

# Effects of rotation and stratification on magnetic flux concentrations

[turbulence] [stratification] [magnetic field]

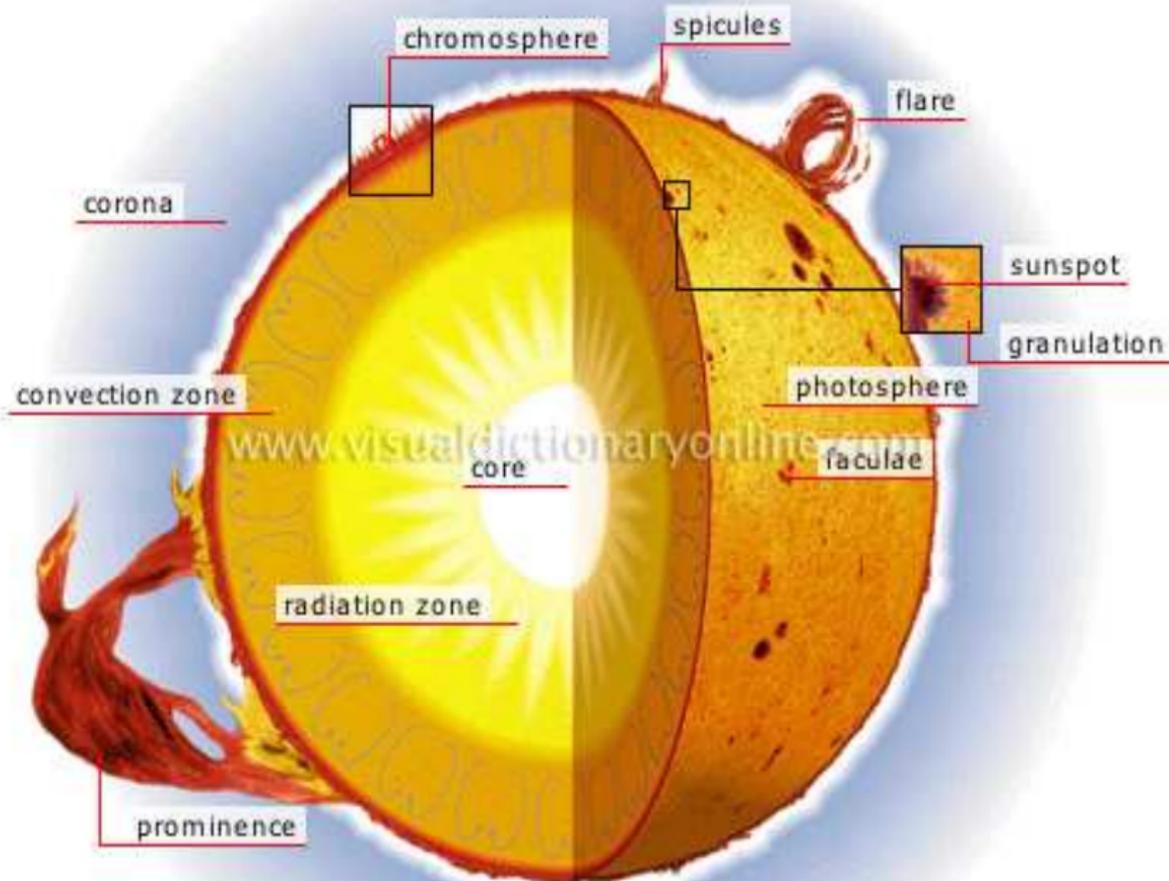
Illa R. Losada

Nordita and Stockholm University

Sunspot formation

11 Mar 2015

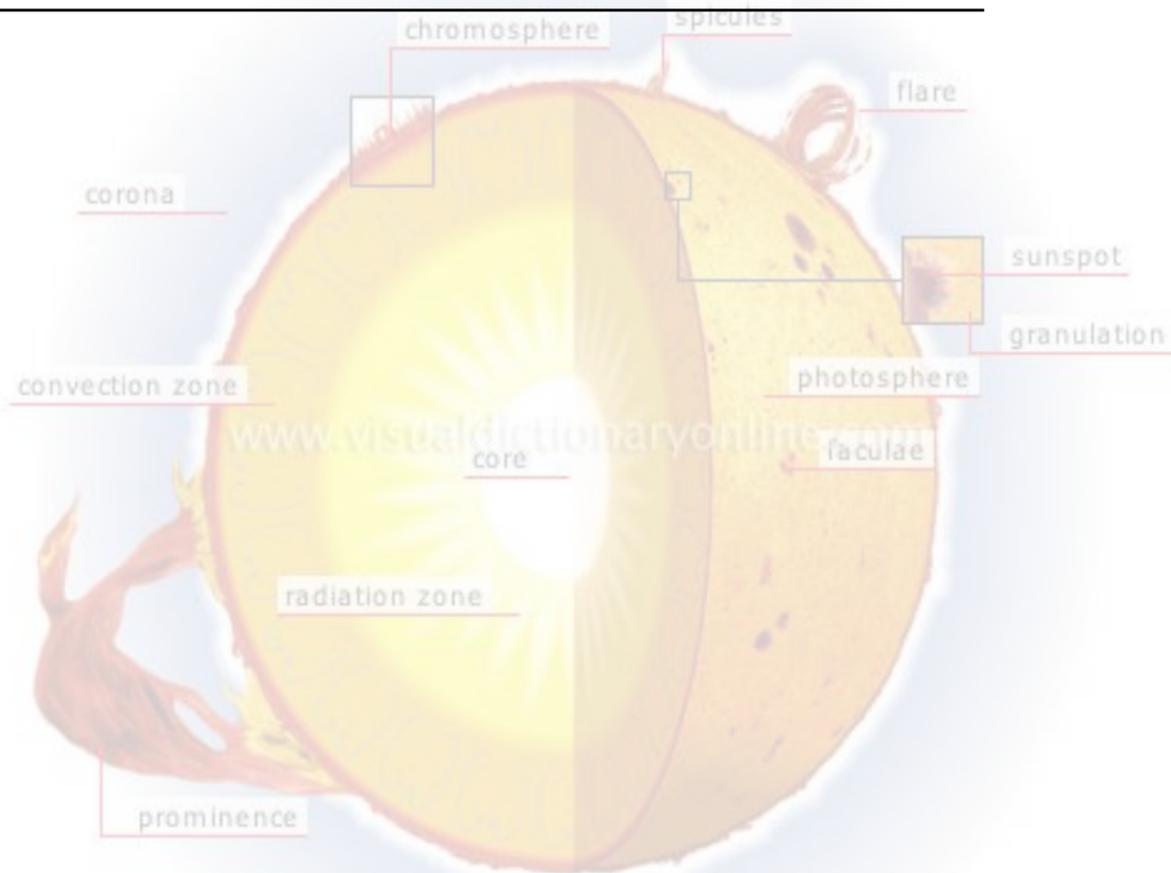




Inside

Surface

Outside



Inside

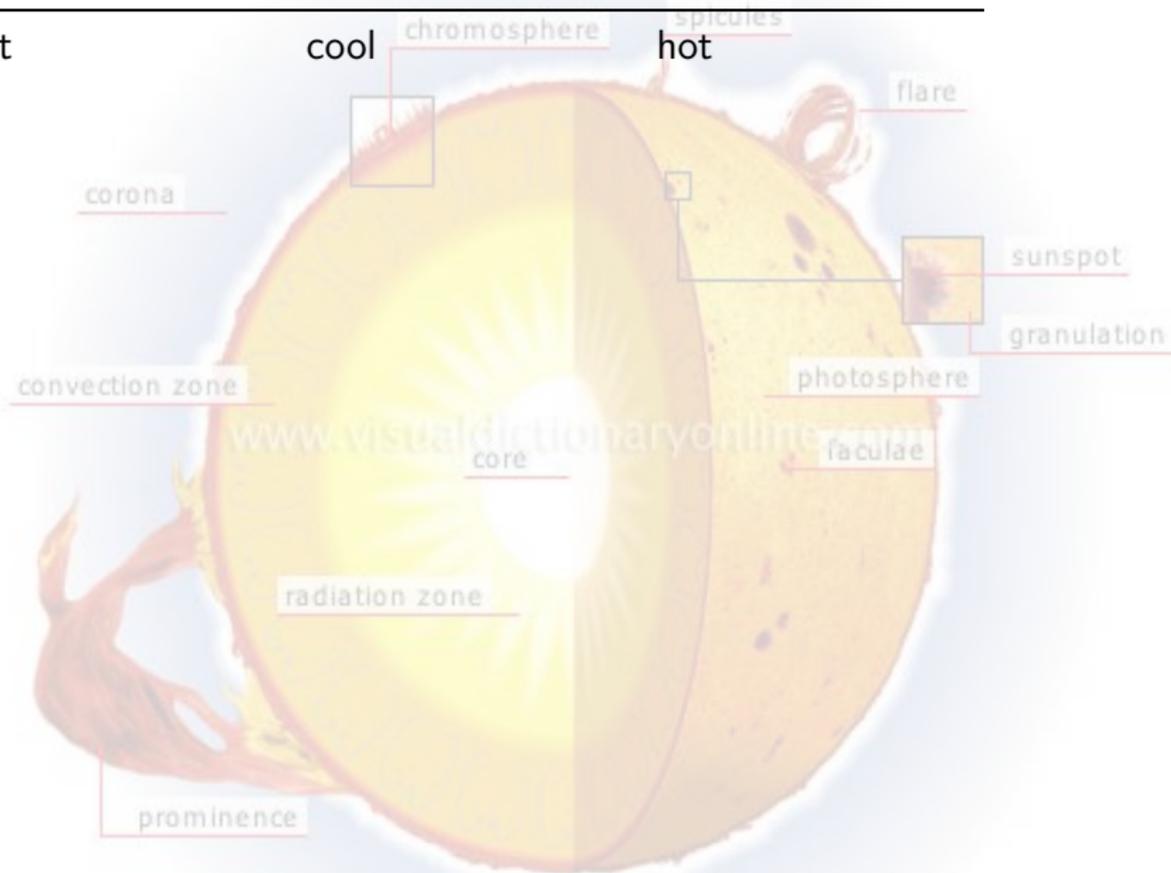
Surface

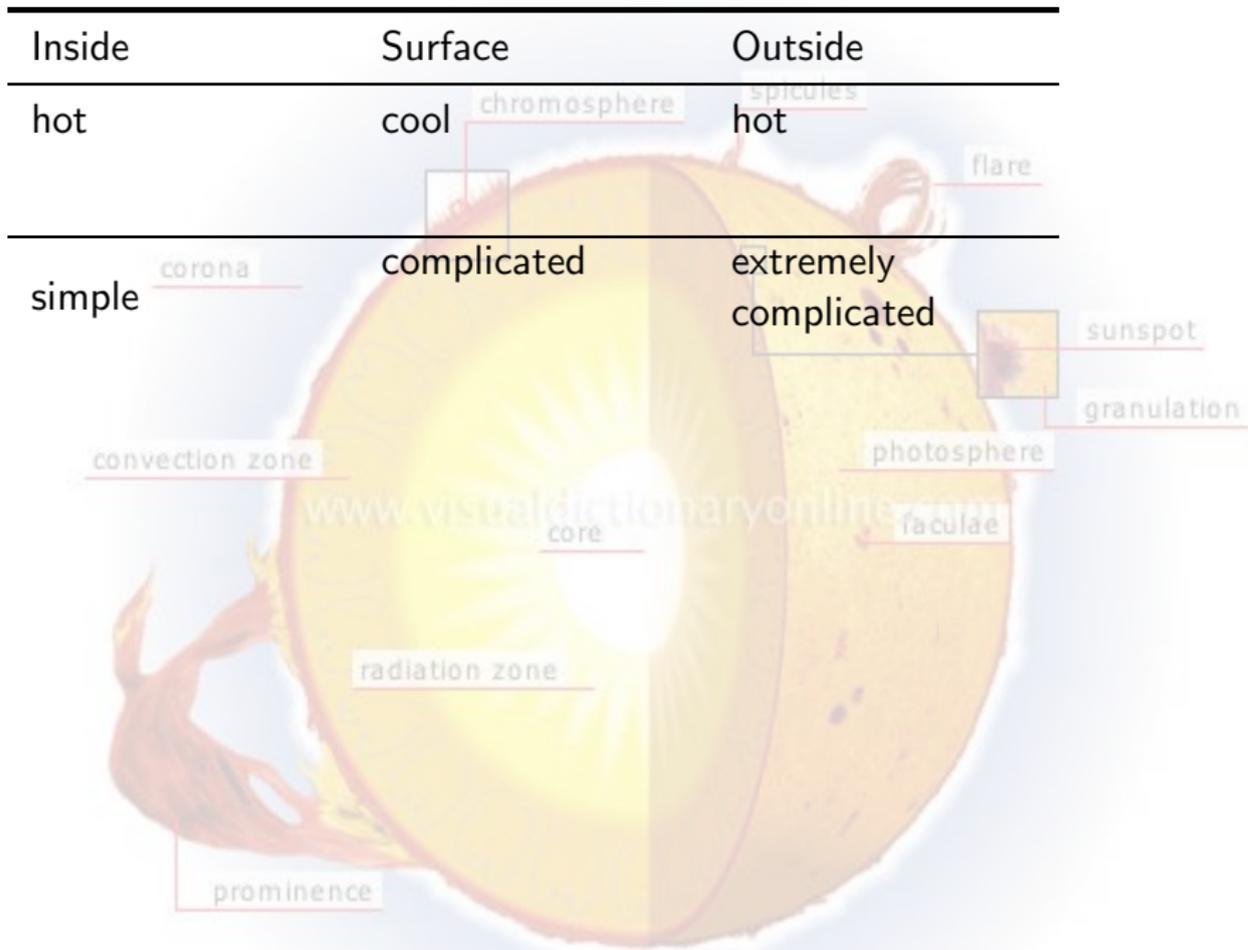
Outside

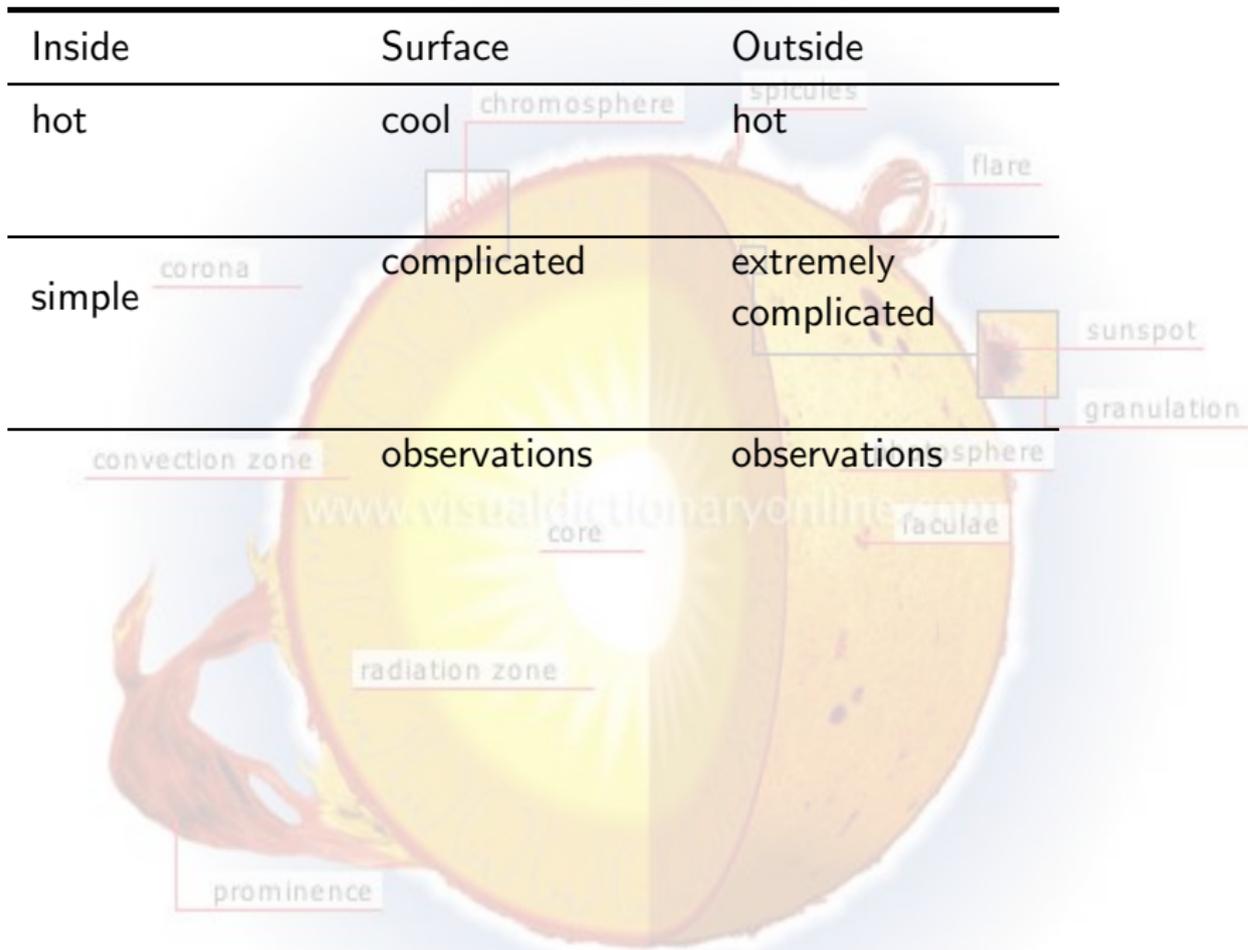
hot

cool

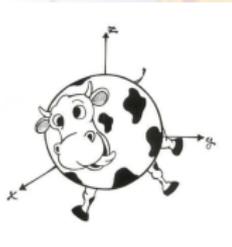
hot







Inside	Surface	Outside
hot	cool	hot
simple	complicated	extremely complicated
model	observations	observations



“It is also a good rule not to put overmuch confidence in the observational results that are put forward until they are confirmed by theory.” Arthur Eddington

# Flux tubes

Parker instability (1955)

# Flux tubes

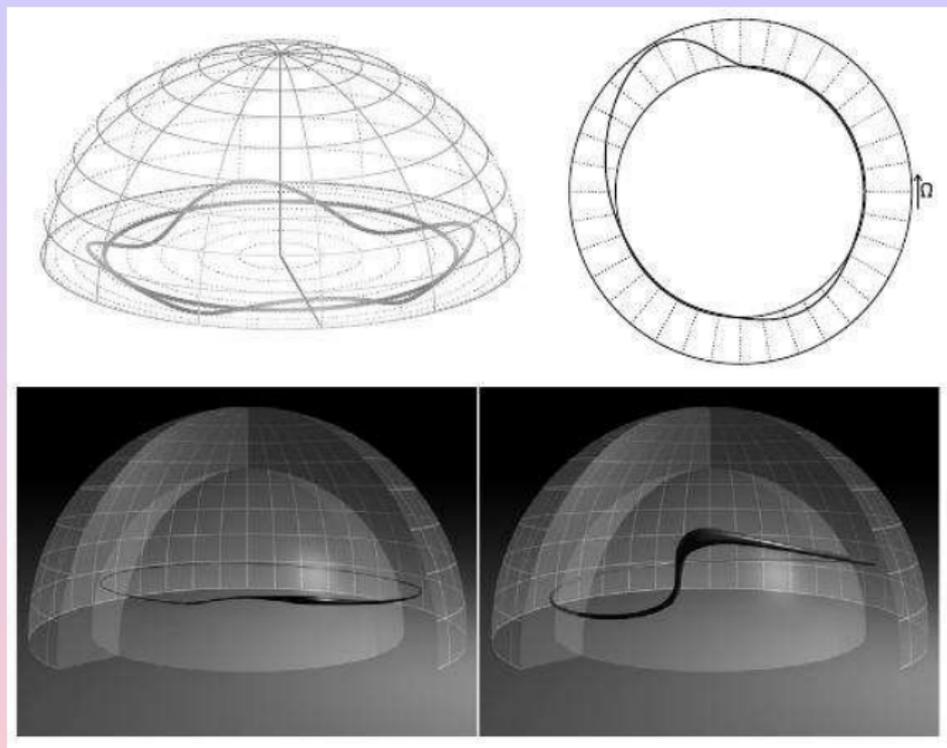


Figure 1 : Caligari et al. (1995)

## Flux tubes

### NEMPI

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DNS

MFS

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2-layers model + rotation

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# Flux tubes

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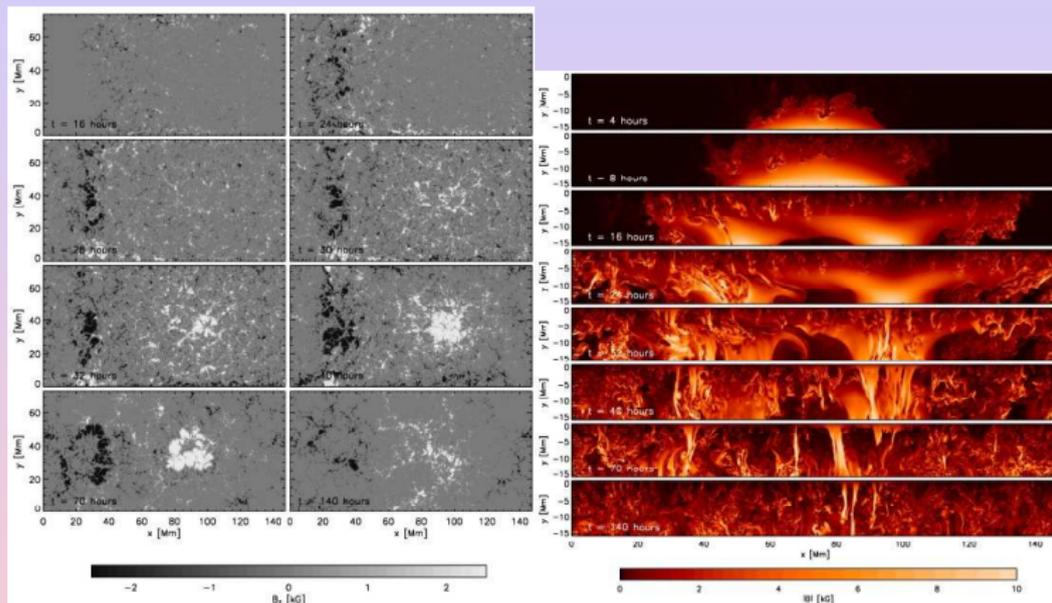


Figure 2 : Time evolution of  $|B|$  at the surface and along a vertical cut. Rempel et al. (2014)

# Negative Effective Magnetic Pressure Instability

# Vertical field. Sunspot-like structure

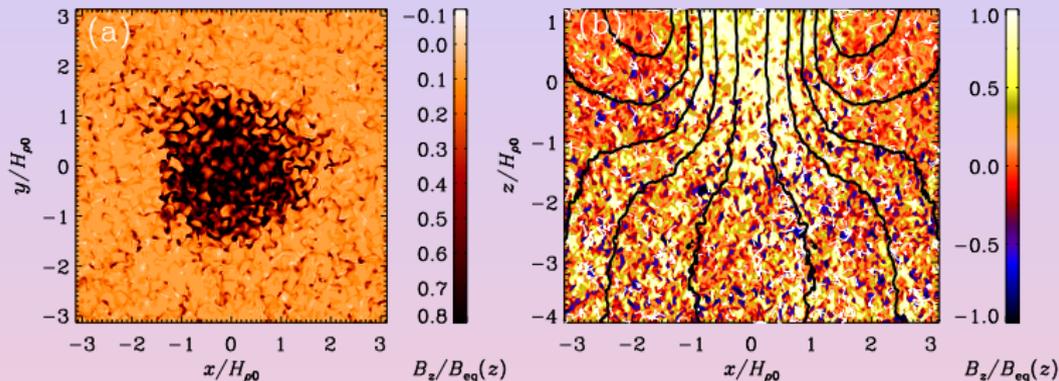


Figure 3: Cuts of  $B_z/B_{eq}(z)$  in the  $xy$  plane at the top boundary ( $z/H_{\rho 0} = 1.2$ ) and the  $xz$  plane through the middle of the spot at  $y = 0$  for  $\gamma = 5/3$  and  $\beta_0 = 0.05$ . In the  $xz$  cut, we also show magnetic field lines and flow vectors obtained by numerically averaging in azimuth around the spot axis.

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# Vertical field. Sunspot-like structure

Magnetic flux concentrations  
Illa R. Losada

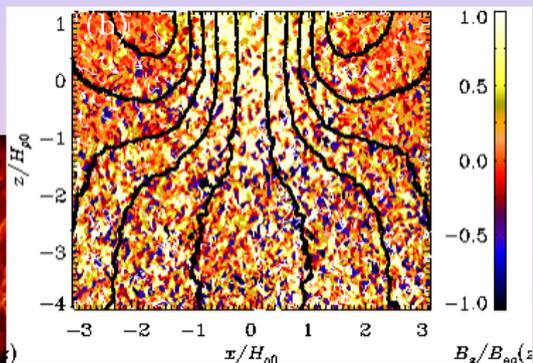
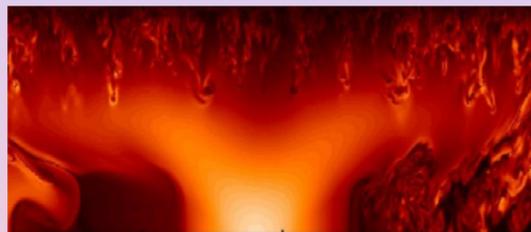


Figure 4 : Rempel et al. (2014) and Losada et al. (2014)

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# Velocity field and NEMPI

## Forced Velocity field

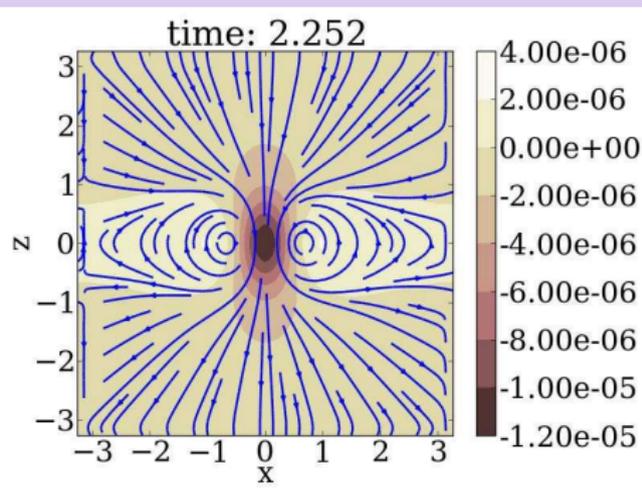


Figure 5 : hydro simulation with imposed velocity

## NEMPI.

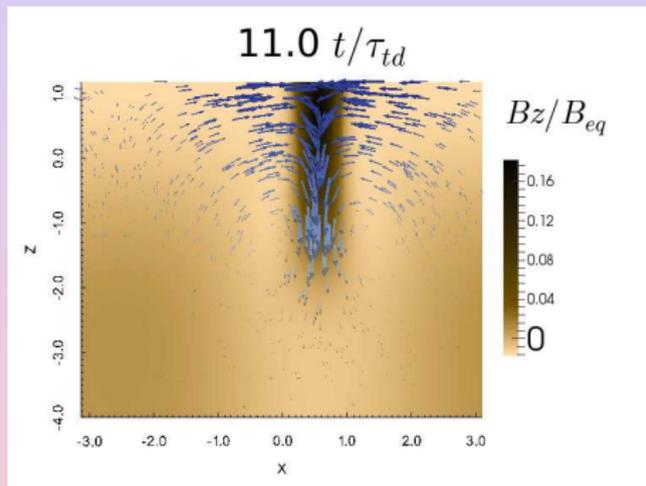


Figure 6 : MHD simulation.  $P = P_{gas} + P_{eff}$

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Let's go back to the equations!

Magnetohydrodynamics (MHD): equations for the dynamics of the plasma.

- ▶ Maxwell equation:  $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E}$
- ▶ Mass conservation:  $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0$
- ▶ Momentum conservation:  $\rho \frac{D\mathbf{U}}{Dt} = -\nabla p + \mathbf{J} \times \mathbf{B}$
- ▶ Low frequency Maxwell eq. :  $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$
- ▶ Energy conservation:  $\frac{d}{dt} \left( \frac{p}{\rho^\gamma} \right) = 0$
- ▶ Ohm's law:  $\mathbf{E} + \mathbf{U} \times \mathbf{B} = \eta \mu_0 \mathbf{J}$

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Missing: viscosity, heating, conduction, radiation, gravity, rotation, ionisation, etc

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# Solve MHD equations

## MHD simulations

- ▶ Goal of simulations: solve MHD equations.
- ▶ Not possible to use solar parameters:  
$$\text{Re} \approx \frac{UL}{\nu} \approx 10^{10} - 10^{15}$$

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## Simulations

- ▶ Direct Numerical Simulations (DNS)
- ▶ Mean-field Simulations (MFS)
- ▶ Large Eddy Simulations (LES)

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## MFS and DNS: How to cook paella!

### Mean Field Simulations (MFS)

- ▶ Quantities = averaged + fluctuations:  $F = \overline{F} + f$
- ▶ Approximations: add or subtract terms in the equations.
- ▶ Control the physics.

### Direct Numerical Simulations (DNS)

- ▶ Solve full equations.
- ▶ Approximations: only in resolution.
- ▶ No control.

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Goal: Obtain DNS results with known MFS physics.

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## MFS and DNS: How to cook paella!

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Simulations done with Pencil Code

(<http://pencil-code.googlecode.com>)

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# Full MHD equations.

## Direct Numerical simulations (DNS):

- ▶ Continuity equation:  $\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{U})$
- ▶ Momentum equation:  
$$\rho \frac{D\mathbf{U}}{Dt} = -\nabla p + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g} + \rho \mathbf{F}_\nu + \rho \mathbf{f}$$
- ▶ Induction equation:  $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} - \eta \mu_0 \mathbf{J})$

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# Full MHD equations.

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- ▶ Induction equation:  $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} - \eta \mu_0 \mathbf{J})$

## Setup:

- ▶ Forcing,  $k_f/k_1 = 15$   
(control scale separation)
- ▶ Strong stratification:  
density contrast  $\approx 535$
- ▶  $B_0/B_{\text{eq}0} = 0.05$  (in range)
- ▶  $64^3 \times 128$  mesh-points
- ▶  $\text{Re}_M \approx \frac{UL}{\eta} = 6$

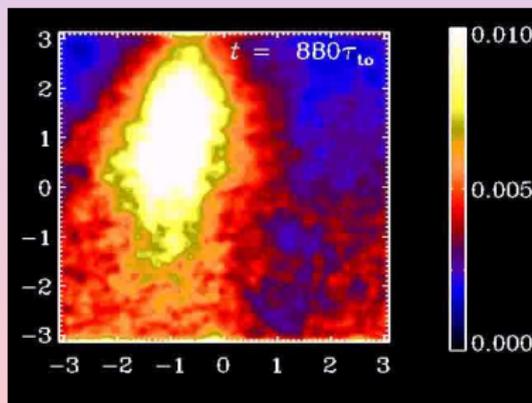


Figure 7:  $\Delta \bar{\mathbf{B}}/B_{\text{eq}0}$  (Brandenburg et al. 2011).

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## Mean-Field MHD equations.

Quantities = averaged + fluctuations:  $F = \bar{F} + f$

## Mean field simulations (MFS):

▶ Continuity equation:  $\frac{D\bar{\rho}}{Dt} = -\bar{\rho}\nabla \cdot \bar{\mathbf{U}}$

▶ Momentum equation:

$$\frac{D\bar{\mathbf{U}}}{Dt} = -c_s^2 \nabla \ln \bar{\rho} + \mathbf{g} + \bar{\mathcal{F}}_M + \bar{\mathcal{F}}_K$$

▶ Induction equation:

$$\frac{\partial \bar{\mathbf{B}}}{\partial t} = \nabla \times (\bar{\mathbf{U}} \times \bar{\mathbf{B}} + \overline{\mathbf{u} \times \mathbf{b}}) + \eta \nabla^2 \bar{\mathbf{B}}$$

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# From induction equation: dynamo instability

$$\frac{\partial \bar{\mathbf{B}}}{\partial t} = \nabla \times (\bar{\mathbf{U}} \times \bar{\mathbf{B}} + \overline{\mathbf{u} \times \mathbf{b}}) + \eta \nabla^2 \bar{\mathbf{B}} \quad (1)$$

$\alpha$ -effect (Steenbeck et al. 1966; Moffat 1978; Krause & Rädler 1980)

- ▶ Isotropic case:  $\overline{\mathbf{u} \times \mathbf{b}} = \alpha \bar{\mathbf{B}} - \eta_t \bar{\mathbf{J}}$
- ▶ If  $\alpha \neq 0 \rightarrow$  generate a  $\bar{\mathbf{B}}$
- ▶  $\alpha \propto$  helicity
- ▶  $\eta_t$ : turbulent diffusivity.

Responsible for the Sun's large-scale field.

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## From momentum equation: NEMPI

$$\frac{D\bar{\mathbf{U}}}{Dt} = -c_s^2 \nabla \ln \bar{\rho} + \mathbf{g} + \bar{\mathcal{F}}_M + \bar{\mathcal{F}}_K \quad (2)$$

$\bar{\mathcal{F}}_M$  : Lorentz Force  $\rightarrow$  magnetic stress tensor

$\bar{\mathcal{F}}_K$  : Reynolds stresses  $\rightarrow$  kinetic stress tensor

(3)

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## From momentum equation: NEMPI

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$\bar{\mathcal{F}}_M$  : Lorentz Force  $\rightarrow$  magnetic stress tensor

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Total pressure:

$$P_T = P_{gas} + \frac{\bar{\mathbf{B}}^2}{2\mu_0} + p_t \quad (3)$$

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From momentum equation: NEMPI

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Total pressure:

$$P_T = P_{gas} + \underbrace{\frac{\overline{\mathbf{B}}^2}{2\mu_0}}_{\text{Effective magnetic pressure}} + p_t \quad (3)$$

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Total pressure:

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Effective magnetic pressure = turbulent pressure =  
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$\bar{\mathcal{F}}_M$  : Lorentz Force  $\rightarrow$  magnetic stress tensor

$\bar{\mathcal{F}}_K$  : Reynolds stresses  $\rightarrow$  kinetic stress tensor

Total pressure:

$$P_T = P_{gas} + P_{eff} \quad (3)$$

Effective magnetic pressure = turbulent pressure = hydrodynamic + magnetic

$$P_{eff} = (1 - q_p(\bar{\mathbf{B}})) \frac{\bar{\mathbf{B}}^2}{2\mu_0} \quad (4)$$

$$\langle b^2 \rangle = f(\bar{\mathbf{B}}) \bar{\mathbf{B}}^2 = q_p 3 \bar{\mathbf{B}}^2 \quad (5)$$

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Total pressure:

$$P_T = P_{gas} + P_{eff} = P_{gas} + (1 - q_p(\bar{\mathbf{B}})) \frac{\bar{\mathbf{B}}^2}{2\mu_0} \quad (3)$$

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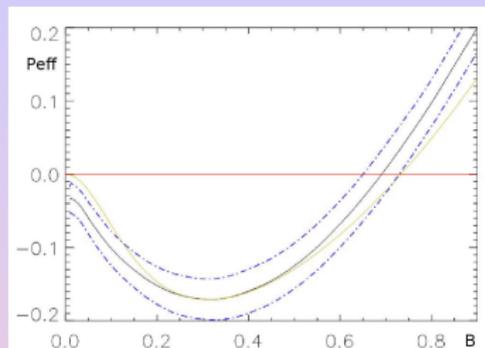
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## Effective magnetic pressure

(Kemel et. al 2012):

(effects of turbulence on the mean Lorentz force)

$$P_{eff} = \frac{1}{2}(1 - q_p)\overline{\mathbf{B}}^2 / \overline{B}_{eq}^2 \quad (6)$$



## Negative Effective Magnetic Pressure Instability (NEMPI)

- ▶ Regions below the minimum value of  $P_{eff} \rightarrow$  NEMP
- ▶ NEMP + strong stratification ( $|\nabla \ln \rho| > |\nabla \overline{\mathbf{B}}|$ )  $\rightarrow$  NEMPI <sup>1</sup>
- ▶ Magnetic field suppress turbulence  $\rightarrow$  structures sink!

<sup>1</sup> Predicted: Kleeorin et al. 1989, 1990; Kleeorin & Rogachevskii 1994; Kleeorin et al. 1996; Rogachevskii & Kleeorin 2007. Confirmed: Brandenburg et al. 2011

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# MFS Parametrization

Normalized effective magnetic pressure:

$$\mathcal{P}_{\text{eff}} = \frac{1}{2}(1 - q_p)\beta^2, \quad (7)$$

$q_p(\beta)$  approximated by (Kemel et al. 2012a):

(empirical, fits to DNS results)

$$q_p(\beta) = \frac{q_{p0}}{1 + \beta^2/\beta_p^2} = \frac{\beta_\star^2}{\beta_p^2 + \beta^2}, \quad (8)$$

$q_{p0}$ ,  $\beta_p$ , and  $\beta_\star = \beta_p q_{p0}^{1/2}$ : constants.

$\mathcal{P}_{\text{eff}}$  has a minimum value  $\mathcal{P}_{\text{min}}$  at  $\beta_{\text{min}}$ , related with the parameters:

$$\beta_p = \beta_{\text{min}}^2 / \sqrt{-2\mathcal{P}_{\text{min}}}, \quad \beta_\star = \beta_p + \sqrt{-2\mathcal{P}_{\text{min}}}. \quad (9)$$

Growth rate:

$$\frac{\lambda}{\eta_t k^2} \approx 3\beta_\star \frac{k_f/k}{kH_\rho - 1} \quad (10)$$

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## Previous results.

- ▶ NEMP and NEMPI described in DNS and MFS <sup>2</sup>.
- ▶ NEMPI observed in DNS and MFS <sup>3</sup>
- ▶ Parameters values studied for maximize the growth strength and time:
  - ▶  $k_f/k_1 = 30$  (scale separation ratio)
  - ▶  $Re \equiv u_{rms}/\nu k_f = 36$
  - ▶  $P_m = \nu/\eta = 0.5$ .
  - ▶  $Re_M = P_m Re = 18$

## My contribution:

- ▶ Effects of rotation
- ▶ Effects of stratification

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<sup>2</sup>Kleeorin et al. 1989, 1989, 1996; Kleeorin & Rogachevskii 1994;  
Rogachevskii & Kleeorin 2007; Brandenburg et al. 2010

<sup>3</sup>Brandenburg et al. 2011, 2012; Kemel et al. 2012

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# Units

## Magnetic field

Equipartition field strength:  $B_{\text{eq}} = \sqrt{\mu_0 \rho} u_{\text{rms}}$

Sun (top-bottom): 300 G - 3 kG

Sunspots: 1 kG

## Time

Turnover time:  $\tau_c = \frac{1}{u_{\text{rms}} k_f}$

Sun:  $10^3 - 10^{-1}$  hours

## Length

Density scale height:  $H_\rho = \frac{c_s^2}{g}$

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# Rotation. Structures evolution

$Co = 0.03$   
DNS

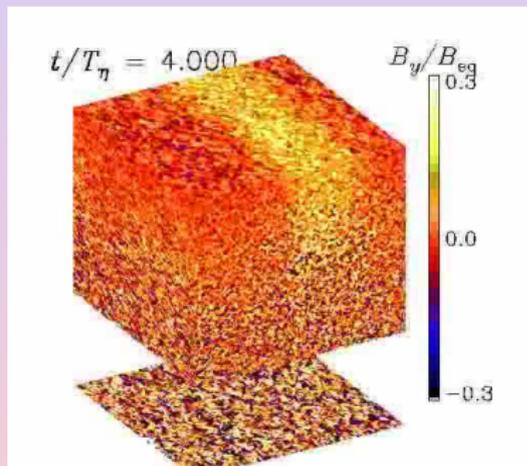


Figure 8 : DNS

MFS

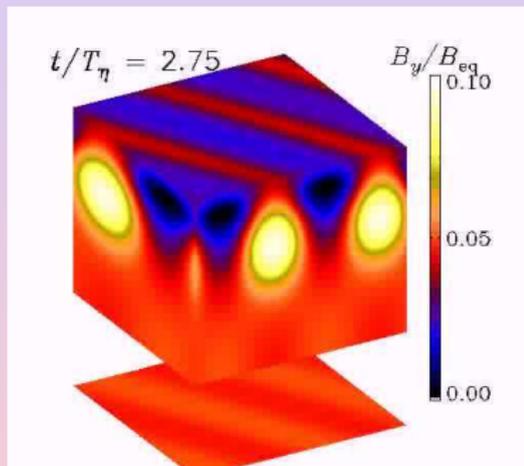


Figure 9 : MFS

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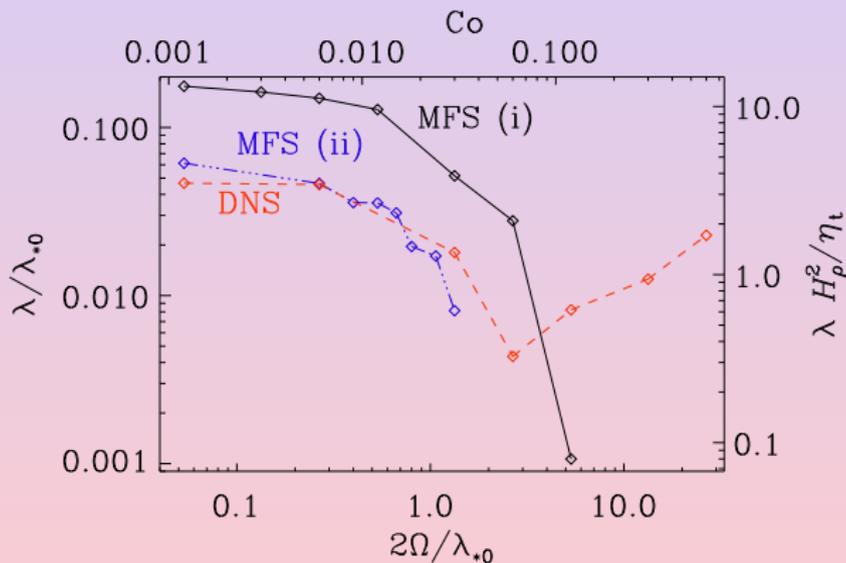
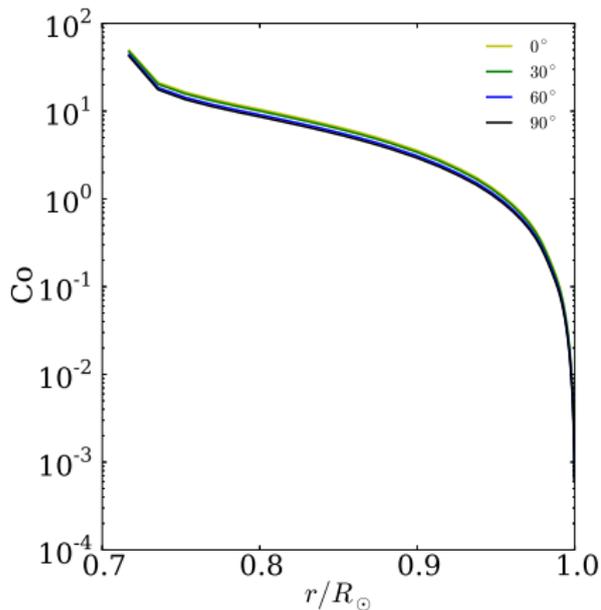
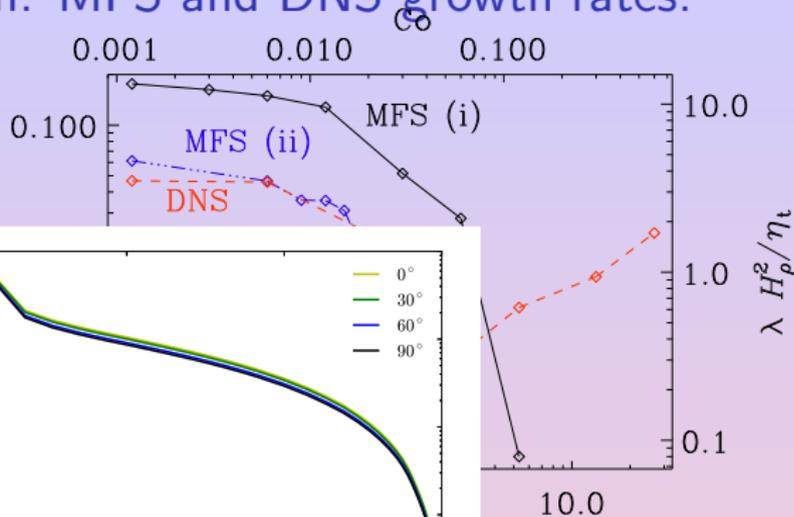


Figure 10 : MFS (i):  $q_{p0} = 20$  and  $\beta_p = 0.167$ ; MFS (ii):  $q_{p0} = 32$  and  $\beta_p = 0.058$ .

$$B_0/B_{eq0} = 0.05. \quad q_p = \frac{\beta_*^2}{(\beta_*^2 + \beta^2)} \quad \beta = \frac{\bar{B}}{B_{eq0}} \quad \lambda_{*0} \equiv \beta_* u_{rms} / H_p$$

# Rotation. MFS and DNS growth rates.



$$\tau_{p0} = 32 \text{ and } \beta_p = 0.058.$$

$$\equiv \beta_* u_{\text{rms}} / H_p$$

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# Polytropic EoS: q-exponential

Isentropic

$$\frac{\rho}{\rho_0} = [1 + (\gamma - 1) (-\Phi/c_{s0}^2)]^{1/(\gamma-1)}$$

Density and Scale Height dependence.

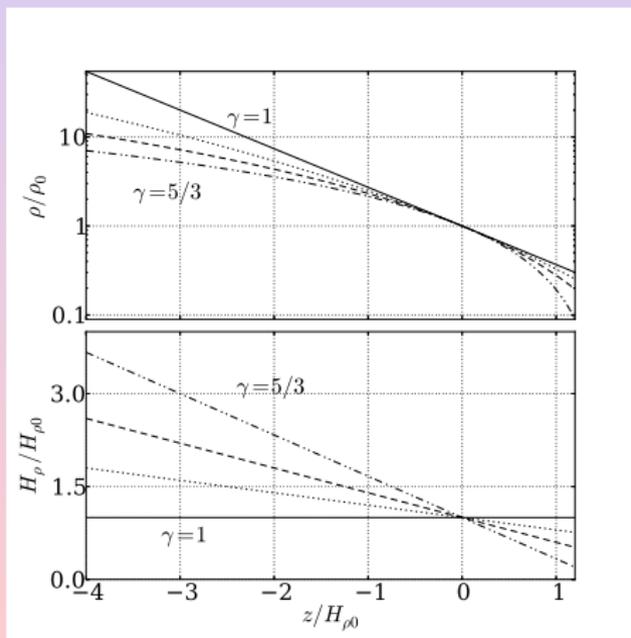


Figure 11 : Polytropes with  $\gamma = 1$  (solid line), 1.2 (dash-dotted), 1.4 (dotted), and 5/3 (dashed) for

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# Stratification. Horizontal imposed field (MFS).

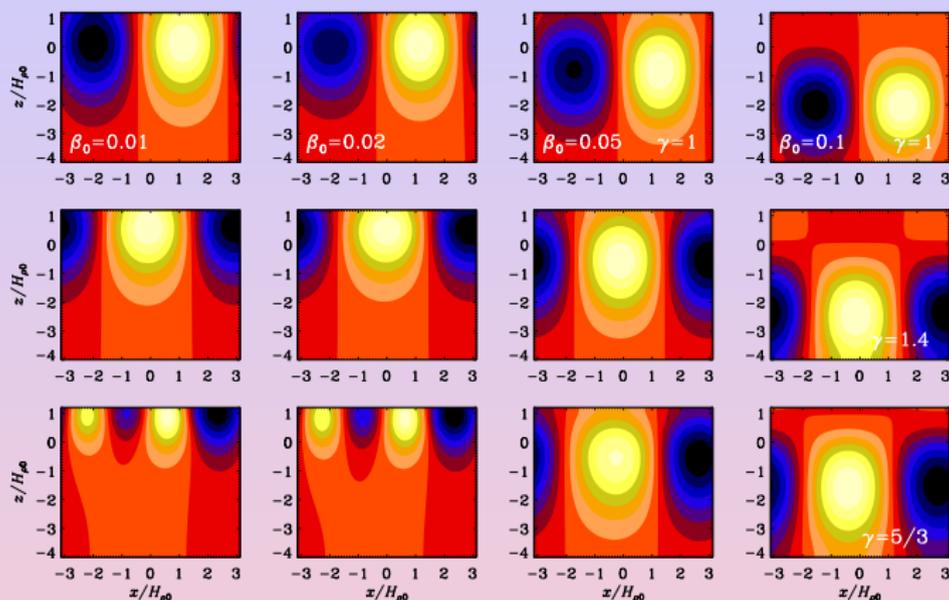


Figure 12 :  $\overline{B}_y$  in the kinematic growth phase for  $\gamma = 1$  (top row), 1.4 (middle row), and 5/3 (bottom row) and  $\beta_0 = 0.01$  (left column), 0.02, (middle column), and 0.05 (right column) in the presence of a horizontal field using the perfect conductor boundary condition.

Horizontal:  $k_{\perp} H_{\rho} \sim 0.8...1$

Vertical:  $k_{\perp} H_{\rho} \sim 0.7...1$

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# Vertical imposed field (DNS vs MFS). Evolution.

DNS

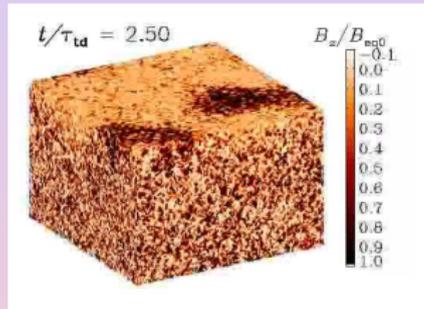


Figure 13 : DNS

MFS

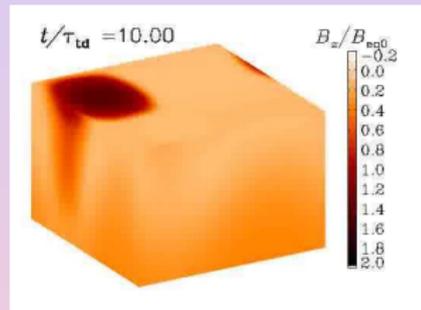


Figure 14 : MFS

$$\beta_0 = 0.05$$

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# Parameters dependence study in polytropic stratification

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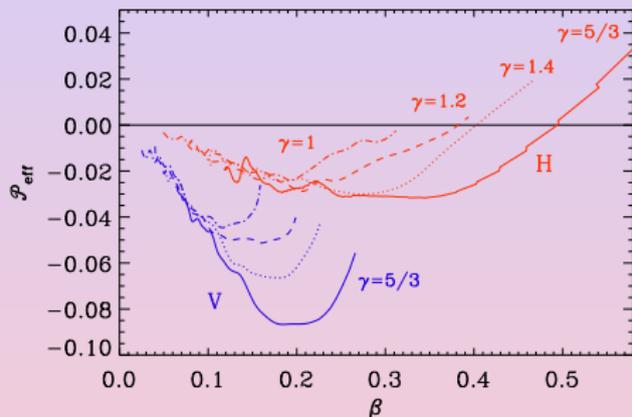


Figure 15 : Effective magnetic pressure obtained from DNS in a polytropic layer with different  $\gamma$  for horizontal (H, red curves) and vertical (V, blue curves) mean magnetic fields.

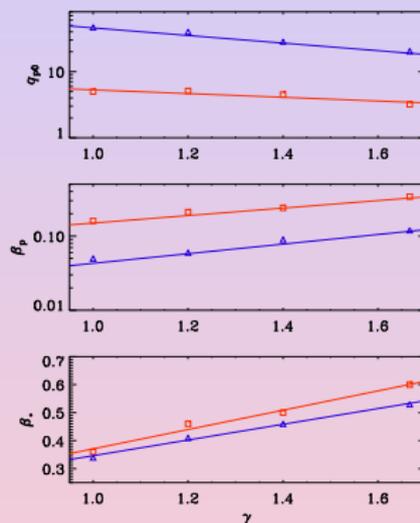
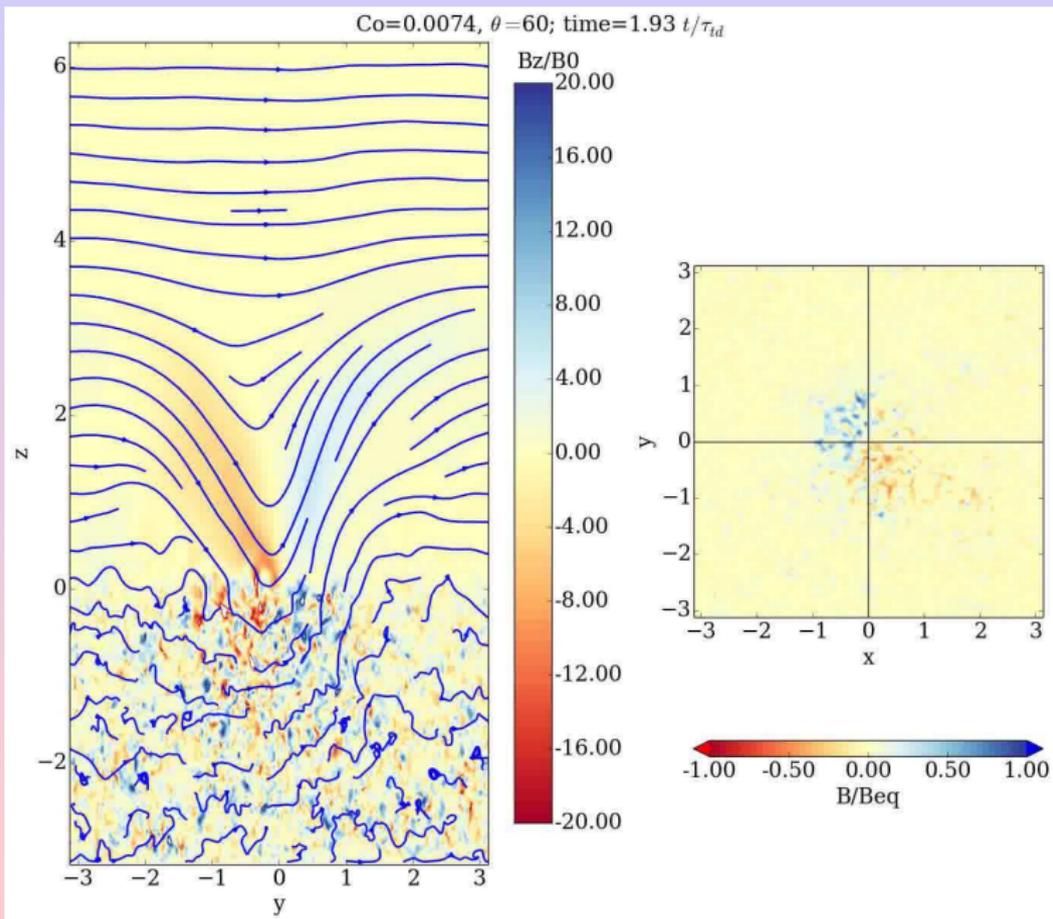


Figure 16 : Parameters  $q_{p0}$ ,  $\beta_p$ , and  $\beta_*$  for the function  $q_p(\beta)$  versus  $\gamma$  for horizontal (red line) and vertical (blue line) mean magnetic fields.

# 2-layers model + rotation



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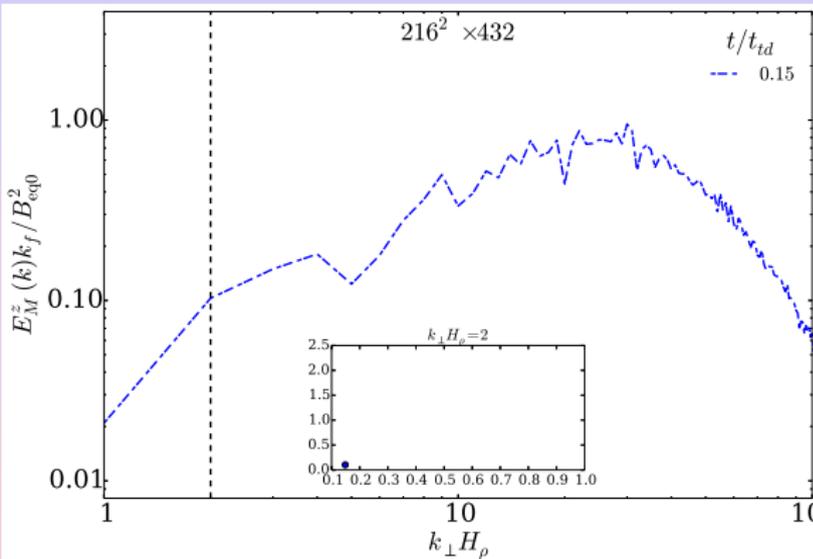


Figure 18 : Magnetic power spectra

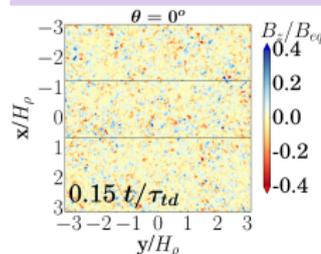


Figure 19 :  $xy$  visualization at the surface

# Energy transfer

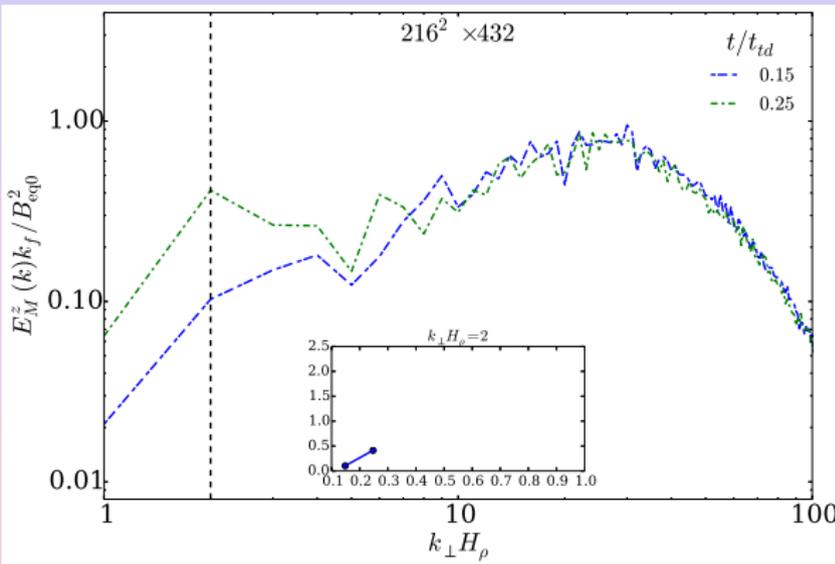


Figure 18 : Magnetic power spectra

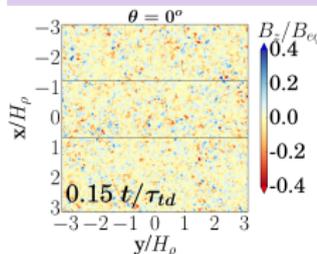


Figure 19 :  $xy$  visualization at the surface

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# Energy transfer

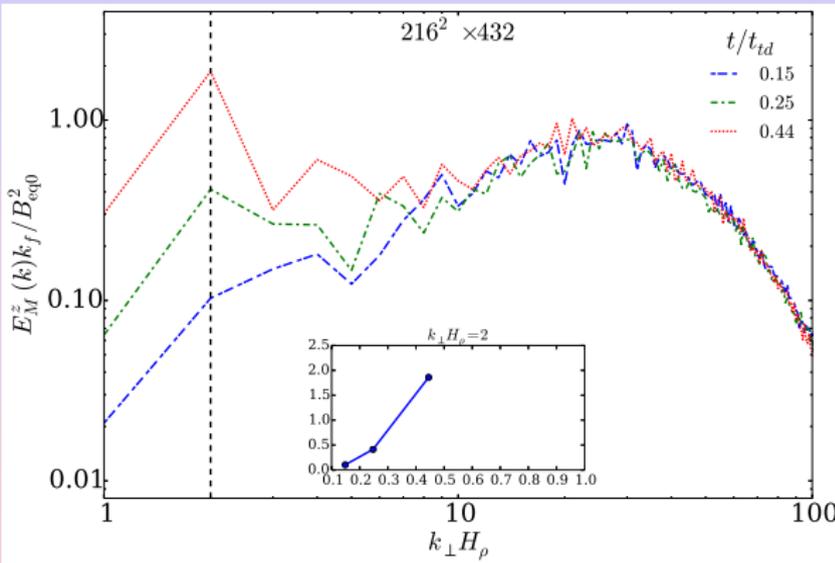


Figure 18 : Magnetic power spectra

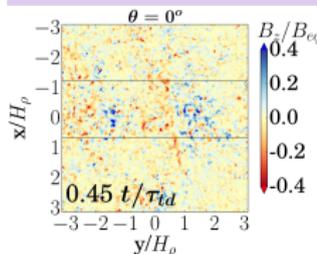


Figure 19 :  $xy$  visualization at the surface

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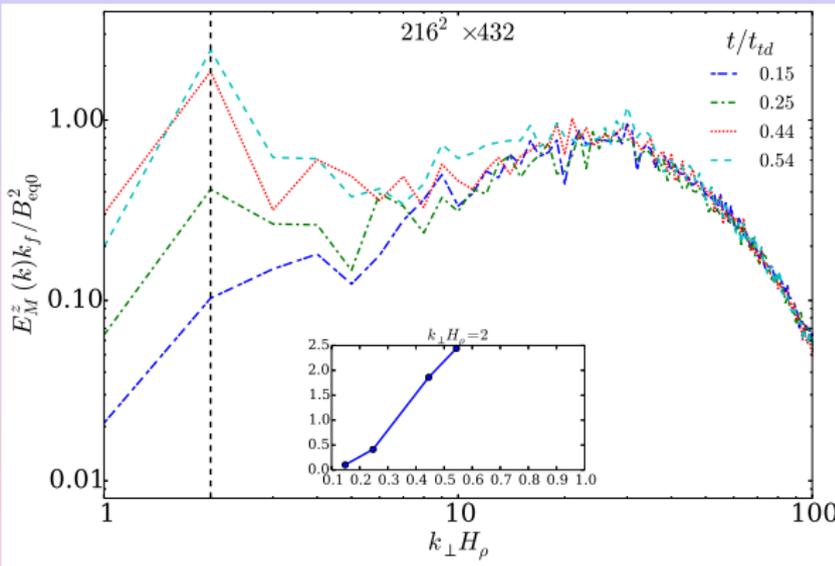


Figure 18 : Magnetic power spectra

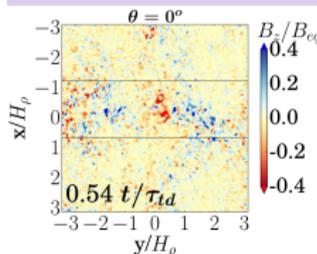


Figure 19 :  $xy$  visualization at the surface

# Energy transfer

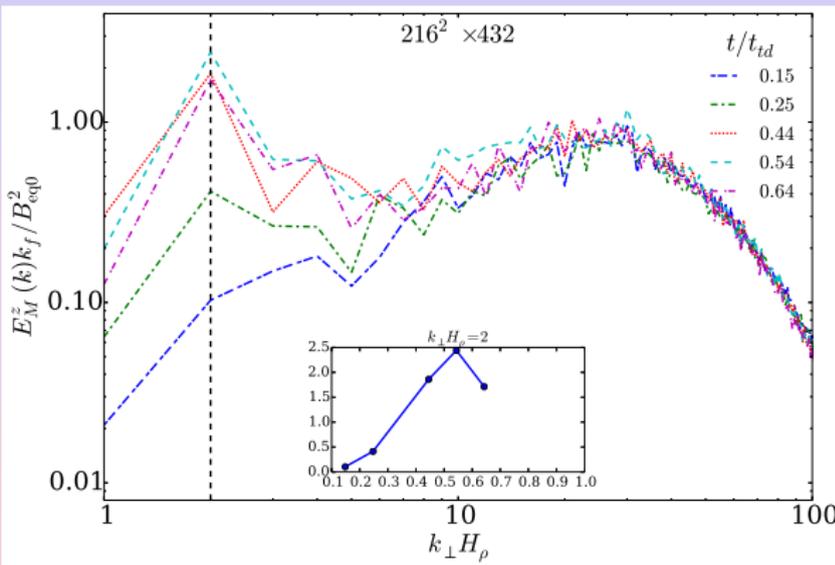


Figure 18 : Magnetic power spectra

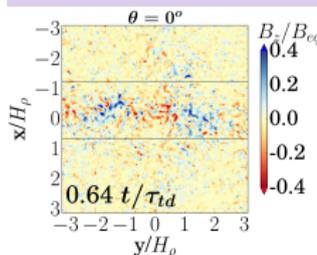


Figure 19 : xy visualization at the surface

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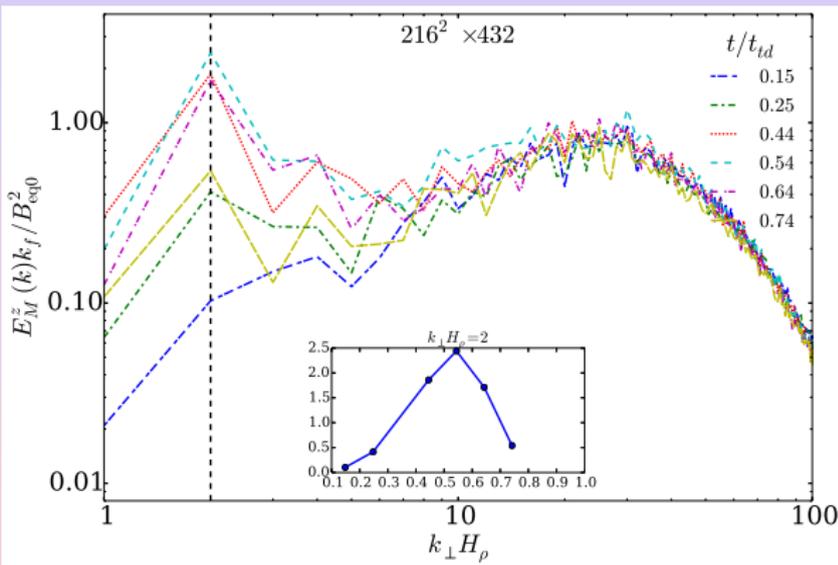


Figure 18 : Magnetic power spectra

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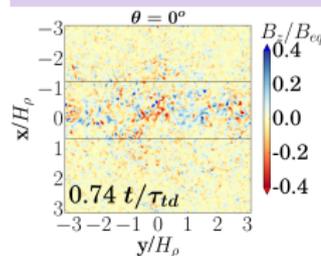


Figure 19 : xy visualization at the surface

# Energy transfer

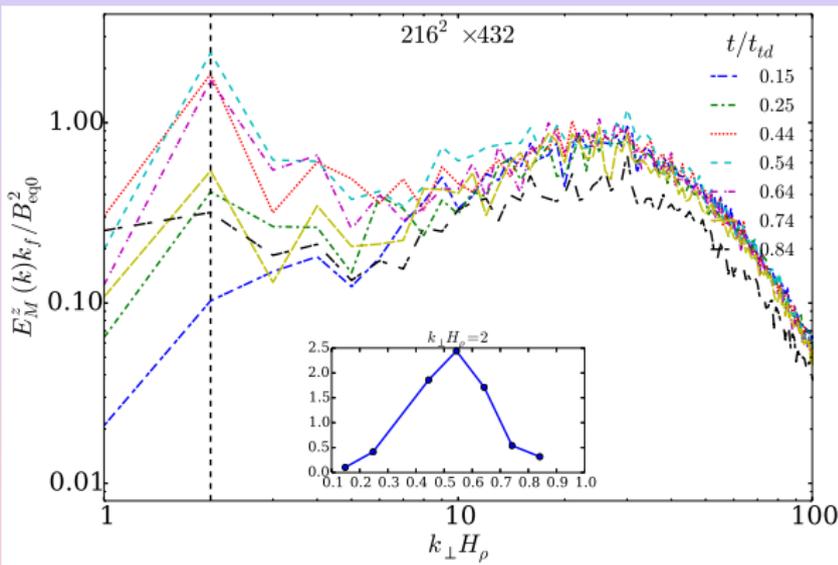


Figure 18 : Magnetic power spectra

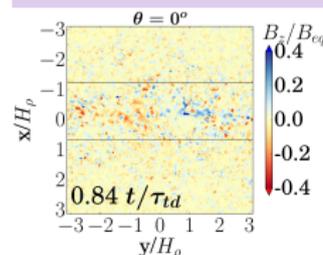


Figure 19 : xy visualization at the surface

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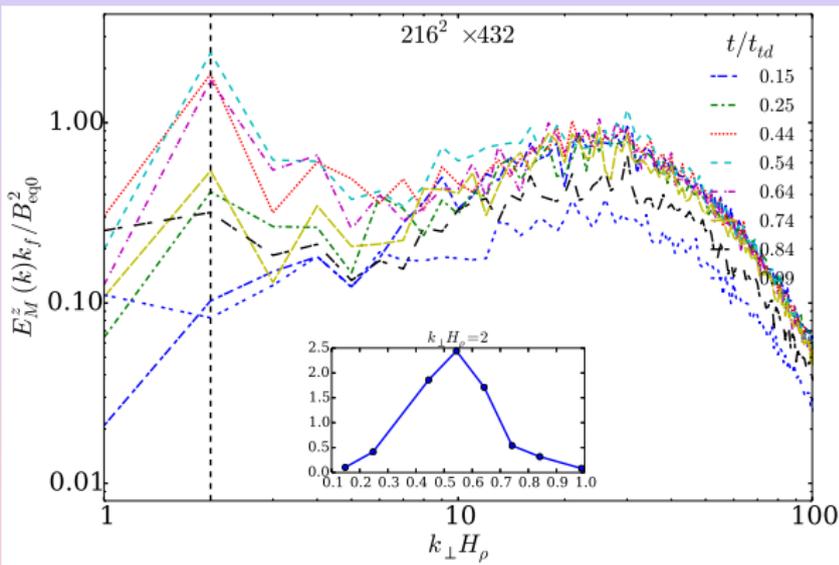


Figure 18 : Magnetic power spectra

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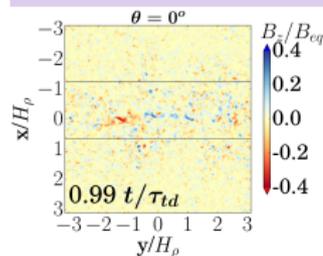


Figure 19 :  $xy$  visualization at the surface

# Conclusions

# Conclusions.

## Rotation.

NEMPI depends on the ratio between rotation and turbulence.

- ▶ Turnover time:  $\tau \approx 2\text{hours}$
- ▶ On the Sun: only upper-most layers (supergranulation layer  $\tau \sim 1\text{day}$ )

## Polytropic EoS:

- ▶ NEMPI develops in the uppermost layers.
- ▶ Isothermal models applicable locally.
- ▶ No “potato-sack” effect with vertical fields.

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# How sunspots/active regions are form?

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- ▶ Flux tube?

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# How sunspots/active regions are form?

## Formation

- ▶ Flux tube?
- ▶ NEMPI?

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# How sunspots/active regions are form?

## Formation

- ▶ Flux tube?
- ▶ NEMPI?
- ▶ Flux tube + NEMPI?

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# How sunspots/active regions are form?

## Formation

- ▶ Flux tube?
- ▶ NEMPI?
- ▶ Flux tube + NEMPI?
- ▶ Other ?

## Observations

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# How sunspots/active regions are form?

## Formation

- ▶ Flux tube?
- ▶ NEMPI?
- ▶ Flux tube + NEMPI?
- ▶ Other ?

## Observations

- ▶ Downflow/upflow

### Flux tubes

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# How sunspots/active regions are form?

## Formation

- ▶ Flux tube?
- ▶ NEMPI?
- ▶ Flux tube + NEMPI?
- ▶ Other ?

## Observations

- ▶ Downflow/upflow
- ▶ Structure depth

### Flux tubes

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Thank you for your time



# From momentum equation: NEMPI

Why the pressure is negative?

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# From momentum equation: NEMPI

Why the pressure is negative?

Total Energy

$$E = E_k + E_m \approx \text{const} \quad (11)$$

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# From momentum equation: NEMPI

Why the pressure is negative?

Total Energy

$$E = E_k + E_m \approx \text{const} \quad (11)$$

Total pressure + mean field considerations

$$P_T = P_{gas} + \frac{\overline{\mathbf{B}}^2}{2\mu_0} + p_t \quad (12)$$

## Appendix

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# From momentum equation: NEMPI

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# From momentum equation: NEMPI

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$$E = E_k + E_m \approx \text{const} \quad (11)$$

Total pressure + mean field considerations

$$P_T = P_{gas} + P_{eff} \quad (12)$$

Turbulent pressure = hydrodynamic + magnetic

$$p_t = \frac{E_m}{3} + \frac{2E_k}{3} = \frac{2}{3}(E_k + E_m) - \frac{1}{3}E_m \quad (13)$$

Change in pressure:

$$\delta p_t = -\frac{1}{3}E_m \quad (14)$$

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## From momentum equation: NEMPI

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Total Energy

$$E = E_k + E_m \approx \text{const} \quad (11)$$

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$$P_T = P_{gas} + P_{eff} = P_{gas} + (1 - q_p(\overline{\mathbf{B}})) \frac{\overline{\mathbf{B}}^2}{2\mu_0} \quad (12)$$

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$$E_m = \left( \frac{\langle b^2 \rangle}{2} \right) \quad (14)$$

$\langle b^2 \rangle$ : magnetic fluctuations of the mean magnetic field

$$\langle b^2 \rangle = f(\overline{\mathbf{B}}) \overline{\mathbf{B}}^2 = q_p 3 \overline{\mathbf{B}}^2 \quad (15)$$

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# From momentum equation: NEMPI

1st approach:

Total stress tensor  
(Isotropic turbulence)

$$\sigma = \delta_{ij} \left( \rho U_i U_j - B_i B_j + \frac{1}{2} \delta_{ij} B^2 \right) \quad (16)$$

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# From momentum equation: NEMPI

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$$\rho U^2 - B^2 + \frac{3}{2} B^2 \quad (17)$$

# From momentum equation: NEMPI

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Total stress tensor  
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$$\sigma = \delta_{ij} (\rho U_i U_j - B_i B_j + \frac{1}{2} \delta_{ij} B^2) \quad (16)$$

$$\rho U^2 - B^2 + \frac{3}{2} B^2 = \rho u^2 + B^2 - \frac{1}{2} B^2 \quad (17)$$

## From momentum equation: NEMPI

1st approach:

Total stress tensor

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Total stress tensor  
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$$\rho U^2 - B^2 + \frac{3}{2} B^2 = \underbrace{\rho U^2 + B^2}_{\text{constant}} - \frac{1}{2} B^2 \quad (17)$$

The magnetic field decrease the turbulent pressure!

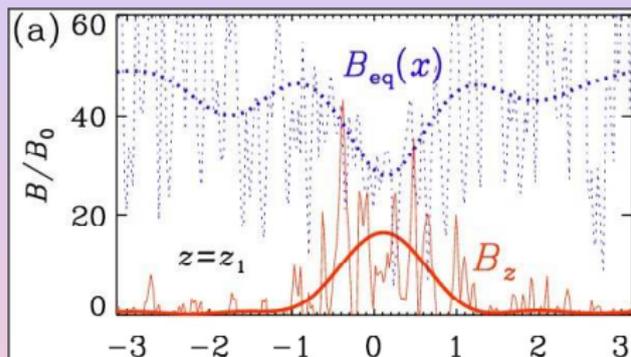
# From momentum equation: NEMPI

1st approach:

Total stress tensor  
(Isotropic turbulence)

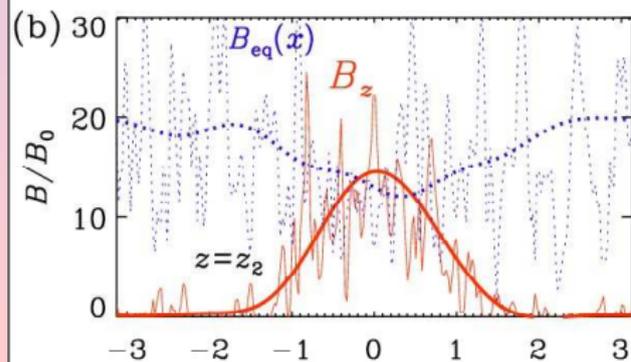
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$$+ \frac{1}{2} \delta_{ij} B^2) \quad (16)$$

$$\underbrace{+ B^2}_{\text{instant}} - \frac{1}{2} B^2 \quad (17)$$



## From momentum equation: NEMPI

1st approach:

Total stress tensor

(Isotropic turbulence)

$$\sigma = \delta_{ij} \left( \rho U_i U_j - B_i B_j + \frac{1}{2} \delta_{ij} B^2 \right) \quad (16)$$

$$\rho U^2 - B^2 + \frac{3}{2} B^2 = \underbrace{\rho U^2 + B^2}_{\text{constant}} - \frac{1}{2} B^2 \quad (17)$$

Approx:

$$p_{turb} = \frac{1}{3} \rho v^2 \approx \frac{\frac{1}{3} \rho v_0^2}{1 + a_p \frac{B^2}{B_{eq}^2}} \quad (18)$$

# From momentum equation: NEMPI

2nd approach: Total pressure + mean field considerations

Total pressure

$$P_T = P_{gas} + \frac{\overline{\mathbf{B}}^2}{2\mu_0} + p_t \quad (19)$$

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Turbulent pressure

Total turbulent pressure = hydrodynamic + magnetic

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2nd approach: Total pressure + mean field considerations

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$$P_T = P_{gas} + P_{eff} = P_{gas} + (1 - q_p(\overline{\mathbf{B}})) \frac{\overline{\mathbf{B}}^2}{2\mu_0} \quad (19)$$

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$$p_t = \frac{E_m}{3} + \frac{2E_k}{3} = \frac{2}{3}(E_k + E_m) - \frac{1}{3}E_m \quad (20)$$

## Stress tensor.

Momentum equation:

$$\frac{D\bar{\mathbf{U}}}{Dt} = -c_s^2 \nabla \ln \bar{\rho} + \mathbf{g} + \bar{\mathcal{F}}_M + \bar{\mathcal{F}}_K \quad (21)$$

Magnetic stress tensor: Lorentz Force,  $\mathcal{F}_M$ :

$$\mathcal{F}_M = \mathbf{J} \times \mathbf{B} = -\frac{1}{2} \nabla B^2 + (\mathbf{B} \cdot \nabla) \mathbf{B} = -\nabla_j \left[ \frac{1}{2} B^2 \delta_{ij} - B_i B_j \right] = \nabla_j \sigma_{ij} \quad (22)$$

Mean Lorentz Force,  $\bar{\mathcal{F}}_M$ :

$$\bar{\mathcal{F}}_M = -\nabla_j \left[ \frac{\langle b^2 \rangle}{2} \delta_{ij} - \langle b_i b_j \rangle \right] = \nabla_j \sigma_{ij}^m \quad (23)$$

Isotropic turbulence:

$$\begin{aligned} \sigma_{ij}^m &= -\frac{\langle b^2 \rangle}{2} \delta_{ij} + \langle b_i b_j \rangle = -\frac{\langle b^2 \rangle}{2} \delta_{ij} + \frac{\langle b^2 \rangle}{3} \delta_{ij} \\ &= -\frac{1}{3} \left( \frac{\langle b^2 \rangle}{2} \right) \delta_{ij} = -\frac{W_m}{3} \end{aligned} \quad (24)$$

Kinetic stress tensor: Reynolds stresses

$$\langle v_i v_j \rangle = \frac{\langle v^2 \rangle}{3} \delta_{ij} = \frac{2}{3} \frac{\langle v^2 \rangle}{2} \delta_{ij} = \frac{2}{3} W_k \delta_{ij} \quad (25)$$

(isotropic turbulence)