Fluid Modeling of Reconnection

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Outline

- MHD: equations, Ohm's law. Why MHD?
- Sweet-Parker, Petschek reconnection
- Kinetic Ohm's law
- iPIC-MHD: Sweet-Parker, Petschek reconnection simulations
- Visualizations using Paraview

MHD equations, Ohm's law and frozen-in constraint

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MHD equations (1)

The system of non-ideal MHD equations is written in conservative form in Cartesian coordinates

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$ (continuity)

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v}\rho\mathbf{v} - \mathbf{B}\mathbf{B}) + \nabla(p + \frac{\mathbf{B}^2}{2}) = 0$$ (momentum)

$$\frac{\partial e}{\partial t} + \nabla \cdot (\mathbf{v}e + \mathbf{v}(p + \frac{\mathbf{B}^2}{2}) - \mathbf{B}\mathbf{B} \cdot \mathbf{v}) = S_e$$ (internal energy)

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0$$ (Faraday)

$$e = \frac{p}{\gamma - 1} + \frac{\rho \mathbf{v}^2}{2} + \frac{\mathbf{B}^2}{2},$$ (total energy density)

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}, \quad \mathbf{J} = \nabla \times \mathbf{B}$$ (Ohm, Ampére)

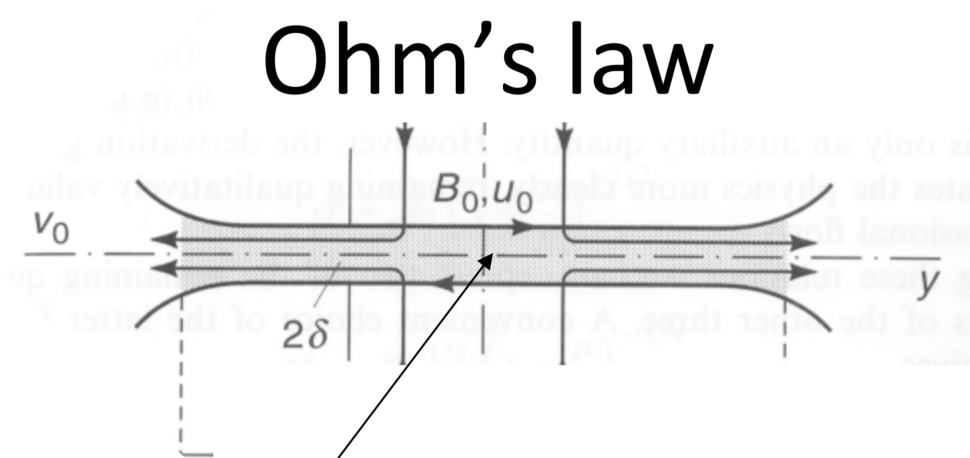
MHD equations (2)

- Continuity, momentum, internal energy equations are truncated moments of Boltzmann equation (**textbooks**).
- For MHD description to be valid, size, duration, magnetic field strength should be large enough to average out microscopic phenomena: $L_{MHD} >> \rho_i$, $\tau >> \Omega_i^{-1}$
- MHD description is questionable if applied to collisionless systems (solar wind, magnetosphere), since non-Maxwellian d.f. are generated easily. *Interpretation of results is needed!*
- If formally collisional resistivity $\eta \sim 0$, then *anomalous* η (e.g. $\eta \sim j^{\alpha}$) resistivity can approximate the dynamics.

Models of magnetic reconnection

- ✔ Fluid classics: Sweet-Parker, Petschek (steady/nonsteady), annihilation (SP: Solar corona, Petschek: solar wind)
- Other fluid models: Priest, Forbes, etc.
- Beyond MHD: Hall magnetic reconnection (magnetopause, magnetotail)
- 3D null-point reconnection
- Multifluid models, Full kinetic models (mostly simulation-based)
- Onset (tearing) problem: how to start reconnection in a quiet force-balance current layer?

Ohm’s law



- Why need resistivity/diffusion?
  If $\mathbf{E}+[\mathbf{v}\times\mathbf{B}]=0$ everywhere, then **E**=-[**V**x**B**]=0 in the stagnation point, no flow across the diffusion region.
- Diffusive term (~η**J**) allows magnetic field lines to slip through the DR
- In collisionless plasmas, current layer size is comparable to ion gyroradius or ion skin depth, full kinetic Ohm’s law is required (see iPIC-kinetic lectures)

$$\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}=\eta\mathbf{J}+\frac{1}{nec}\mathbf{J}\times\mathbf{B}-\frac{1}{ne}\nabla\cdot\mathbf{P}_e+\frac{m_e}{e^2}\frac{\mathrm{d}\mathbf{J}/n}{\mathrm{d}t}$$

Frozen-in flux constraint (1)

- Boltzmann equation: $$\frac{\partial f_s}{\partial t}+\mathbf{v}\frac{\partial f_s}{\partial x}+\frac{\mathbf{F}_s}{m_s}\frac{\partial f_s}{\partial v}=\left(\frac{df_s}{dt}\right)_{coll},\ \Re^6$$

  Force: $$\mathbf{F}_s=e_s\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)$$

- Moments of the Boltzmann equation: fluid equations

$$\frac{\partial n_s}{\partial t}+\nabla\cdot(n_s\mathbf{V_s})=0$$

$$n_s m_s\left(\frac{\partial\mathbf{V_s}}{\partial t}+(\mathbf{V_s}\cdot\nabla)\mathbf{V_s}\right)=-\nabla\cdot\mathbf{P}_s+n_s e_s\left(\mathbf{E}_s+\frac{1}{c}\mathbf{V_s}\times\mathbf{B}_s\right)+\mathbf{M}_{coll}=0$$

- Ohm's law: equations for ions, electrons combined:

$$\mathbf{E}+\frac{1}{c}\mathbf{V}\times\mathbf{B}=-\frac{1}{n_e e}\nabla\cdot\mathbf{P}_e-\frac{m_e}{e}\frac{d\mathbf{v}_e}{dt}+\frac{1}{n_e ec}\mathbf{j}\times\mathbf{B}+\eta\mathbf{j}$$

RHS terms

Frozen-in flux constraint (2)

- Ohm's law: equations for ions, electrons combined:

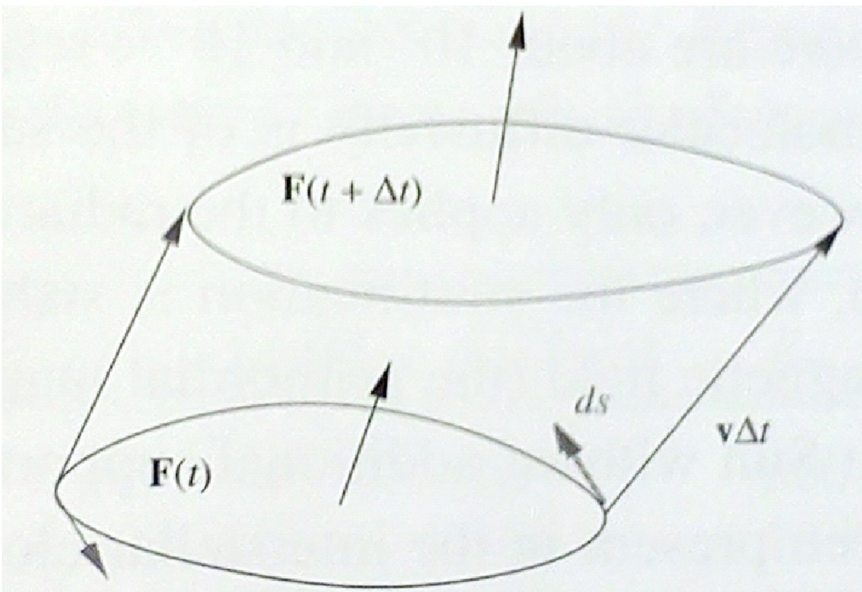
$$\mathbf{E} + \frac{1}{c}\mathbf{V}\times\mathbf{B} = \underbrace{-\frac{1}{n_e e}\nabla\cdot\mathbf{P}_e - \frac{m_e}{e}\frac{d\mathbf{v}_e}{dt} + \frac{1}{n_e e c}\mathbf{j}\times\mathbf{B} + \eta\mathbf{j}}$$

- When the RHS terms are important?
- Pressure divergence $\nabla\cdot\mathbf{P}_e$: typical scale $L \sim \rho_e\left(\frac{m_i}{m_e}\right)^{1/2}\beta_e^{1/2} \sim \rho_i\,\beta_e^{1/2}$
- Electron inertia $\frac{d\mathbf{v}_e}{dt}$: typical scale $L \sim d_e$
- Hall term $\frac{1}{n_e e c}\mathbf{j}\times\mathbf{B}$ scale: $L \sim d_i$
- Collisional effects: $L \sim d_i\,\frac{\nu_{ei}}{\Omega_{ce}}$
- Microturbulence: $\langle n\rangle\langle\mathbf{E}\rangle + \frac{1}{c}\langle n\mathbf{v}_e\rangle\times\langle\mathbf{B}\rangle = -\langle\delta n\,\delta\mathbf{E}\rangle - \frac{1}{c}\langle\delta(n\mathbf{v}_e)\times\delta\mathbf{B}\rangle$

Frozen-in flux constraint (3)

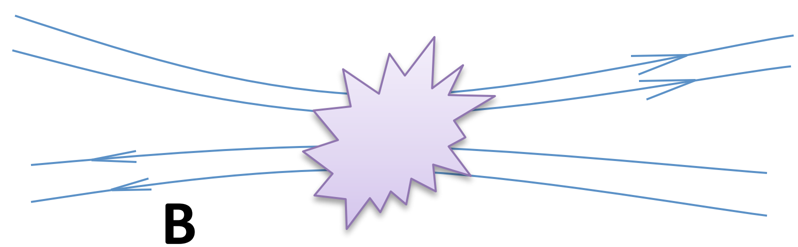
Ideal MHD Ohm's law: neglect RHS

$$\mathbf{E} + \frac{1}{c}\mathbf{V} \times \mathbf{B} = 0$$



Courtesy Schrijver and Siscoe (2009)

1) Magnetic field lines are convected with the flow
2) Flux tubes are m.f.l. "loaded" with plasma
3) Regions having high gradients of **j**, **v**, ρ can form spontaneously:

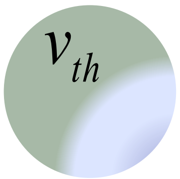


Thin current sheets displays more dynamics, than the global convective flow.

Pressure anisotropy

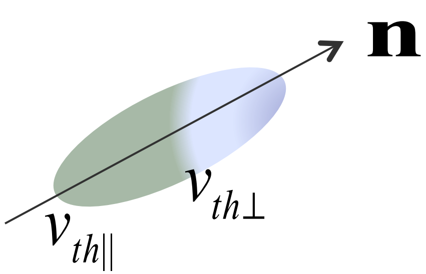
Maxwellian, isotropic:

$$f(\mathbf{v}) = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \exp\left\{-\frac{m\mathbf{v}^2}{2kT}\right\}$$



$$\hat{\mathrm{P}} = \mathrm{I}p = \begin{pmatrix} nkT & & 0 \\ & nkT & \\ 0 & & nkT \end{pmatrix}$$

$$f(\mathbf{v}) \sim \exp\left\{-\frac{\mathbf{v}^2}{v_{th\|}^2} - \frac{\mathbf{v}^2}{v_{th\perp}^2}\right\}$$



$$\hat{\mathrm{P}} = p_{\|}\mathbf{nn} + p_{\perp}(\mathbf{I} - \mathbf{nn})$$

$$f_N(x_1, x_2, \ldots, x_N) =$$
$$\frac{1}{(2\pi)^{\frac{N}{2}}(det\Sigma)^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\mathbf{x}-\mu)^{\mathrm{T}}\mathbf{\Sigma}^{-1}(\mathbf{x}-\mu)\right)$$

$\hat{\mathrm{P}} = mn\Sigma$, arbitrary

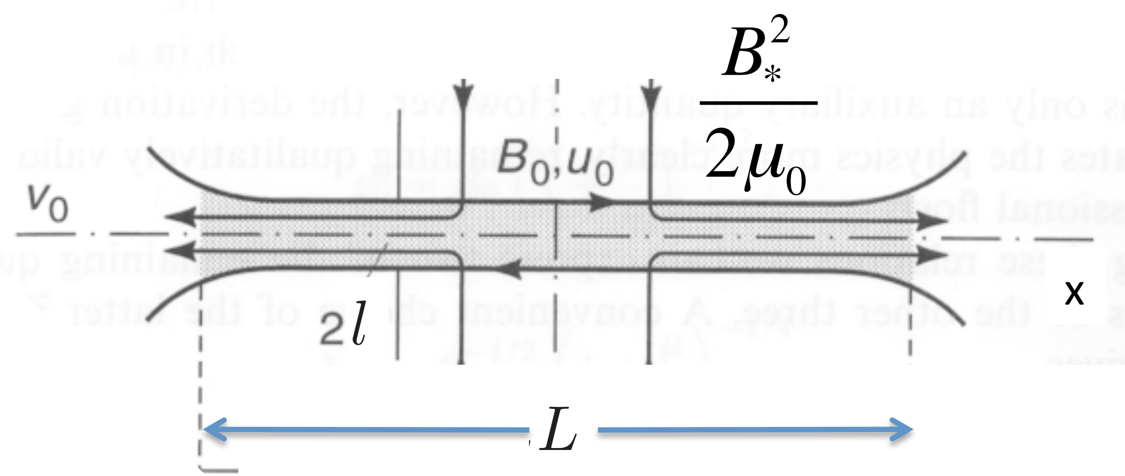
Σ - covariance matrix

μ - mean <**x**>

Sweet, Parker, Petschek and others

Sweet-Parker model

- 2D, d/dt=0, antiparallel B, plasma is squeezed by $B^2/2$



$V_x l = V_y L$

$j_z = B_x / l$

$|E_z| = V_x B_y = V_y B_x$

$\nabla \cdot \mathbf{B} = 0$

$\varrho(\mathbf{V} \cdot \nabla) V_x \sim \varrho \frac{V_x^2}{L} \sim \frac{B_x}{l} B_y$

$v_y = \eta / l$

$E_z = \eta B_x / l$

$\frac{B_x}{L} = \frac{B_y}{l}$

$V_x = B_x / \sqrt{\rho} \equiv V_A$

$V_y^2 = \frac{\eta V_x}{L} \equiv \frac{\eta V_A}{L}$

Sweet-Parker model: summary

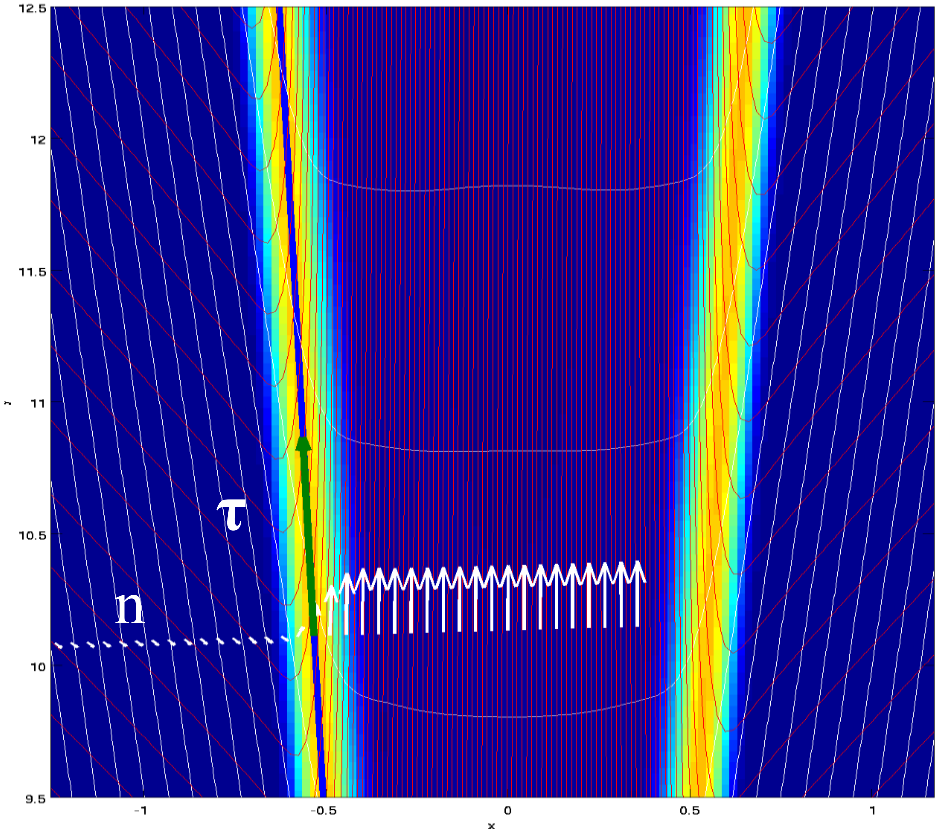
- Outflow velocity: $V_x \sim V_A$
- Reconnection rate **ε** is usually defined as a ratio of inflow velocity V_y to Alfven velocity V_A
- Lundquist number, $S = LV_A/\eta$
- Reconnection rate is $\boldsymbol{\varepsilon} = S^{-1/2}$, which makes the current layer thin and long in nearly collisionless environment. E. g., in solar flares $S \sim 10^8$, $V_A \sim 100$ km/s, $L \sim 10^4$ km, time scale ~ tens of days.
- In reality, 1m…1h. Need faster mechanism!
- Sweet-Parker scaling analysis is a common ansatz for diffusion region study in kinetic reconnection

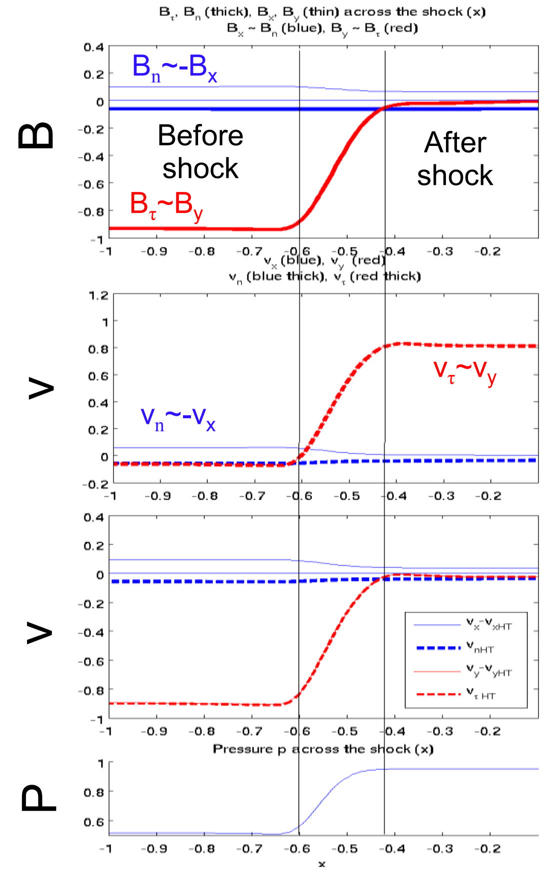
Petschek's steady model: waves

- Assuming that the diffusion region (DR) size L << the system size, DR can be viewed as an obstacle in the flow, which launces ***waves***

Petschek's steady model

Shocks: detailed view





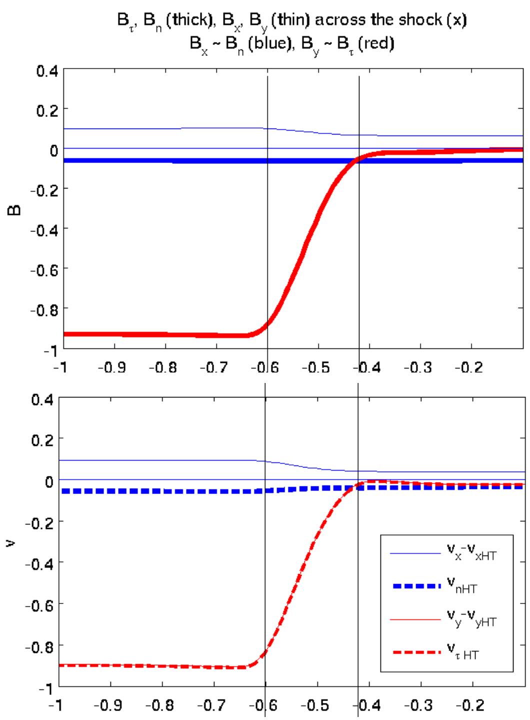
B_n conserves across the shock

Shift to dHT frame

- 2D MHD simulation with localized resistivity

Petschek's steady model

Slow shocks



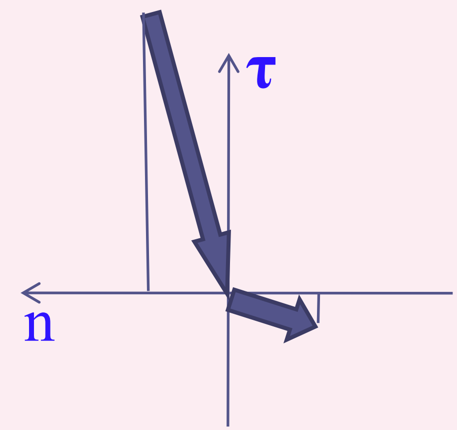
Before shock

After shock

- We use de Hoffman-Teller frame in which the shock is stationary (*Kivelson*, *Russel*, p.157) and **v**||**B**, what simplifies greatly the study of shock properties

$$\mathbf{v}_{\mathrm{HT}} = \frac{\mathbf{n}\times(\mathbf{v}_u \times \mathbf{B}_u)}{\mathbf{n}\cdot\mathbf{B}_u}, \quad \mathbf{v}' = \mathbf{v} - \mathbf{v}_{\mathrm{HT}}$$

- Here $\mathbf{v}_u$, $\mathbf{B}_u$ correspond to inflow region (upstream). Note that $\mathbf{v}_{HT}$ is parallel to $\boldsymbol{\tau}$ thus the shock moves 'parallel to itself'. Motional electric field **E**=- **v'**x**B**=0, rendering **v'**||**B**



Before shock (inflow, upstream)

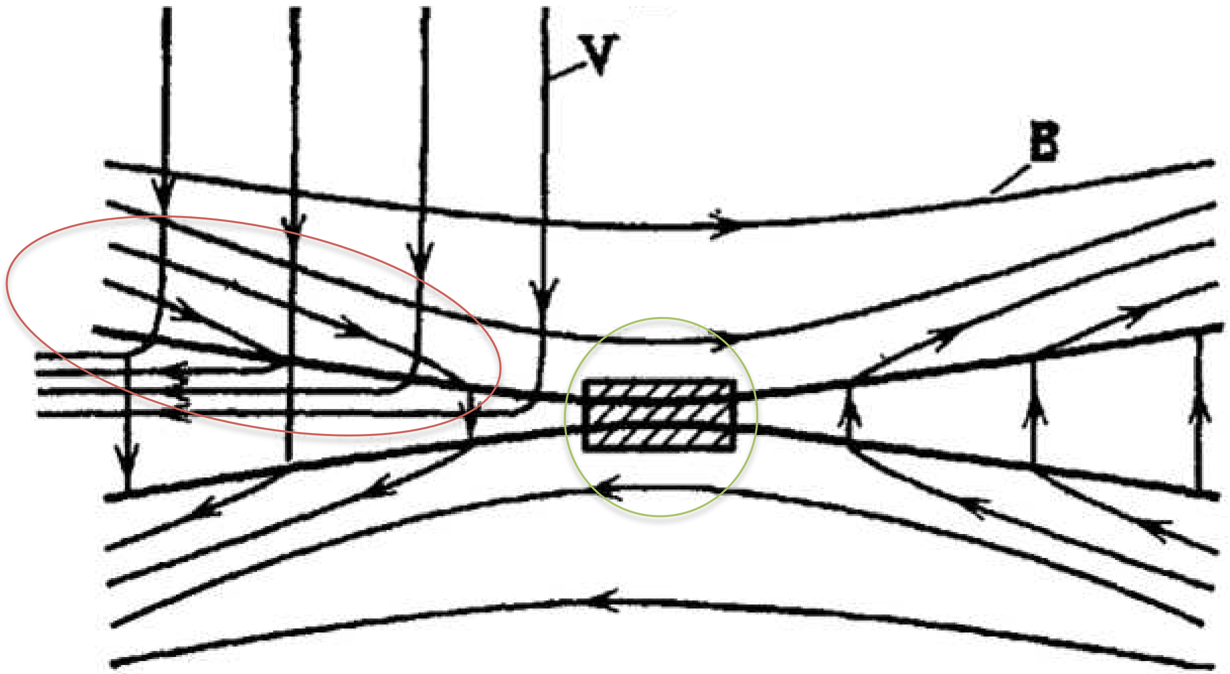
After shock (outflow, dowstream)

B_τ, v_τ reduces in downstream
$\{B_n\}\sim 0, \{v_n\}\sim 0$
Clear slow shock features
(see *Kivelson*, *Russel*, p.142)

- 2D MHD simulation with localized resistivity

Petschek's steady model

- Energy conversion occurs on slow-mode shocks attached to diffusion region



(skip derivation)

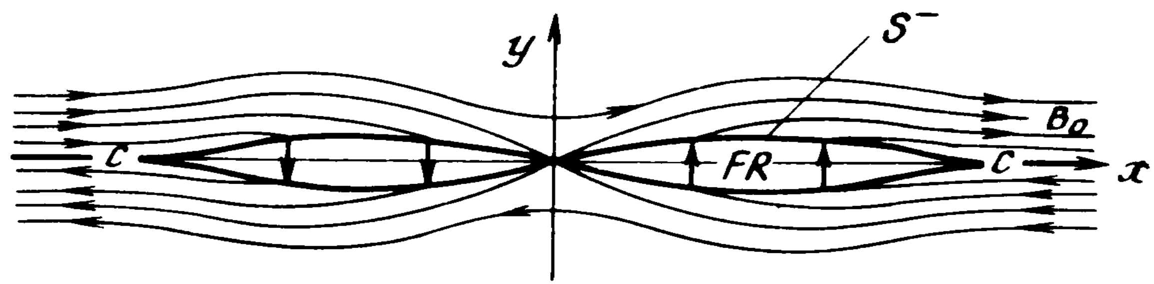
- B field drops, plasma velocity jumps across the shock

Petschek model: summary (skip derivation)

- Outflow velocity: $V_x \sim V_A$
- Reconnection rate $\boldsymbol{\varepsilon}=\pi/(8 \ln S)$ is 0.01…0.1 for various heliospheric parameters
- The model needs a diffusion region (Sweet-Parker-like), which can be provided by kinetic physics.

Petschek model extensions: time

- Unsteady case: supposing a brief pulse of $E^{(rec)}$
- Curved jet fronts, bounded by slow mode shocks



Shape:

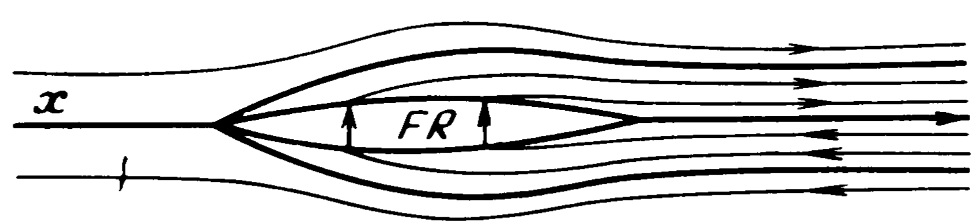
$Y \approx |X| E^{(REC)} (t - x / V_{A0}) / (B_0 V_{A0})$

Fields:

$E_z (x,t) \approx E^{(REC)} (t - x/V_{A0})$

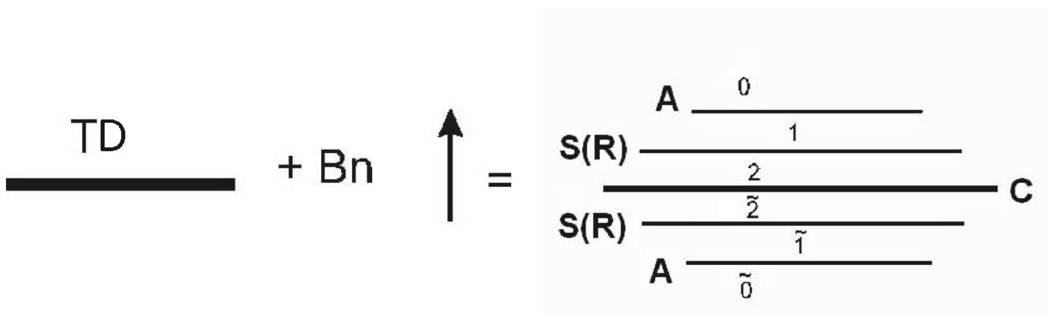
$B_y \approx E^{(REC)} (t - x/V_{A0})/V_{A0}$; $Bx \approx 0$

$|Vx| \approx V_{A0}$; $|Vz| \approx 0$;



Petschek model extensions: asymmetry

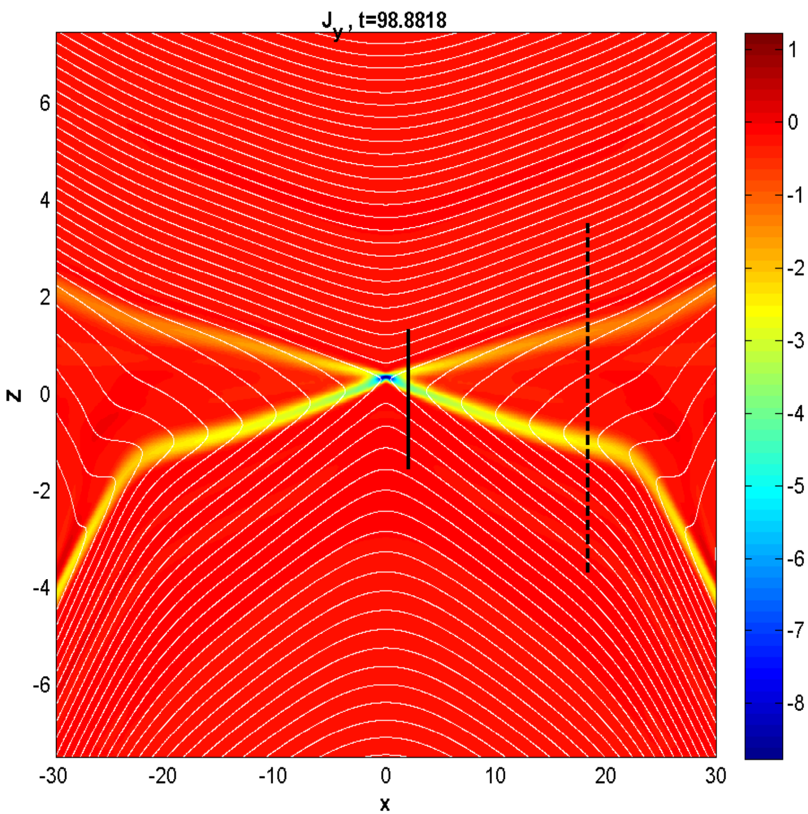
- Asymmetric reconnection needs more rigorous computation of shock wave train
- Electric field E(REC) introduces reconnected magnetic field Bn which leads to decay



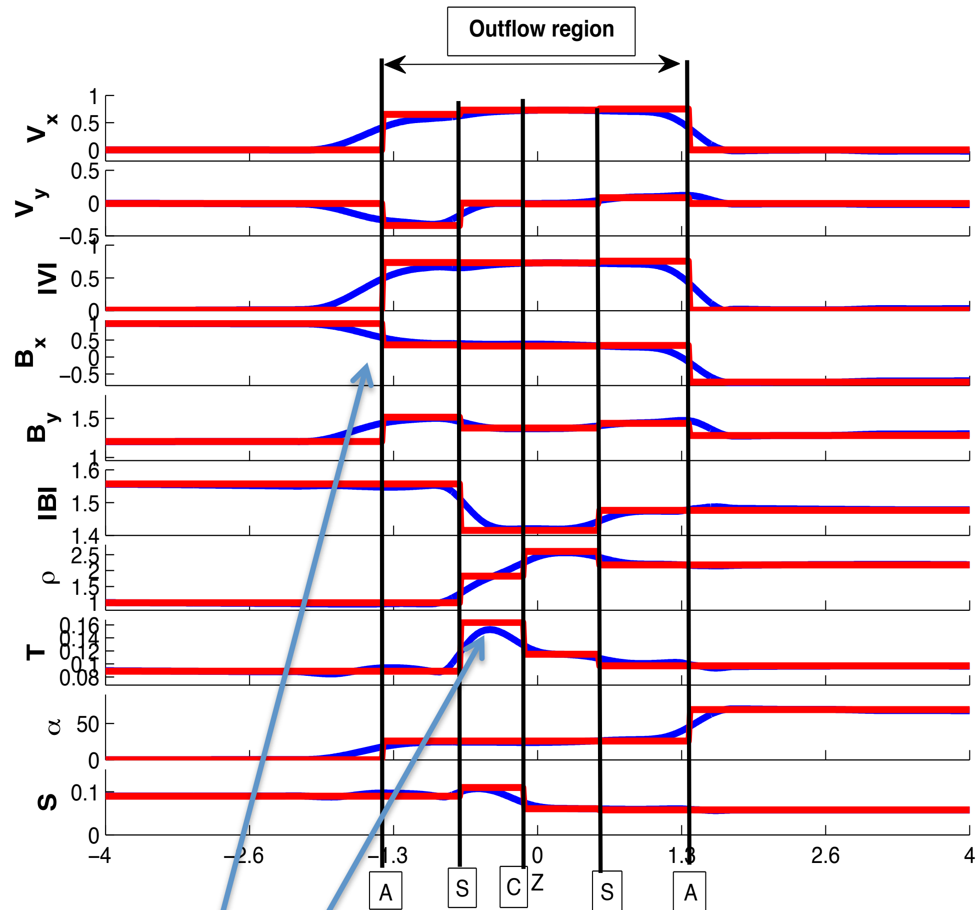
Assume that the current layer is a tangential discontinuity, infinitely thin at t=0

A: rotational discontinuity,
S: slow shock wave,
C: contact discontinuity

Petschek model extensions: asymmetry



2D MHD, asymmetric reconnection simulation



Comparison with theory

Smoothing due to numerical resistivity

iPIC-MHD: overview, installation, run

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What are MHD Simulations ?

- Solving numerically a set of MHD PDEs on a grid.
- Variables are (time/space) differenced/ discretized.
    - Space → Dx, Time → DT
    - U (time)→ U^n (space)→ U^n_j
- Discretization introduces errors proportional to Dx and Dt (Accuracy defines how big is this error)
- If this error remains bounded in time then the scheme is stable, if it grows it is unstable.
- If conservation laws are not satified, some spurious effect is introduced in the simulation

What are MHD simulations

- These PDEs describe the evolution on time of a fluid (plasma) density, momentum and energy and magnetic field. These equations are coupled.
- These PDEs are hyperbolic in nature → waves (local perturbation moving at a certain velocity)

Variables

- rho = mass density
- rho **v** (in code **p**) = momentum density
- e (in code U) = total energy density
- p = pressure
- gamma = polytropic index
- eta = resistivity

Equations

- Conservation of mass
- Conservation of momentum (3x)
- Conservation of total energy
- Equation of State (Closure)
- Induction equation for the evolution of magnetic field (3x)

Equations

The system of non-ideal MHD equations are written in conservative form in Cartesian coordinates

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v} \rho \mathbf{v} - \mathbf{B}\mathbf{B}) + \nabla (p + \frac{\mathbf{B}^2}{2}) = 0$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (\mathbf{v} e + \mathbf{v}(p + \frac{\mathbf{B}^2}{2}) - \mathbf{B}\mathbf{B} \cdot \mathbf{v}) = S_e$$

$$\frac{\partial B}{\partial t} + \nabla \cdot (\mathbf{v}\mathbf{B} - \mathbf{B}\mathbf{v}) = S_B$$

$$e = \frac{p}{\gamma - 1} + \frac{\rho \mathbf{v}^2}{2} + \frac{\mathbf{B}^2}{2},$$

Diad product:

$$(\nabla \cdot \mathbf{AB})_j = \sum_k \frac{\partial (\mathbf{A}_k \mathbf{B}_j)}{\partial x_k}$$

Sources:

$$S_e = \mathbf{B} \times \eta \mathbf{J},$$

$$S_B = -\nabla \times \eta \mathbf{J}$$

Polytropic exponent γ=5/3

Conservative form

- The conservative form is preferred because the discretization of governing equations provides better numerical properties
- Since Rankine-Hugoniot conditions across shocks express the continuity of the flux of mass, momentum and energy, if one does not explicitly conserve such quantities, there is no reason to obtain a physically acceptable speed of **propagation for shocks** .

Finite Volume Method

- "Finite volume" refers to the small volume surrounding each node point on a mesh
- Volume integrals in a partial differential equation that contain a divergence term are converted to surface integrals, using the divergence theorem.
- These terms are then evaluated as fluxes at the surfaces of each finite volume.

$$\left(\int_{\Delta V} \mathrm{U}_j^{\mathrm{t}2} dV - \int_{\Delta V} \mathrm{U}_j^{\mathrm{t}1} dV \right) + \int_{t1}^{t2} \int_{S} (\vec{\mathrm{F}}_{x(j)}\ \vec{\mathrm{F}}_{y(j)}\ \vec{\mathrm{F}}_{z(j)}) \cdot \vec{\mathrm{n}}\, dS dt = \int_{t1}^{t2} \int_{\Delta V} \vec{\mathrm{S}}_j\, dV\, dt$$

Explicit Discretization in Time

- We discretize explicitly in time the set of equations $dU/dt = S \rightarrow (U^{n+1} – U^n)/DT = S^n$
- This might result in numerical stability issues
- What about “implicit” ?

CFL condition

- CFL is a necessary condition for stability while solving hyperbolic PDEs
- It arises in the numerical analysis of explicit time integration schemes
- $U\ DT/\ DX < 1$
- In the code MHD-iPIC3D, DT is chosen to satisfy the CFL condition → DT non constant in time but adjusted

Time Discretization - Strang Splitting

It divides initial multidimensional task into following "system": $$\partial_t \vec{\mathrm{U}} + \partial_x \vec{\mathrm{F}}_x + \partial_y \vec{\mathrm{F}}_y + \partial_z \vec{\mathrm{F}}_z = \vec{\mathrm{S}}(\vec{\mathrm{U}}) \qquad 1)$$

If L_x is an operator that solves

$$\partial_t \vec{\mathrm{U}} + \partial_x \vec{\mathrm{F}}_x + \partial_y \vec{\mathrm{F}}_y + \partial_z \vec{\mathrm{F}}_z = \vec{\mathrm{S}}(\vec{\mathrm{U}})$$

discretized (2) for cell-averages from t to t + Delta t , L_y solve (3) and L_z solve (4) then multidimensional evolution of original problem (1) is well approximated by $U^{n+1} = L_z L_y L_z U^n$

$$\partial_t \vec{\mathrm{U}} + \partial_x \vec{\mathrm{F}}_x = \frac{1}{3}\vec{\mathrm{S}}(\vec{\mathrm{U}}) \qquad (2)$$

$$\partial_t \vec{\mathrm{U}} + \partial_y \vec{\mathrm{F}}_y = \frac{1}{3}\vec{\mathrm{S}}(\vec{\mathrm{U}}) \qquad (3)$$

$$\partial_t \vec{\mathrm{U}} + \partial_z \vec{\mathrm{F}}_z = \frac{1}{3}\vec{\mathrm{S}}(\vec{\mathrm{U}}), \qquad (4)$$

In the code you will see these Lx, Ly, Lz

Divergence Cleaning

- div(**B**) = 0 not always satisfied in MHD codes.
- A general way is to use divergence cleaning method, e.g. projection method.
- Solution of elliptic Poisson equation of the form $\nabla^2 \phi = \nabla \cdot \mathbf{B}^*$
  is required: we use Conjugate Gradient (CG) to invert the sparse matrix.
- Finally, the corrected magnetic field $\mathbf{B} = \mathbf{B}^* - \nabla \phi$
- In the MHD-iPIC3D code we do divergence cleaning every 10 cycles

The MHD-iPIC3D code

- C++ code and serial
- No libraries are needed.
- This stripped out version of a full 3D parallel version of the MHD-iPIC3D code.
- Today, 2D simulations but the code is 3D.
- We use uniform grid.

Software Requirements

- g++ or any other c++ compiler
- If g++ not available, we provide the results from a previous simulation.
- Autotool make available. Otherwise compilation from command line possible.
- Paraview software for visualization
- (Eventually Visit program can substitute Paraview)

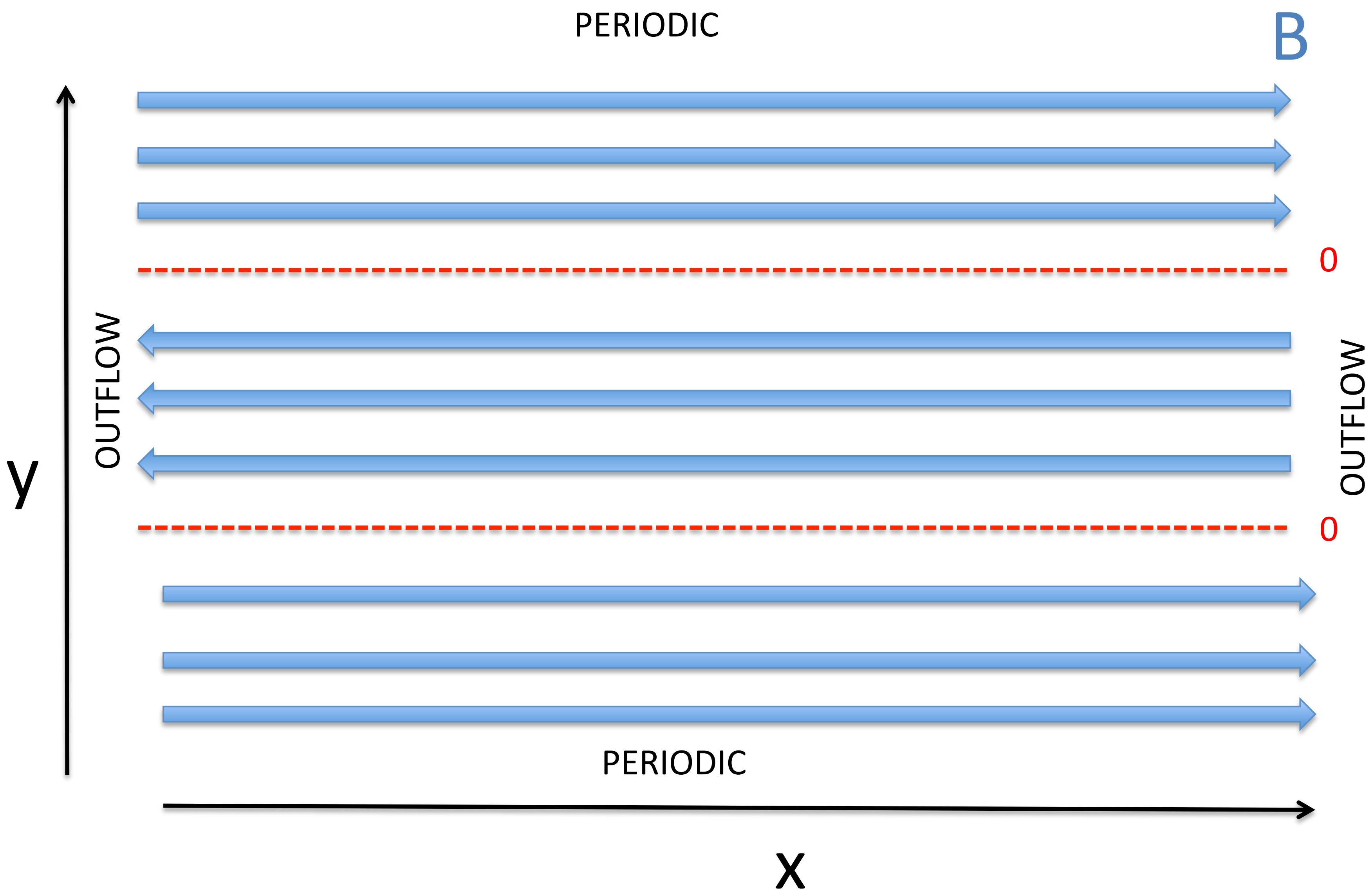
Input File (Sample)

```
# 2D Reconnection Double-periodic configuration
# Directories (should be without "/" at the end)
SaveDirName = results
RestartDirName = results
# Select the reconnection case
Case            = PETSCHEK
# Magnetic field amplitudes.
B0x = 1.0
B0y = 0.0
B0z = 0.0
# current sheet thickness
delta = 0.5
# Time
dt = 0.15       # Initial Time step
ncycles = 3001    # Number of cycles
# Simulation Box dimensions
Lx =   20.0         # Lx = simulation box length - x direction
Ly =   20.0         # Ly = simulation box length - y direction
nxc =  128          # nxc = number of cells - x direction
nyc =  128          # nyc = number of cells - y direction
FieldOutputCycle = 20
# Output for diagnostics
DiagnosticsOutputCycle = 5
```

Which Units the Code is Using ?

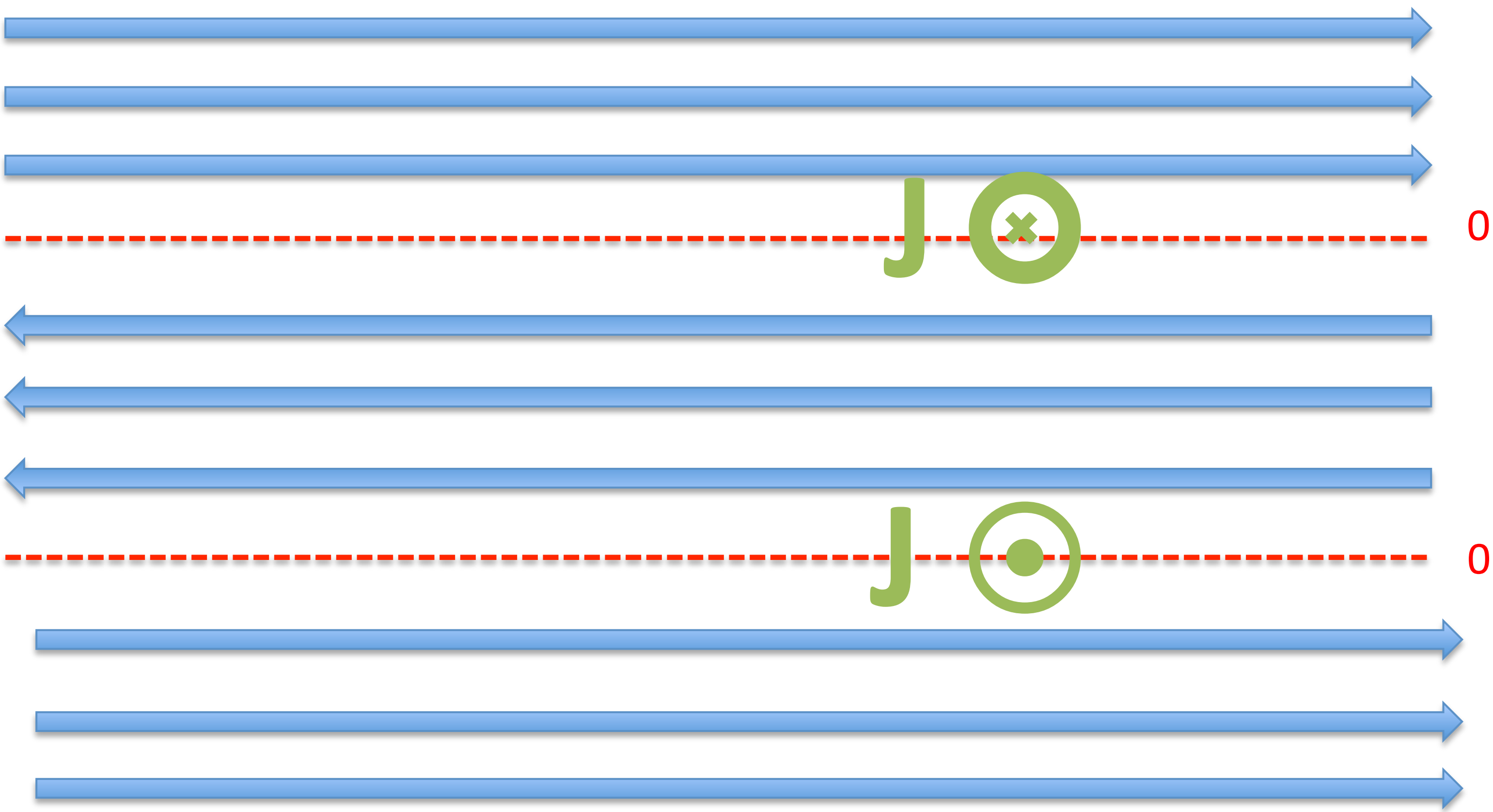
- All variables are dimensionless and are normalized as follows:
  - density is normalized by the characteristic current sheet peak density ρ_0
  - magnetic field units are given by ambient magnetic field $B_{0.}$
  - Velocities are normalized to the Alfven velocity
  - coordinates are normalized by some characteristic length L_0,

Initial Configuration: 2 current sheets



Initial Configuration

Two opposite currents to satisfy Ampere's law



+ Force balance → initialize p

Initial Configuration

- We perturb the magnetic field lines only on the top current sheet layer
- We add resistivity (eta) only on the top current sheet

- We want reconnection occurring only on the top current layer

Different Cases

- 2Cases: Petschek, Sweet-Parker
- They different how resistivity (eta) is distributed: Petschek (localized diffusion region → localized in the middle), Sweet-Parker (extended diffusion region → uniform resistivity)
- You can check put_resistivity in fields/ EMFields3D.h

Installation of iPIC3D

- unzip MHD-iPIC3D.zip
- cd MHD-iPIC3D
- make

```
n139-p162:MHD-iPIC3D markidis$ make
g++  -O2 -c ./ConfigFile/src/ConfigFile.cpp
g++  -O2 -c ./fields/BCStructure.cpp
g++ -O2 -o  MHD-iPIC3D \
        MHD-iPIC3D.cpp ConfigFile.o BCStructure.o\
```

- Download paraview from http://www.paraview.org/

Running iPIC3D-MHD code

- Type "./MHD-iPIC3D inputfiles/Recon2D-MHD-P

...

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Computational cycle: 1418

Simulation time: 19.9381

Total Energy: 9935.84

Magnetic Field Energy: 3375.99

Kinetic Energy: 61.828

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…

Output

- ConservedQuantities.txt

…

cycle time tot_en magnetic_en kinetic_en

…

VTK files in the results directory every 20 cycles (from inputfile): **B**, rho, **p**, U and eta .

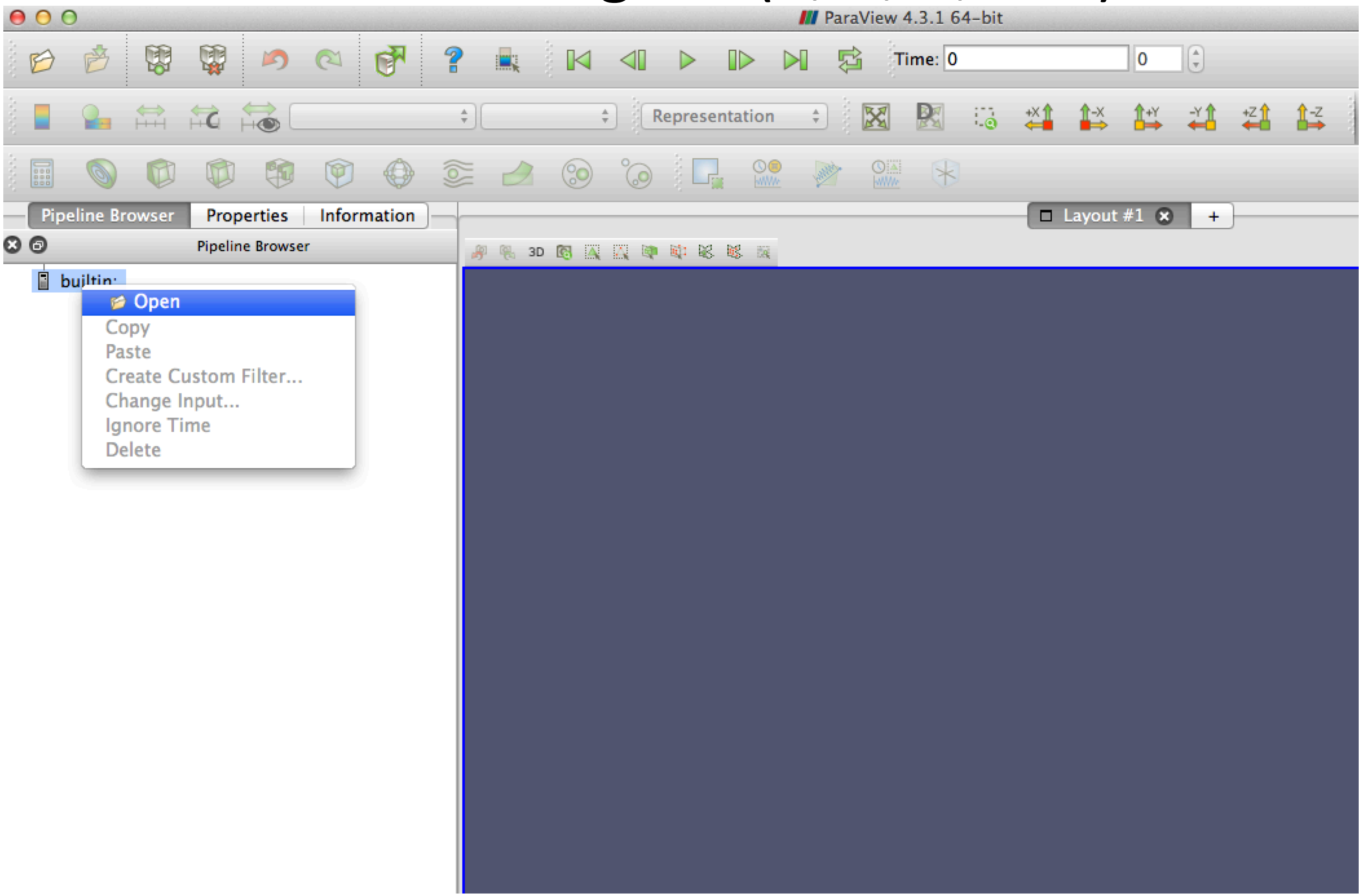
We visualize the VTK files using Paraview

Exercise – Petschek Reconnection

- Install MHD-iPIC3D code
- Run it with inputfile Recon2D-MHD-P.inp (inputfiles folder)
- Check total energy (should it change it? Is magnetic field energy decreasing? What about kinetic energy?)

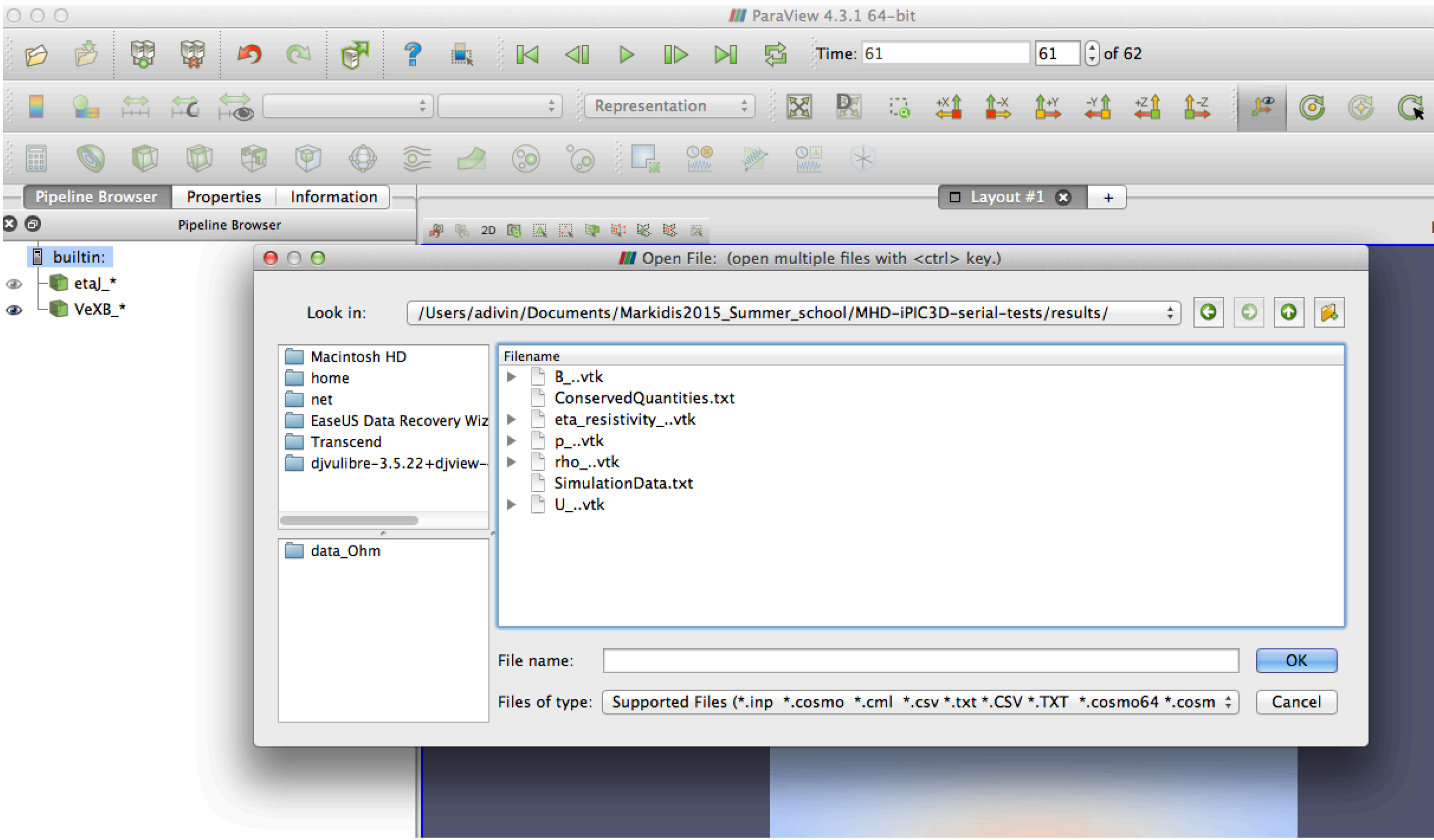
Exercises (1): using Paraview

Paraview: visualize grids (n, B, V, etc.)



Exercises (1): Petschek

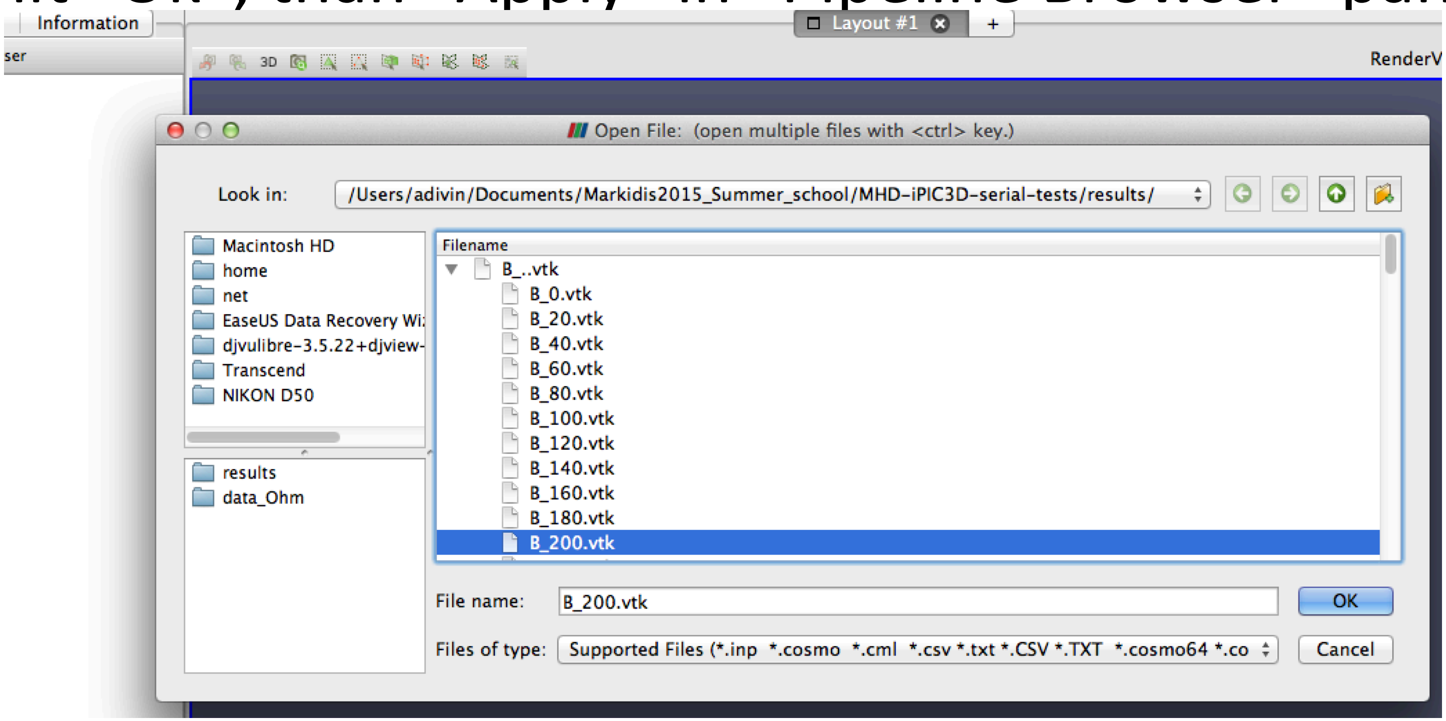
Paraview: visualize grids (n, B, V, etc.)



Exercises (1): Petschek

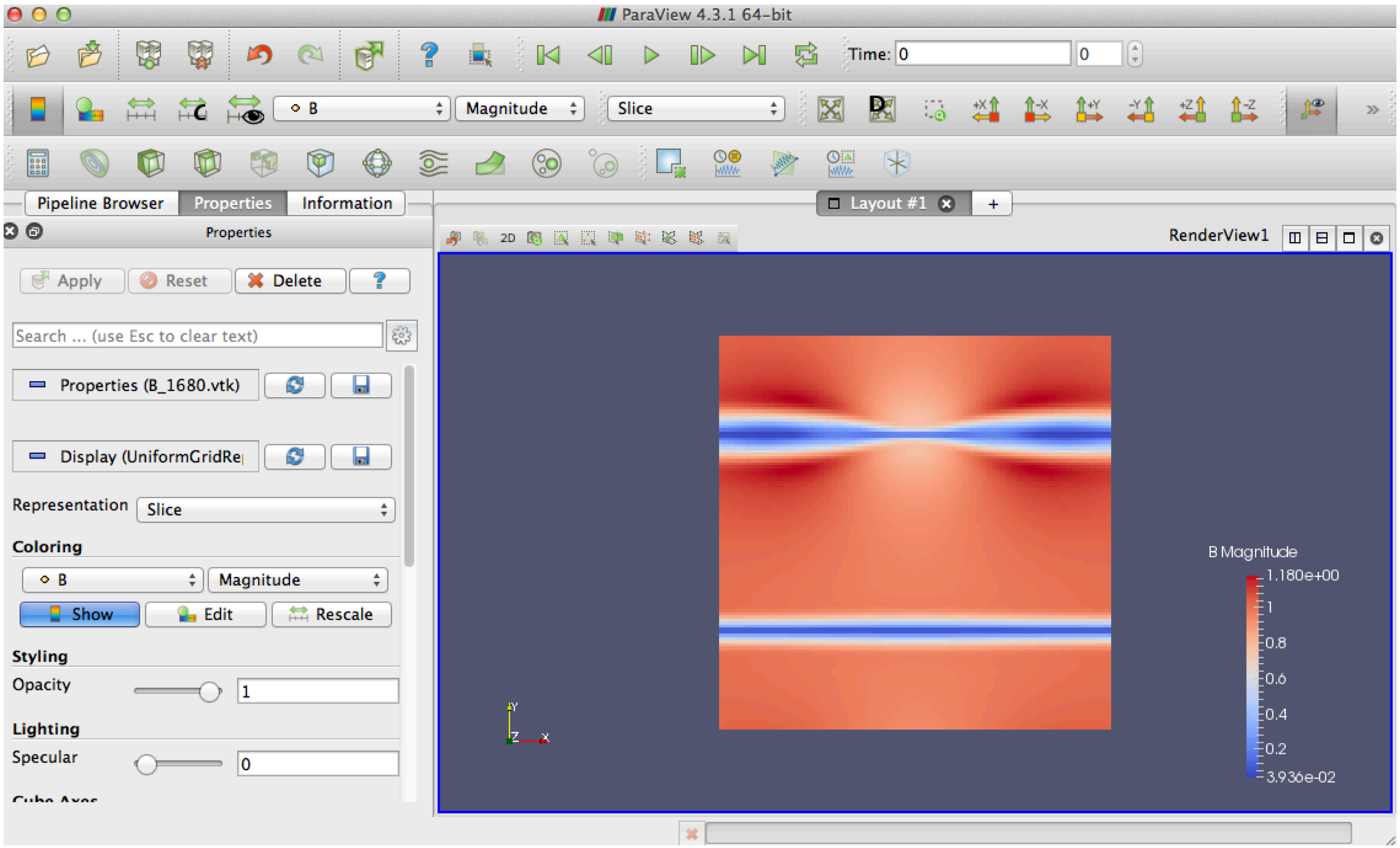
Paraview: visualize grids (n, B, V, etc.)

- It is possible to load the array of grids to plot the animation
- You can load and plot grids from separate time steps (hit “Ok”, than “Apply” in “Pipeline Browser” panel)



Exercises (1): Petschek

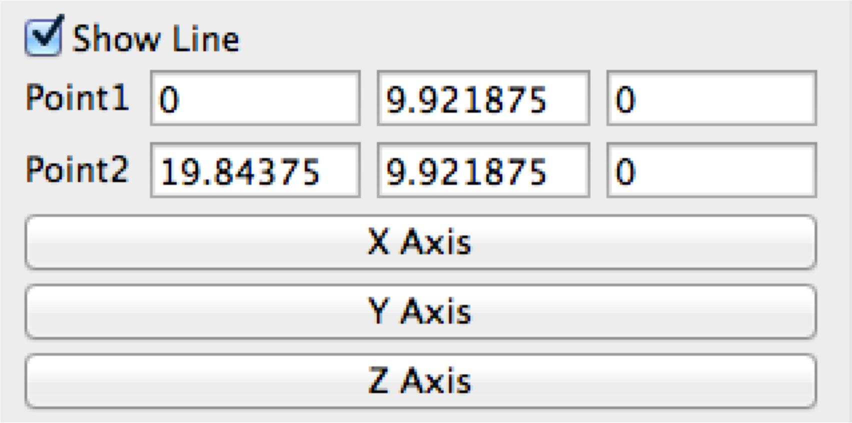
- Did you get this picture?



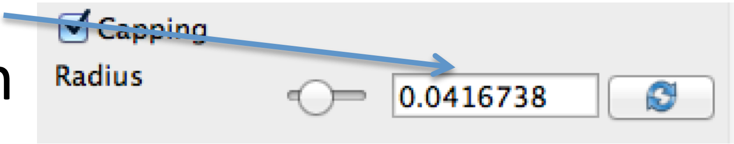
Exercises (1): Petschek

- Applying a filter to visualize magnetic field lines

1) Click on the file name in “Pipeline Browser”
2) In the menu, select “Filters->Common->Stream Tracer
3) Go to “Properties panel”
4) Select “Seed type”: “High Resolution Line Source”
5) Hit “X Axis”,put here the value of (3Ly/4)-1, e.g. 14
6) Hit “Apply”
7) In the menu, select “Filters->Alphabetical ->Tube

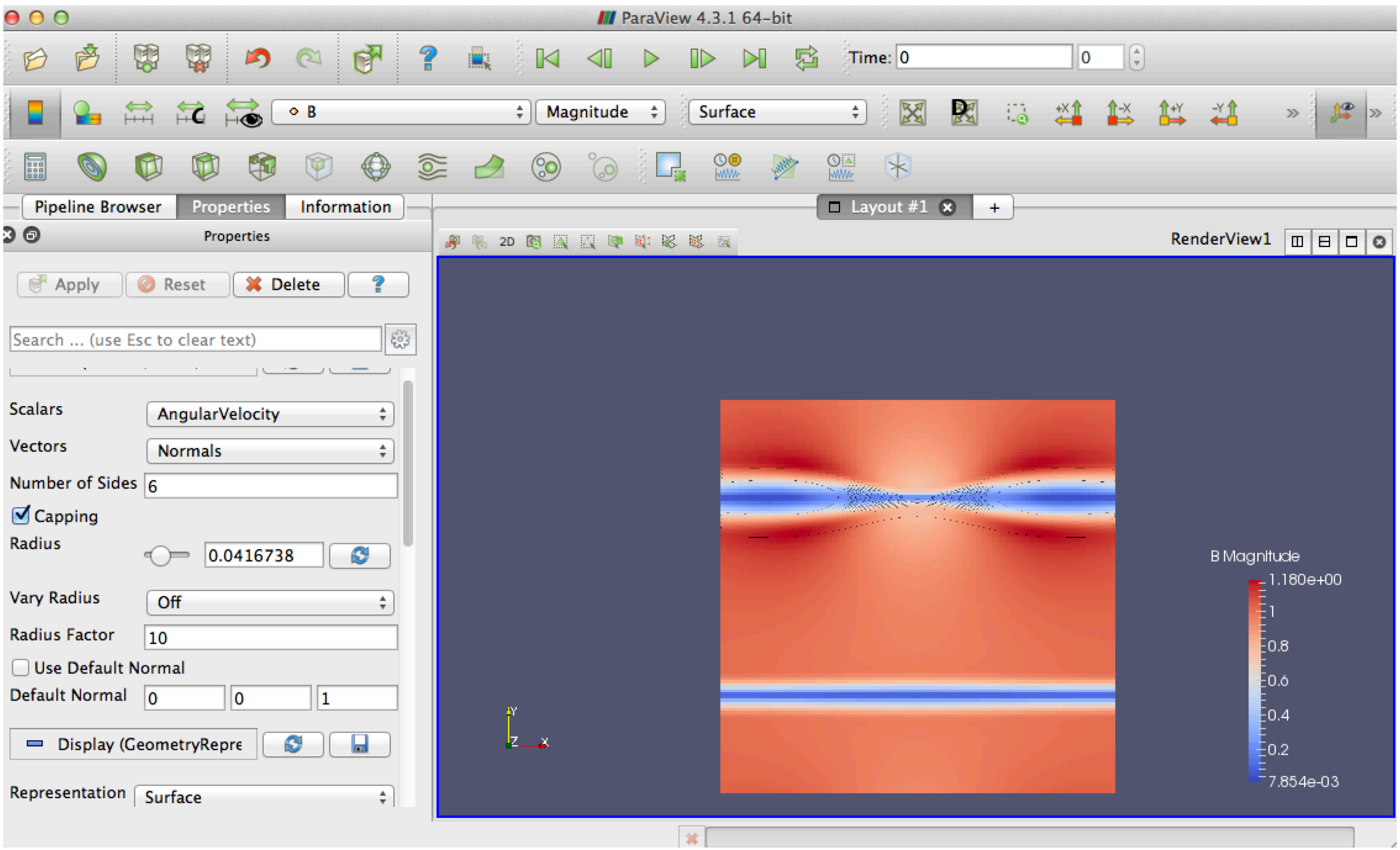


8) In “properties”, change this into something smaller, than “Apply”



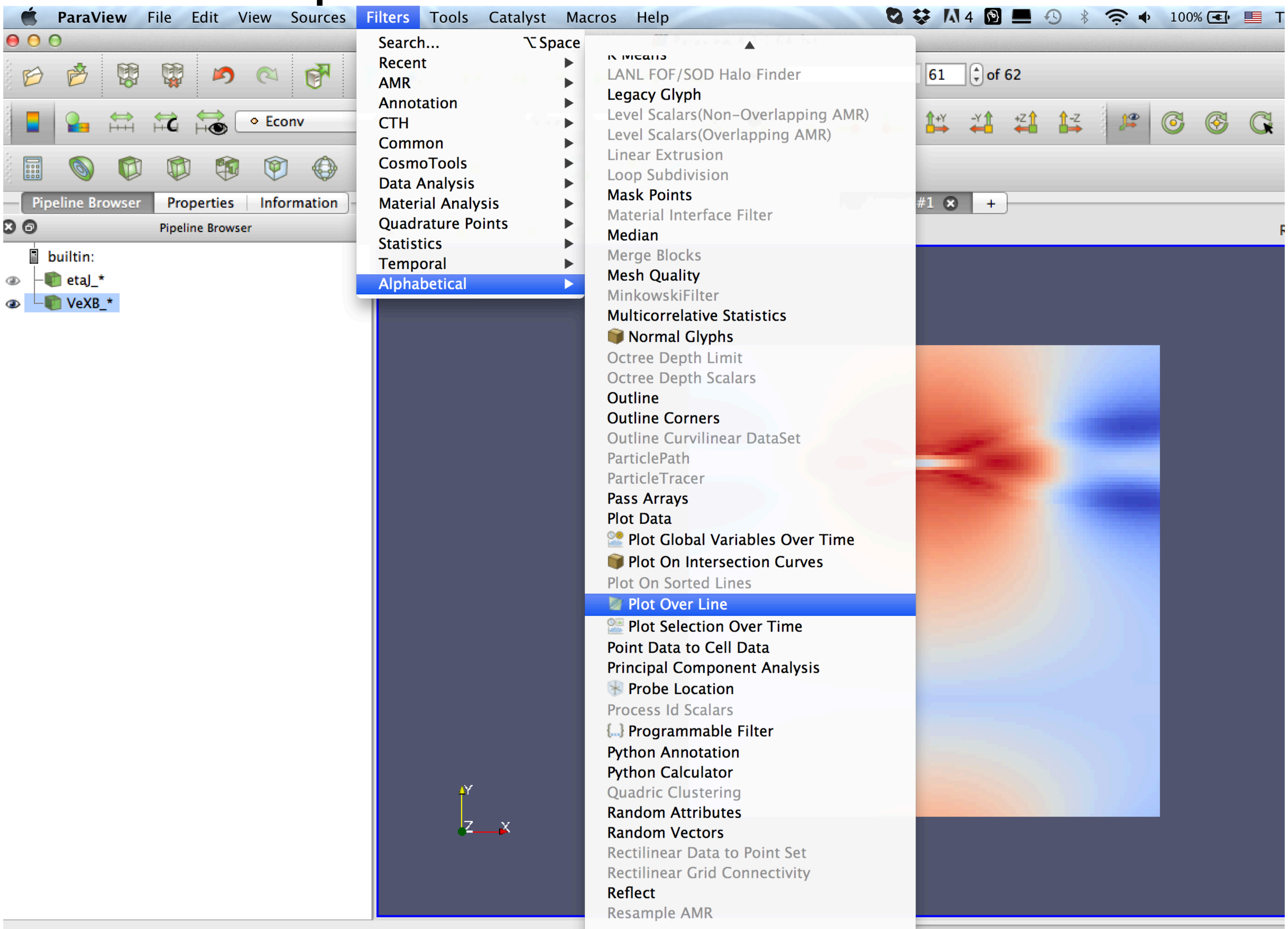
Exercises (1): Petschek

- Did you get this picture?

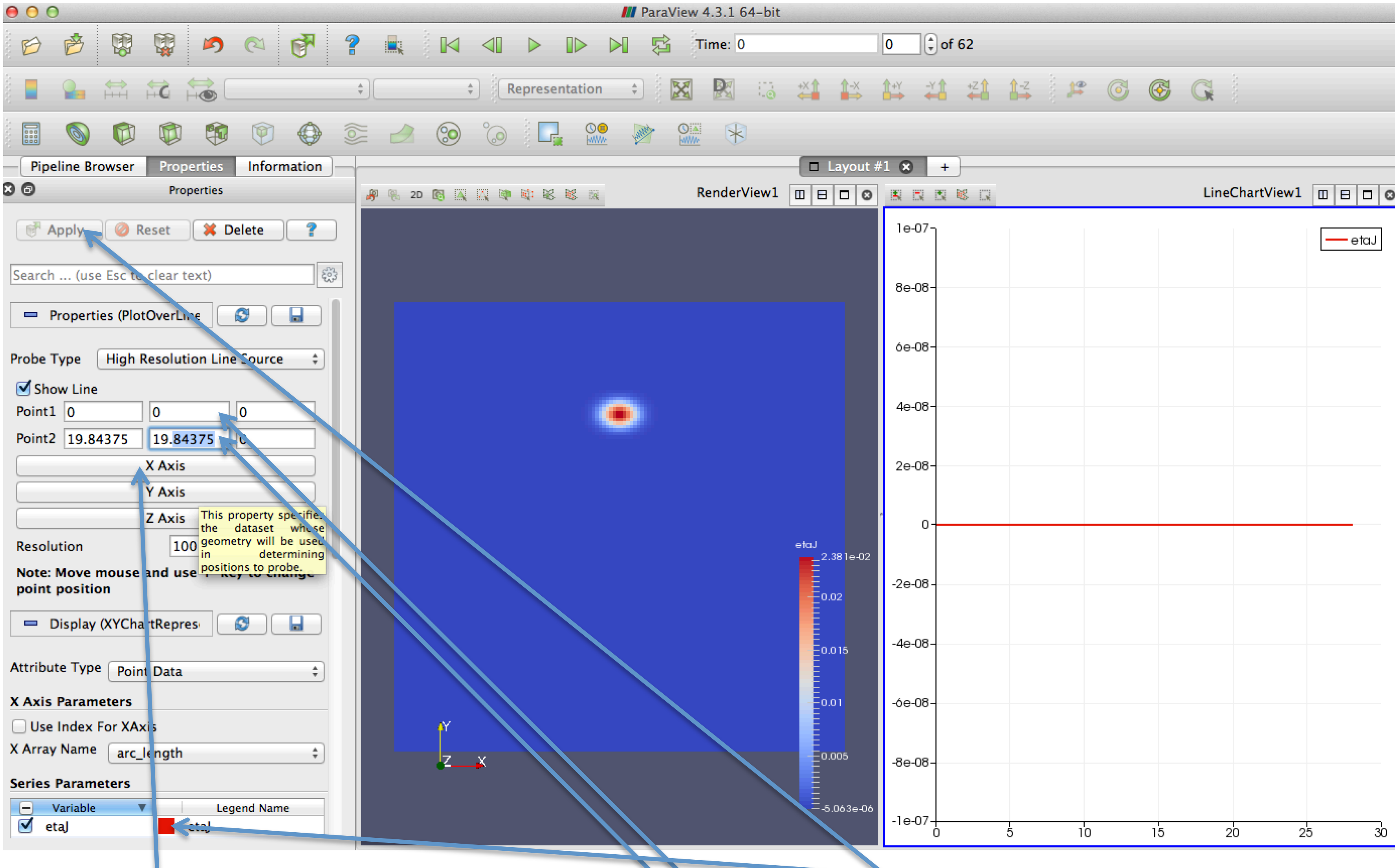


Exercises (1): Petschek

Paraview: plot line



Exercises (1): Petschek



1) Click ‘X axis’, 2) Type here the value of (Ly*3/4), 4) Hit‘Apply’, 4) double-click to change

Exercises (1): Petschek

- Add more grids, you should get this (interpret this figure).