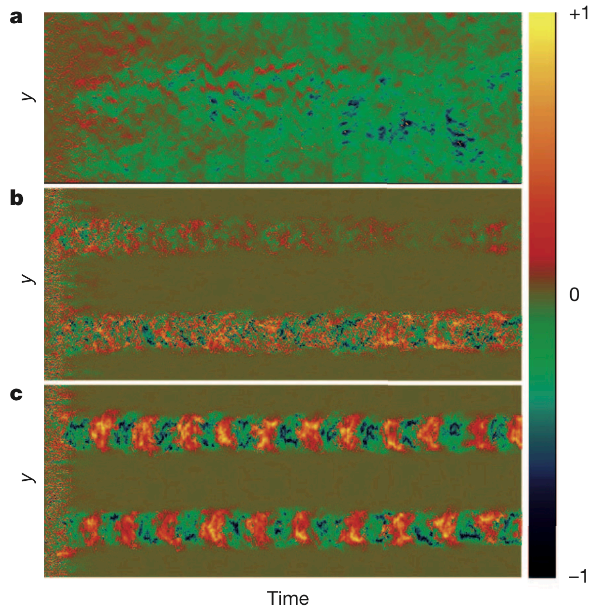
2D kinematic dynamo in high $R_m \sim 2500$(Cattaneo+2013, 2014)

Mean B normalized for removing the exponential growth.

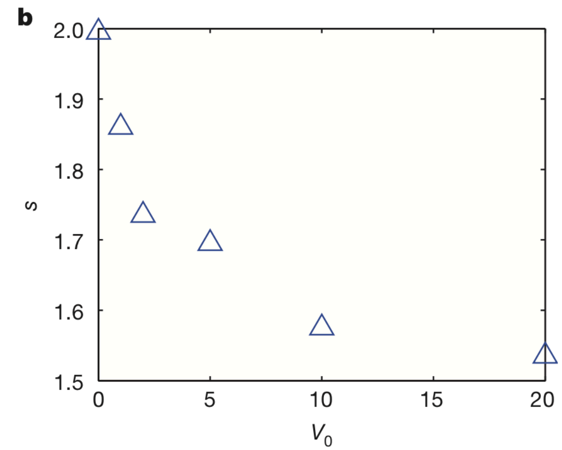
helical no shear

non-helical with shear

helical with shear



Growth rate



Amplitude of shear

The shear suppresses the growth rate of the SSD. Note that the non-linear effect is ignored.

Interesting argument is that suppressing the small-scale dynamo is required for achieving the large-scale dynamo.

Open questions

1. Possibility of small-scale dynamo in the convection zone
   - ✓ Strength of the turbulent magnetic field in the convection zone.
   - ✓ Influence on the convective flow.
   - ✓ Influence on the energy transport.

2. Influence of the small-scale dynamo on the large-scale dynamo
   - ✓ Does small-scale dynamo destroy large-scale dynamo?
   - ✓ Does large-scale dynamo require the destruction of small-scale dynamo?

Investigations for small- and large-scale dynamo

We carry out two series of calculations:

1. HD and MHD calculations in restricted Cartesian geometry exploring the possibility of small-scale dynamo in the convection zone without the rotation in high resolution which currently cannot be achieved in any global settings.

2. MHD calculations in full spherical geometry exploring the interaction between small- and large-scale dynamos using rather low resolution.

Numerical setting (1/2)

Full set of MHD equations are solved with the reduced speed of sound technique (RSST: Hotta+2012,2015).

$$\frac{\partial \rho_1}{\partial t} = -\frac{1}{\xi^2}\nabla\cdot(\rho\mathbf{v})$$

The equations remain fully explicit. Only local communication is required.
Slope-limited diffusion is used in order to maximize the Reynolds numbers (Rempel, 2009, 2014). We use a "solar setting".
The solar standard model is adopted for background stratification and linearized equation of state considering the ionization of H and He is used (This is almost one for the perfect gas in this talk).

$$p_1 = \left(\frac{\partial p}{\partial \rho}\right)_s \rho_1 + \left(\frac{\partial p}{\partial s}\right)_\rho s_1$$

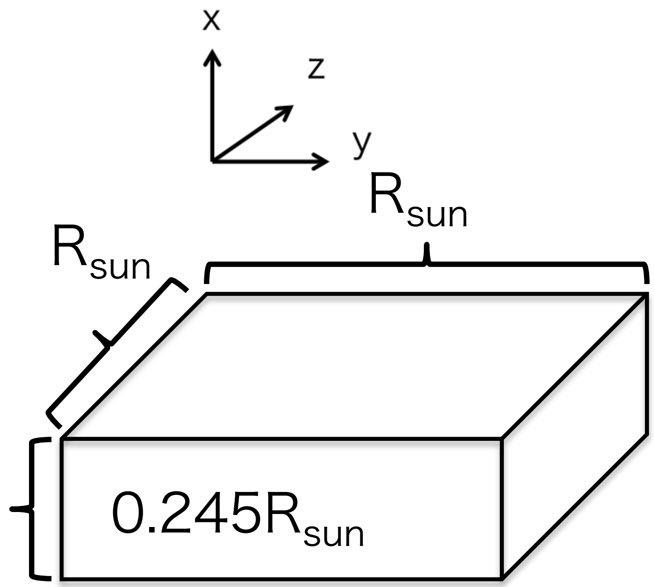
Numerical setting (2/2)

- Calculation domain

$(0.715,0,0)<(x,y,z)/R_{sun}<(0.96,1,1,1)$

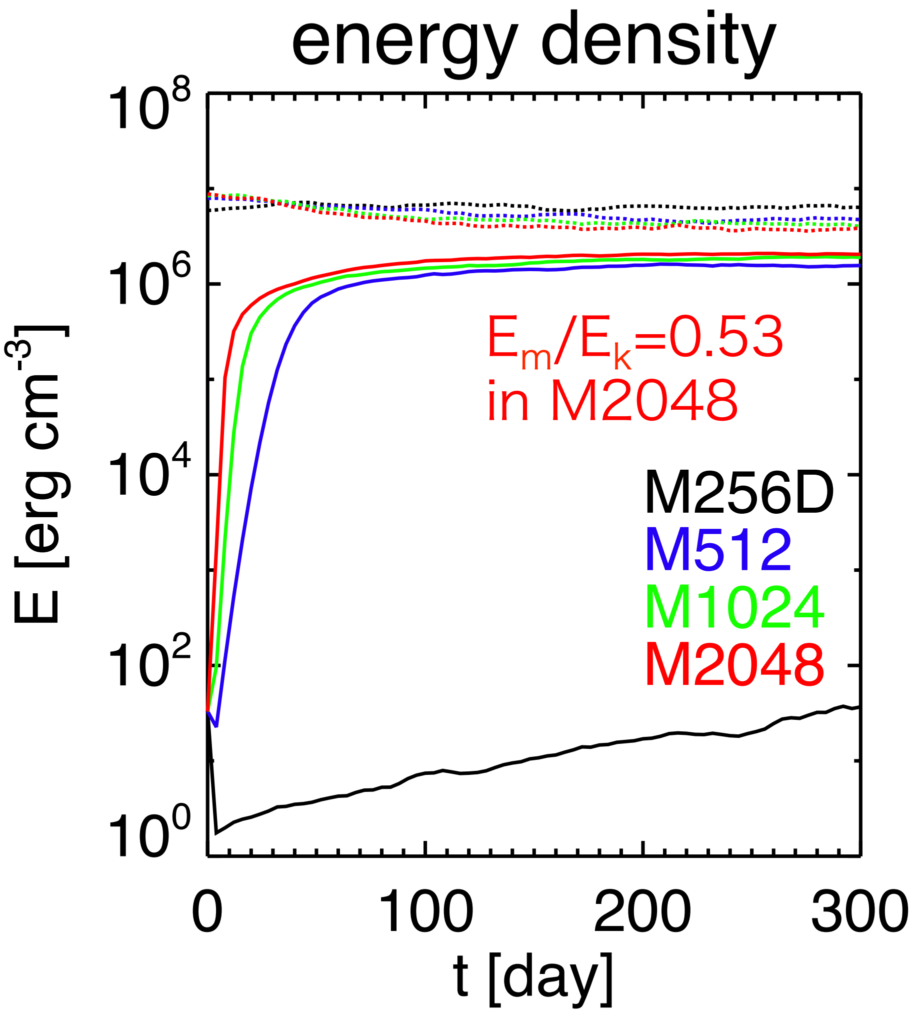
Less diffusive ↓

Cases	$N_x \times N_y \times N_z$
H256D,M256D	72x256x256
H512, M512	144×512×512
H1024, M1024	288×1024×1024
H2048, M2048	576×2048×2048



Solar luminosity at the base of the convection zone is adopted. Only H(M)256D have explicit diffusivities $\kappa=\nu=\eta=1\mathrm{x}10^{12}\ \mathrm{cm}^2\ \mathrm{s}^{-1}$ in order to compare it with ordinary global calculations (Fan+2014). In the highest resolution, the grid spacing is smaller than 350 km. Hydrodynamic cases (H****) are calculated 100 days. Then weak random magnetic field is added with no net magnetic flux (M****).

Energy evolution



Dashed: Kinetic energy
Solid: Magnetic energy

Cases	τ [day]
M256D	112
M512	3.24
M1024	1.97
M2048	0.99

Saturation magnetic energy for M256D is 2×10^5 erg cm^{-3}, which is 10 times smaller than that in M2048.

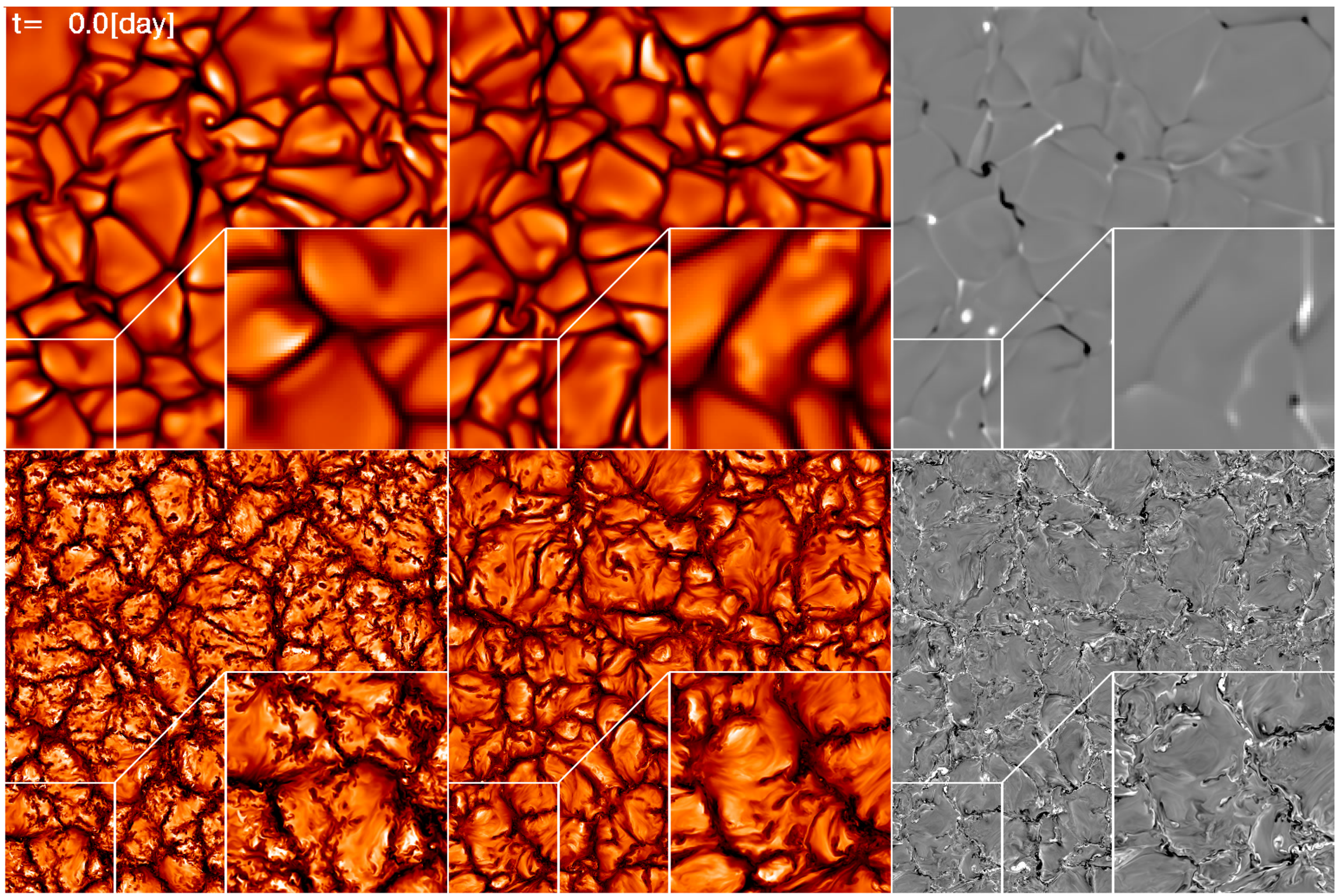
Contours for two extreme cases (1/2)

Values at $r=0.95R_{sun}$

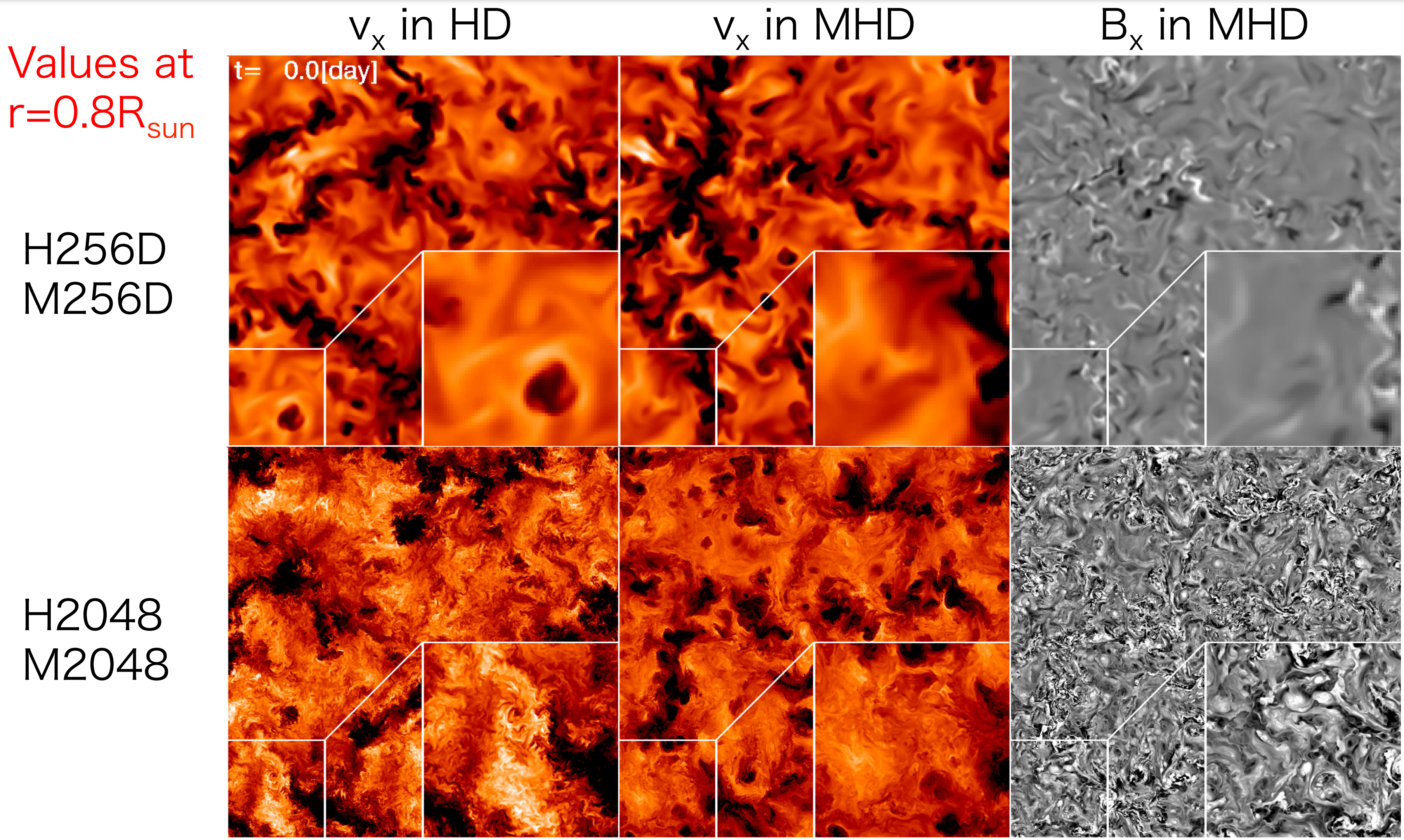
v_x in HD | v_x in MHD | B_x in MHD

H256D
M256D

H2048
M2048

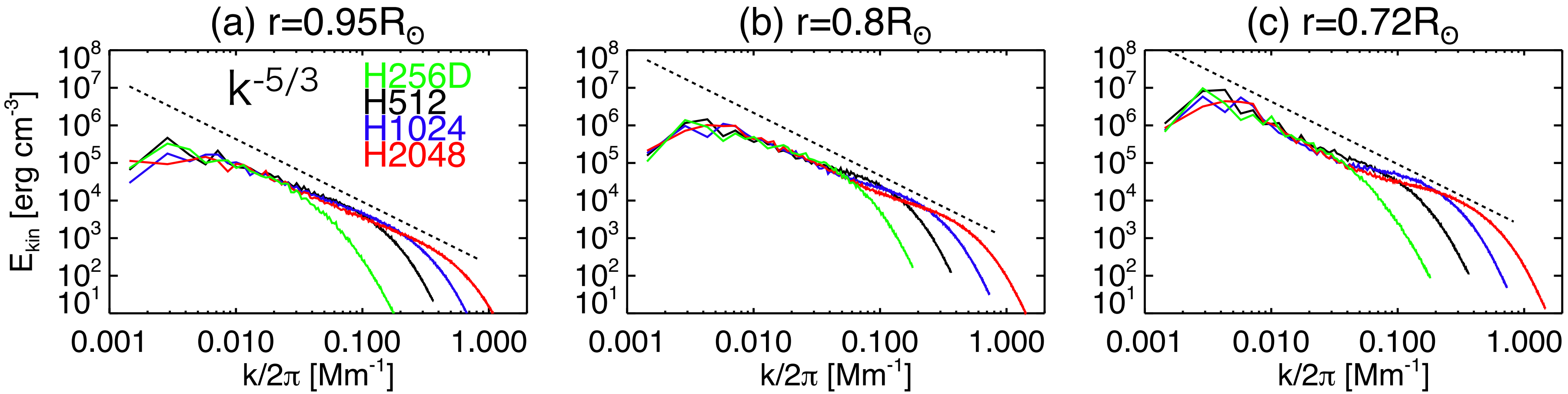


Contours for two extreme cases (2/2)

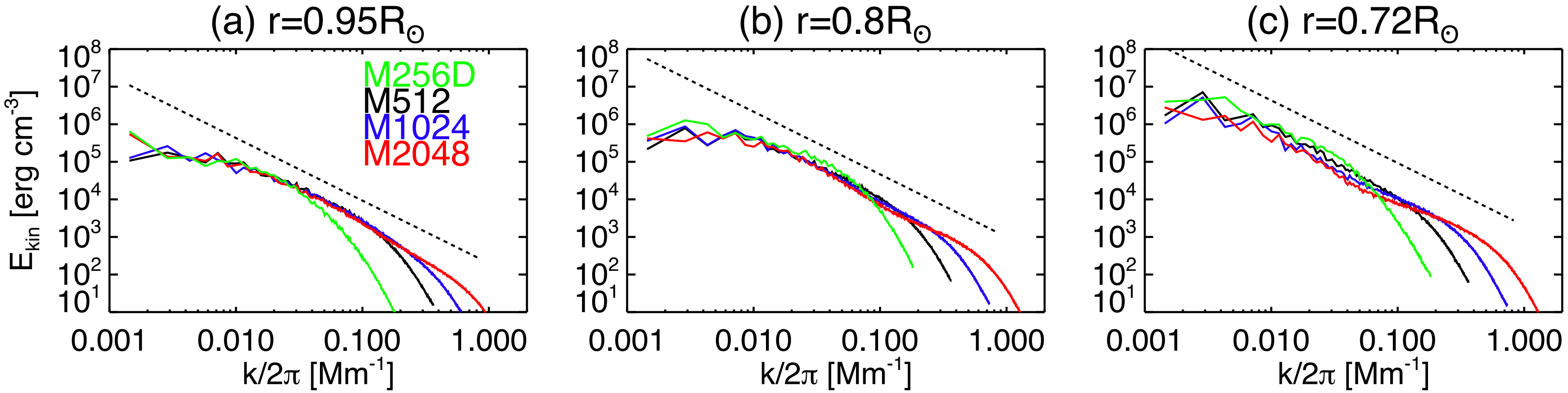


Kinetic energy spectra

Spectra in HD

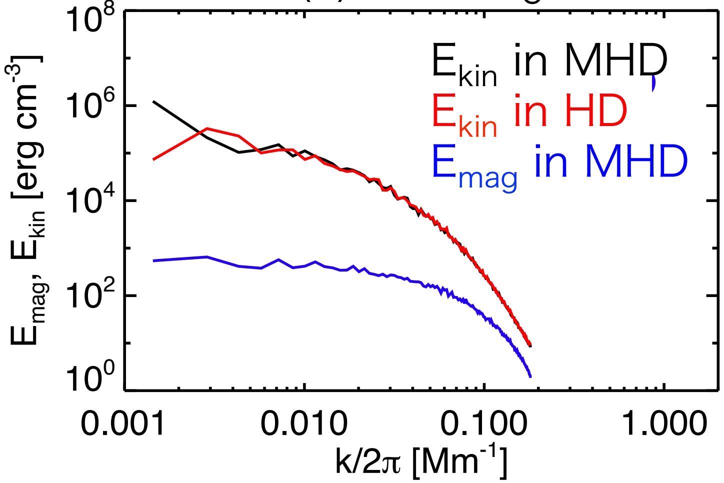


Spectra in MHD

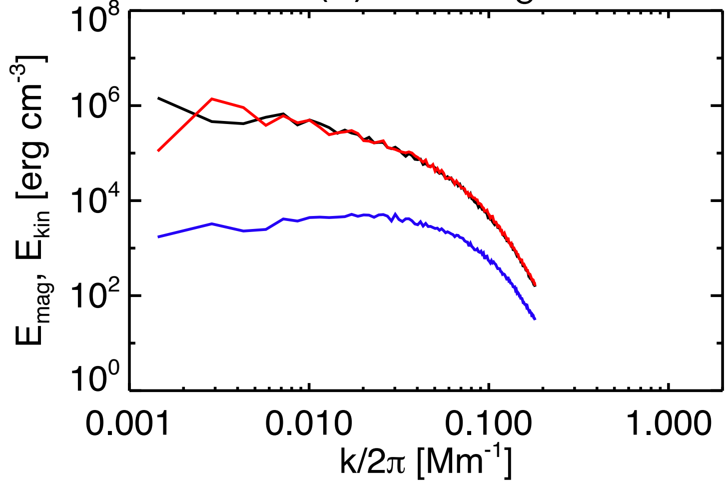


Spectra in HM256D (typical global setting)

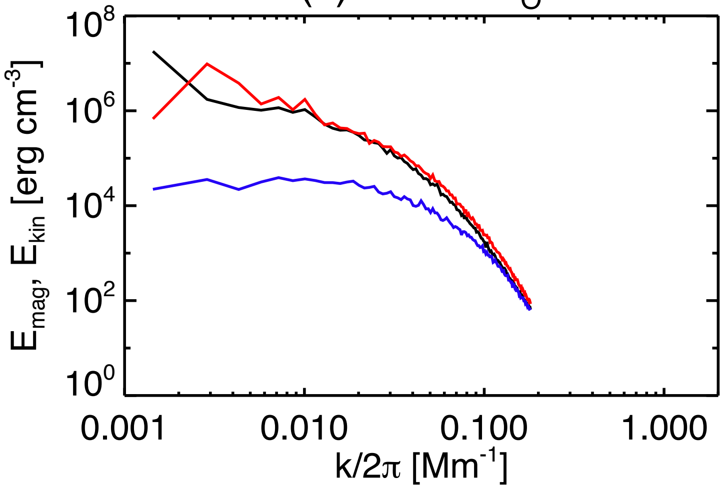
(a) r=0.95R$_\odot$



(b) r=0.8R$_\odot$



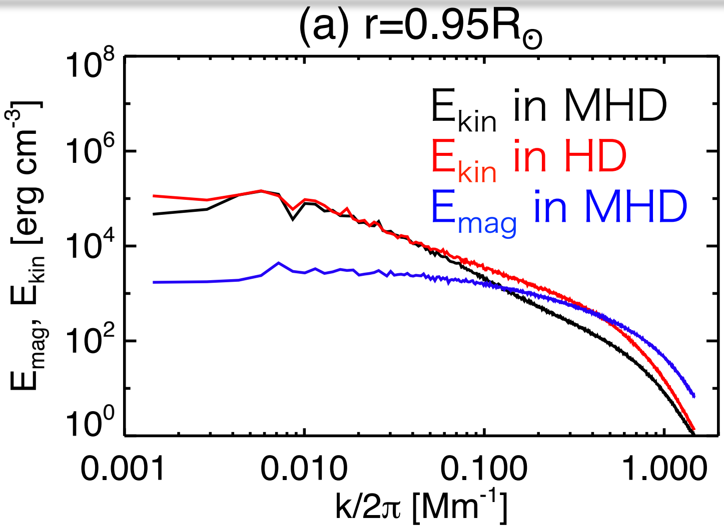
(c) r=0.72R$_\odot$

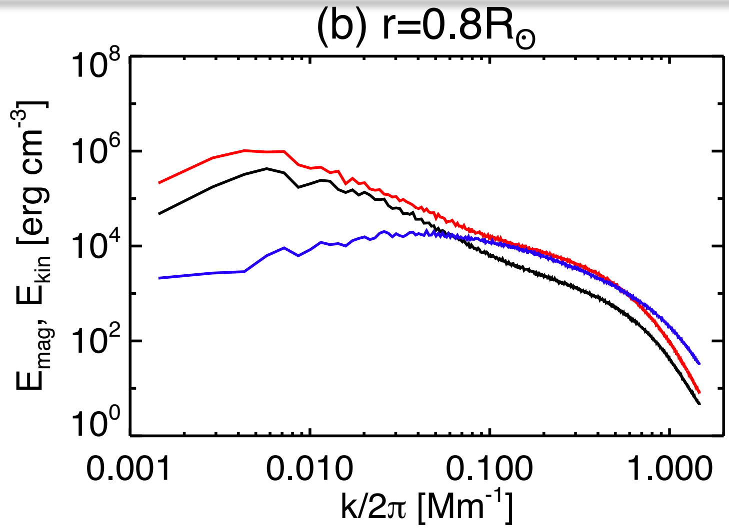


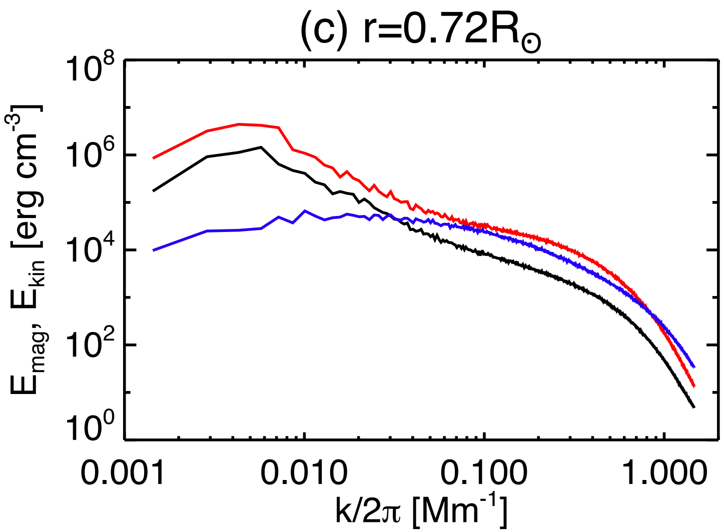
The magnetic energy does not reach the kinetic energy indicating a less efficient small scale dynamo.

The Lorentz feedback to the flow is insignificant.

Spectra in HM2048 (Highest resolution)



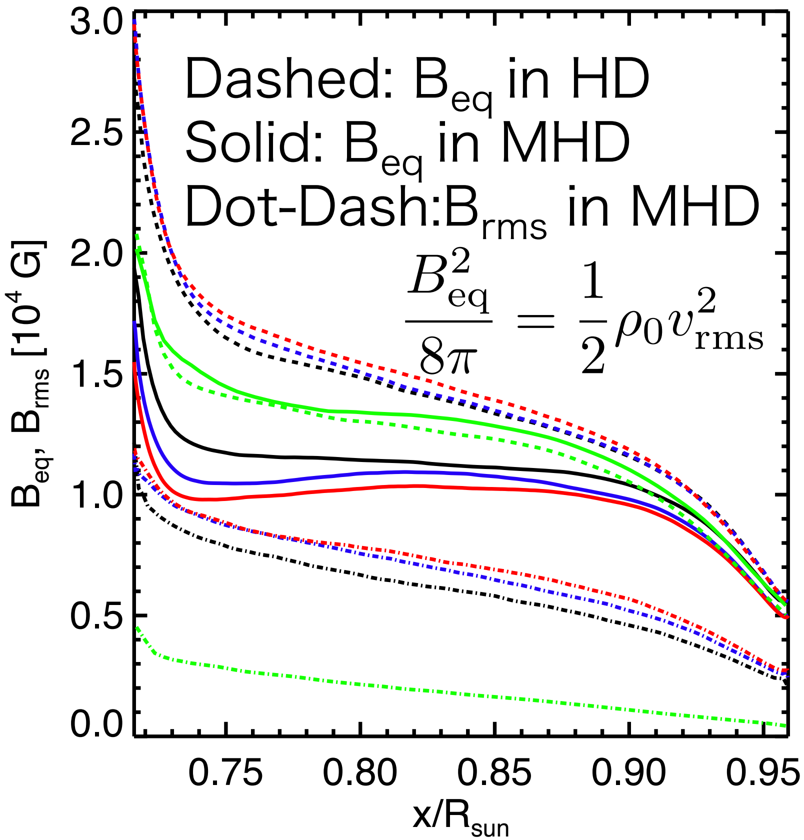


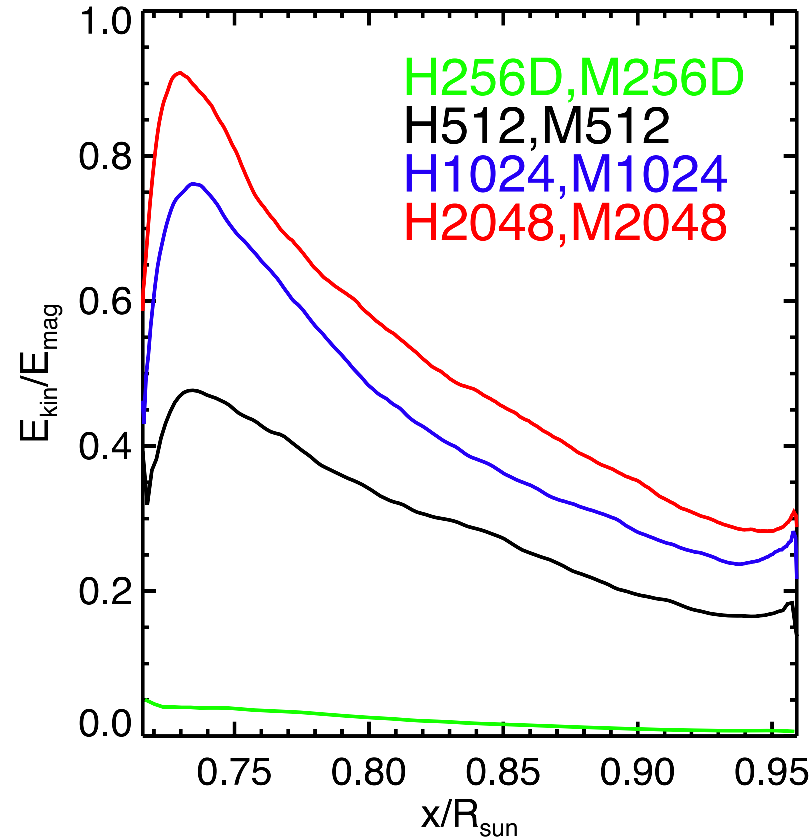


Spectra for the highest resolution. In the upper layer, small-scale velocity is selectively suppressed. In the deeper layer, all the scale is suppressed.

Super equipartition is seen in different scale in different depth.

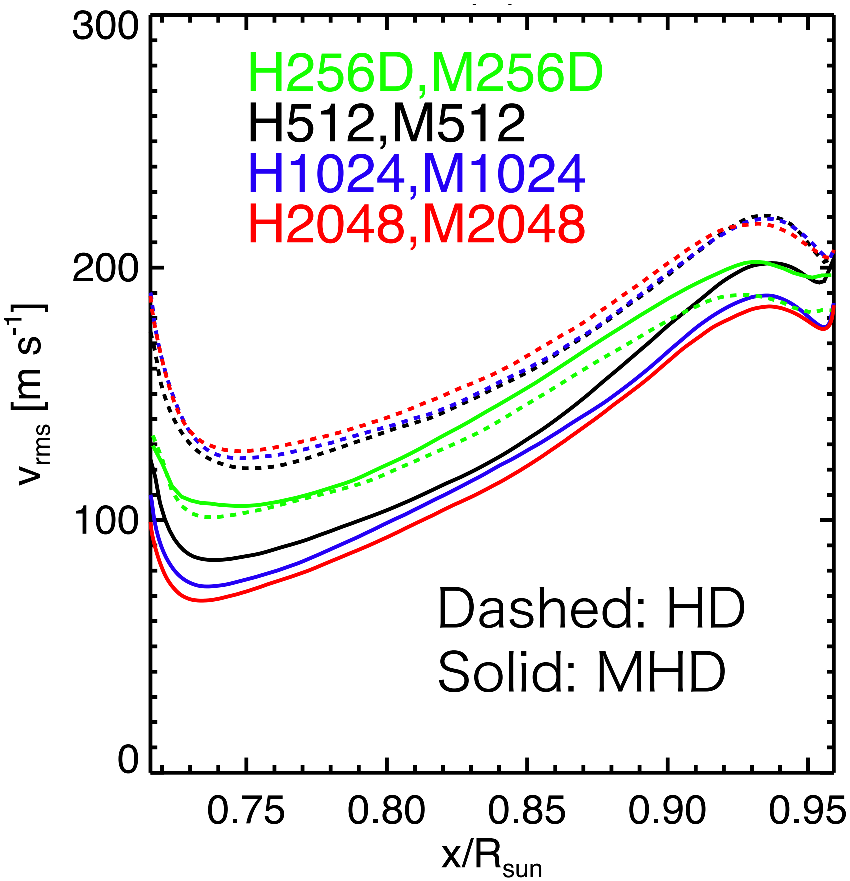
B_{rms} vs. B_{eq}

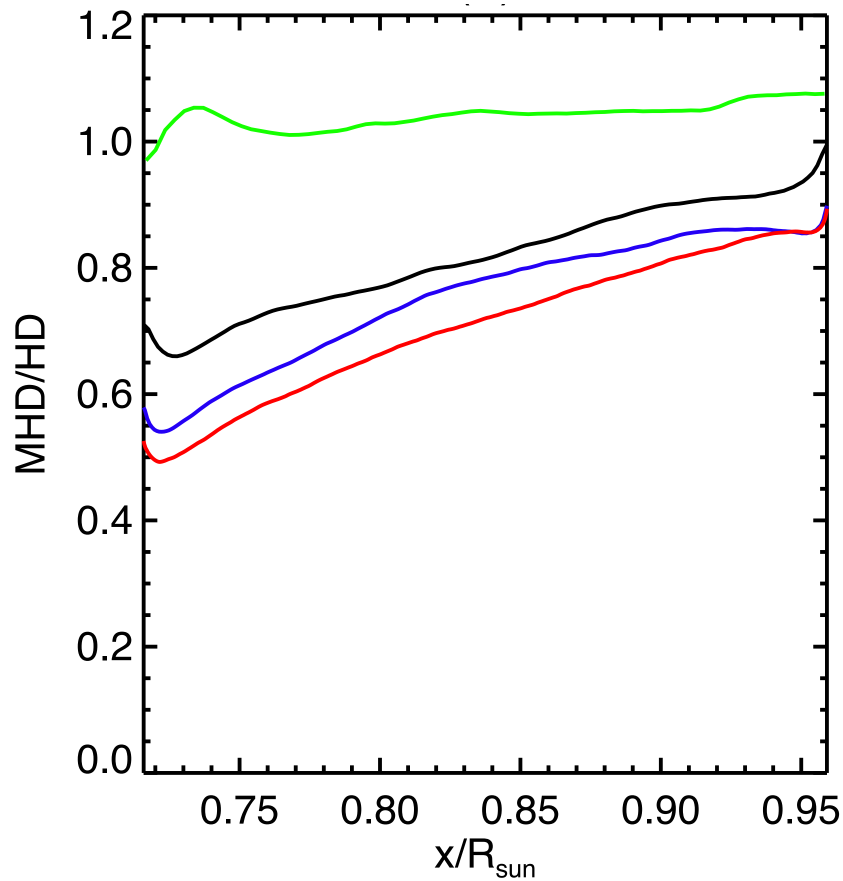




The magnetic energy reaches more than 80% ($0.95B_{eq}$) of kinetic energy at the convection zone in M2048, while M256D can maintain 5% of kinetic energy.

Lorentz feedback on flow

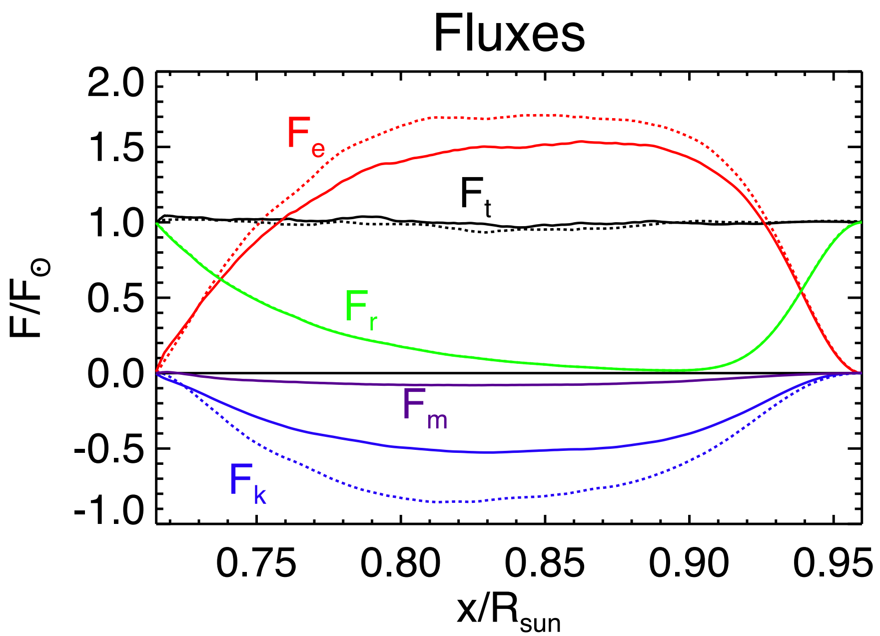




The RMS velocity at the base is reduce by factor of 2 in M2048.
The result has not converged yet.

Influence on energy transport

H2048 and M 2048



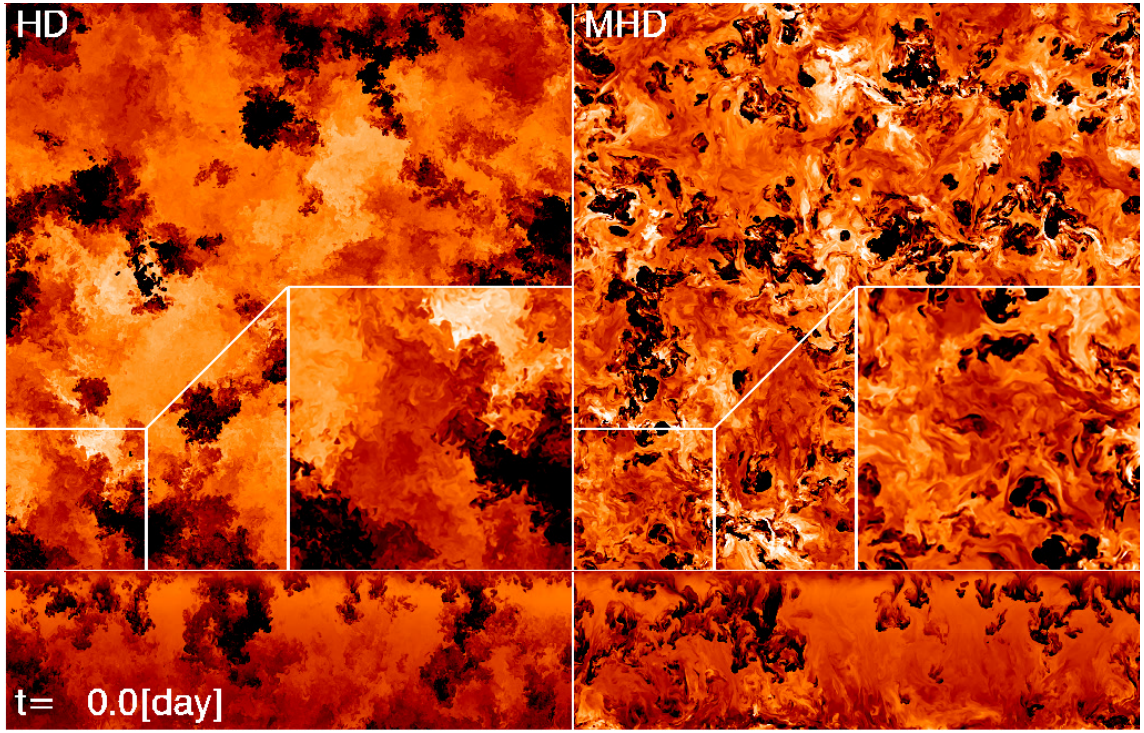
Dotted lines are for hydro.
Solid lines are for MHD.

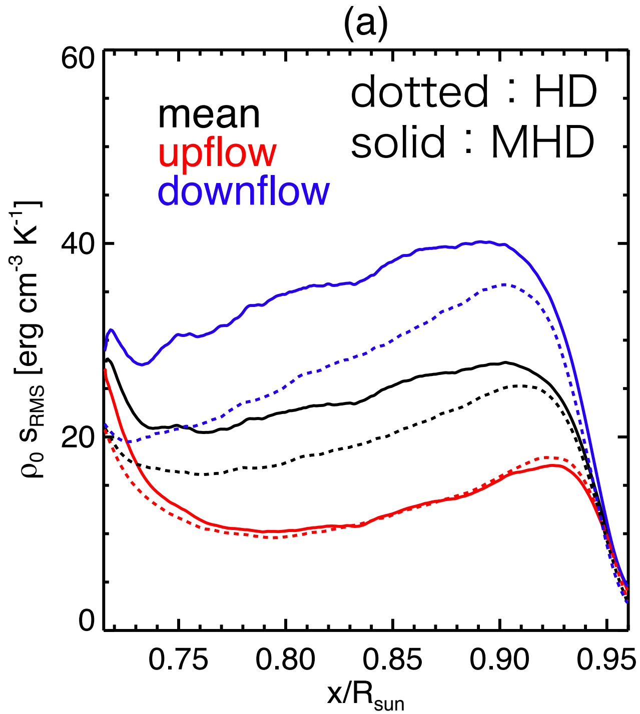
$$F_\mathrm{e} = \int \rho_0 c_p T_1 v_r dS$$

Although the RMS velocity in the middle of the CZ has 30% reduction, the enthalpy flux is reduced only 12% at maximum. This is opposite sense to the previous study, in which RMS velocity is reduced 23% and the enthalpy flux is reduced 40% (Fan & Fang, 2014)

Entropy perturbation

Entropy at $r=0.8R_{sun}$





Magnetic field suppresses the mixing of heat between up- and downflow regions. This makes upflow hotter and downflow cooler than HD case. As a result the energy transport is not significantly suppressed even with strong Lorentz feedback.

Connection between small- and large-scale dynamo

We found significant influences of efficient small-scale dynamo on the convective flow, the entropy mixing and the energy transport.

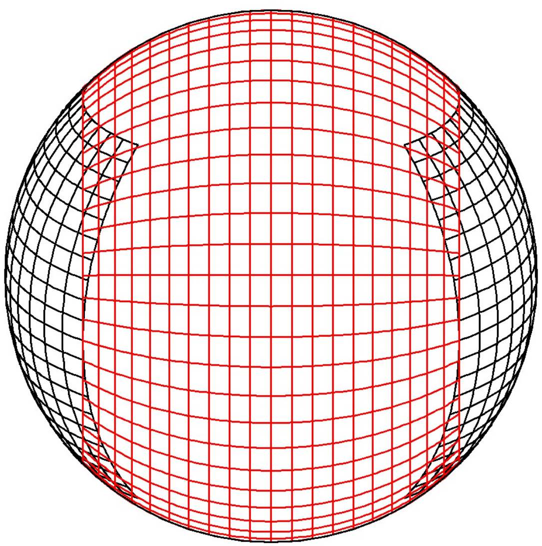
It is currently difficult to carry out the global dynamo calculation with using the same resolution as shown here, since the global dynamo requires large computational domain and long time scale.

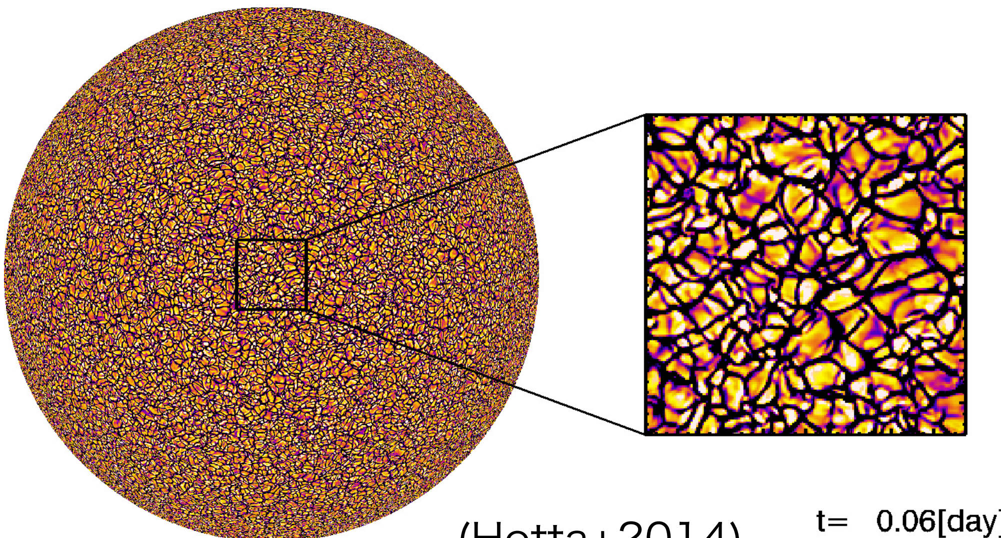
576(N_r)x6072(N_θ)x12144(N_ϕ) for 15 million time step for 50 years calculation requiring 450 million core hours with my code (e.g. 4 million core hours for M. Rempel's largest sunspot calculation.)

Thus we are doing calculations as large as possible using our current numerical resource, with hoping the help of the rotation on the small-scale dynamo.

Numerical setting (1/2)

Most of settings are same as small-scale dynamo runs.
Calculation domain is full sphere using Yin-Yang grid.
Solar model and solar rotation rate are used.
Radial extent is $0.715<r/R_{sun}<0.96$ for focus on dynamo study.
In order to keep solar-like differential rotation, large thermal conductivity ($\kappa = 2\times10^{13}\ \mathrm{cm^2\ s^{-1}}$) is always used.
(see Käpylä+2011,2014, Matt+2011, Gastine+2014, Fan+2014, Featherstone+2015, Karak+2015)





(Hotta+2014)

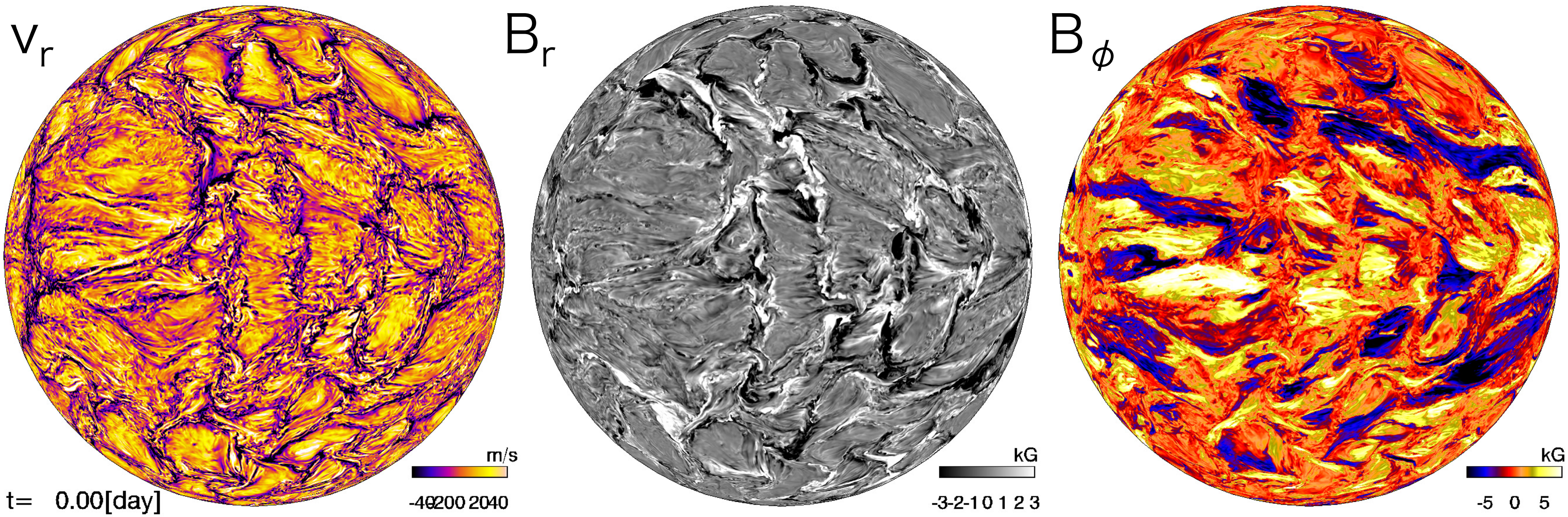
Numerical setting (2/2)

Less diffusive ↓

Cases	$N_r \times N_\theta \times N_\phi$	η, ν [$cm^2\ s^{-1}$]	Note
M64D1	64x192x384	1×10^{12}	Fan+2014
M64D2	64x192x384	3×10^{11}	
M64	64x192x384	N/A	
M128	128x384x768	N/A	
M256	256x768x1536	N/A	
M512S	512x1536x3072	N/A	only 500 days

In initial condition, we put random and small fluctuation on the entropy and weak (100 G) antisymmetric toroidal field. Then integrate the equation for 50 years. When without the character D, we only use slope limited diffusion. M256 costs 800,000 core hours.

Highest resolution calculation



<u>M512: 512x1536x3072</u>

Since the top boundary is at $0.96R_{sun}$, typical convection scale is large (several ten Mm). Each large cell has small-scale turbulent flow and magnetic field. Sometimes nice flux emergences are seen! (I skip this topic in this talk.)

Comparison of resolutions

Less diffusive →

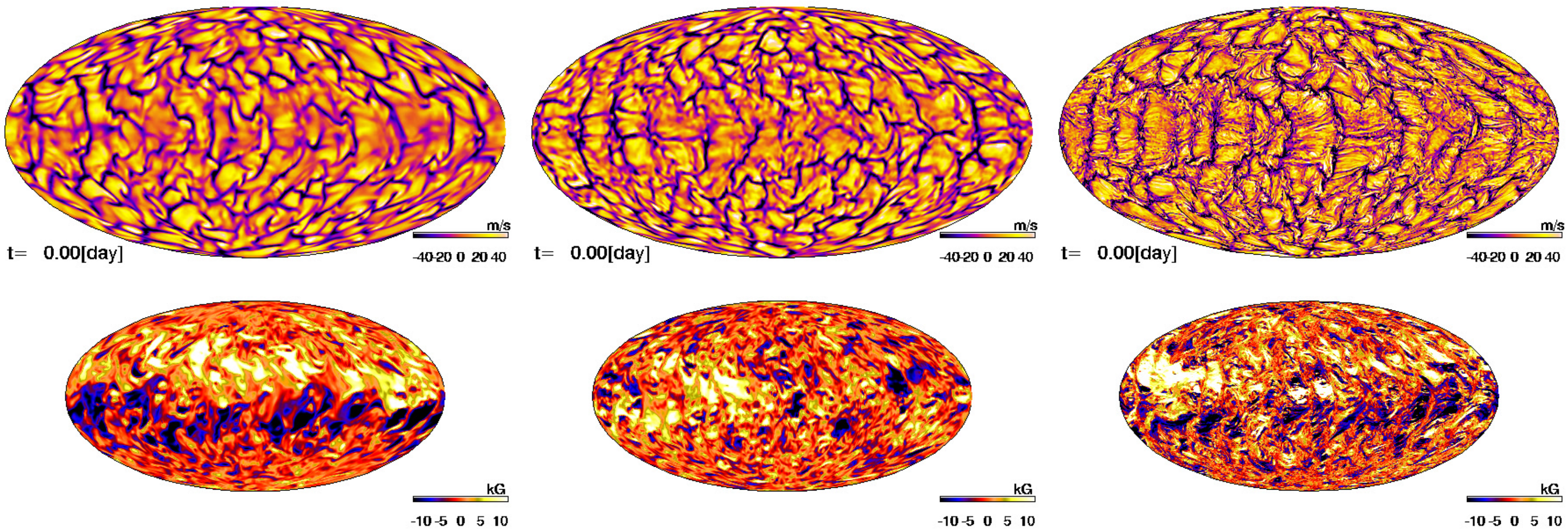
M64D1 $\nu = \kappa = 1 \times 10^{12}\ \mathrm{cm^2\ s^{-1}}$

M64 64x192x384

M256 256x768x1536

v_r at 0.95R

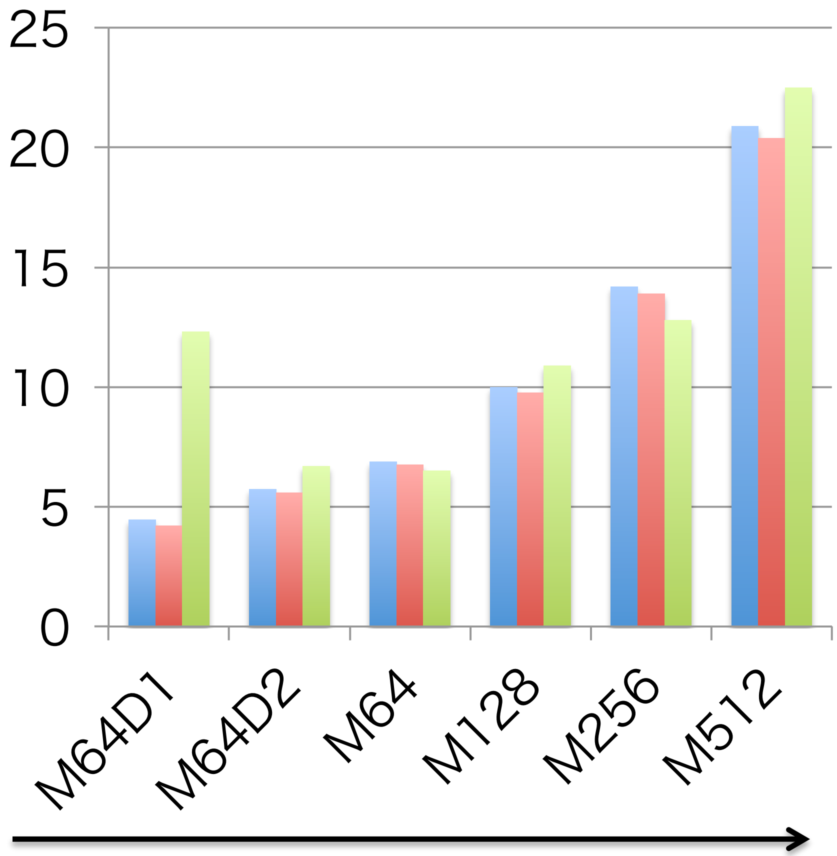
B_ϕ at 0.72R



In M64D1 (Fan+2014), the coherent magnetic field is generated and concentrated around the base of the convection zone. Using higher resolution (M64) large scale magnetic field looks destructed. In the highest resolution calculation, we again see the indication of large-scale magnetic field at the bottom of the convection zone.

Turbulent and mean magnetic energy

10^5 erg cm^{-3}



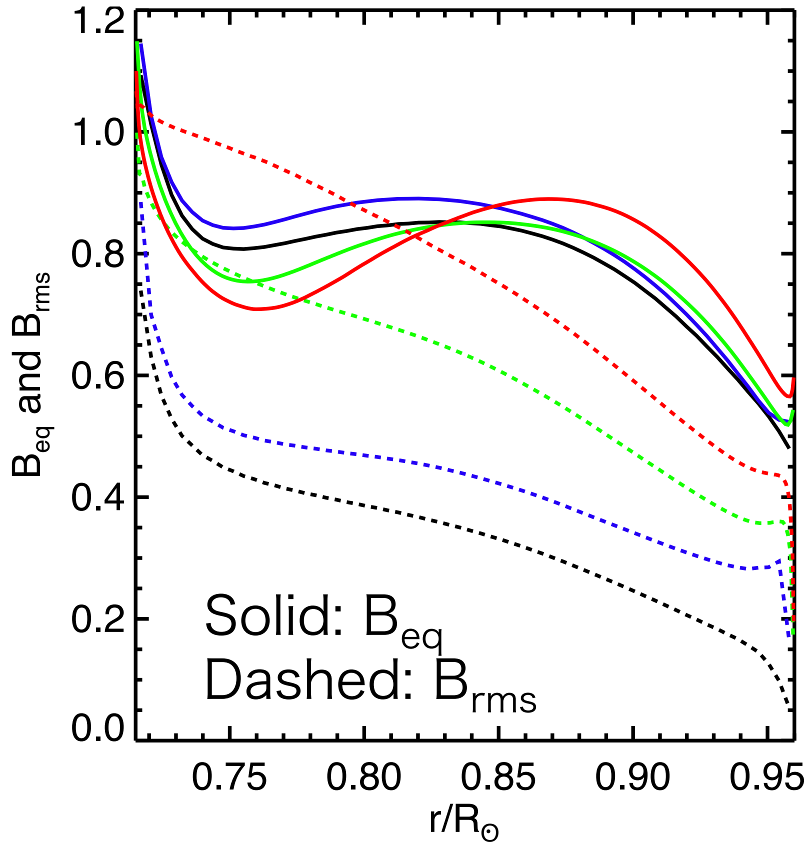
Less diffusive

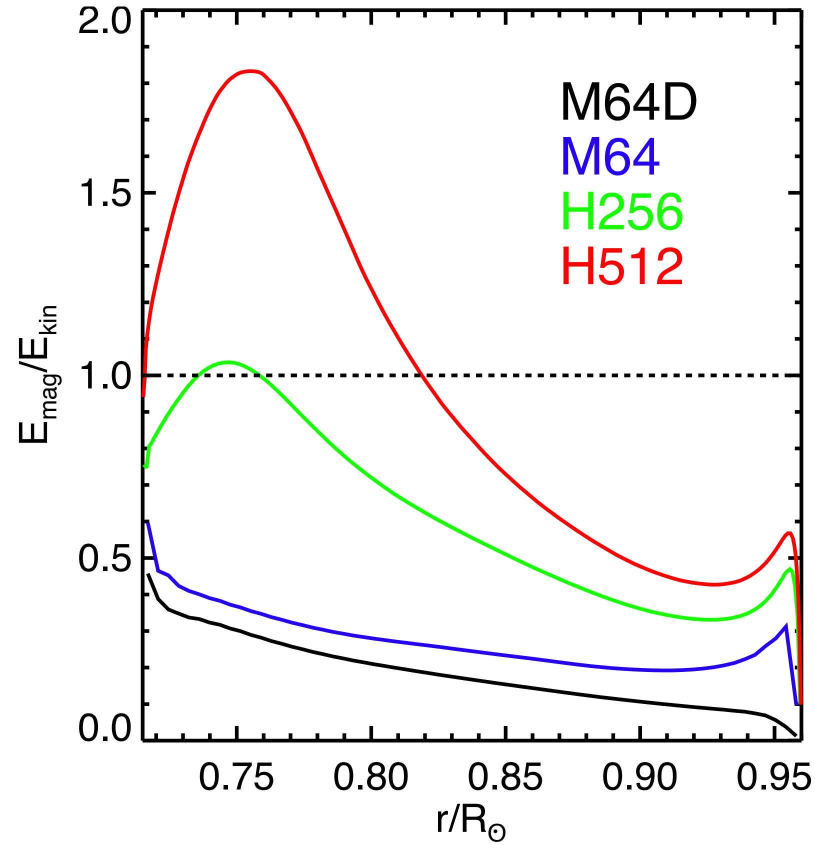
Zonal average

$$\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{B}'$$

Increasing the resolution, (decreasing the diffusivity) causes the large total and turbulent energy.
Mean magnetic energy has interesting behavior.

B_{eq} vs. B_{rms}

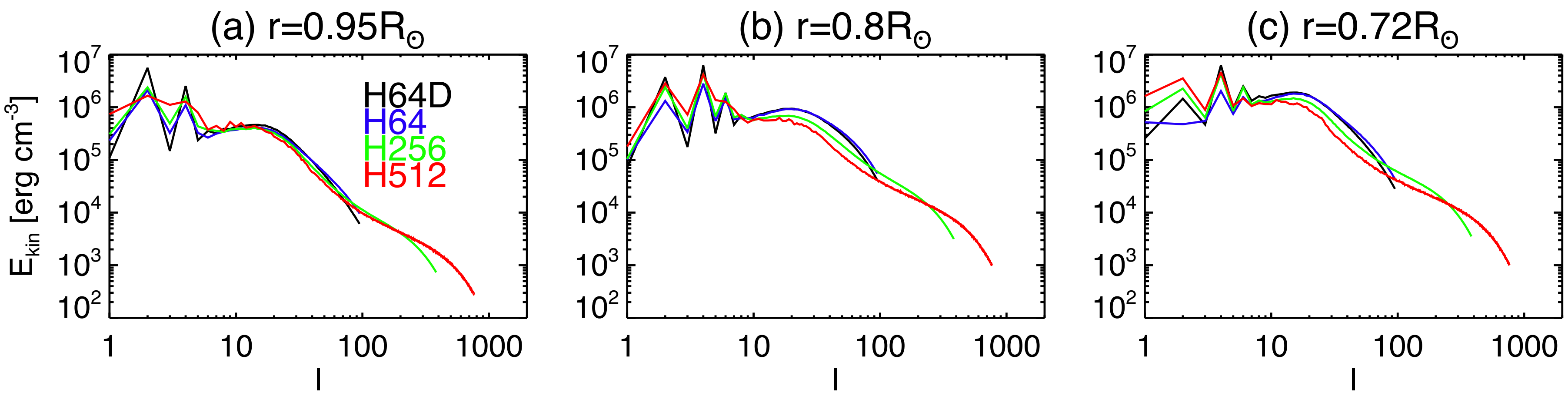




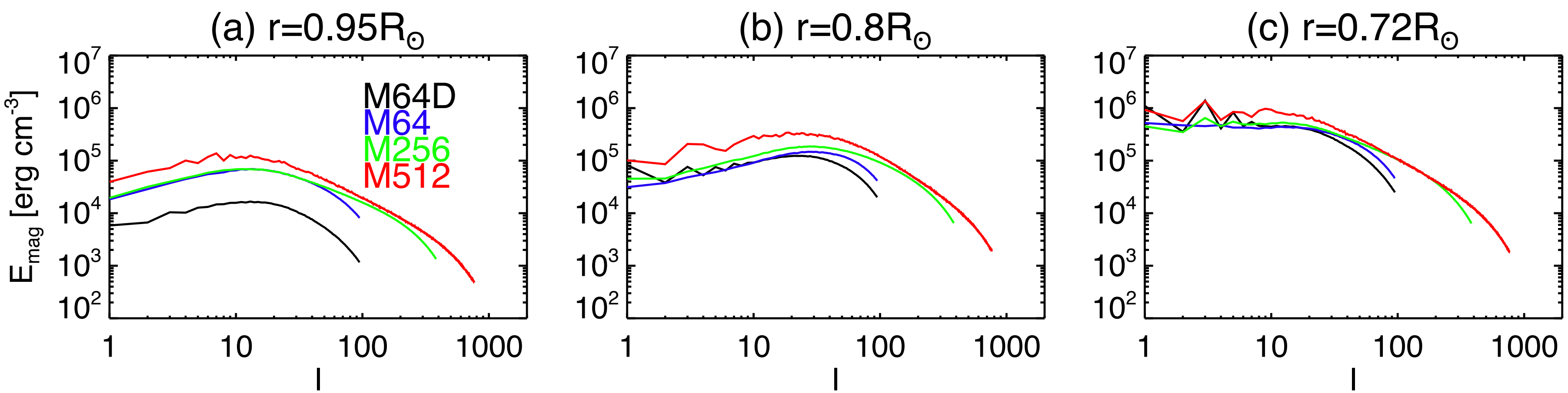
At maximum, the turbulent magnetic energy reach the value almost twice larger than the turbulent kinetic energy. It seems that the large-scale dynamo (the rotation) helps the small-scale dynamo.

Spectra

Kinetic energy spectra

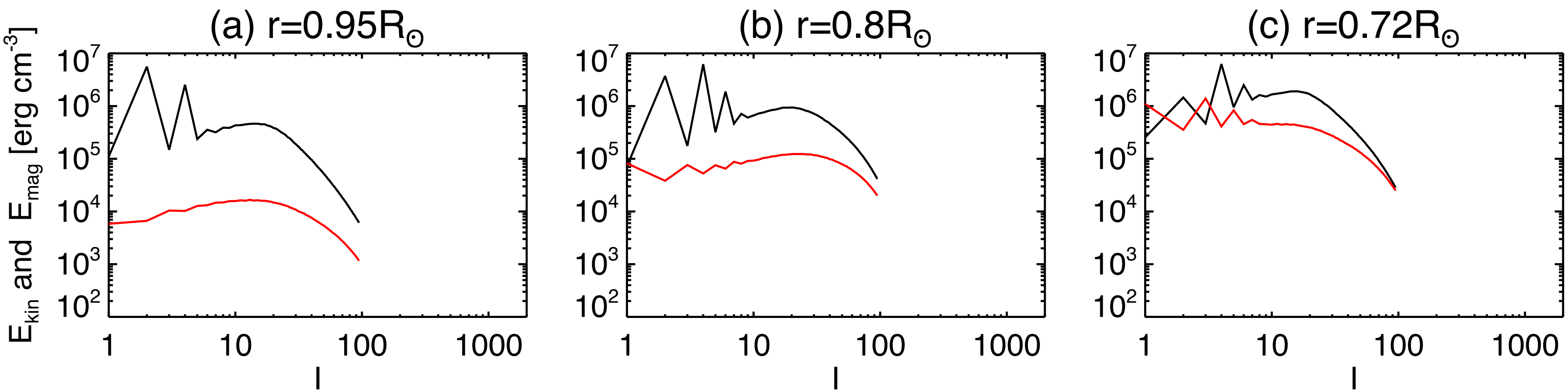


Magnetic energy spectra

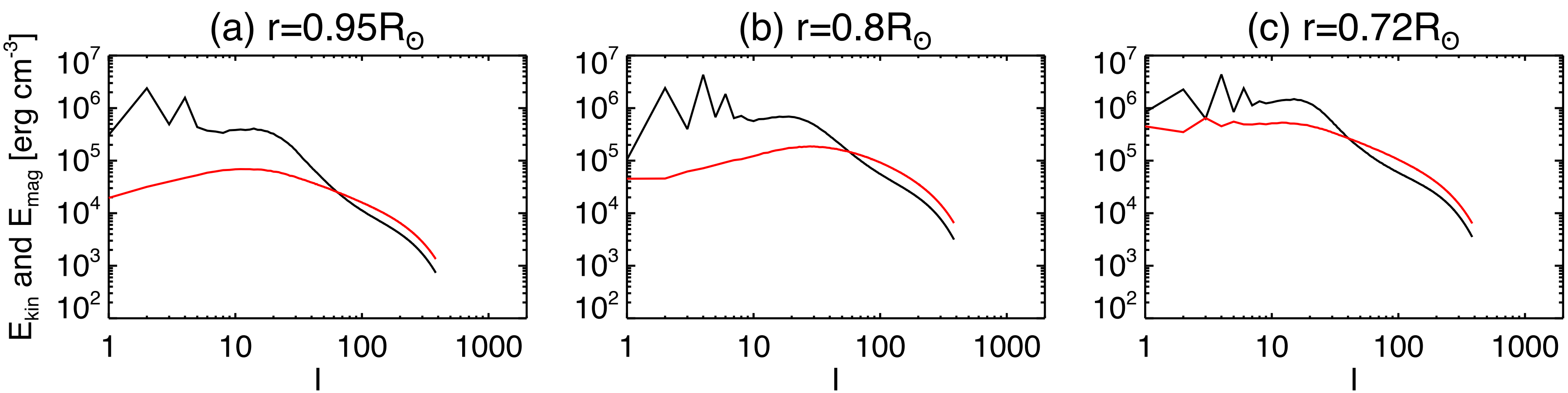


Spectra in M64D1 (Fan+2014) and M512 (Highest resolution)

Spectra at M64D1 Black: kinetic energy, Red: magnetic energy



Spectra at M512, suggesting a nice small-scale dynamo!



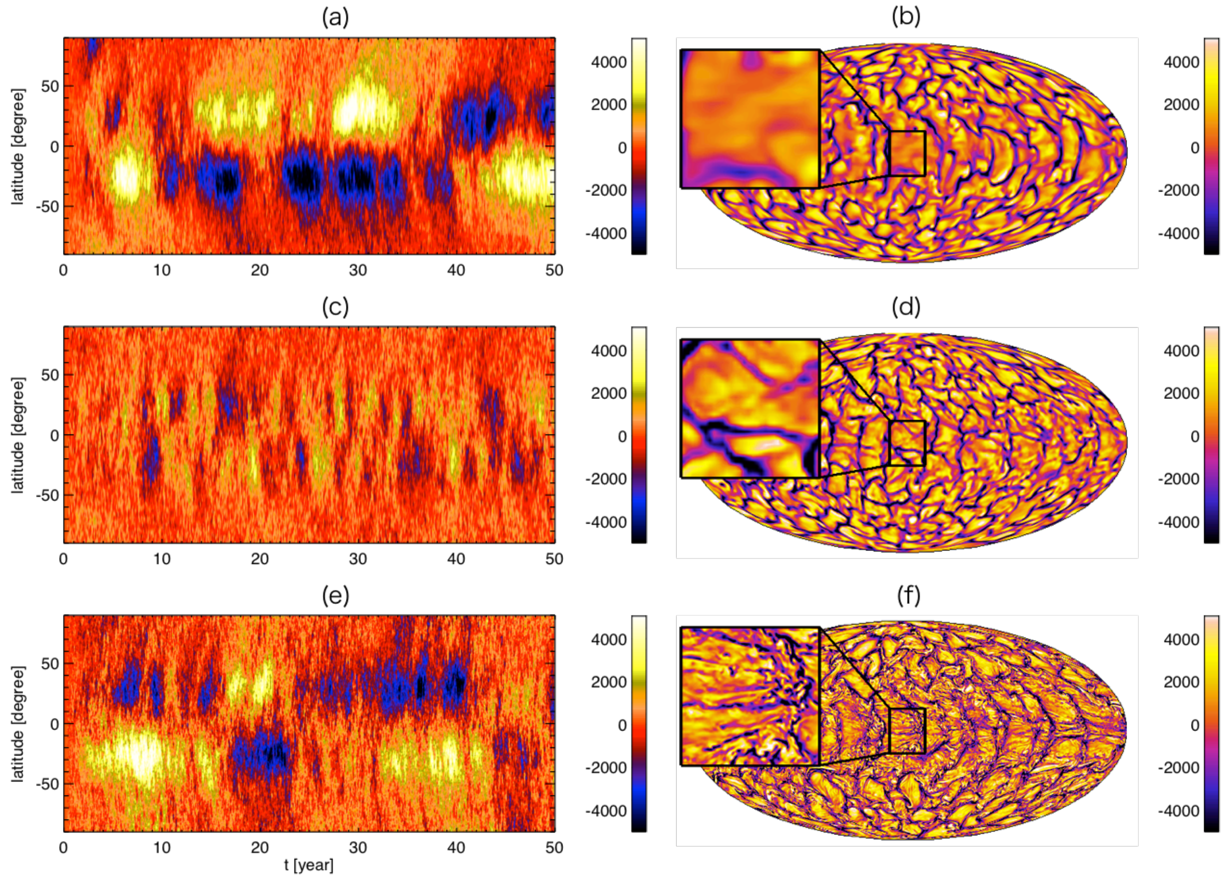
Mean magnetic field and cycle

Less diffusive

M64D1
$\nu = \eta = 1\times10^{12}$ cm^2 s^{-1}

M64
64x192x384

M256
256x768x1536



In the highest resolution calculation, the coherent cycle is recovered even at the large Rm regime.

Generation of toroidal field

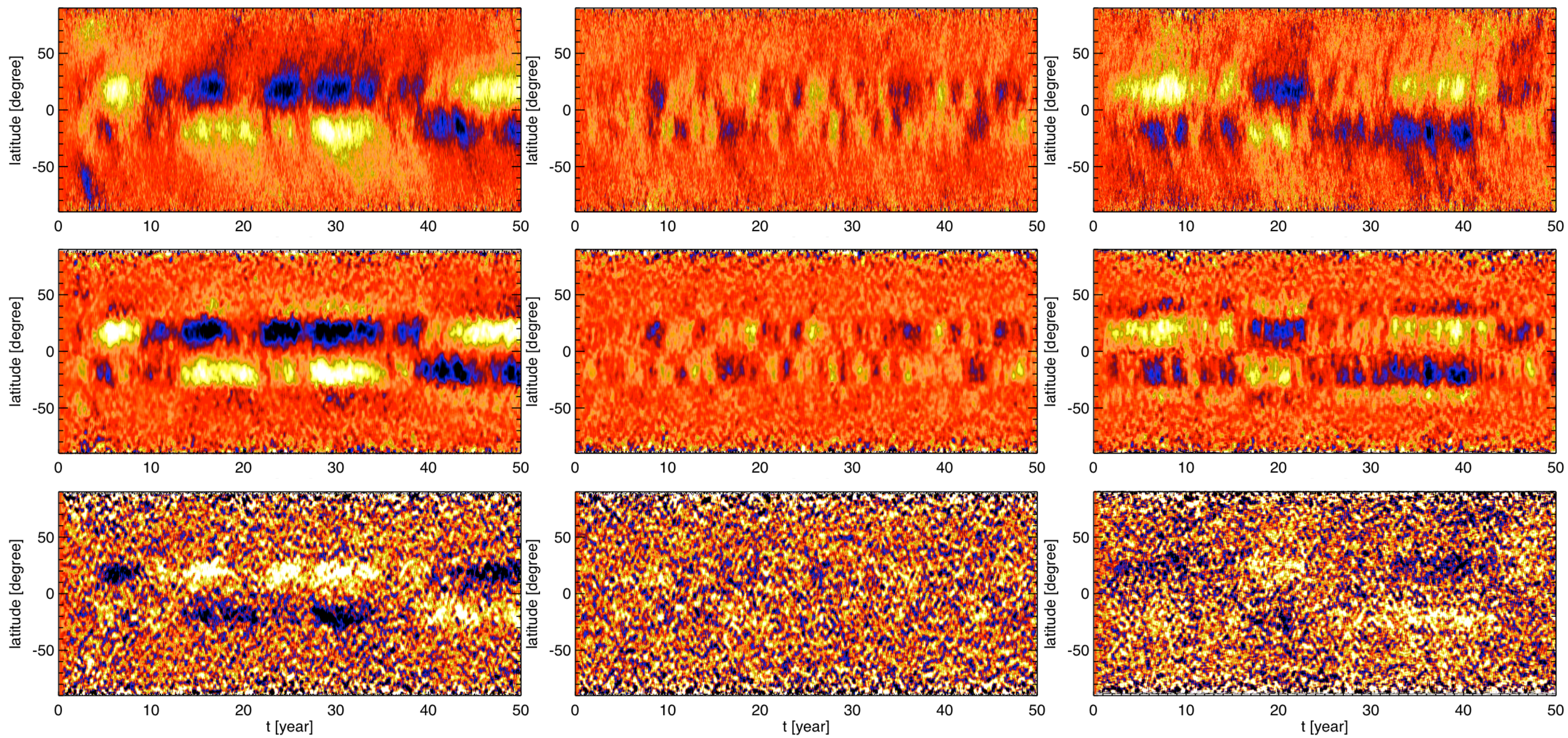
M64D1 $\nu = \kappa = 1\times10^{12}\ \mathrm{cm^2\ s^{-1}}$ | **M64** 64x192x384 | **M256** 256x768x1536

$\langle B_\phi \rangle$

$[(\langle \mathbf{B} \rangle \cdot \nabla)\langle \mathbf{v} \rangle]_\phi$

Ω-effect

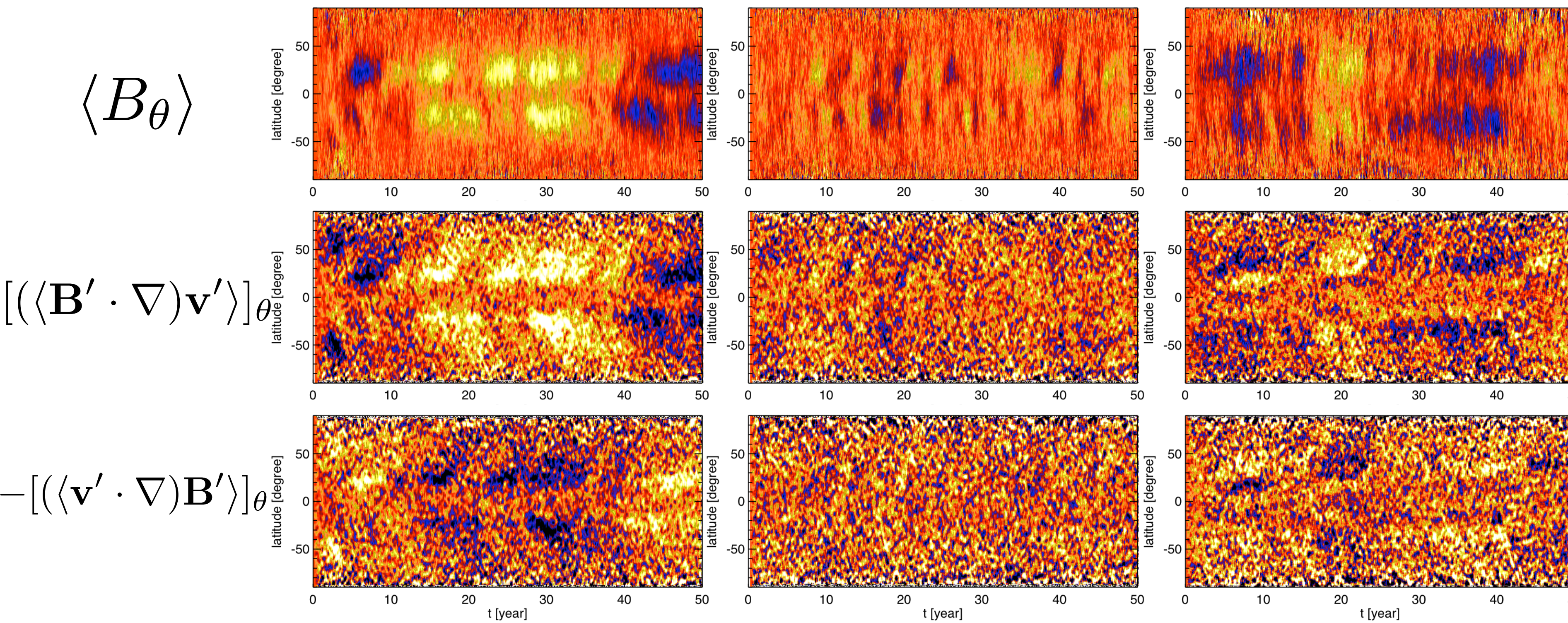
$-[(\langle \mathbf{v}' \cdot \nabla)\mathbf{B}' \rangle]_\phi$



The toroidal magnetic field is generated by mean shear (Ω-effect) if mean poloidal field exists. And It is destructed by turbulent diffusivity

Generation of poloidal field

M64D1 $\nu = \kappa = 1\times10^{12}$ cm^2 s^{-1} | **M64** 64x192x384 | **M256** 256x768x1536



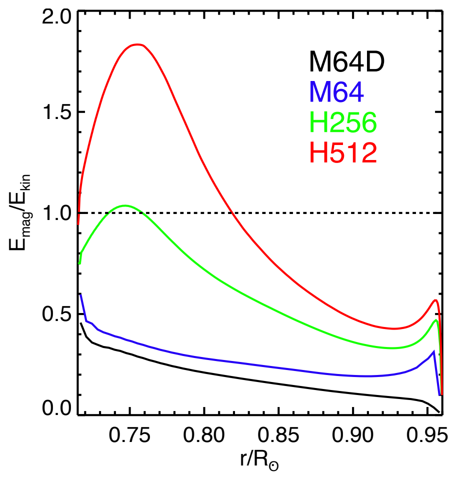
Mean field is generated by turbulent stretching $[(\langle \mathbf{B}' \cdot \nabla)\mathbf{v}' \rangle]_\theta$, which has coherent structure in the high resolution case (M256) and destructed by turbulent diffusivity.

Interaction between small- and large-scale dynamos

Our high resolution calculations indicate that the small- and large-scale dynamos help each other.

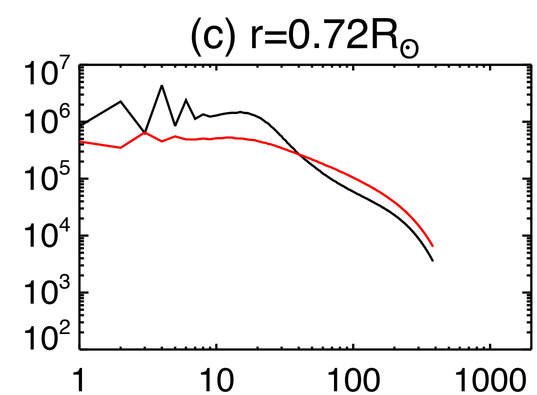
Large-scale → Small-scale

Helical turbulence and mean-shear enhance the magnetic field in “middle-scale” which is included in small-scale in our definition.
(see also Brandenburg+2001 and Haugen+2004 about critical Rm for helical and non-helical turbulence.)



Small-scale → Large-scale

We have not understood well about the construction mechanism of the large-scale field in our high-resolution calculation.
(This needs more analyses, advice from turbulent specialist is welcome).



Summary

1. Small-scale dynamo (SSD) is very efficient with small magnetic diffusivity throughout the convection zone.
   - ✓ $0.95B_{eq}$ is achieved at the base of the convection zone.
   - ✓ The flow is reduced by the factor of 2.
   - ✓ The small-scale magnetic field suppresses the mixing between up- and downflows.
2. Large-scale dynamo (LSD) is influenced by the small-scale dynamo
   - ✓ LSD is suppressed by SSD with using "medium" turbulent diffusivity.
   - ✓ LSD is supported by SSD in the higher resolutions.
3. Our high resolution calculations indicates that the small- and large-scale dynamos help each other.
   - ✓ We need some more analyses to understand it.