The Understanding the equatorward migration of the Sun's magnetic field

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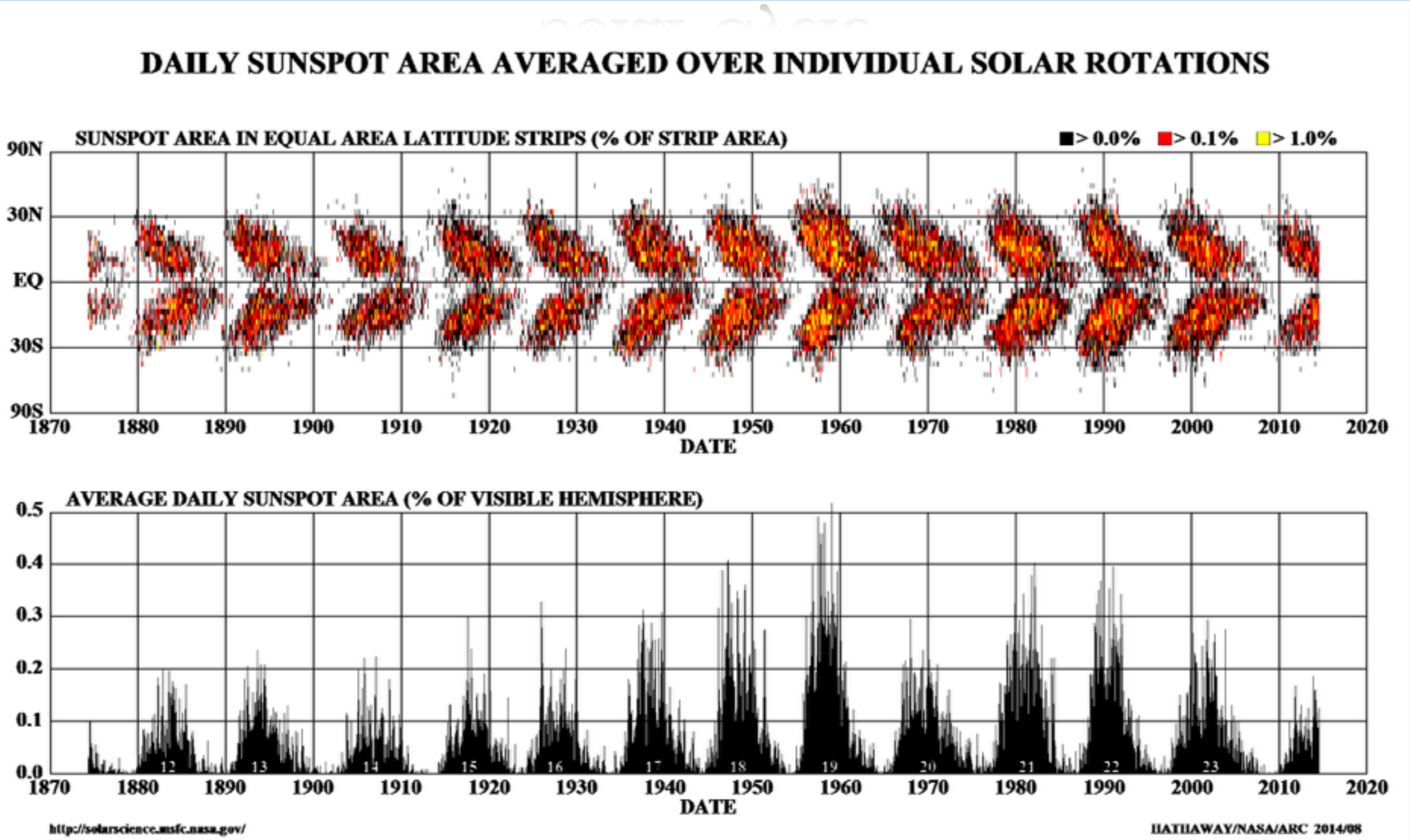
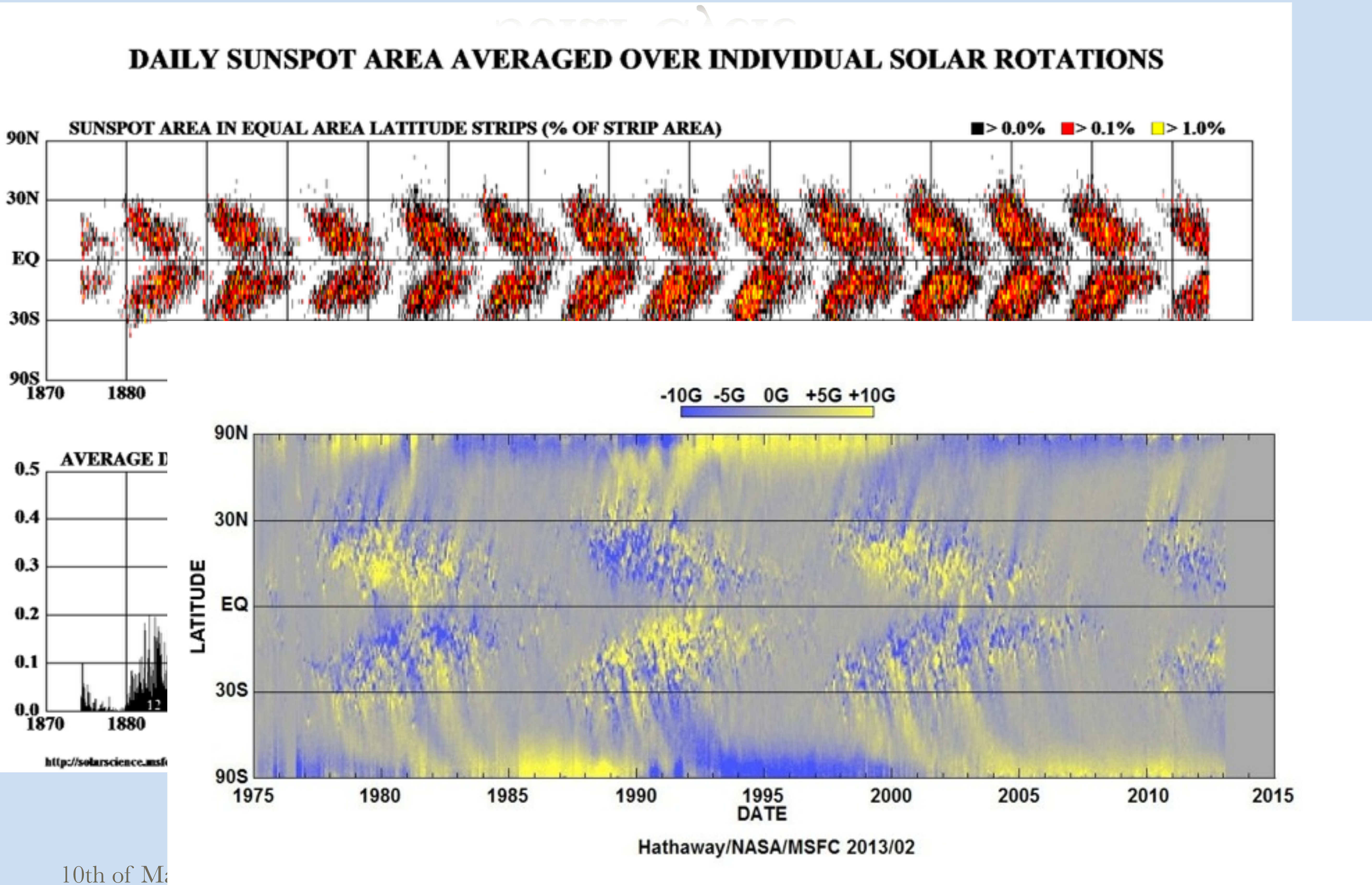
Solar Cycle


DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS
SUNSPOT AREA IN EQUAL AREA LATITUDE STRIPS (% OF STRIP AREA)
■> 0.0% ■> 0.1% ■> 1.0%
90N
30N
EQ
30S
90S
1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000 2010 2020
DATE
AVERAGE DAILY SUNSPOT AREA (% OF VISIBLE HEMISPHERE)
0.5
0.4
0.3
0.2
0.1
0.0
12 13 14 15 16 17 18 19 20 21 22 23
1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000 2010 2020
DATE
http://solarscience.msfc.nasa.gov/
HATHAWAY/NASA/ARC 2014/08

Solar Cycle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90N
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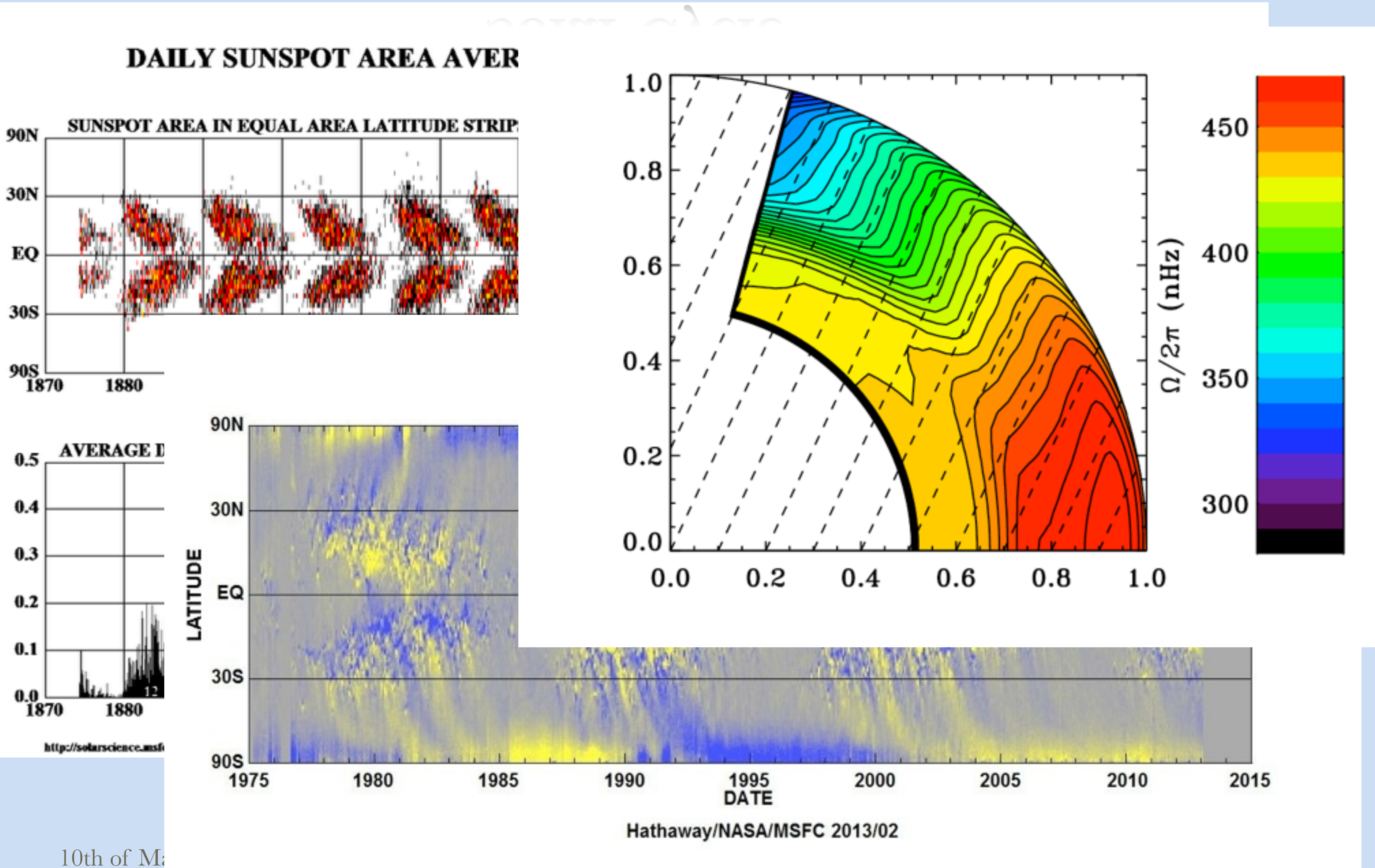
1870
1880
AVERAGE D
0.5
0.4
0.3
0.2
0.1
0.0
1870
1880
12
http://solarscience.msf
-10G -5G 0G +5G +10G
90N
30N
EQ
30S
90S
LATITUDE
1975
1980
1985
1990
1995
2000
2005
2010
2015
DATE
Hathaway/NASA/MSFC 2013/02

Solar Cycle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1.0
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0.0
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0.8
1.0
Ω/2π (nHz)
450
400
350
300

Global convective dynamo simulations

$$\frac{\partial A}{\partial t} = u \times B + \eta \nabla^2 A$$

$$\frac{D \ln \rho}{Dt} = -\nabla \cdot u$$

$$\frac{Du}{Dt} = g - 2\Omega_0 \times u + \frac{1}{\rho}\left(J \times B - \nabla p + \nabla \cdot 2\nu\rho S\right)$$

$$T\frac{Ds}{Dt} = \frac{1}{\rho}\nabla \cdot (K\nabla T + \chi_t \rho T \nabla s) + 2\nu S^2 + \frac{\mu_0\eta}{\rho}J^2 - \Gamma_{\rm cool}(r),$$

http://pencil-code.google.com/

Global convective dynamo simulations

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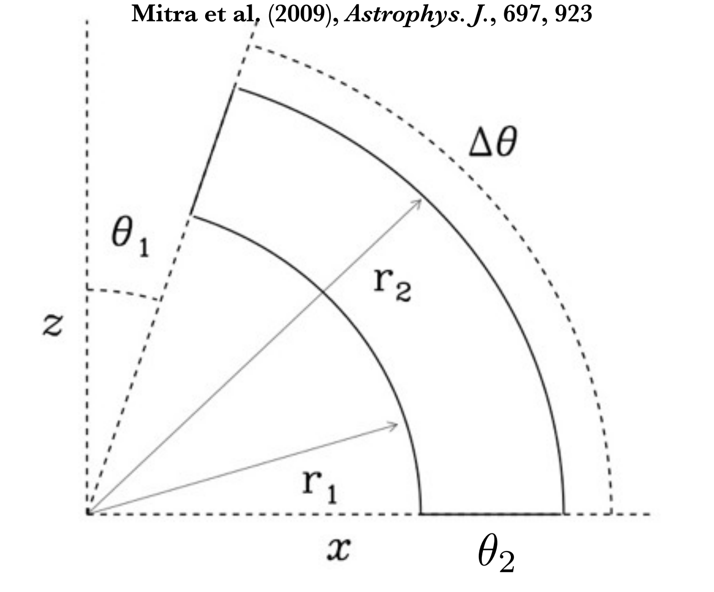
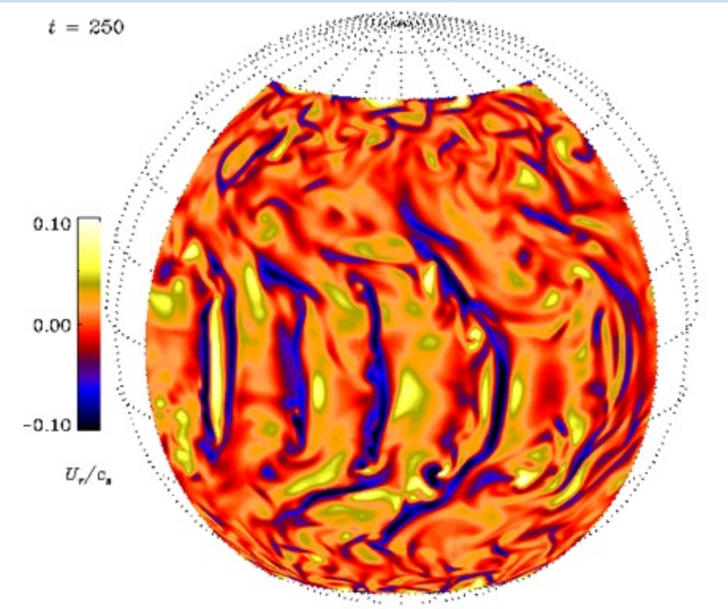
- high-order finite-difference code
- scales up efficiently to over 60.000 cores
- compressible MHD

http://pencil-code.google.com/

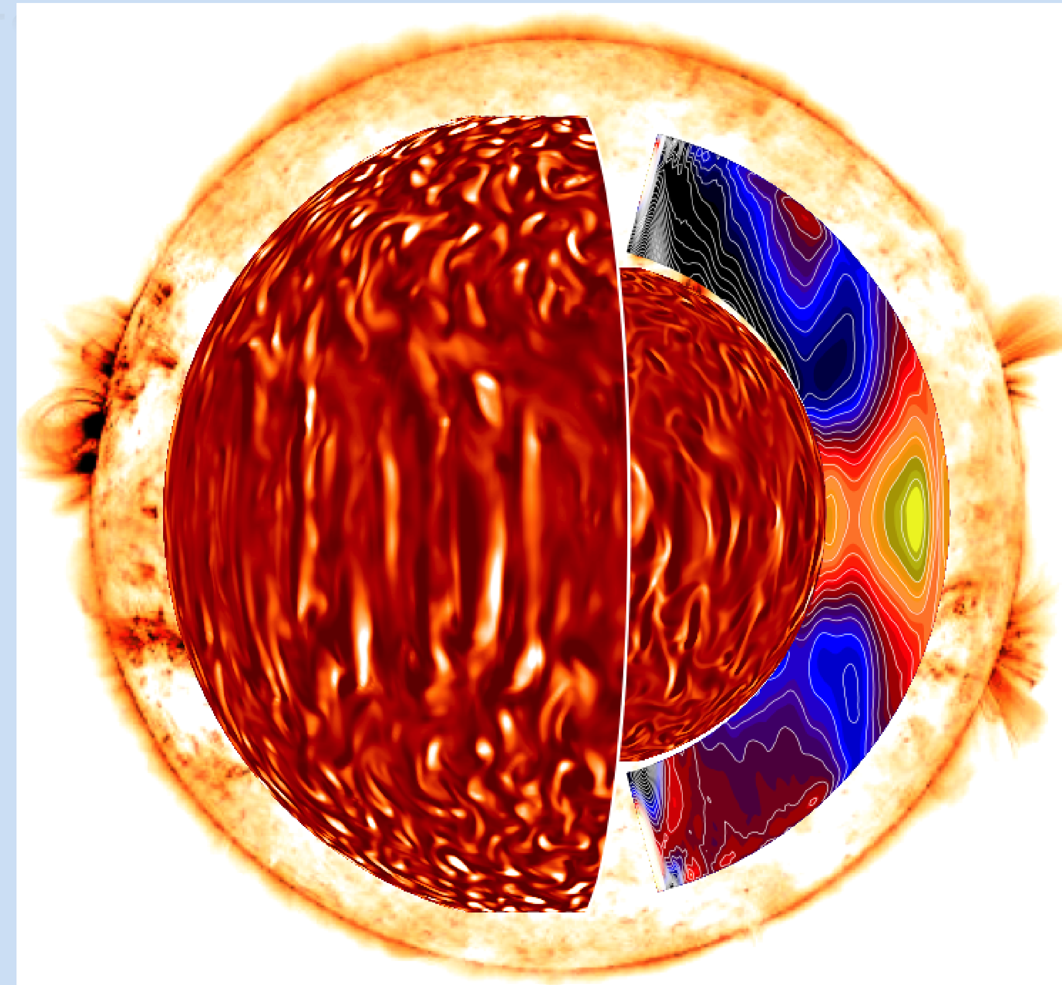
Global convective dynamo simulations



Mitra et al, (2009), *Astrophys. J.*, 697, 923



Käpylä et al. (2010b), *Astron. Nachr.*, 331, 73

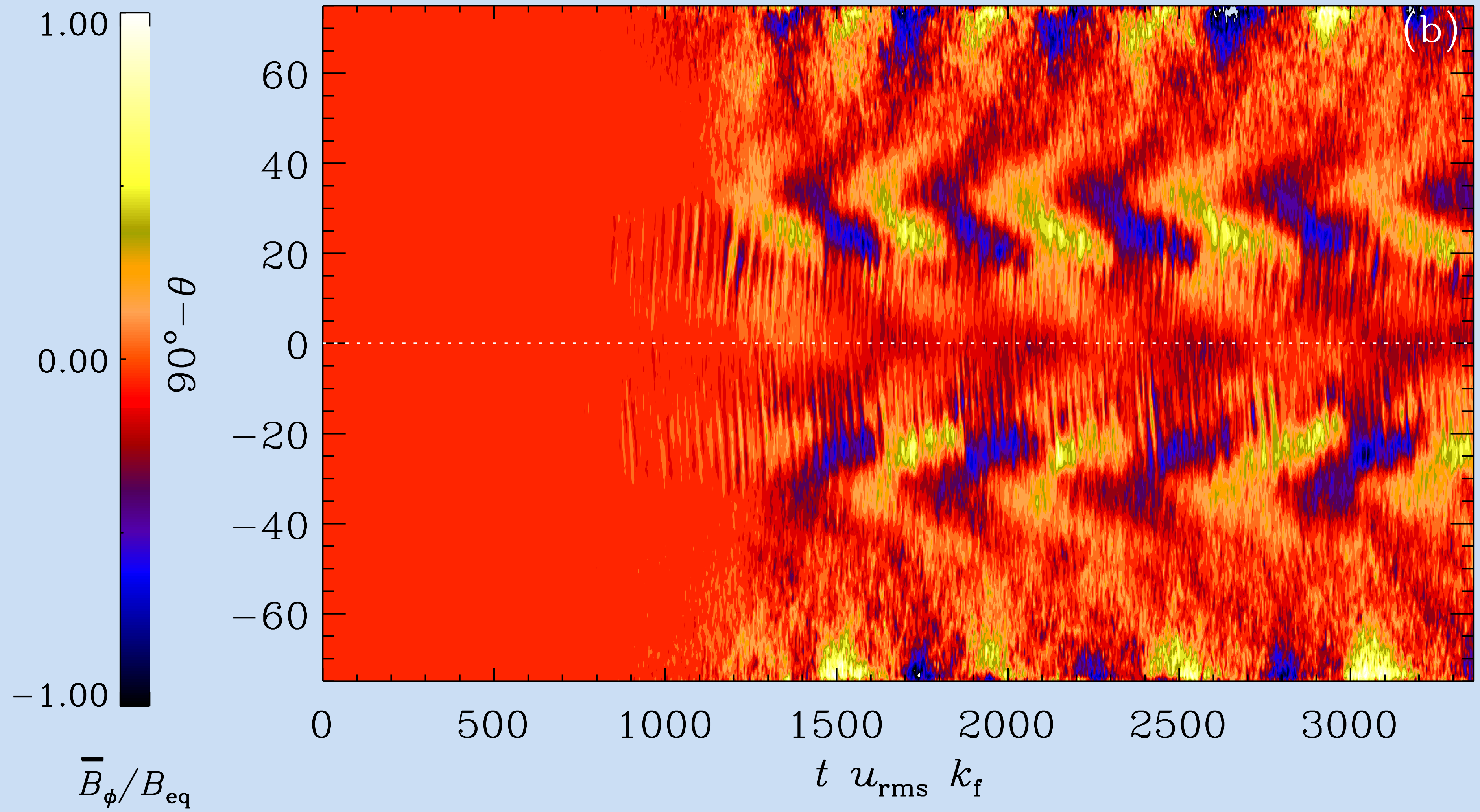


$0.7R < r < R$ $\theta_1 < \theta < \theta_2$ $0 < \phi < \Delta\phi$ $k_{\rm f} = 2\pi/\Delta R$

We model a spherical sector (`wedge') where only parts of the latitudinal and longitudinal extents are taken into account.

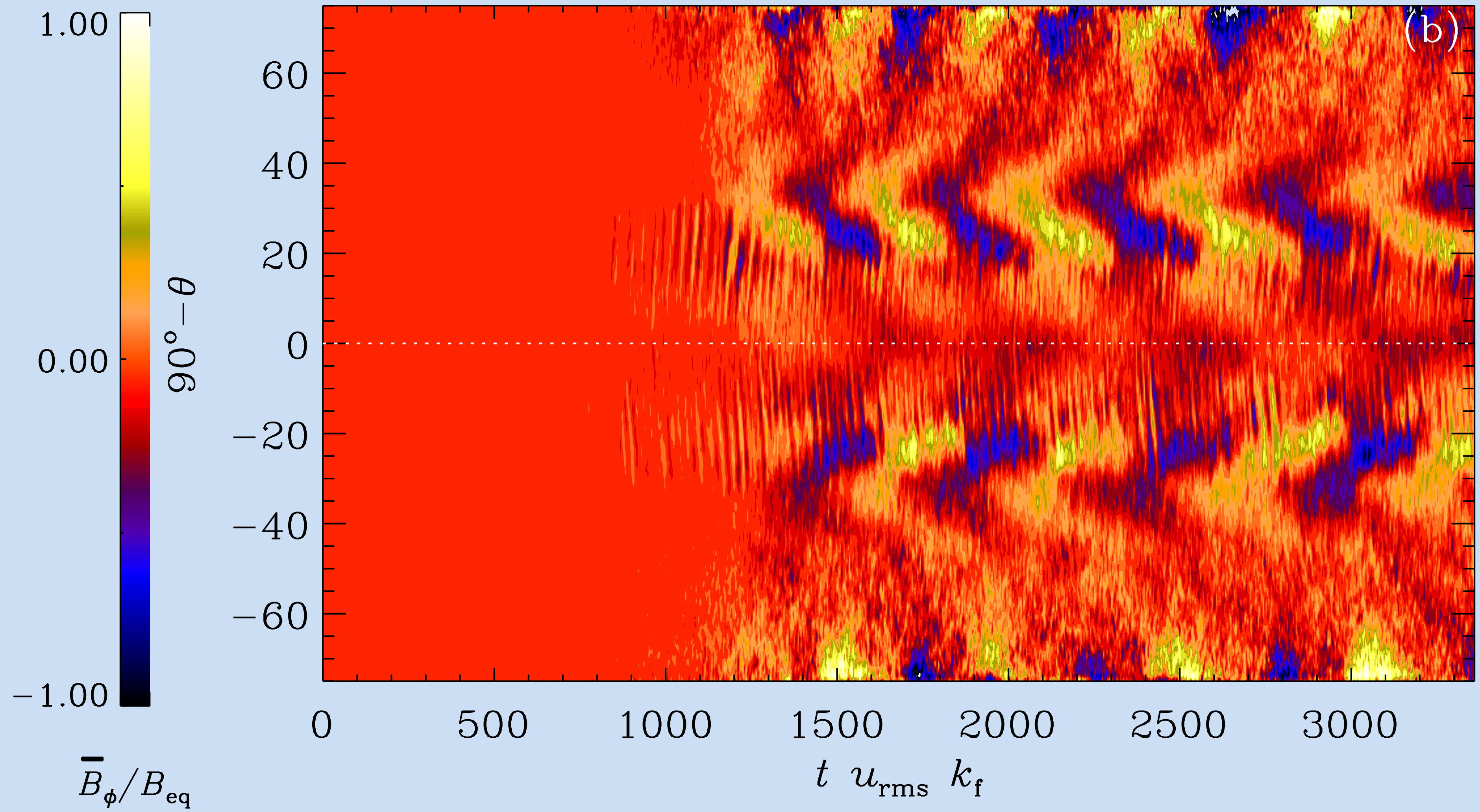
Normal field condition for *B* at the outer radial boundary and perfect conductor at all other boundaries. Impenetrable stress-free boundaries on all boundaries.

Equatorward Migration I



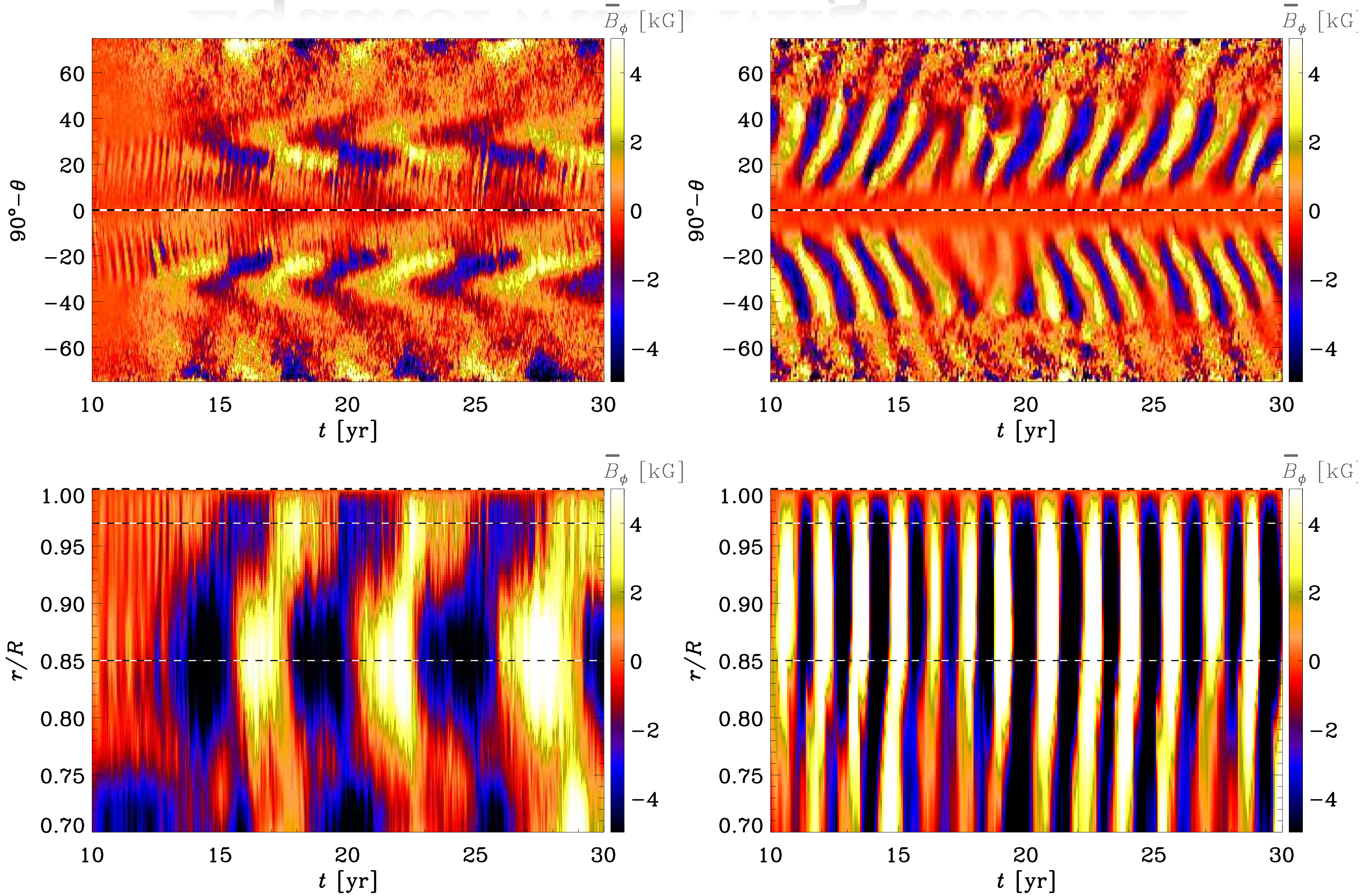
Käpylä, Mantere & Brandenburg 2012 (ApJL 755, L22)

Equatorward Migration I



Käpylä, Mantere & Brandenburg 2012 (ApJL 755, L22)

Equatorward Migration II



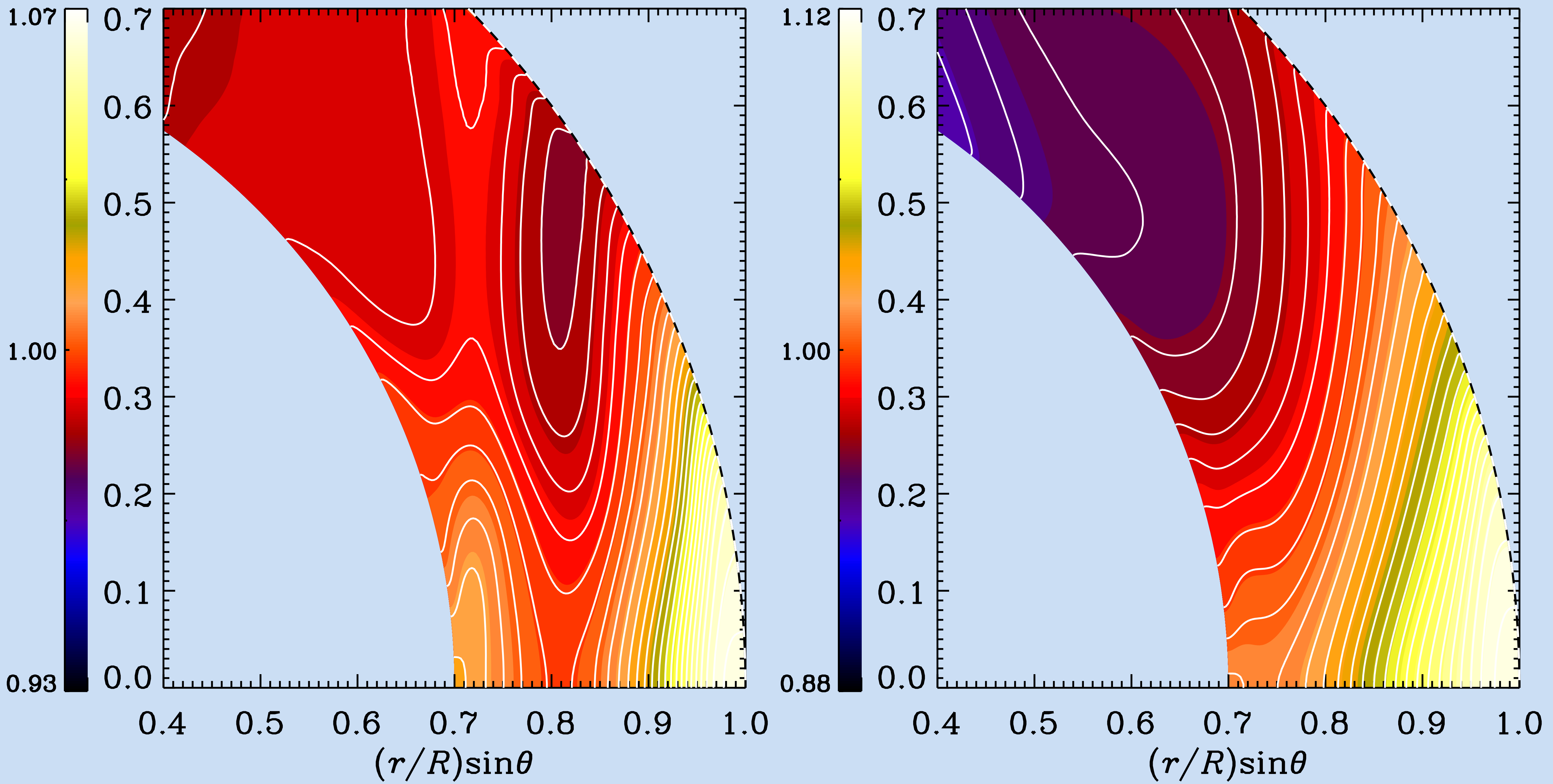
Pr=ν/χ=2.5

Pm=ν/η=1

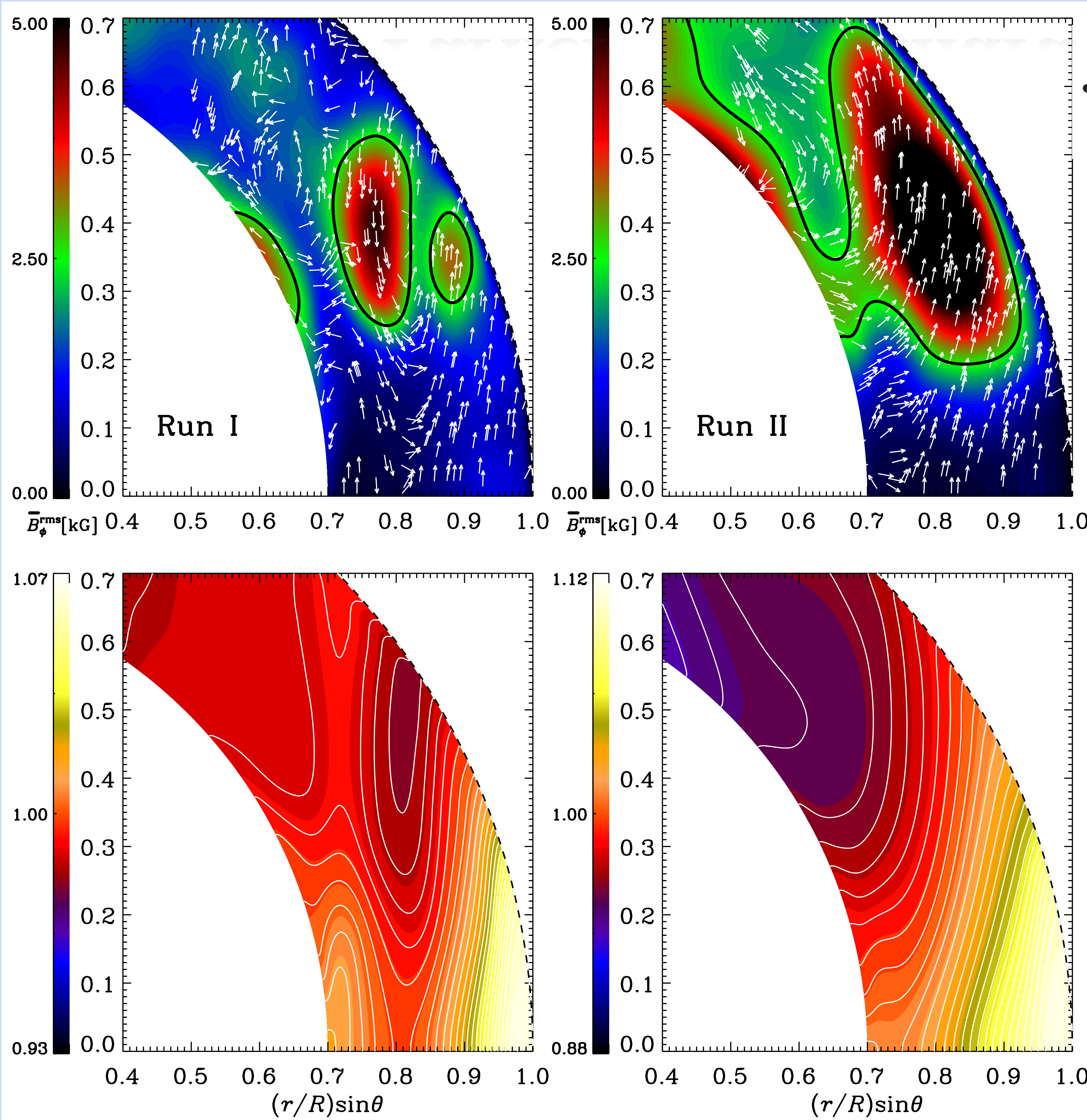
Pr=0.5

Pm=0.5

Differential rotation



Parker—Yoshimura—Rule



$$\boldsymbol{s}_{\rm mig}(r,\theta) = -\alpha \hat{\boldsymbol{e}}_\phi \times \boldsymbol{\nabla}\Omega,$$

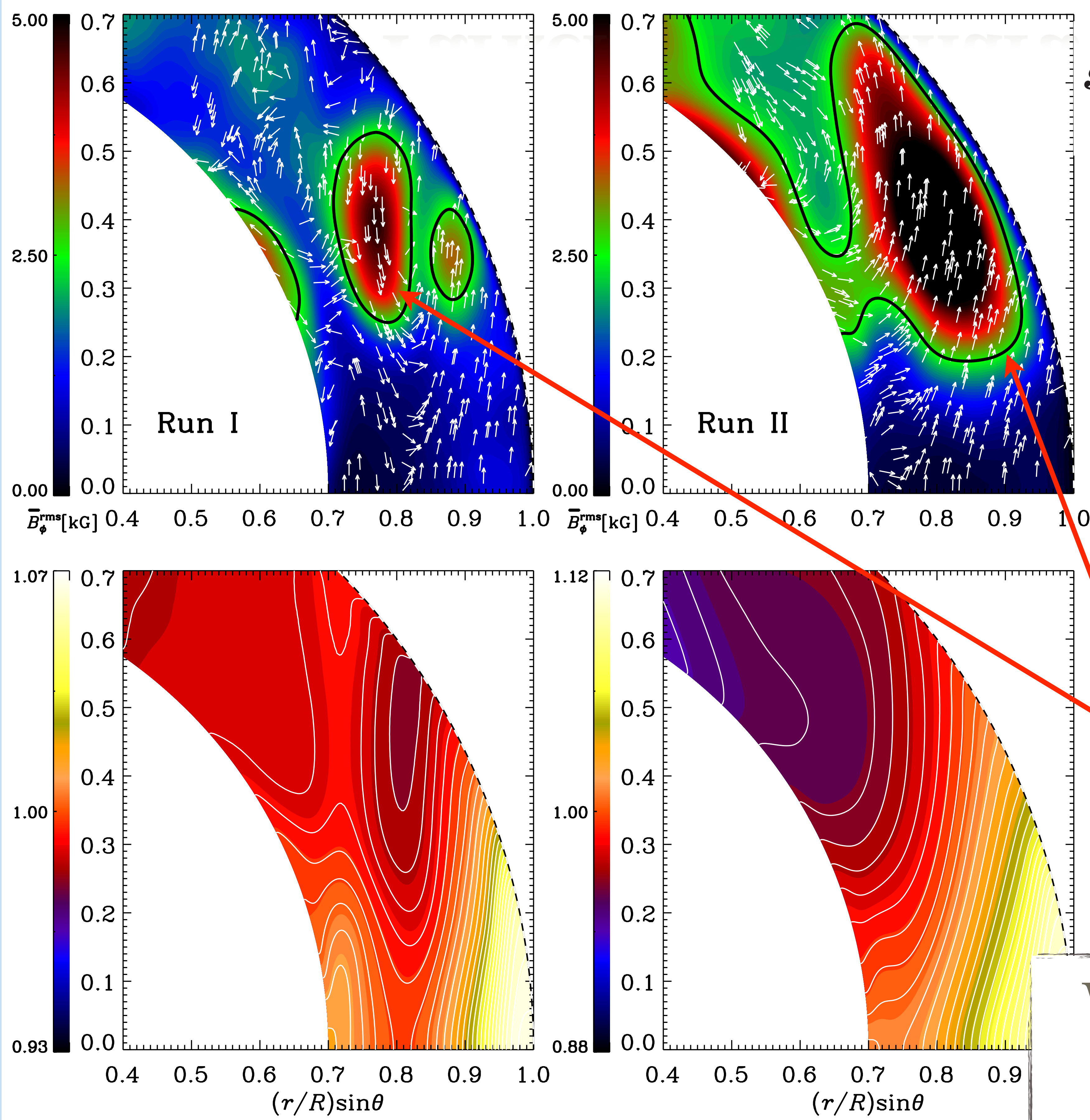
Parker 1955

Yoshimura 1975

$$\alpha = \frac{\tau_{\rm c}}{3}\left(-\overline{\boldsymbol{\omega}\cdot\boldsymbol{u}} + \frac{\overline{\boldsymbol{j}\cdot\boldsymbol{b}}}{\overline{\rho}}\right)$$

Pouquet et al. 1976

Parker—Yoshimura—Rule



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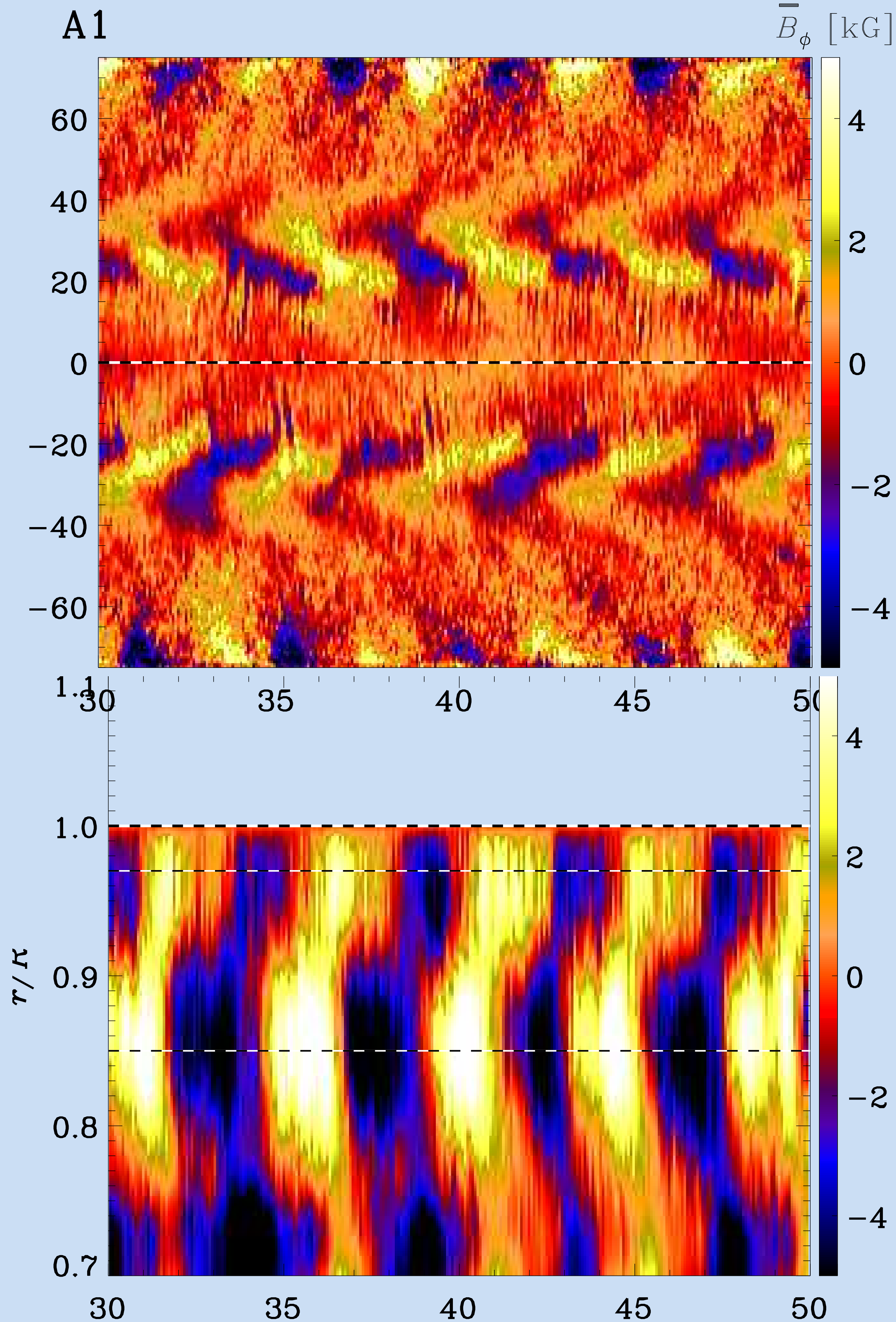
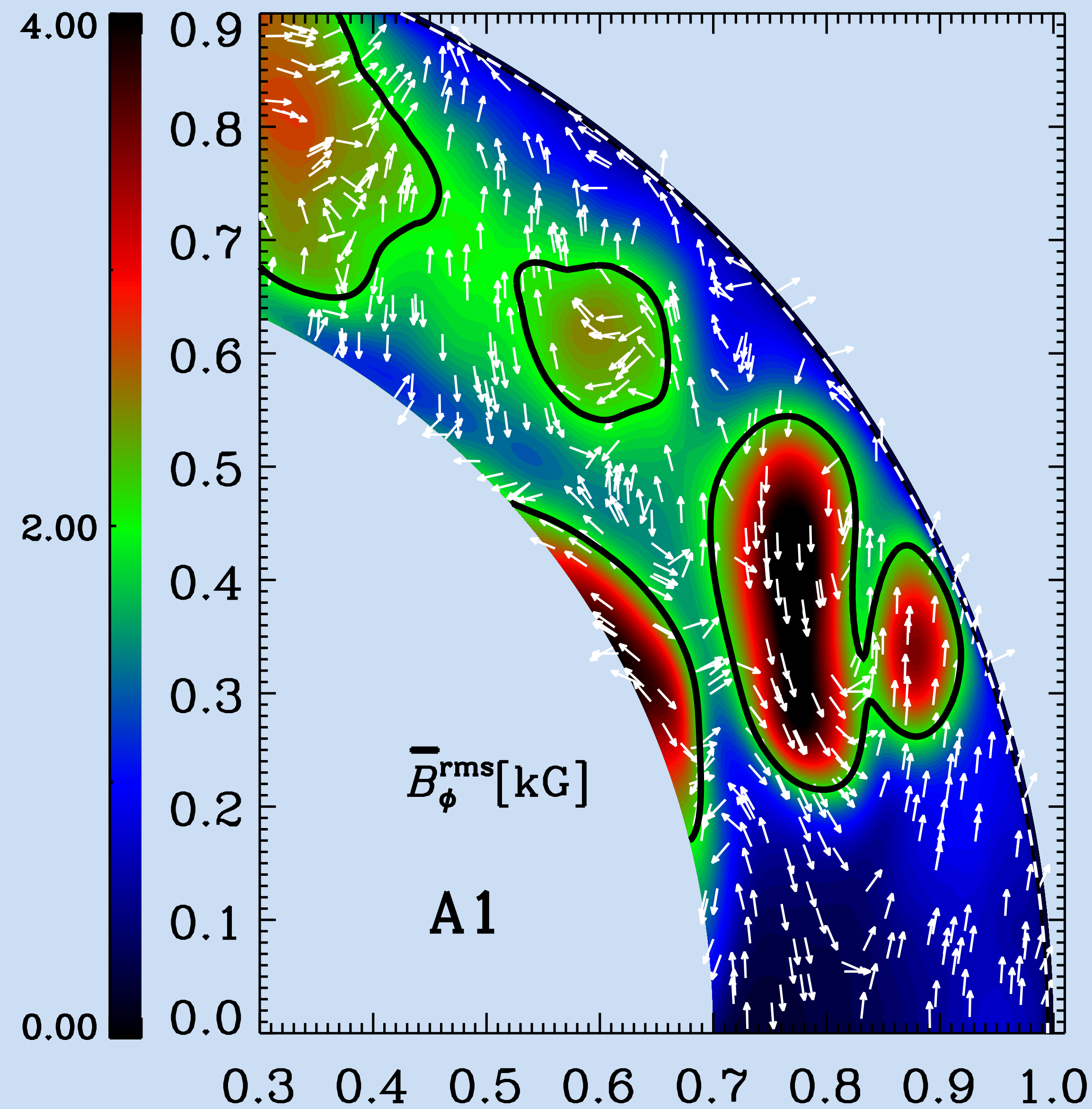
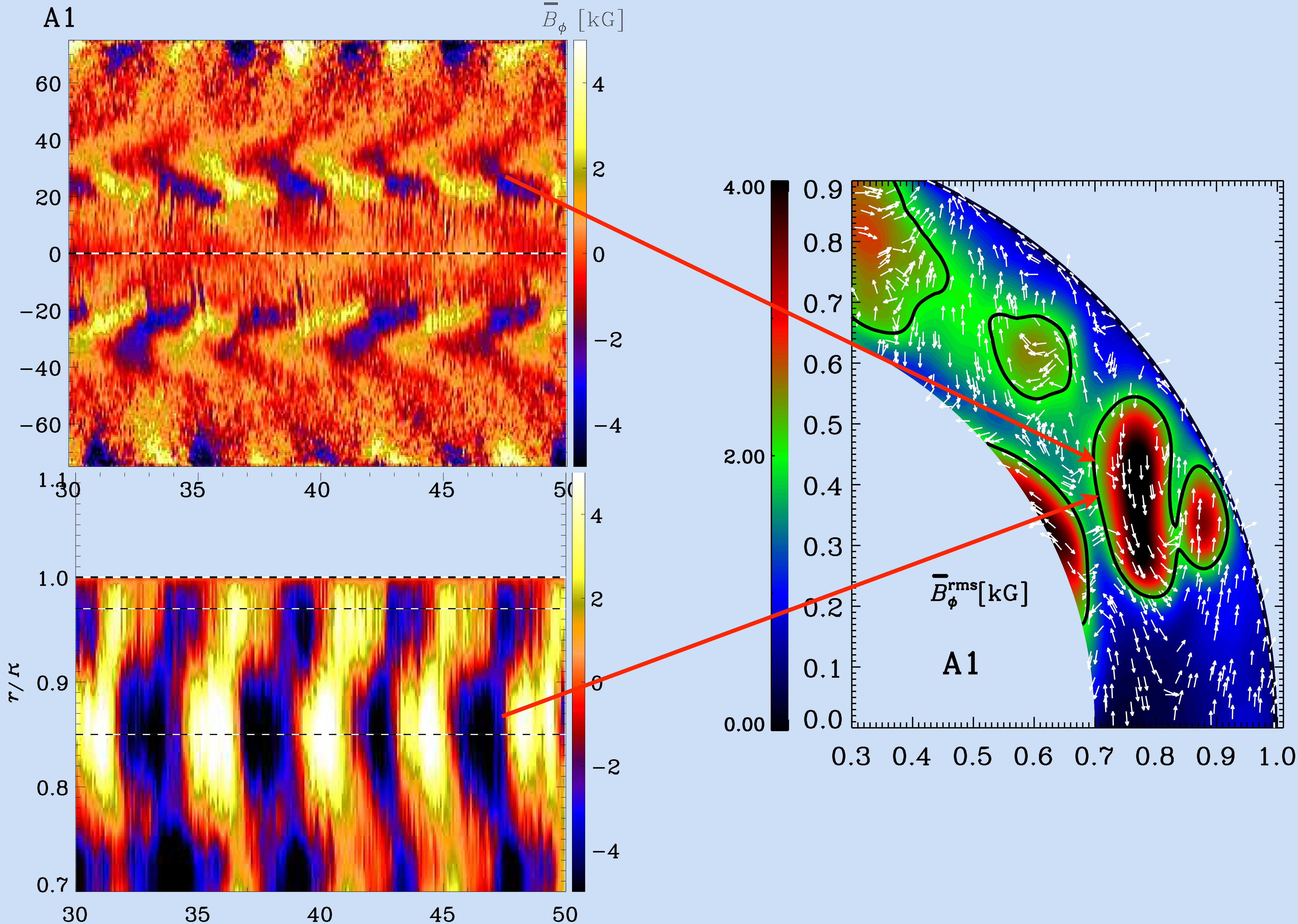
Pouquet et al. 1976

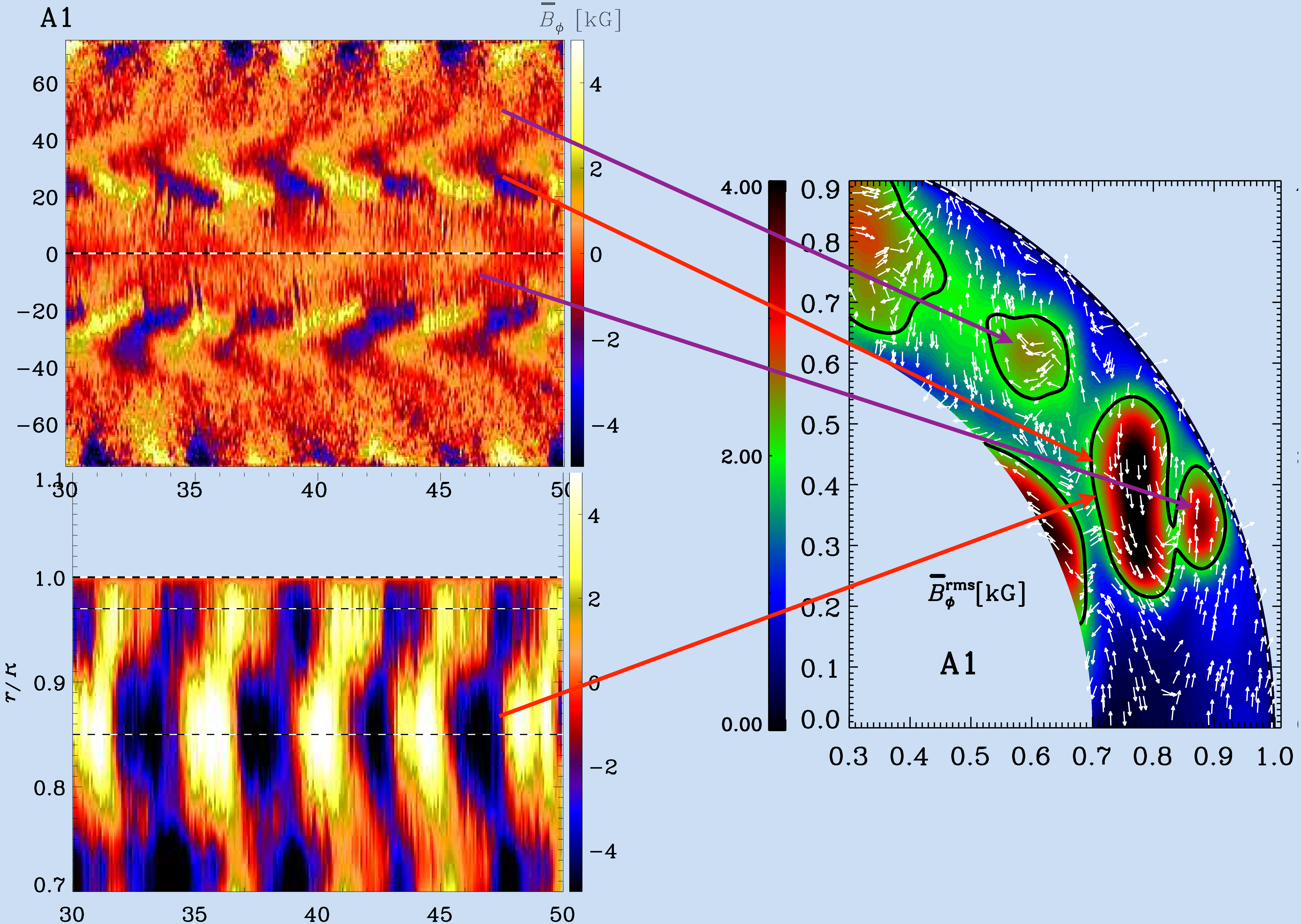
Strong toroidal field

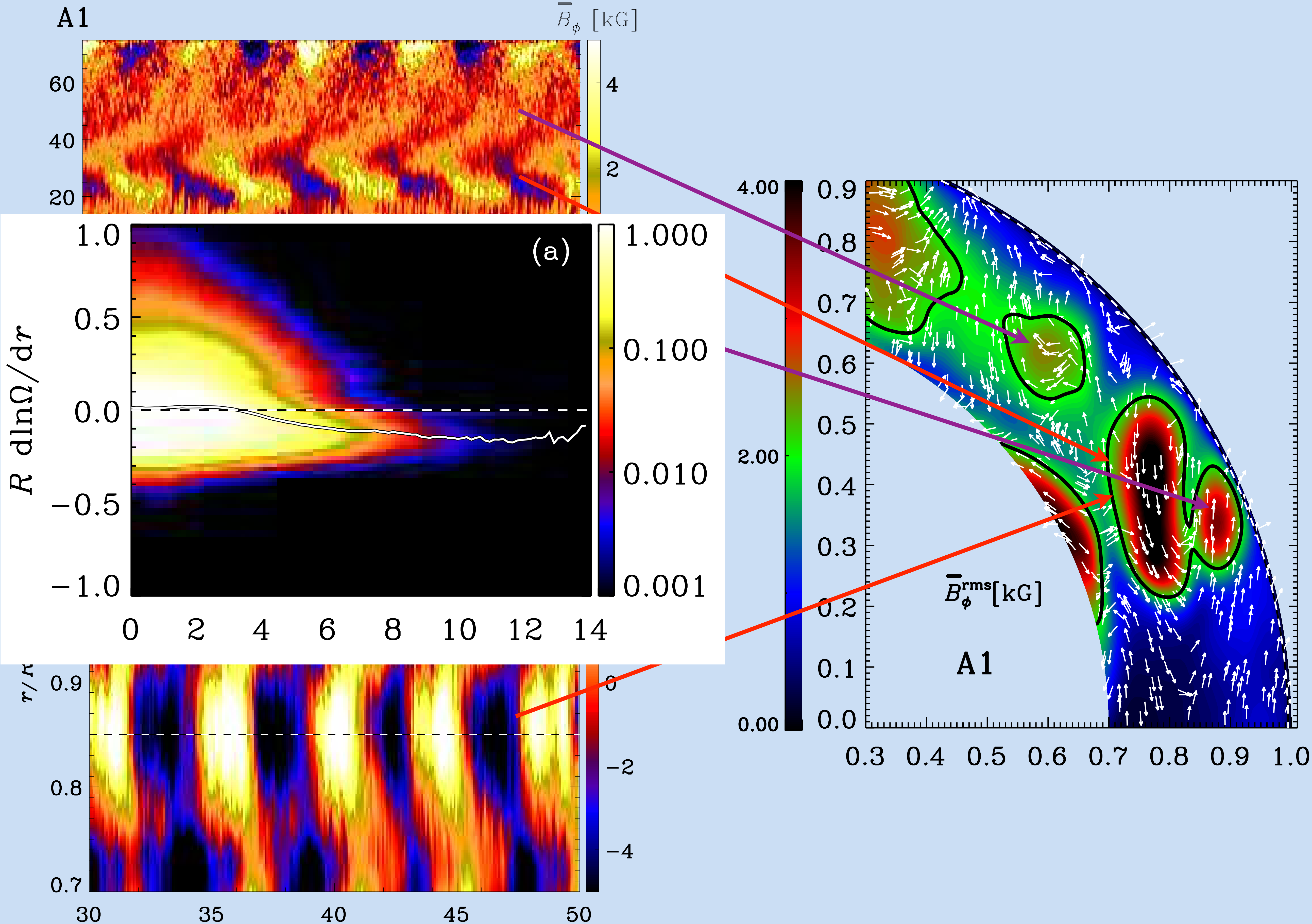
Warnecke et al. 2014 (ApJL 796, L12)

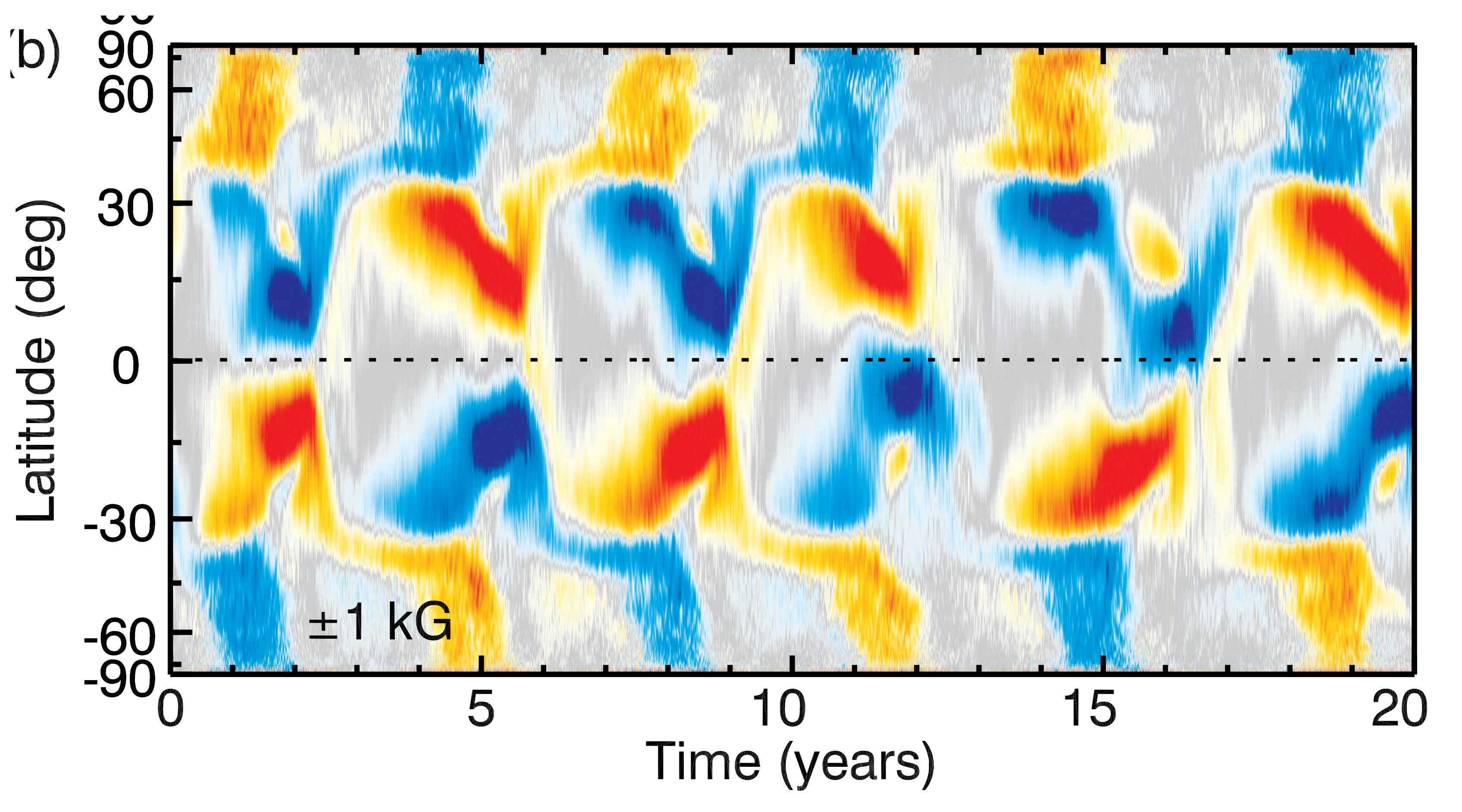

A1
$\overline{B}_\phi$ [kG]
60
40
20
0
−20
−40
−60
4
2
0
−2
−4
1.1
30
35
40
45
50
1.0
0.9
0.8
0.7
r/R

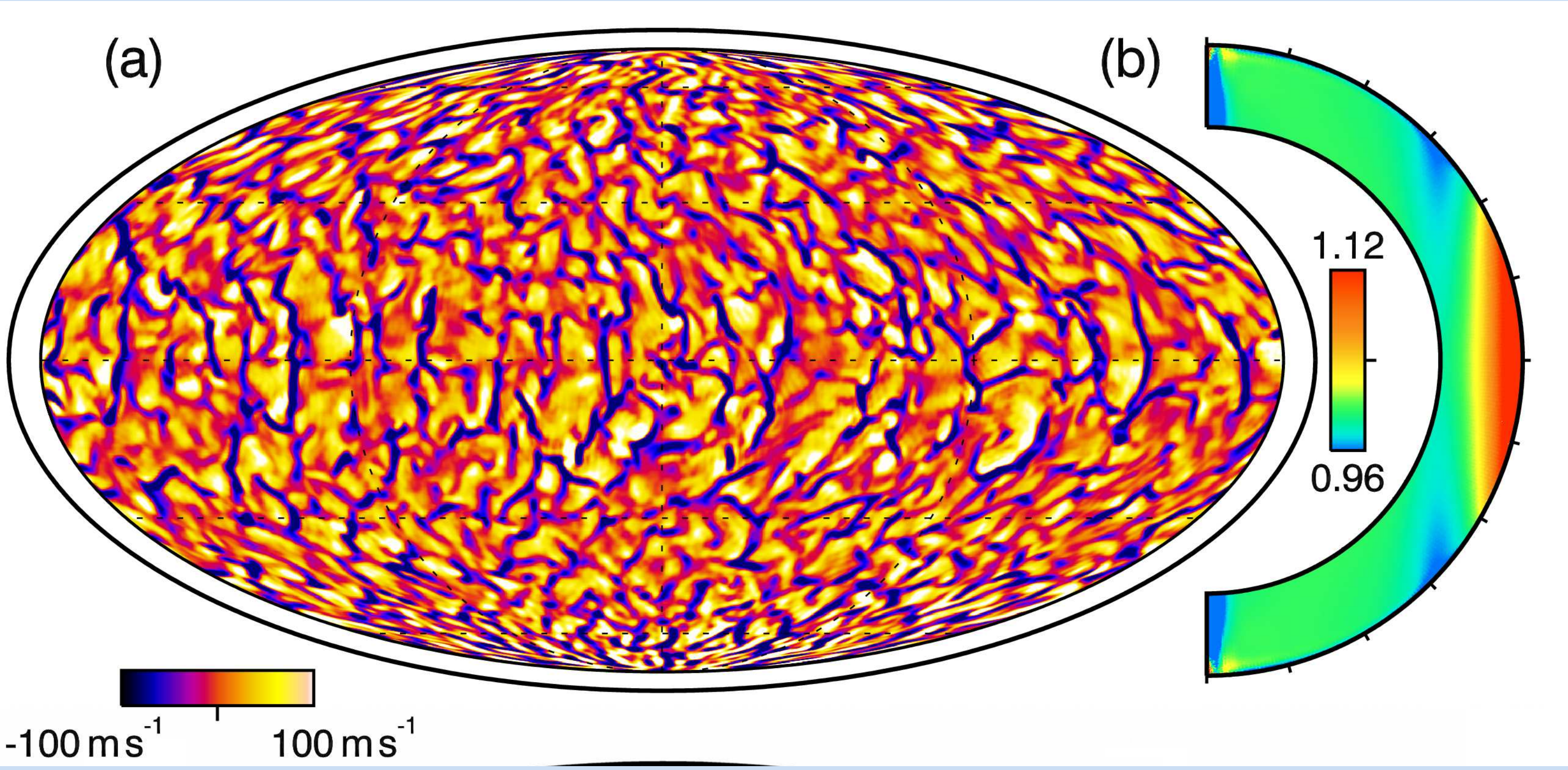

4.00
2.00
0.00
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$\overline{B}_\phi^{rms}$[kG]
A1
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

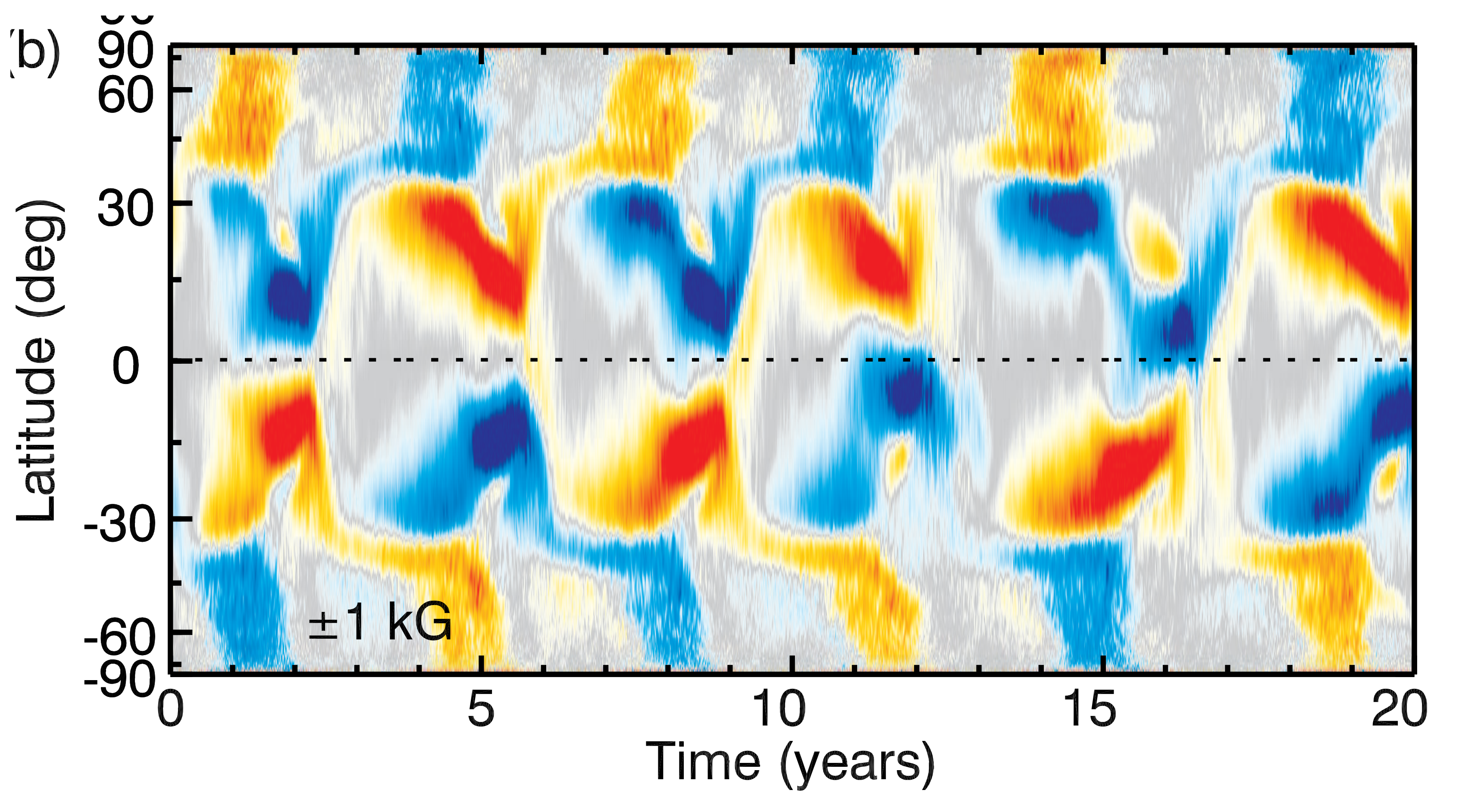

A1
$\overline{B}_\phi$ [kG]
r/R
$\overline{B}_\phi^{rms}$[kG]
A1

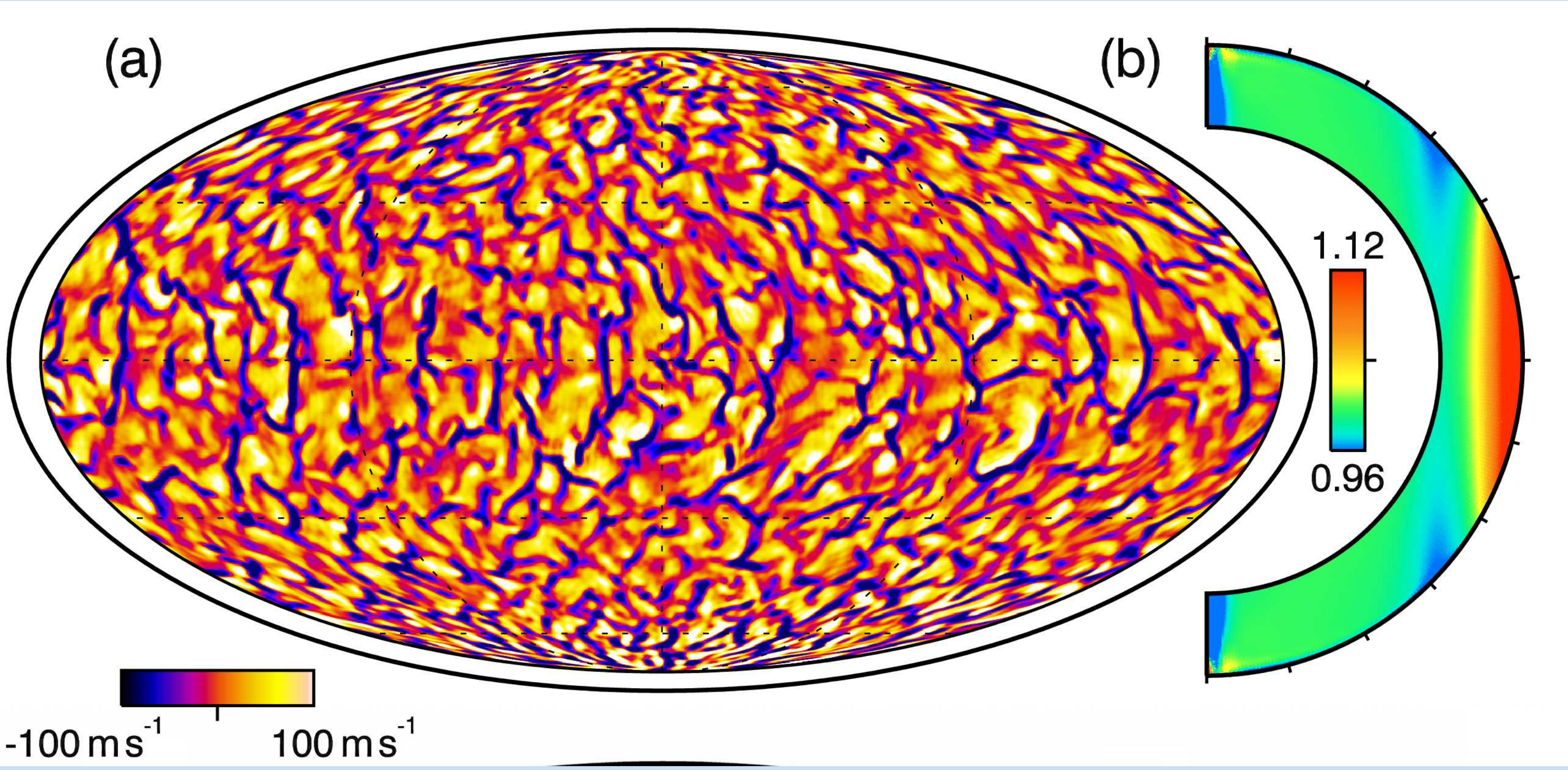

A1
$\overline{B}_\phi$ [kG]
60
40
20
0
−20
−40
−60
30
35
40
45
50
4
2
0
−2
−4
1.1
1.0
0.9
0.8
0.7
r/R
4.00
2.00
0.00
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$\overline{B}_\phi^{rms}$[kG]
A1


A1
$\overline{B}_\phi$ [kG]
60
40
20
4
2
R dlnΩ/dr
(a)
1.0
0.5
0.0
−0.5
−1.0
0 2 4 6 8 10 12 14
1.000
0.100
0.010
0.001
r/R
0.9
0.8
0.7
0
−2
−4
30 35 40 45 50
4.00
2.00
0.00
0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0
0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
$\overline{B}_\phi^{\rm rms}$[kG]
A1


(b)
Latitude (deg)
90
60
30
0
-30
-60
-90
±1 kG
0
5
10
15
20
Time (years)

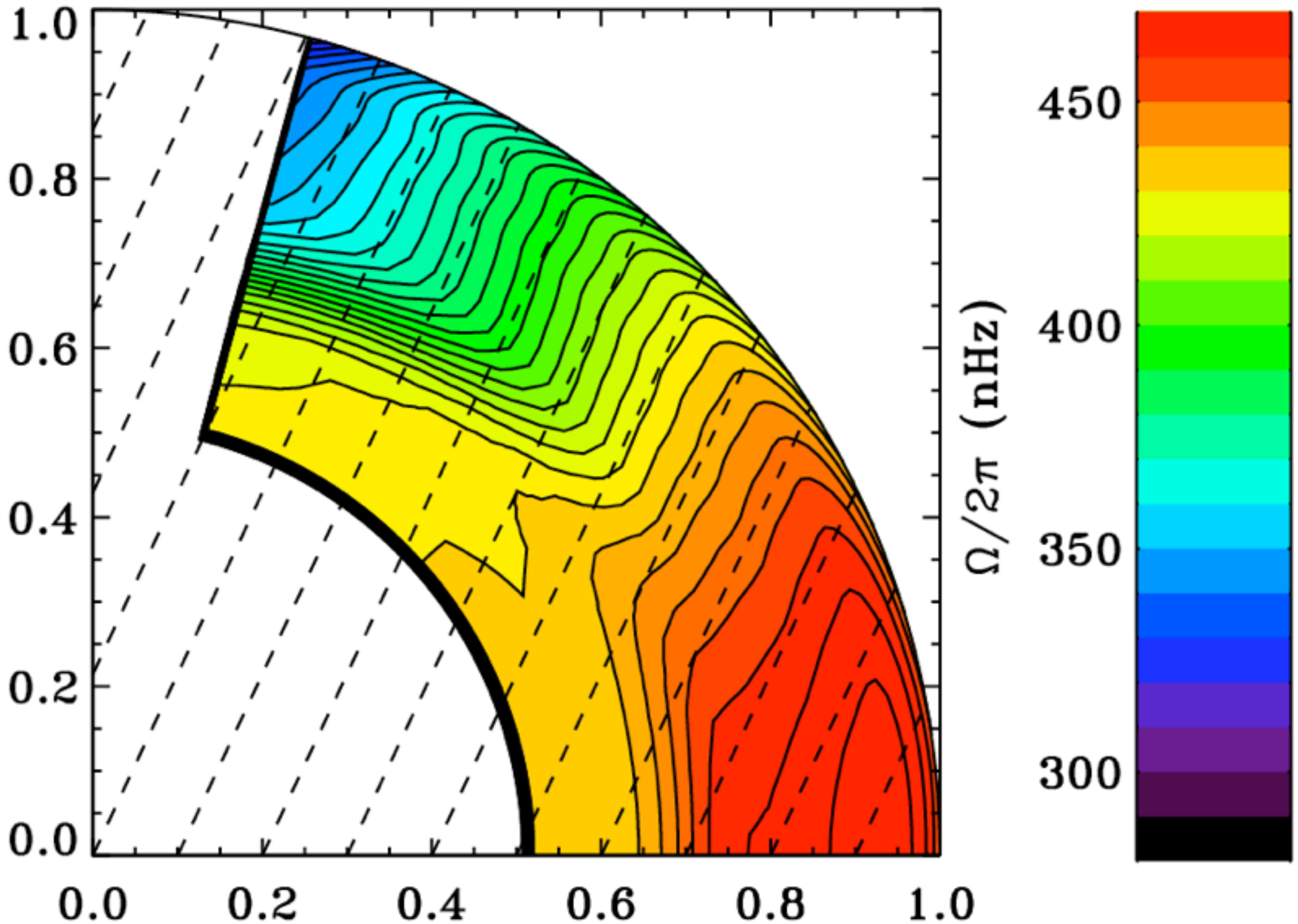

(a)
(b)
1.12
0.96
-100 m s^-1
100 m s^-1





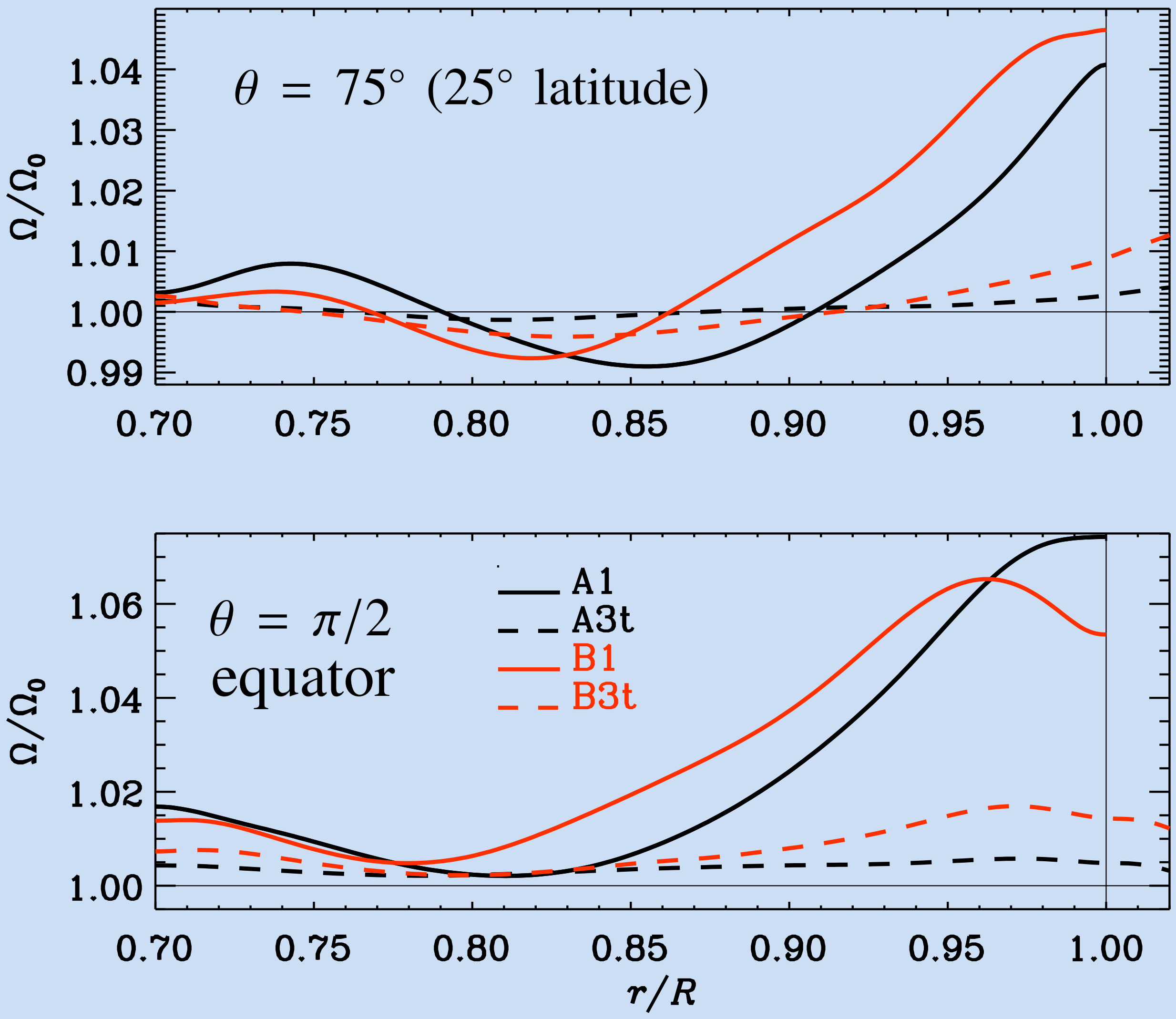
Augustson et al. 2014

Propagation direction of mean toroidal magnetic field can be entirely explain by the Parker—Yoshimura—Rule

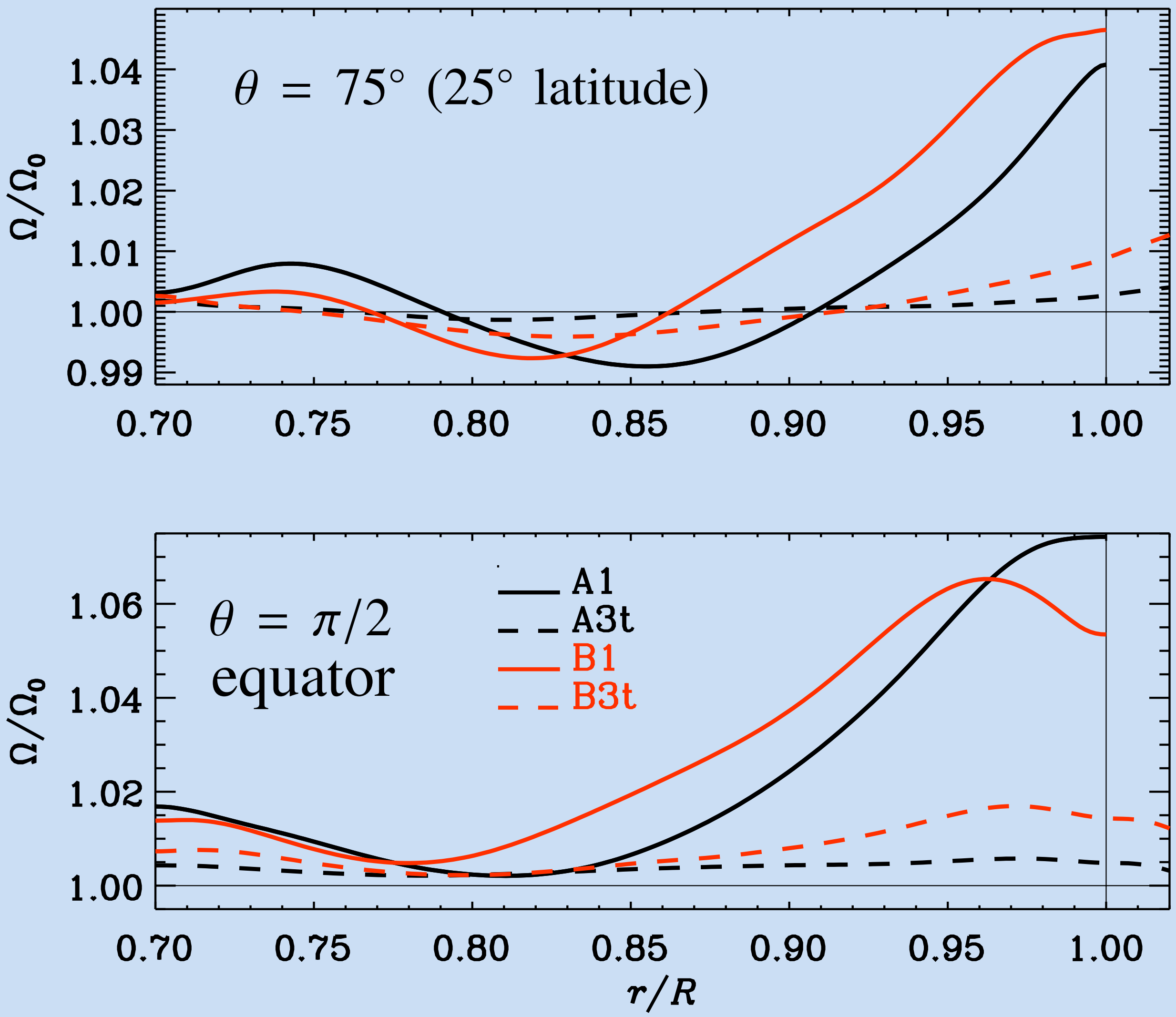


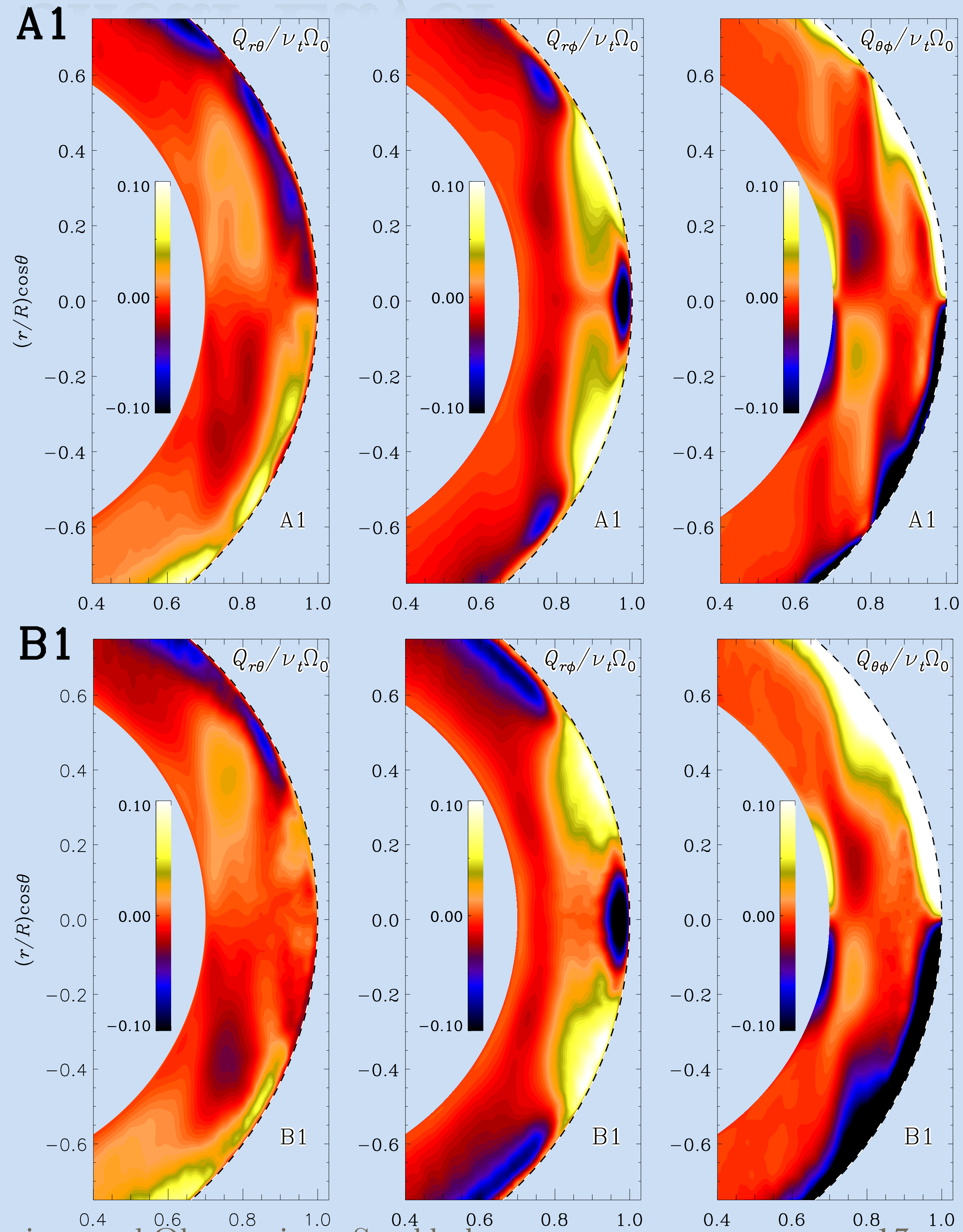
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Ω/2π (nHz)
450
400
350
300

Near-Surface Shear Layer



Near-Surface Shear Layer



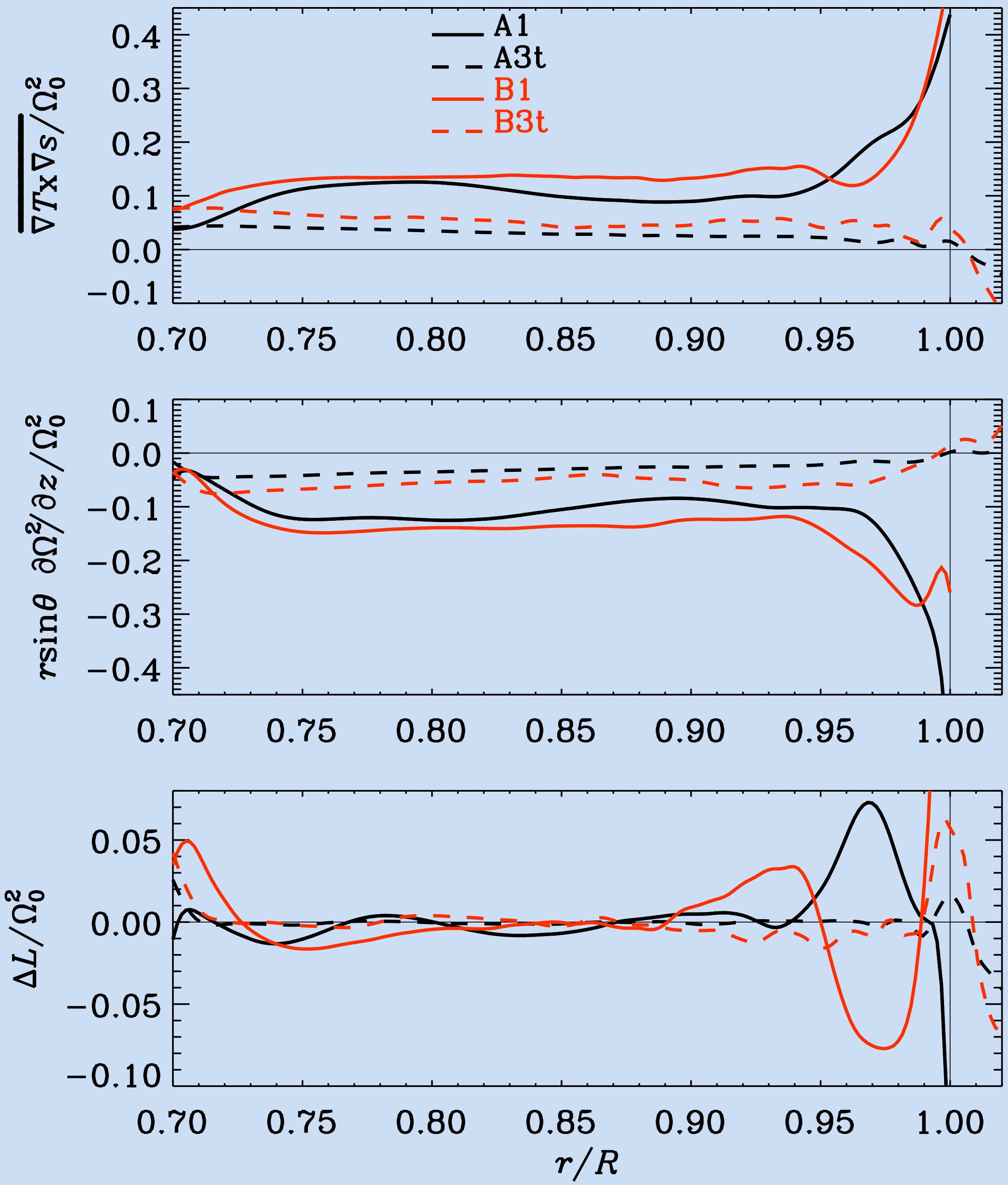


Thermal wind balance

$$\frac{\partial \overline{\omega}_\phi}{\partial t} = r \sin\theta \frac{\partial \Omega^2}{\partial z} + \left[\overline{\boldsymbol{\nabla} T \times \boldsymbol{\nabla} s}\right]_\phi - \left[\boldsymbol{\nabla} \times \left(\frac{1}{\overline{\rho}} \boldsymbol{\nabla} \cdot \overline{\rho}\, \overline{\boldsymbol{u}'\boldsymbol{u}'}\right)\right]_\phi$$

Thermal wind balance

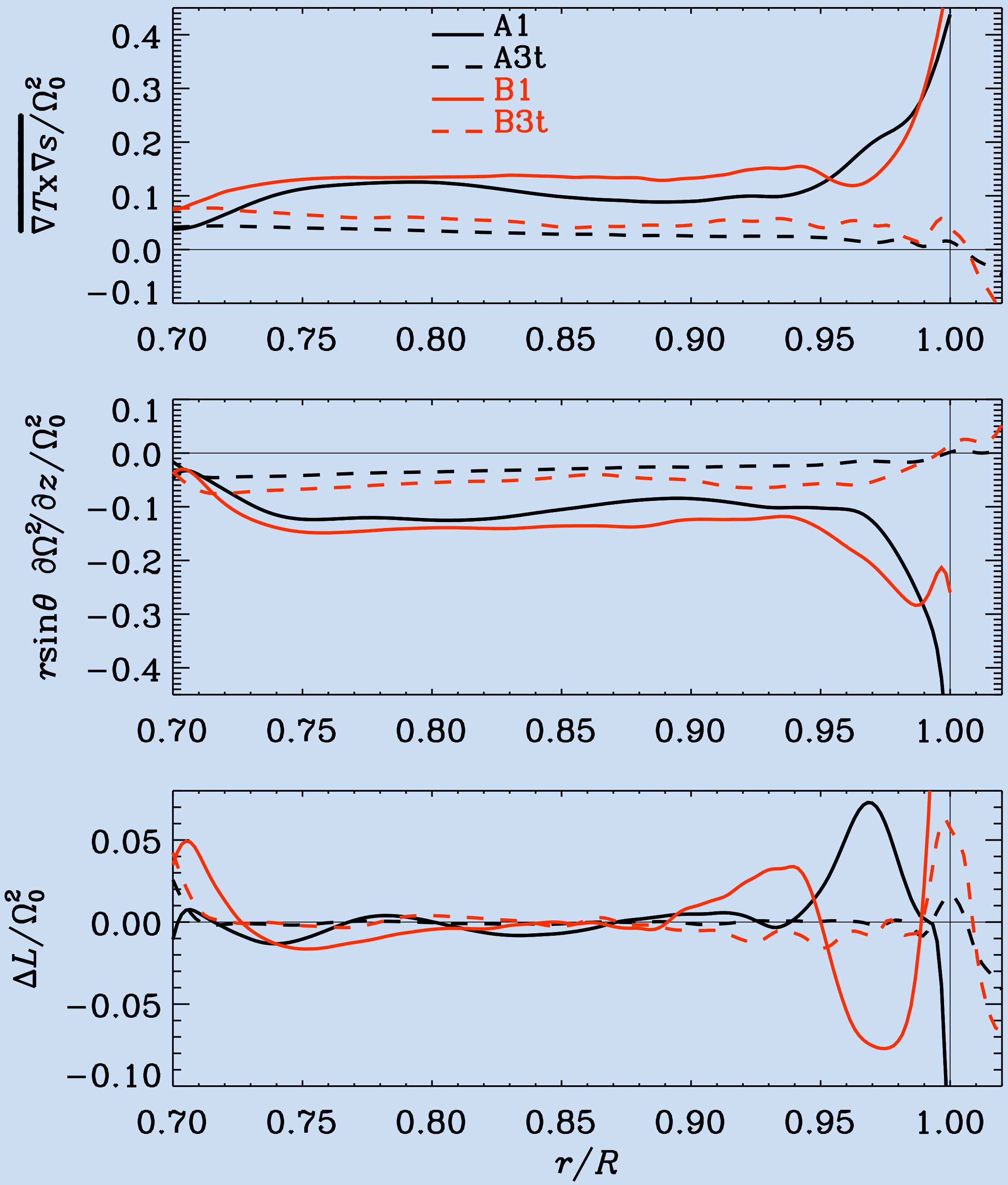
$$\frac{\partial \overline{\omega}_\phi}{\partial t} = r\sin\theta \frac{\partial \Omega^2}{\partial z} + \left[\overline{\nabla T \times \nabla s}\right]_\phi - \left[\nabla \times \left(\frac{1}{\overline{\rho}} \nabla \cdot \overline{\rho}\, \overline{\boldsymbol{u}'\boldsymbol{u}'}\right)\right]_\phi$$



Thermal wind balance

$$\frac{\partial \overline{\omega}_\phi}{\partial t} = r\sin\theta \frac{\partial \Omega^2}{\partial z} + \left[\overline{\boldsymbol{\nabla} T \times \boldsymbol{\nabla} s}\right]_\phi - \left[\boldsymbol{\nabla} \times \left(\frac{1}{\overline{\rho}} \boldsymbol{\nabla}\cdot\overline{\rho}\,\overline{\boldsymbol{u}'\boldsymbol{u}'}\right)\right]_\phi$$

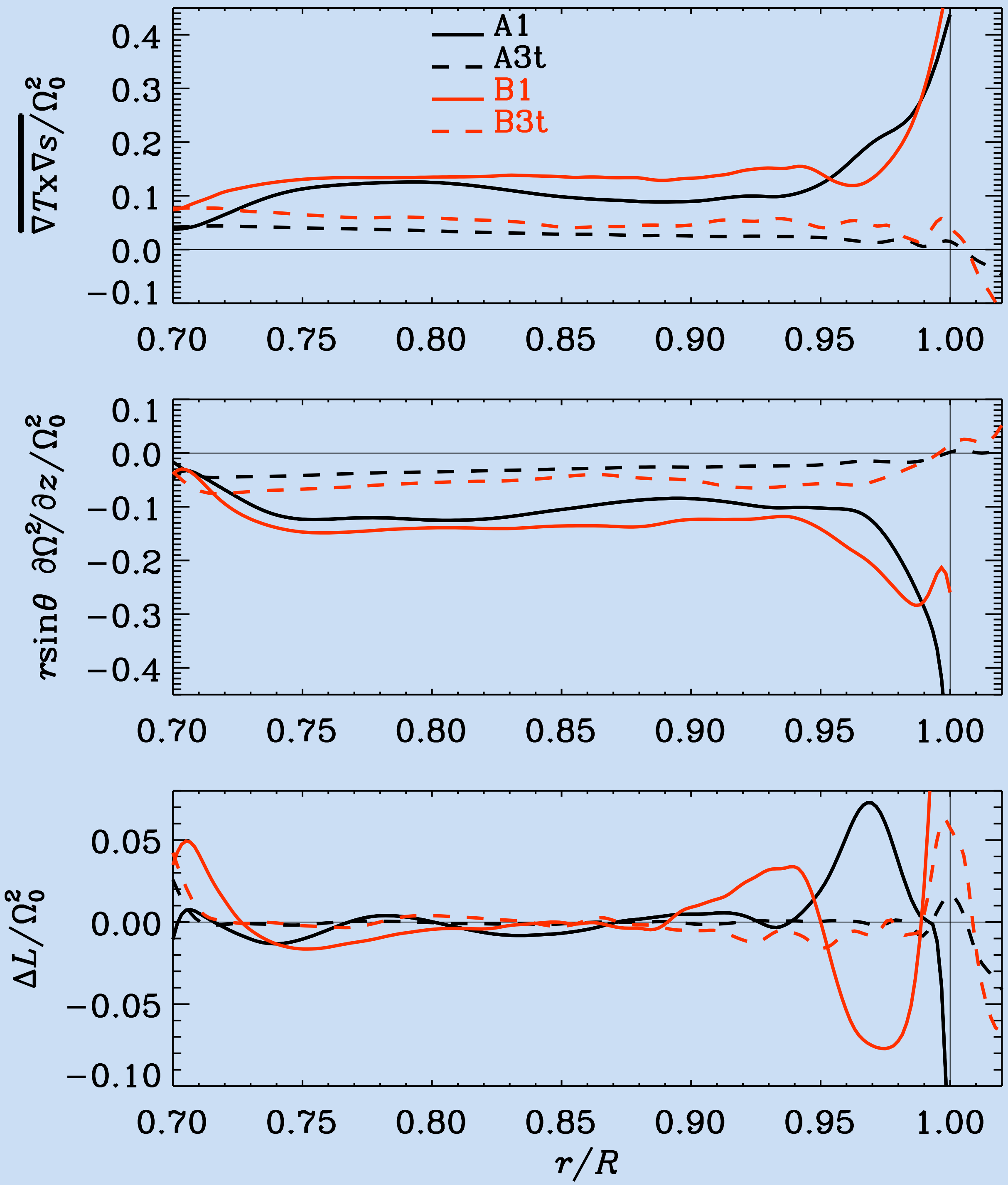
$$-\frac{1}{\overline{\rho}^2}\left(\partial_r\overline{\rho}\left[\boldsymbol{\nabla}\cdot\overline{\rho}\,\overline{\boldsymbol{u}'\boldsymbol{u}'}\right]_\theta - \frac{1}{r}\partial_\theta\overline{\rho}\left[\boldsymbol{\nabla}\cdot\overline{\rho}\,\overline{\boldsymbol{u}'\boldsymbol{u}'}\right]_r\right).$$



Thermal wind balance

$$\frac{\partial \overline{\omega}_\phi}{\partial t} = r\sin\theta\frac{\partial \Omega^2}{\partial z} + \left[\overline{\boldsymbol{\nabla} T \times \boldsymbol{\nabla} s}\right]_\phi - \left[\boldsymbol{\nabla}\times\left(\frac{1}{\overline{\rho}}\boldsymbol{\nabla}\cdot\overline{\rho}\,\overline{\boldsymbol{u}'\boldsymbol{u}'}\right)\right]_\phi$$

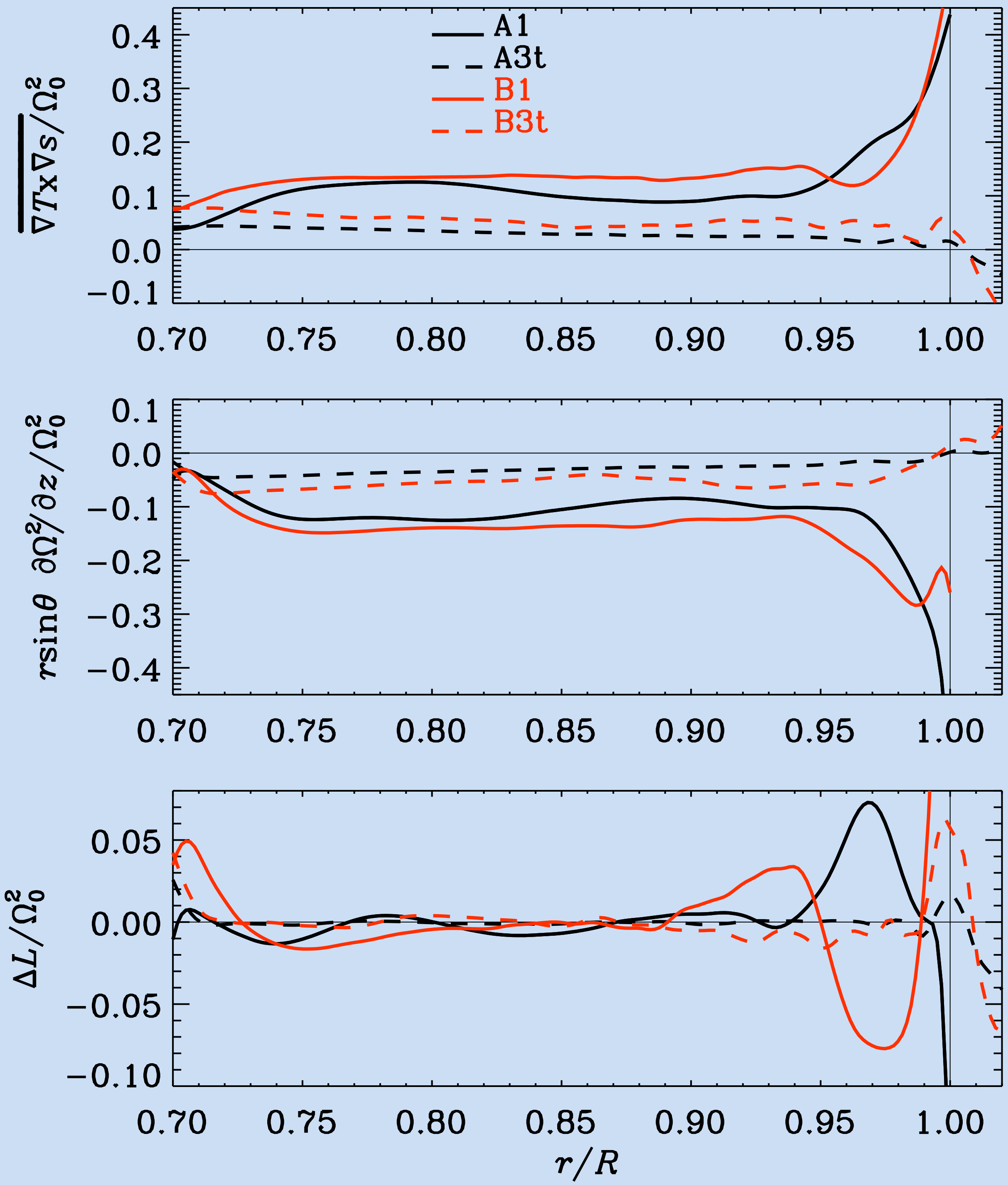
$$-\frac{1}{\overline{\rho}^2}\left(\partial_r\overline{\rho}\left[\boldsymbol{\nabla}\cdot\overline{\rho}\,\overline{\boldsymbol{u}'\boldsymbol{u}'}\right]_\theta - \frac{1}{r}\partial_\theta\overline{\rho}\left[\boldsymbol{\nabla}\cdot\overline{\rho}\,\overline{\boldsymbol{u}'\boldsymbol{u}'}\right]_r\right).$$



Thermal wind balance

$$\frac{\partial \overline{\omega}_\phi}{\partial t} = r\sin\theta \frac{\partial \Omega^2}{\partial z} + \left[\overline{\boldsymbol{\nabla} T \times \boldsymbol{\nabla} s}\right]_\phi - \left[\boldsymbol{\nabla} \times \left(\frac{1}{\overline{\rho}} \boldsymbol{\nabla}\cdot \overline{\rho}\, \overline{\boldsymbol{u}'\boldsymbol{u}'}\right)\right]_\phi$$

$$-\frac{1}{\overline{\rho}^2}\left(\partial_r\overline{\rho}\left[\boldsymbol{\nabla}\cdot\overline{\rho}\,\overline{\boldsymbol{u}'\boldsymbol{u}'}\right]_\theta - \frac{1}{r}\partial_\theta\overline{\rho}\left[\boldsymbol{\nabla}\cdot\overline{\rho}\,\overline{\boldsymbol{u}'\boldsymbol{u}'}\right]_r\right).$$



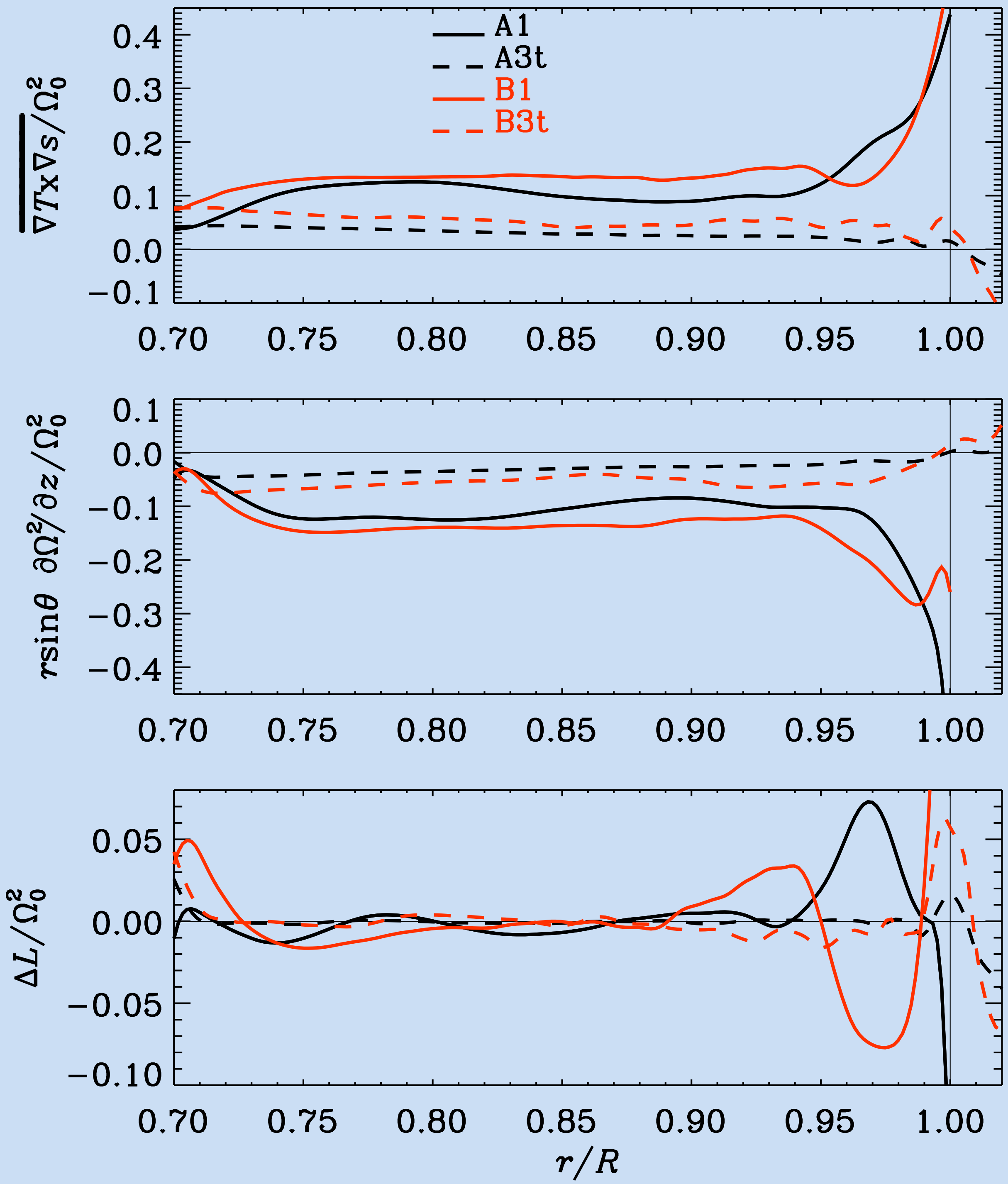
$$-\frac{1}{\overline{\rho}^2}\partial_r\overline{\rho}\left(\frac{1}{r^2}\partial_r\left(r^2\overline{\rho}\,\overline{u'_r u'_\theta}\right) + \frac{\overline{\rho}\,\overline{u'_r u'_\theta}}{r}\right) \equiv Q_{r\theta},$$

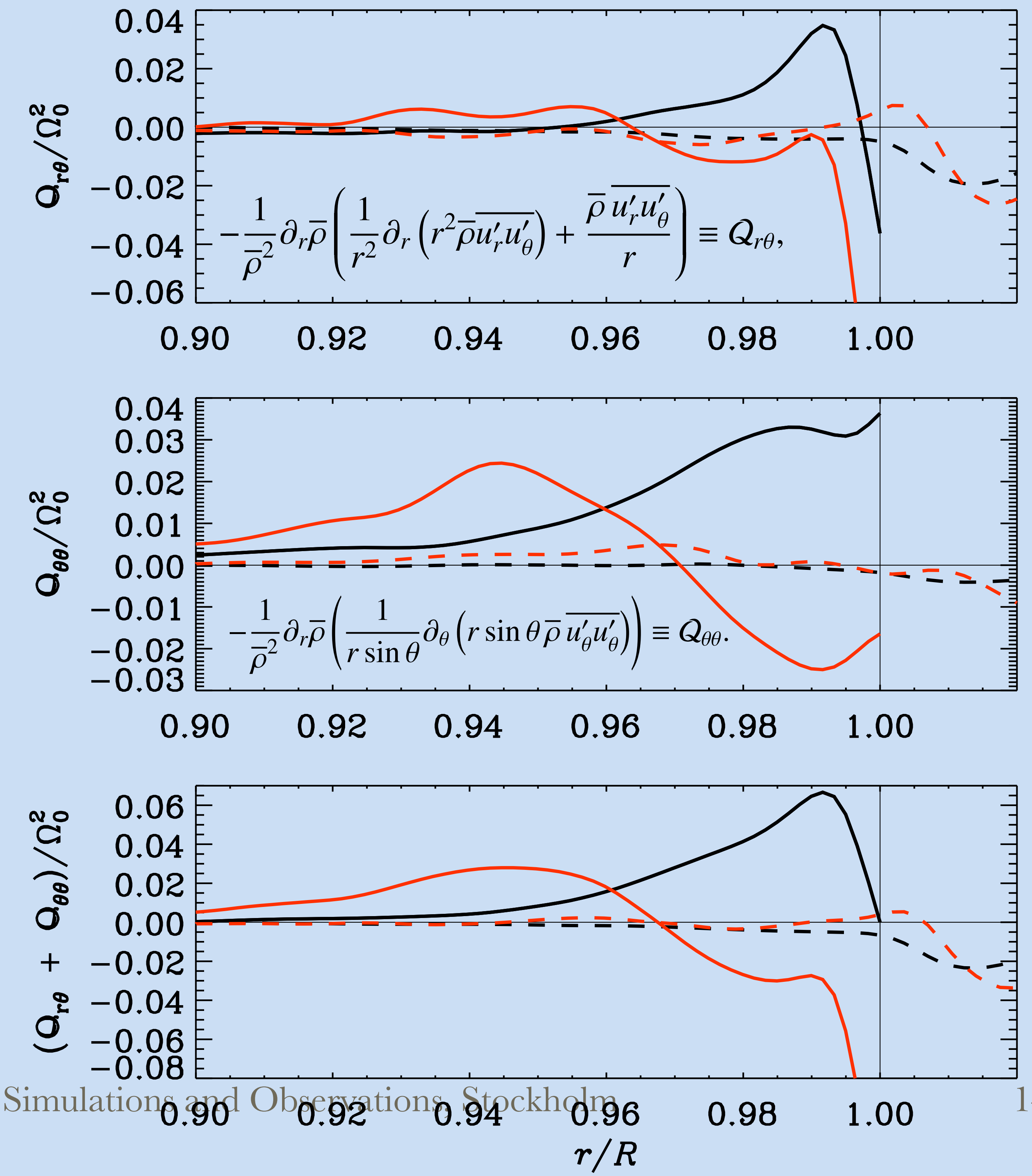
$$-\frac{1}{\overline{\rho}^2}\partial_r\overline{\rho}\left(\frac{1}{r\sin\theta}\partial_\theta\left(r\sin\theta\,\overline{\rho}\,\overline{u'_\theta u'_\theta}\right)\right) \equiv Q_{\theta\theta}.$$

Thermal wind balance

$$\frac{\partial \overline{\omega}_\phi}{\partial t} = r\sin\theta\frac{\partial \Omega^2}{\partial z} + \left[\overline{\boldsymbol{\nabla} T \times \boldsymbol{\nabla} s}\right]_\phi - \left[\boldsymbol{\nabla}\times\left(\frac{1}{\overline{\rho}}\boldsymbol{\nabla}\cdot\overline{\rho}\,\overline{\boldsymbol{u}'\boldsymbol{u}'}\right)\right]_\phi$$

$$-\frac{1}{\overline{\rho}^2}\left(\partial_r\overline{\rho}\left[\boldsymbol{\nabla}\cdot\overline{\rho}\,\overline{\boldsymbol{u}'\boldsymbol{u}'}\right]_\theta - \frac{1}{r}\partial_\theta\overline{\rho}\left[\boldsymbol{\nabla}\cdot\overline{\rho}\,\overline{\boldsymbol{u}'\boldsymbol{u}'}\right]_r\right).$$





Conclusions

- Equatorward propagation in simulation are related to the negative shear.
- Migration of mean magnetic field can be entirely explained by an alpha-omega-dynamo wave
- Parker-Yoshimura-Rule works!
- Near-surface shear layer in the Sun might produce the equatorward migration.
- Change of sign in $Q_{r\theta}$ related to NSSL.
- $\partial_r\ Q_{r\theta}$ and $\partial_\theta\ Q_{\theta\theta}$ balance the thermal wind