Modeling reconnection with particle-assisted magnetohydrodynamics

I. Honkonen, A. Glocer
ilja.honkonen@nasa.gov, alex.glocer-1@nasa.gov
NASA GSFC, Code 673, Greenbelt, MD, USA

Physics-based plasma modeling techniques

- Multi-scale medium, impossible to model all physics of many interesting systems
- Various approximations / optimizations
  - Hybrid PIC/Vlasov, gyro-averaged, bounce-averaged, electron-fluid MHD, (M)HD
    - Usually restricted to predetermined region(s)
    - Always a strict boundary between diff. descriptions
  - e/i mass ratio, dipole strength, spatial symmetry, fewer dims., slower light speed
  - Parallel program, temporal substepping, reduced resolution (adaptive mesh refinement, etc.), implicit solver

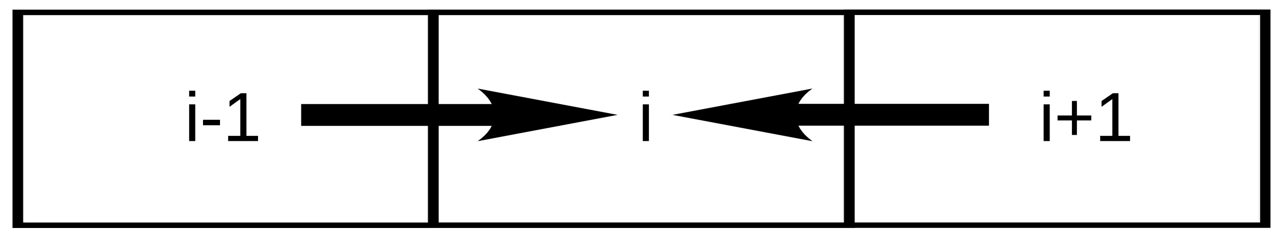
Particle-assisted magnetohydrodynamics

- Plasma represented by both particles and fluid simultaneously
  - Particles and fluid in same simulation cell at same time
- Smooth transition between different physical descriptions
  - No discontinuity in modeled physics
  - No artificial boundaries with interpolation/averaging of variables
- Flexible tradeoff between time to solution and accuracy
  - Allows also other approx. / optimizations (previous slide)
  - More accurate physics not limited to predetermined region or time

Method part 1: standard (M)HD

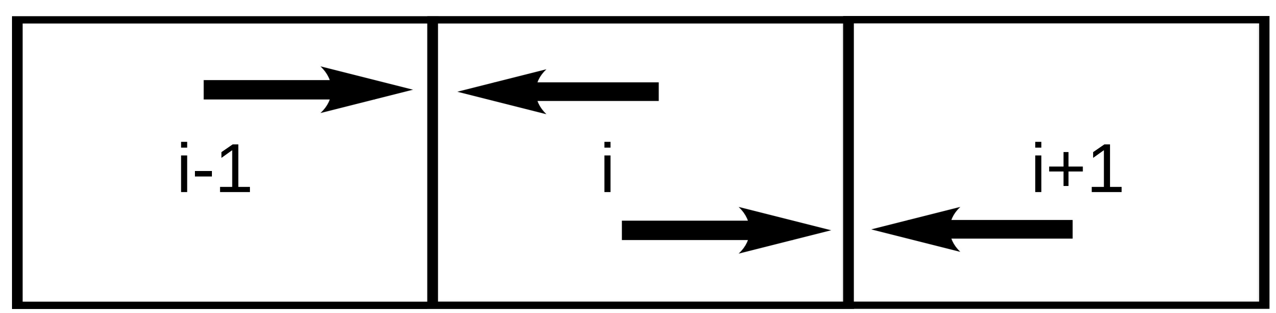
Change in variable C of cell i from time t to t+dt...

$$\Delta C(t,i) = F_C(t,i-1/2) - F_C(t,i+1/2)$$



...with contributions from each cell separated

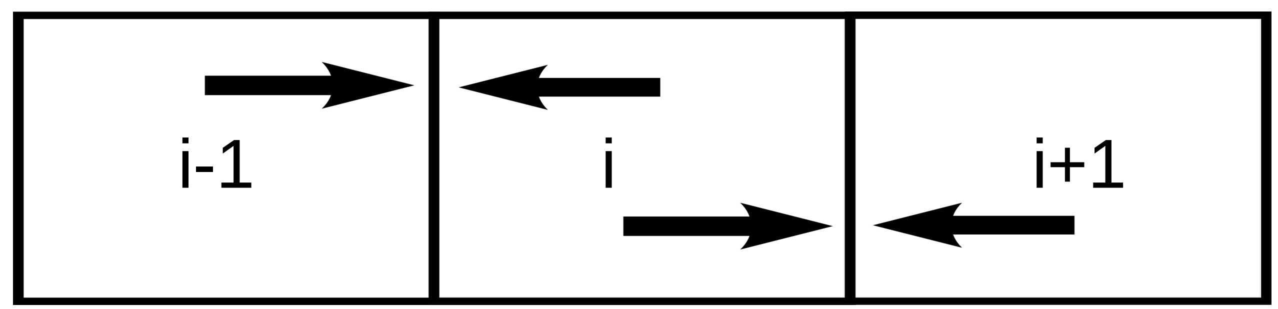
$$\Delta C(t,i) = F_{C-}(t,i-1/2) + F_{C+}(t,i-1/2) - F_{C-}(t,i+1/2) - F_{C+}(t,i+1/2)$$



where F is calculated using e.g. HLLD MHD solver

Method part 2: multi-fluid/species

$$\Delta C(t,i) = F_{C-}(t,i-1/2)+F_{C+}(t,i-1/2)-F_{C-}(t,i+1/2)-F_{C+}(t,i+1/2)$$



Contributions of calculated fluxes are split between different fluids (j) by their fraction of mass compared to the total:

$$\Delta C_j(t,i) = \frac{\rho_j(t,i-1)}{\rho(t,i-1)} F_{C-}(t,i-1/2) + \frac{\rho_j(t,i)}{\rho(t,i)}[F_{C+}(t,i-1/2) - F_{C-}(t,i+1/2)] - \frac{\rho_j(t,i+1)}{\rho(t,i+1)} F_{C+}(t,i+1/2)$$

Method part 3: particle-assisted MHD

Contributions of calculated fluxes are split between different fluids (j) by their fraction of mass compared to the total:

$$\Delta C_j(t,i) = \frac{\rho_j(t,i-1)}{\rho(t,i-1)} F_{C-}(t,i-1/2) + \frac{\rho_j(t,i)}{\rho(t,i)} [F_{C+}(t,i-1/2) - F_{C-}(t,i+1/2)] - \frac{\rho_j(t,i+1)}{\rho(t,i+1)} F_{C+}(t,i+1/2)$$

- One fluid/species replaced with (macro)particle ions
- Fluxes are calculated from total quantities
  - Fluid(s) + accumulated particle density, velocity, pressure
- Ideal MHD Ohm's law is used when advancing B
  - Other terms also possible but not used here
- E in Lorentz force also includes Hall term
  - Otherwise particles modeled in bulk velocity frame

Testing

- We perform basic tests in 1 spatial and 3 velocity dimensions to verify MHD, hybrid PIC and combined algorithms
- Study behavior of the new method in various corner cases
- Test the combined effects of particle noise and numerical diffusion of fluid

Advection part 1

- Sinusoidal magnetic field advected once through a periodic system
- Result should be identical regardless of what fraction of plasma mass is represented by particles
- Numerical diffusion in field solver should not change

System parameters

- Tube length (1d): 1
- Number of simulation cells: 20
- Vacuum permeability: 1
- Boltzmann constant: 1
- Particle mass: 1
- Adiabatic index: 5/3

Initial condition

- Mass density: 1
- Velocity: 1, 0, 0
- Temperature/pressure: 1
- Magnetic field: 0, sin(2*pi*x), cos(2*pi*x)

Advection part 2

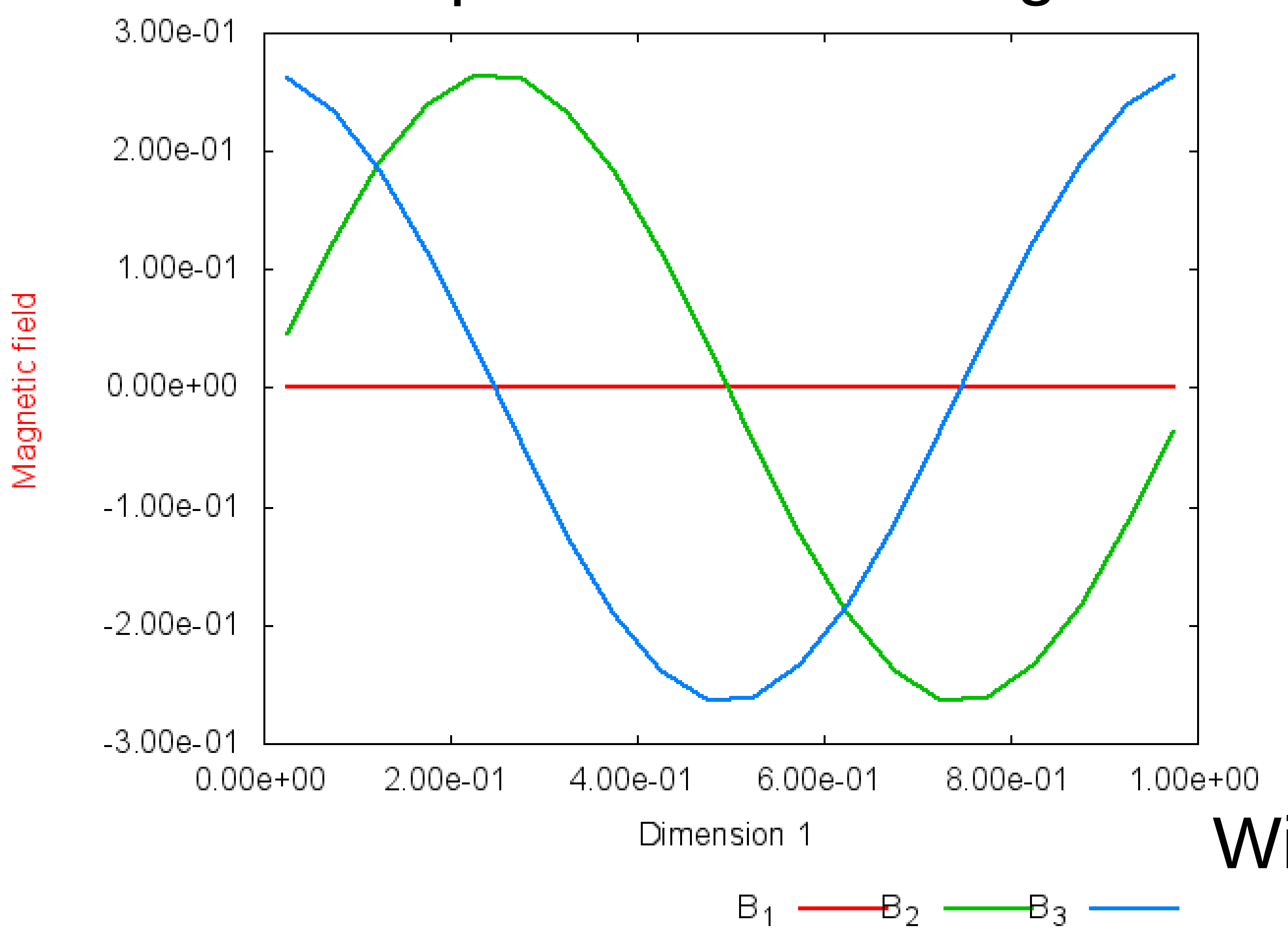
System parameters

Tube length (1d): 1
Number of simulation cells: 20
Vacuum permeability: 1
Boltzmann constant: 1
Particle mass: 1
Adiabatic index: 5/3

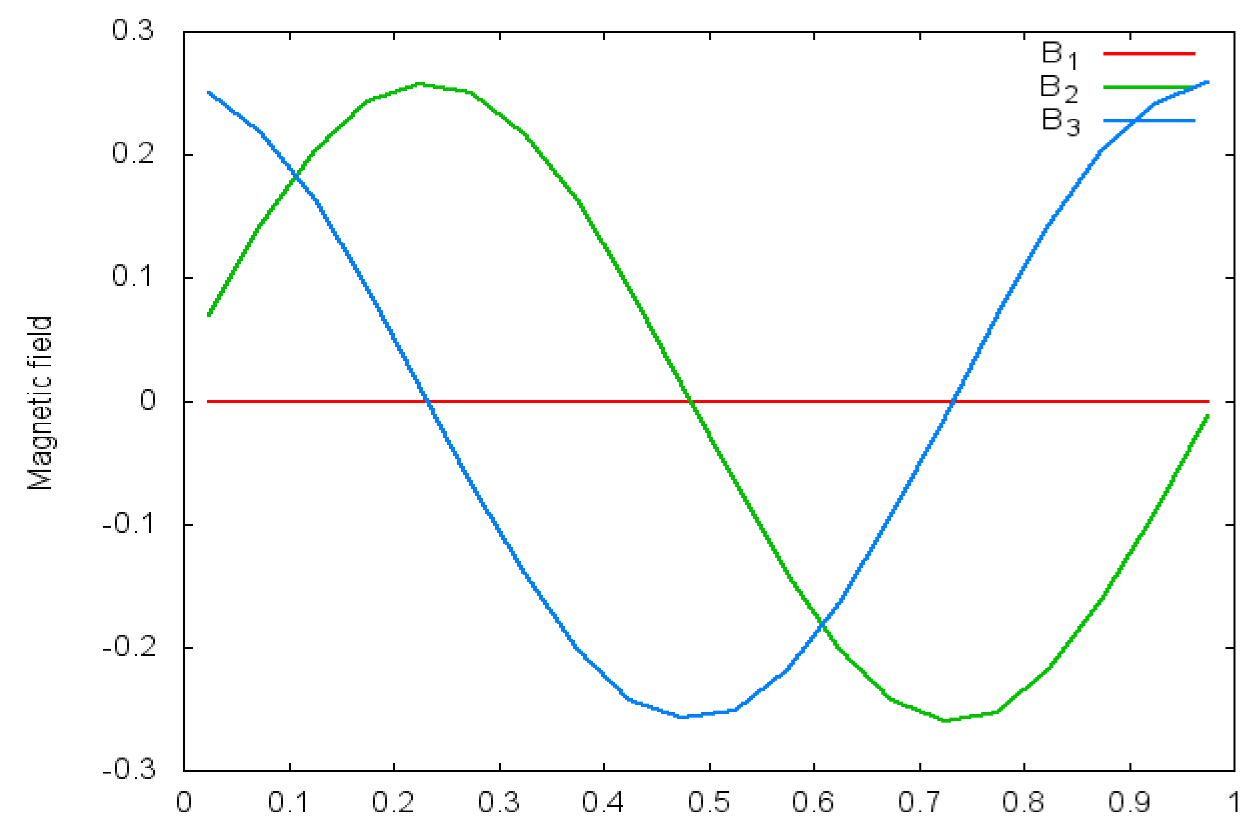
Initial condition

Mass density: 1
Velocity: 1, 0, 0
Temperature/pressure: 1
Magnetic field: 0, sin(2*pi*x), cos(2*pi*x)

Fluid solution after one period: magnetic field amplitude 25% of original



Particle solution with 1e4 particles / cell

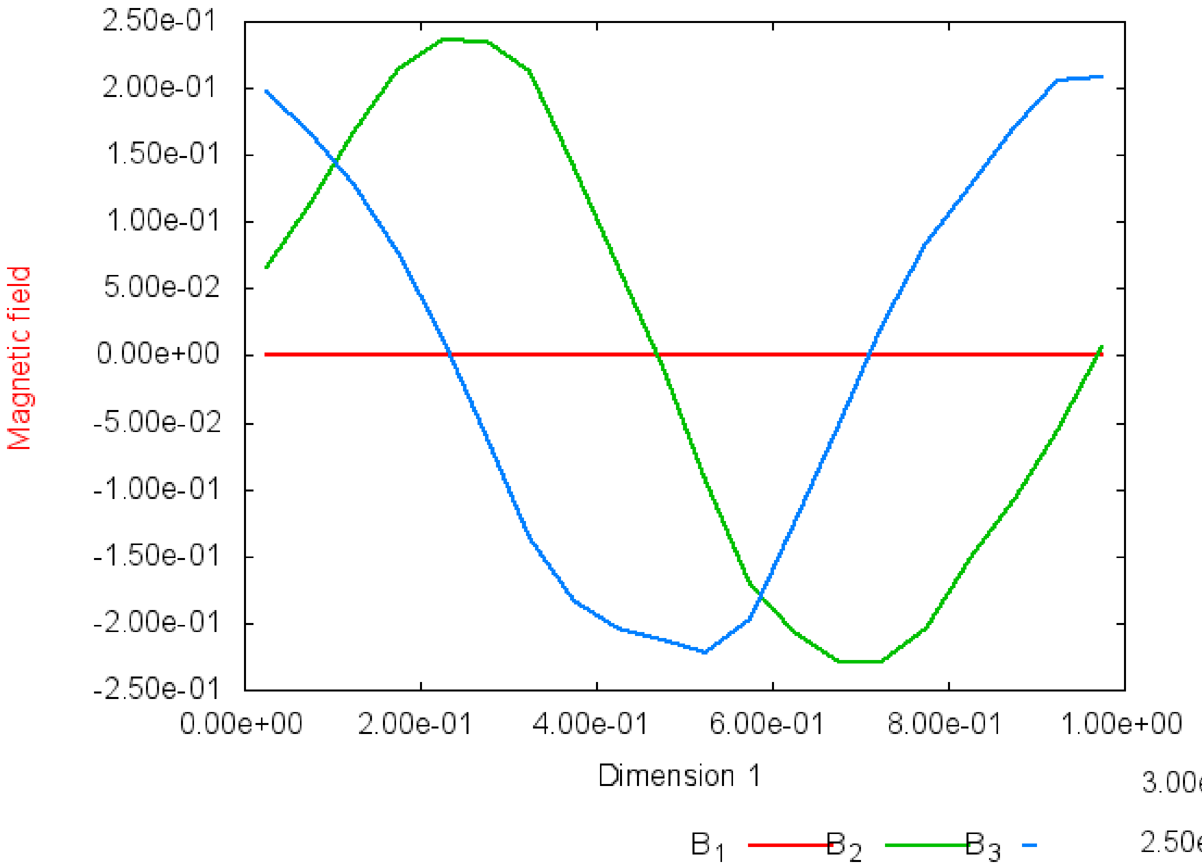


Without particle noise both tests give the same result

Advection part 3

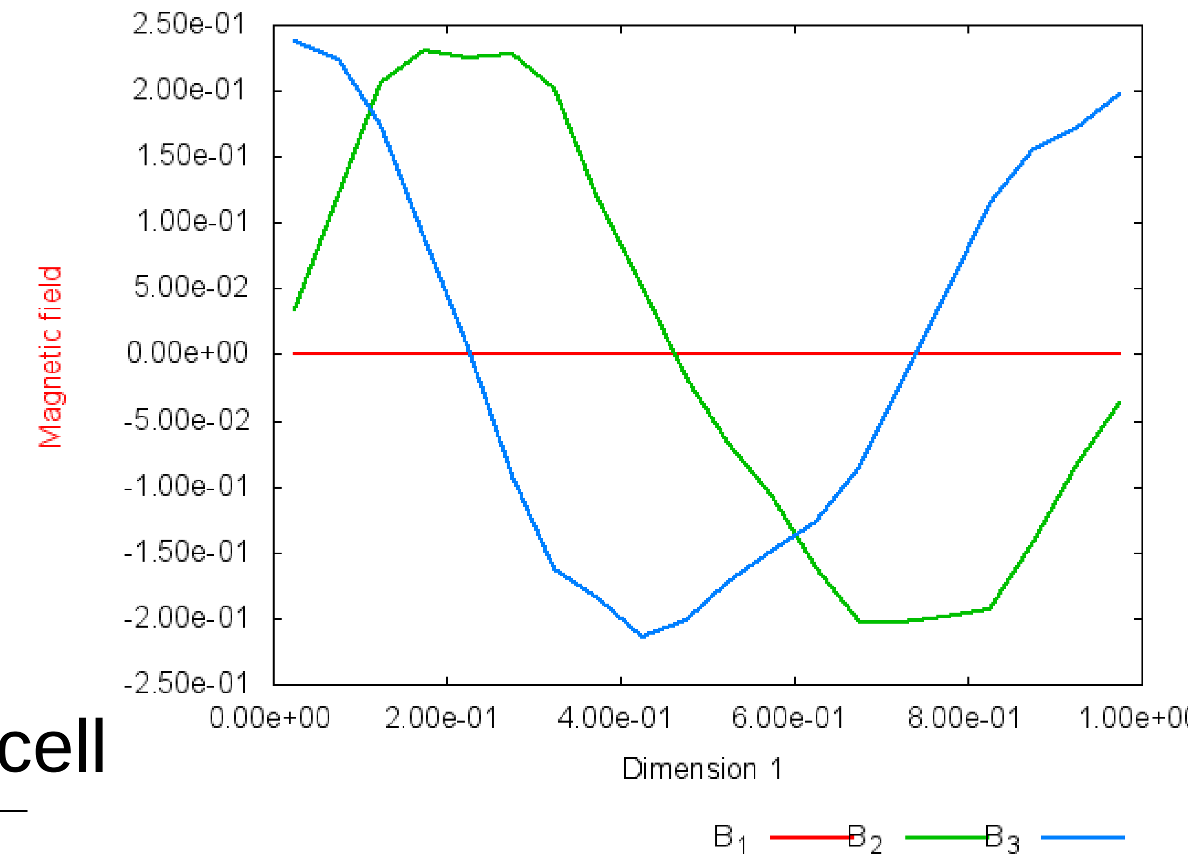
Plasma mass distributed uniformly in space between fluid and particles

80% fluid, 2 particles / cell

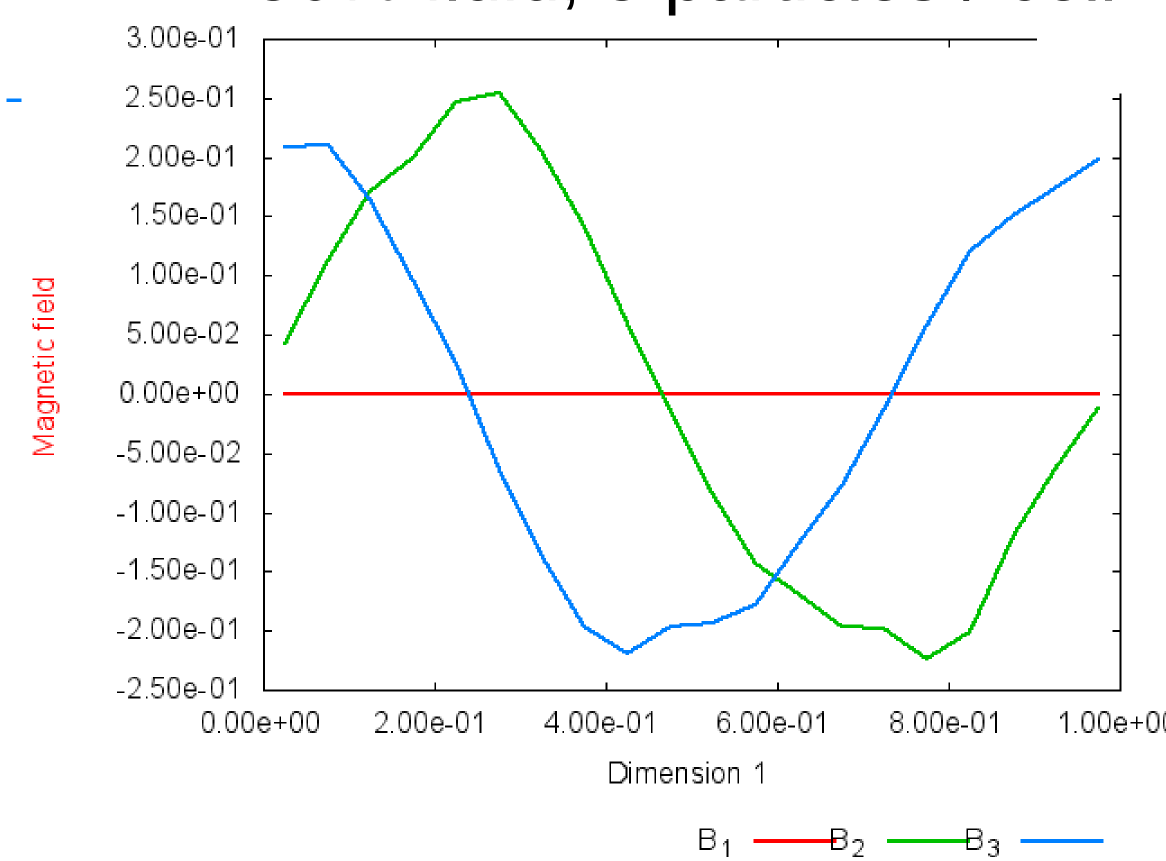


- Particle masses identical in each case
- Fluid solution stabilizes noise from small # of particles

20% fluid, 8 particles / cell



50% fluid, 5 particles / cell

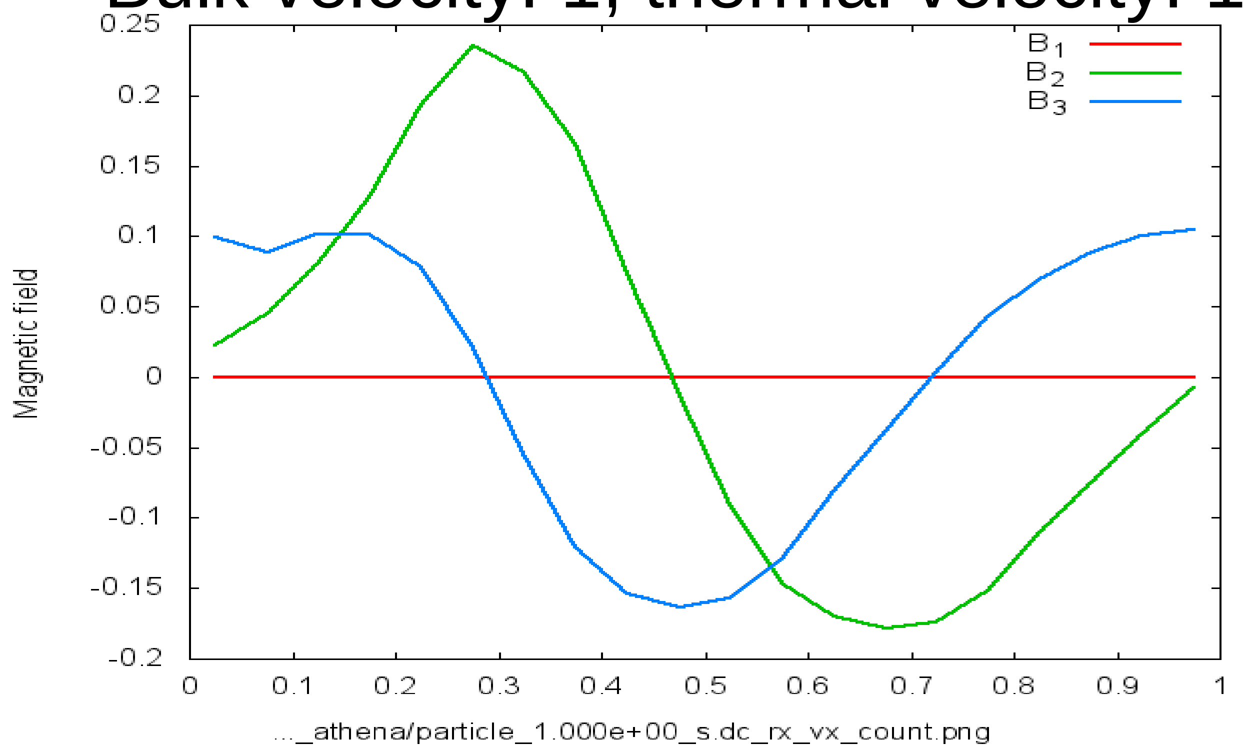


Advection part 4

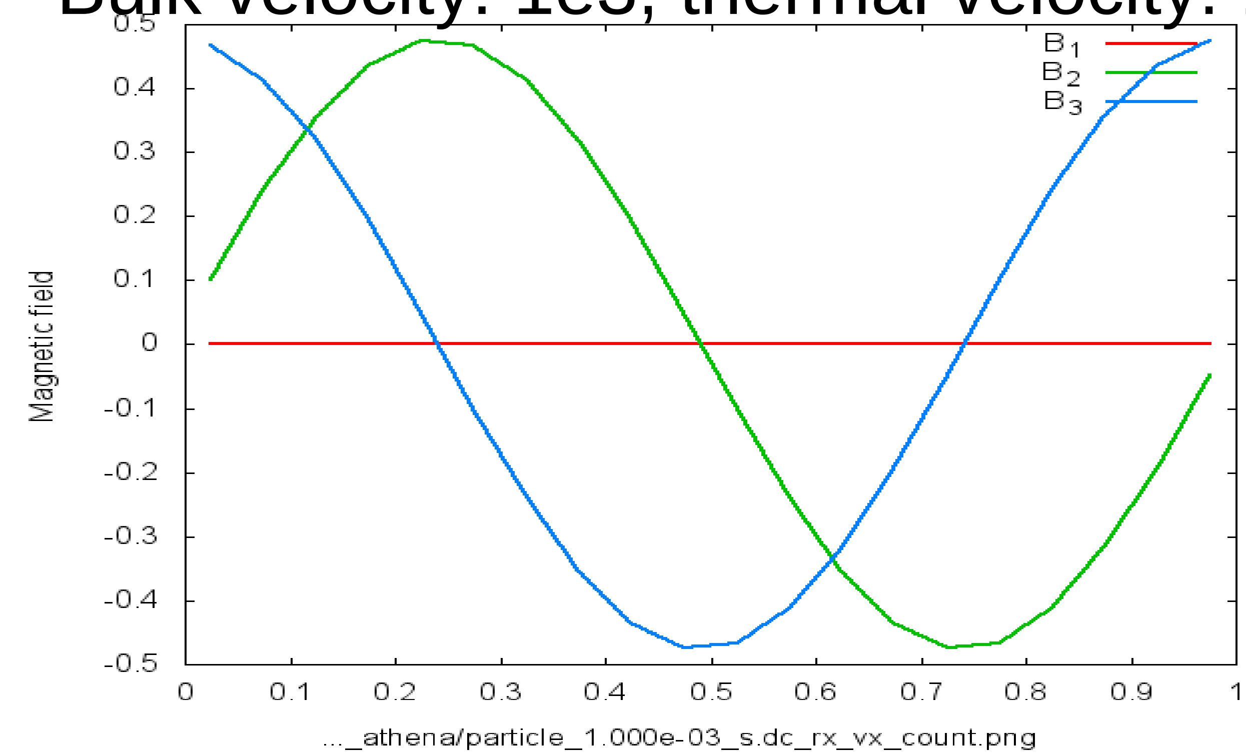
Mass in negative half of tube only particles, in positive half of tube only fluid

Final magnetic field

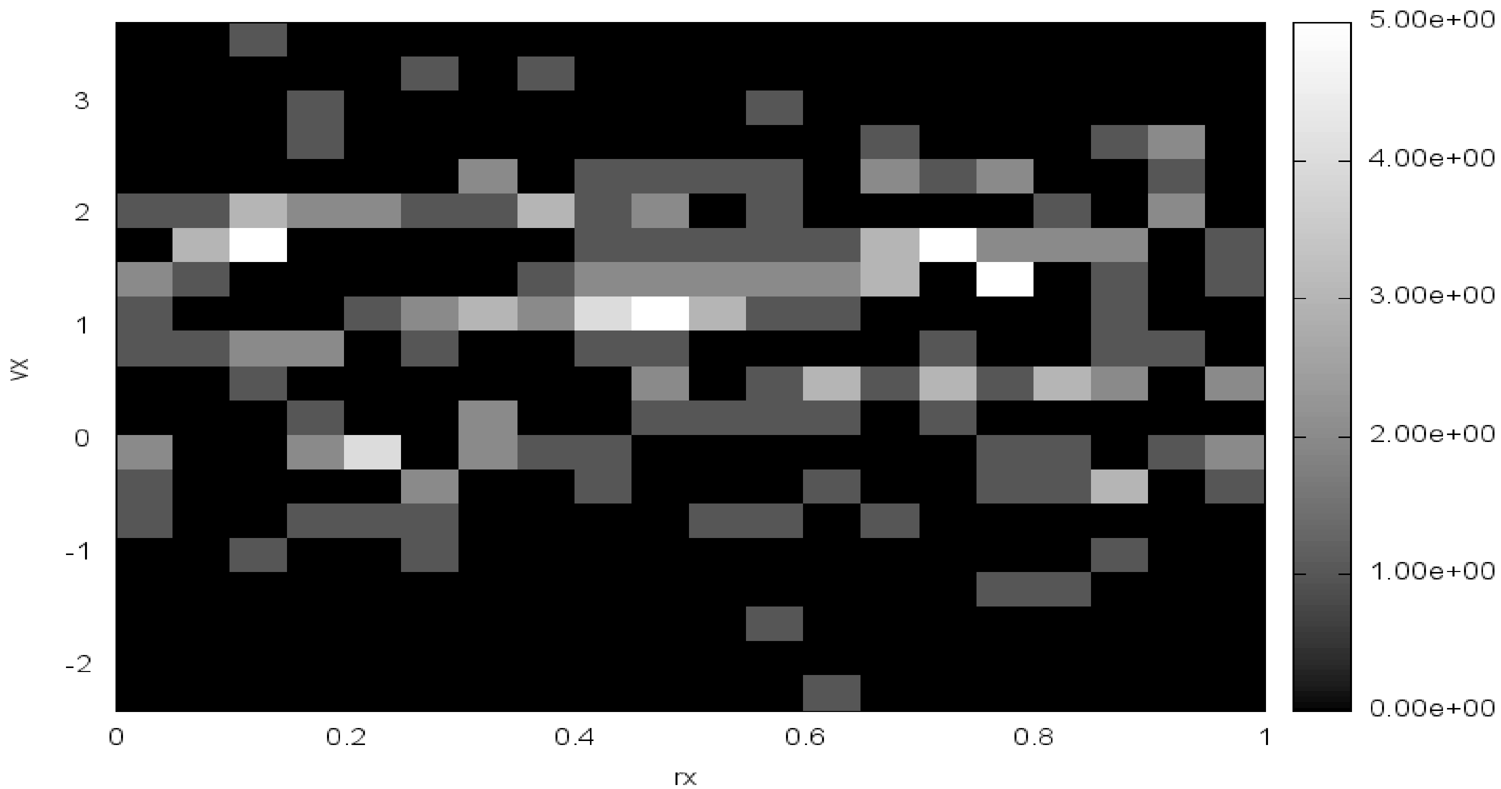
Bulk velocity: 1, thermal velocity: 1

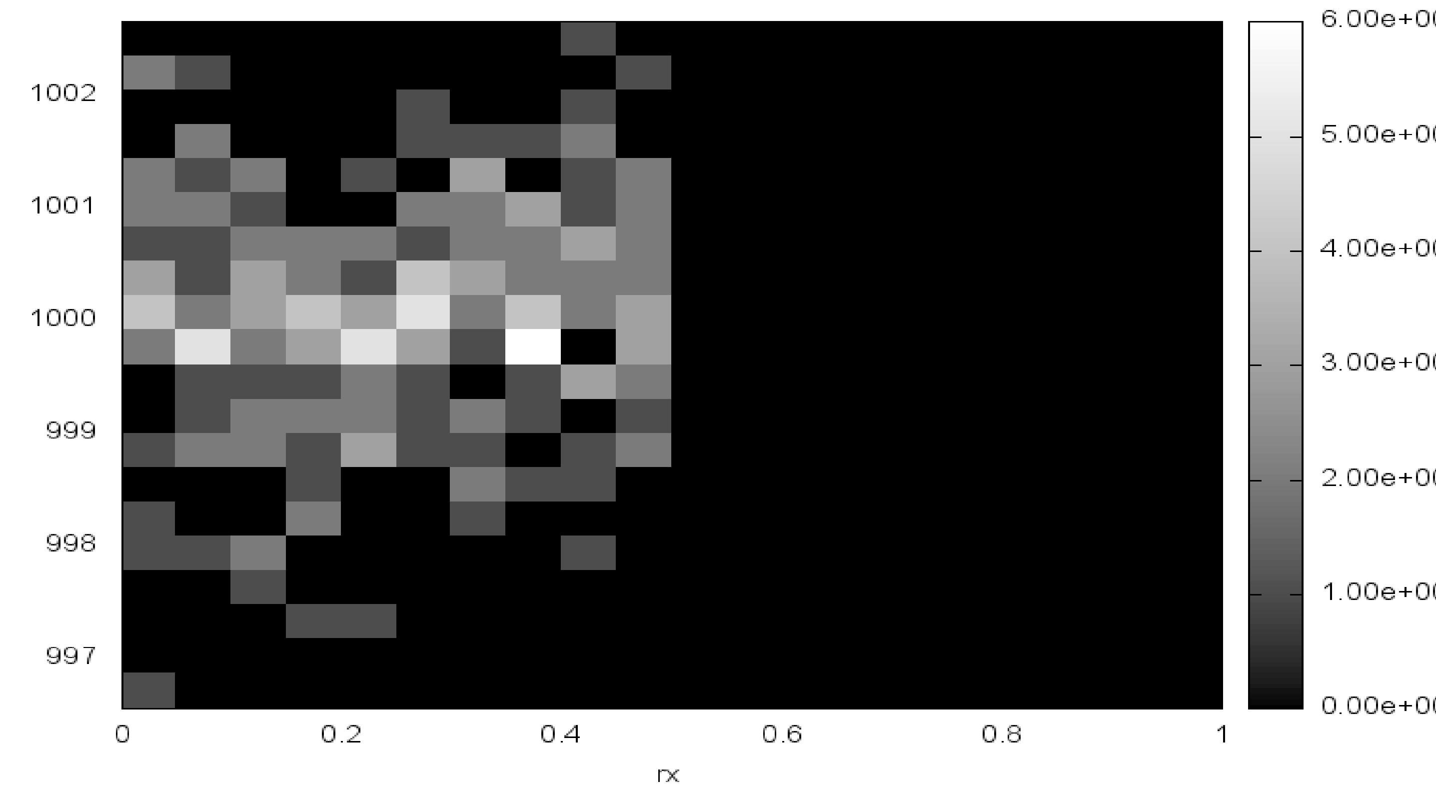


Bulk velocity: 1e3, thermal velocity: 1



Final distribution functions





In fluid regime on right particle-assisted result agrees with pure fluid solution

Ion-ion two-stream instability part 1

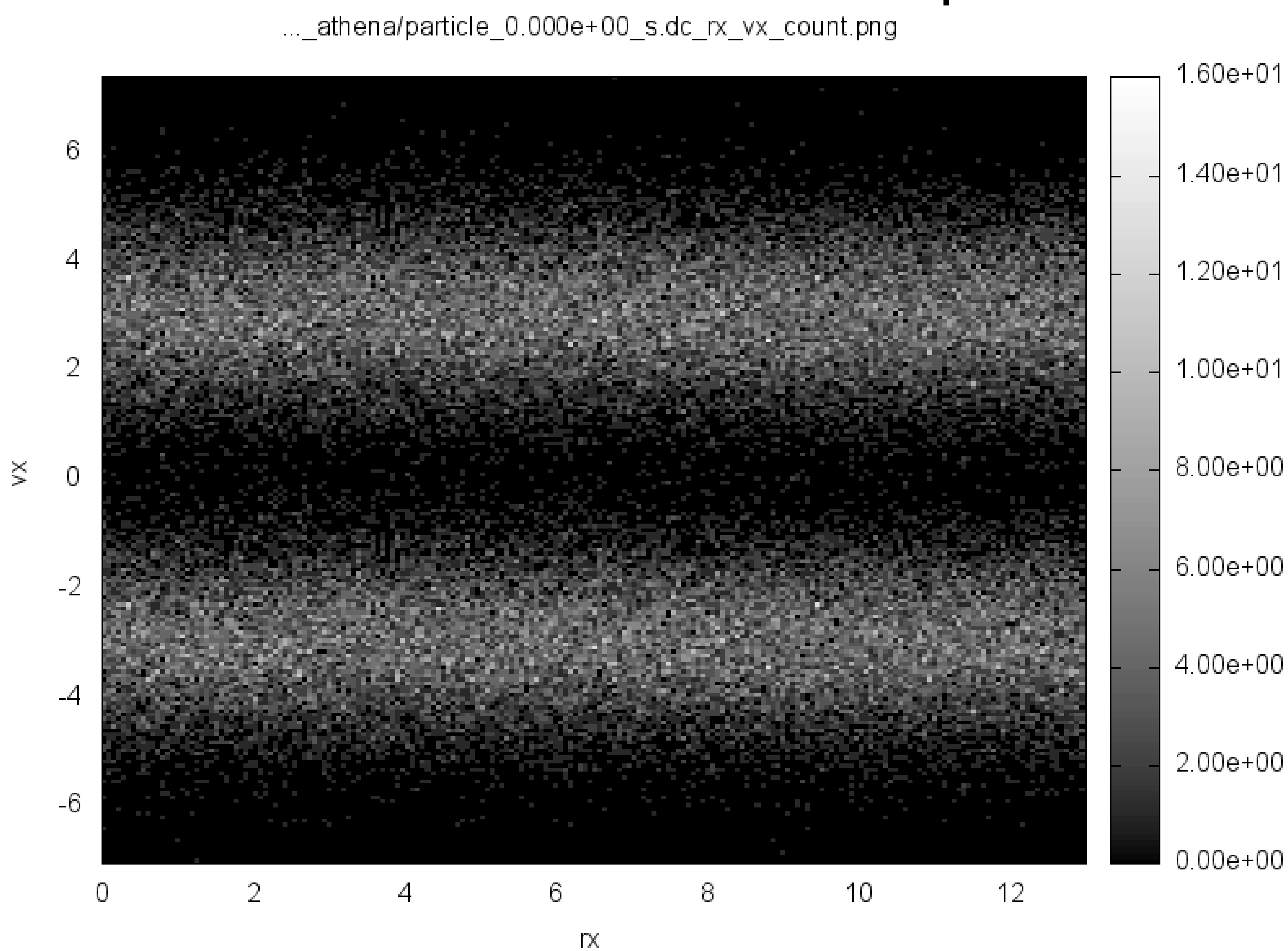
System parameters

Tube length (1d): 13
Number of simulation cells: 300
Every 10% of plasma mass represented by 20 particles/cell where applicable

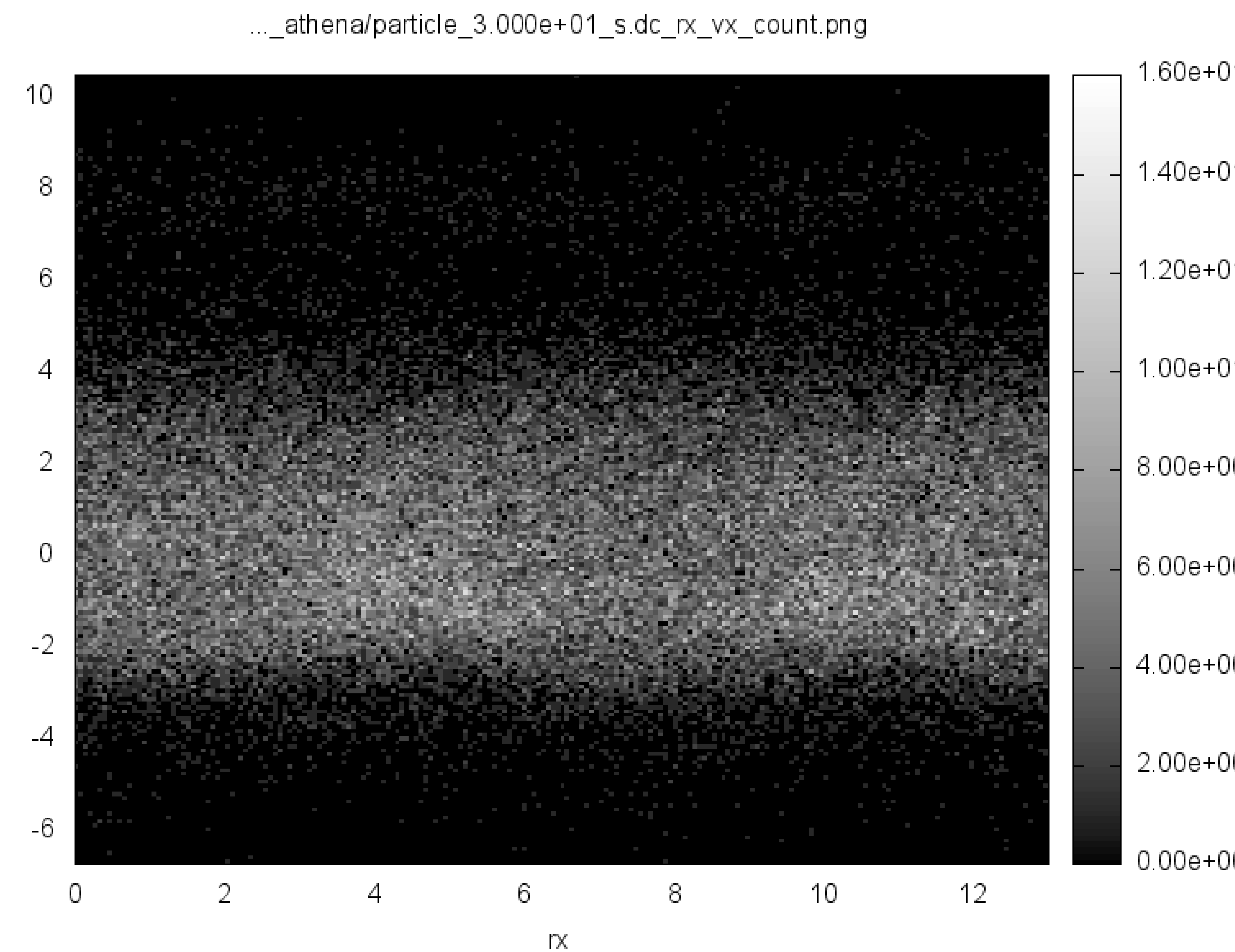
Initial condition

Total mass density: 1
Velocity: +3 & -3, 0, 0
Temperature/pressure: 1
Magnetic field: 1, 0, 0

Initial condition when all mass in particles



Solution at time 30

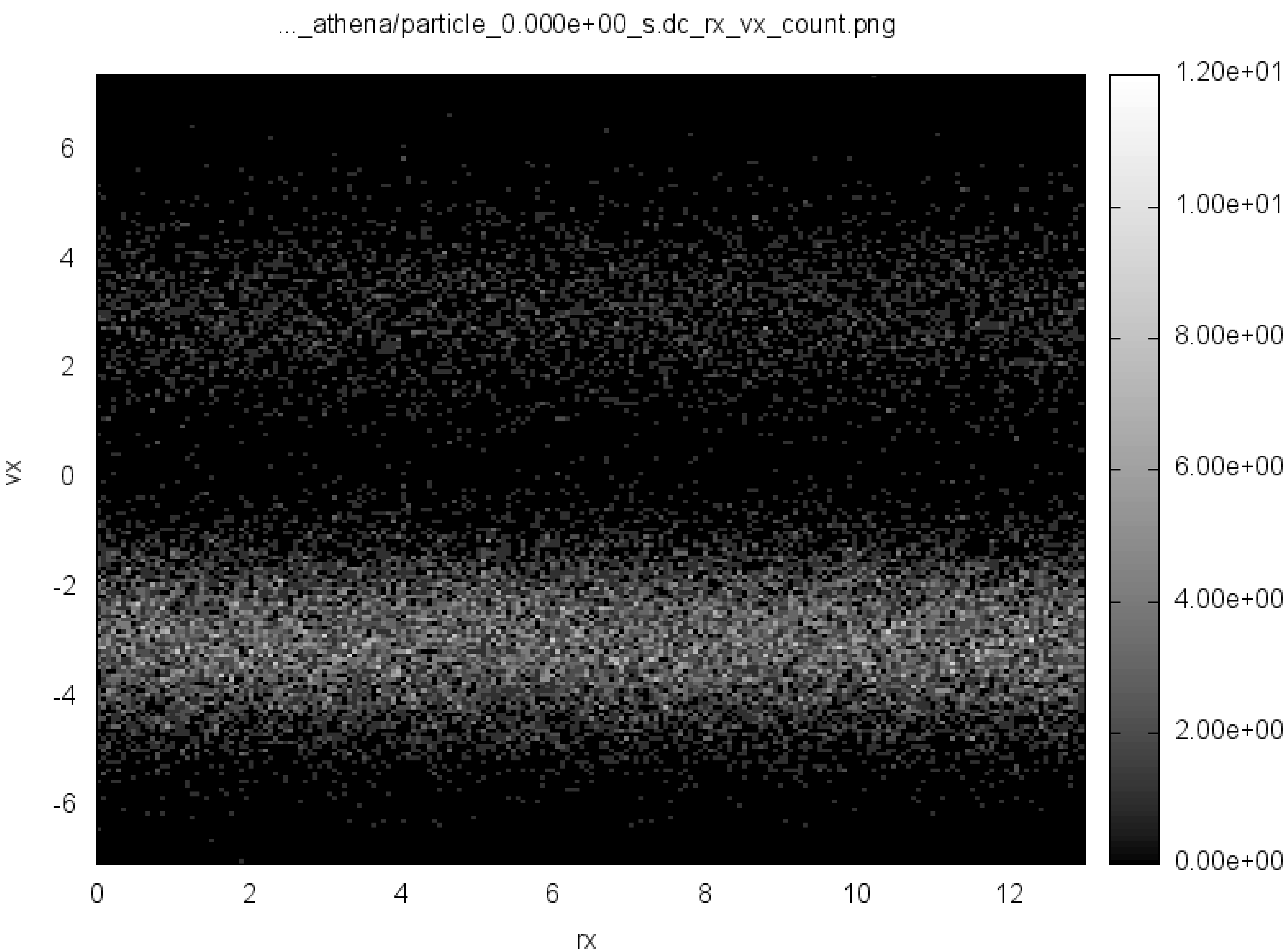


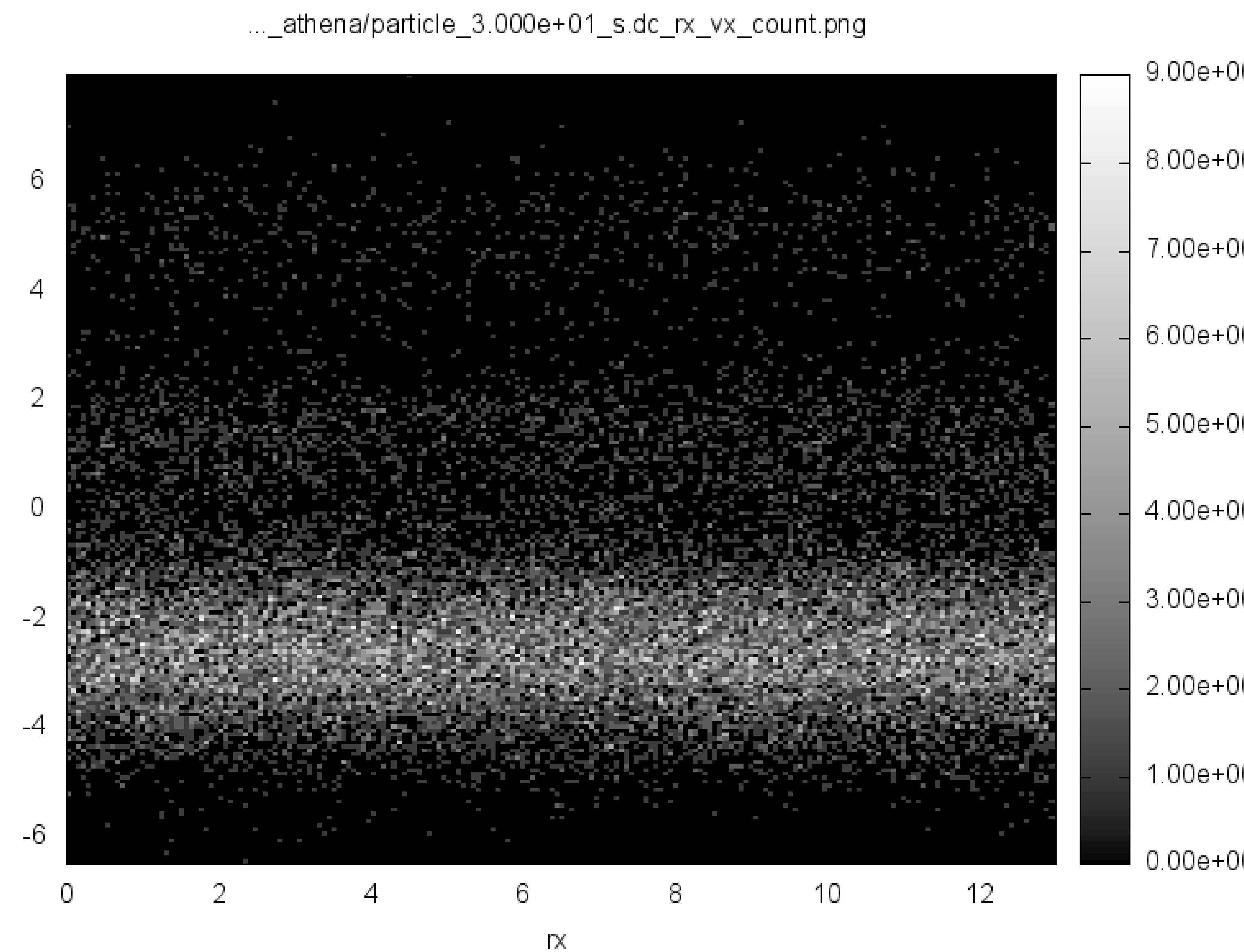
Ion-ion two-stream instability part 2

Initial condition as previously except:

- Vx = -3: particles
- Vx = +3: 80% mass in fluid
  - distributed uniformly in space

Solution at time 30



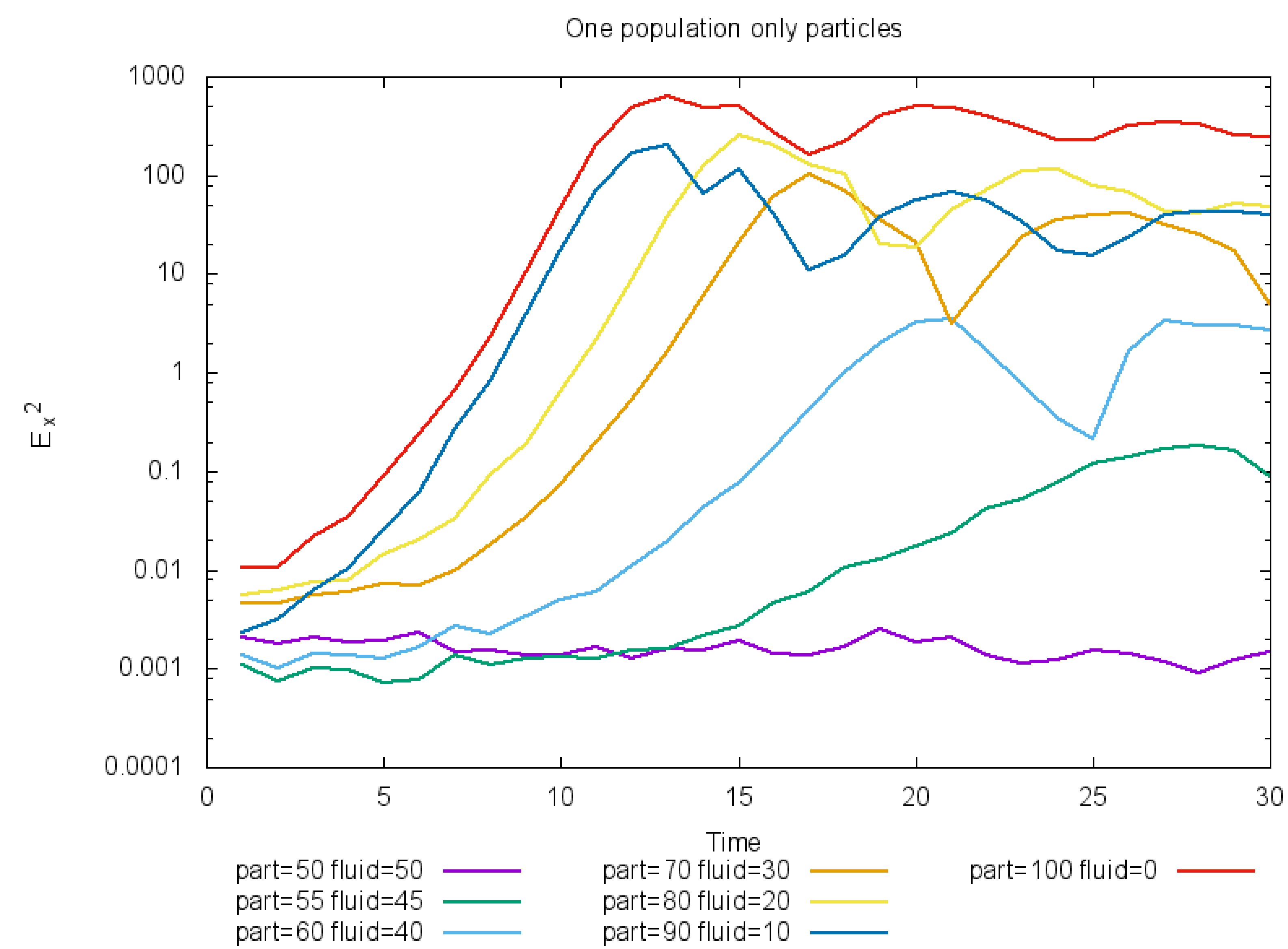


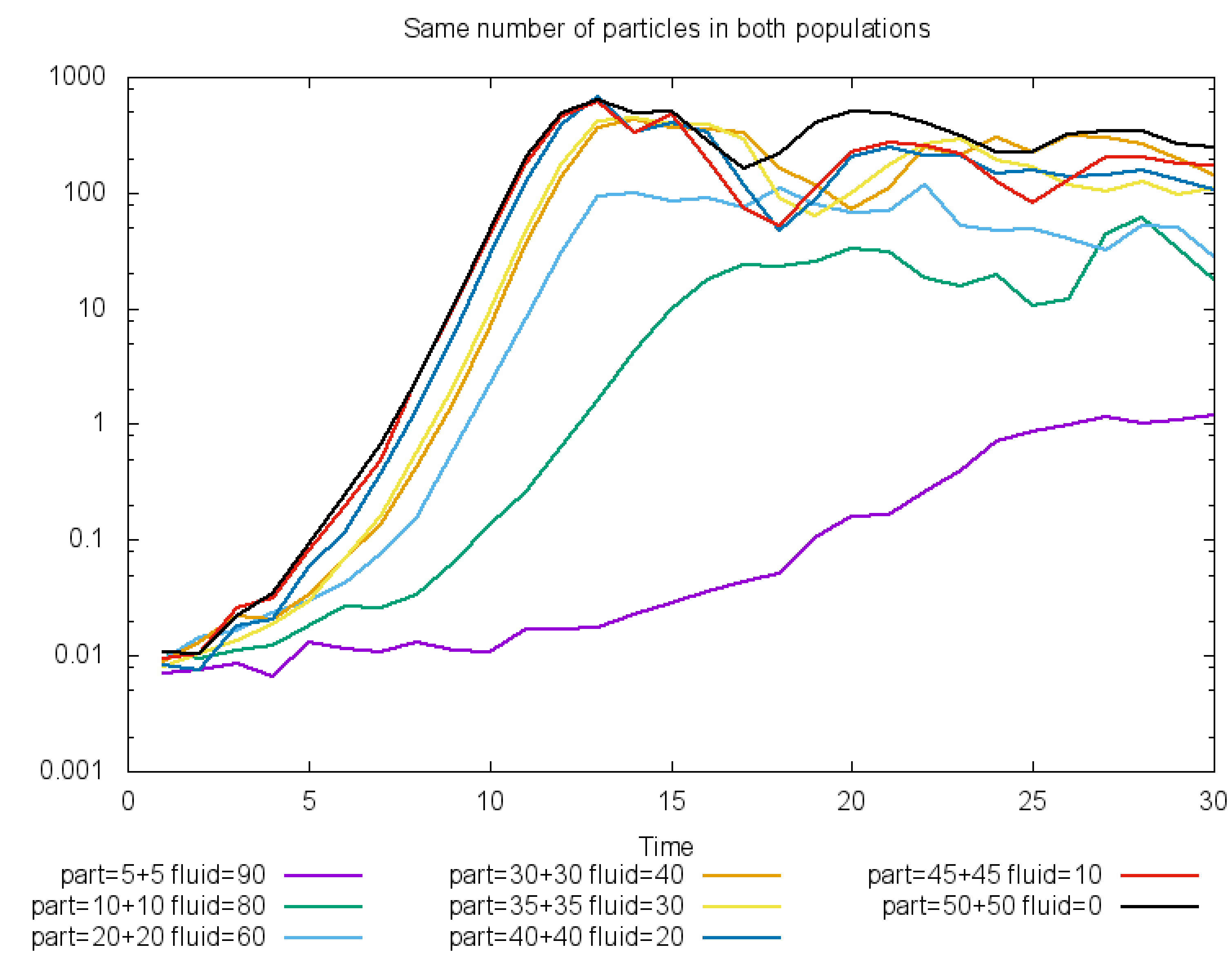
Phase space mixing still present but weaker as more particles seem to be unaffected

Ion-ion two-stream instability part 3

Growth rate of Ex^2:
- Vx = -3: only particles, 50% total mass
- Vx = +3: fraction of mass in particles

Growth rates in bow shock type case:
- Vx = -3: only particles, < 50% total mass
- Vx = +3: as many particles as above





Behavior can be classified as of linear type in mass fraction of particles, as larger fraction increases growth rate and maximum amplitude

Reconnection part 1

System parameters

- Box size (2d): 25.6*12.8
- Periodic in horizontal direction
- Copy boundaries at vertical edges
- Number of simulation cells: 102*(51+2)
- Vacuum permeability: 1
- Boltzmann constant: 1
- Particle mass: 1
- Adiabatic index: 5/3

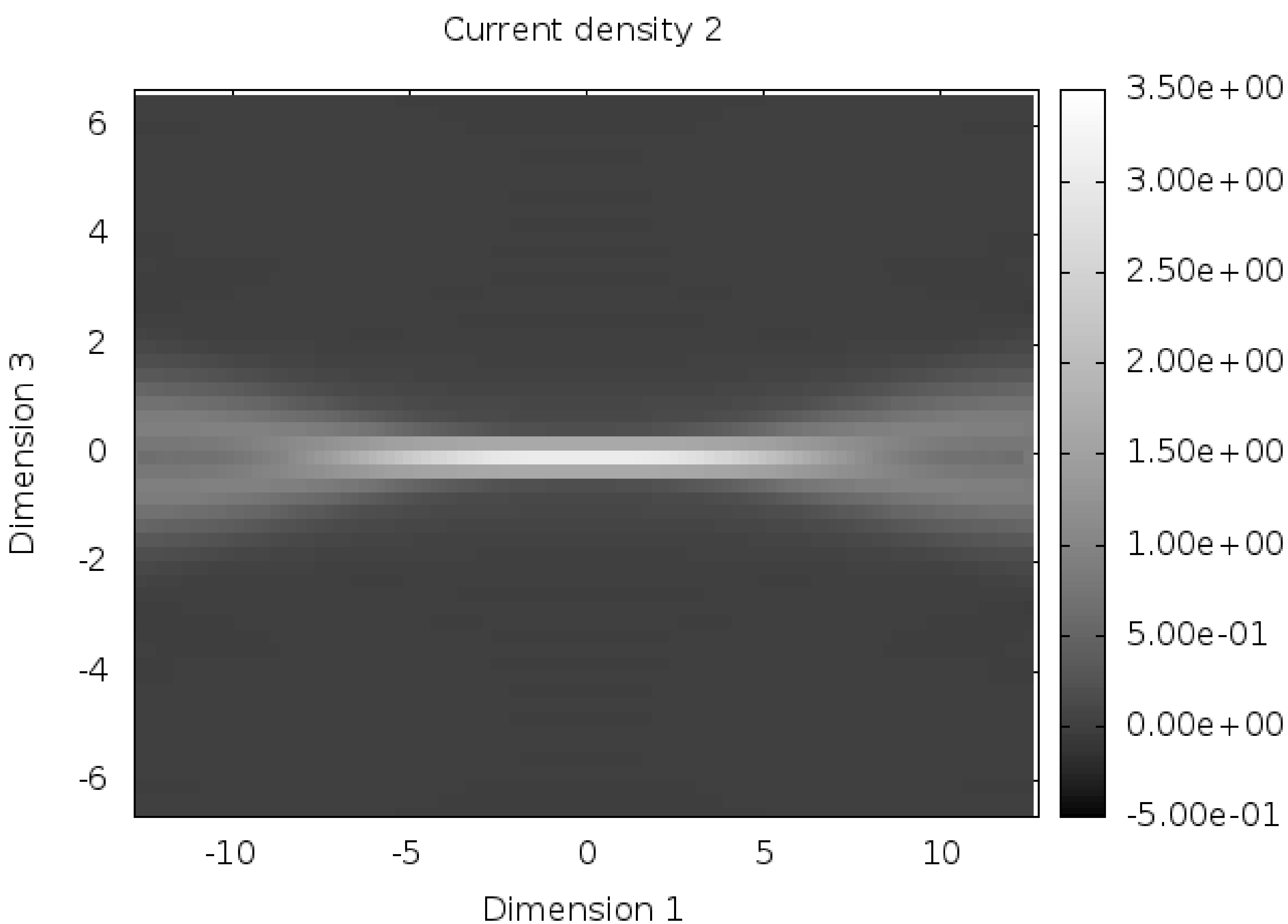
Initial condition

- Mass density: 1/cosh(z/ 0.5)^2 + 0.2
- Velocity: 0, 0, 0
- Temperature/pressure: 10
- Magnetic field
  - X: tanh(z/0.5) - 0.1 * pi/12.8 * cos(2*pi*x/25.6) * sin(pi*z/12.8)
  - Y: 0
  - Z: 0.1 * 2*pi/25.6 * sin(2*pi*x/25.6) * cos(pi*z/12.8)

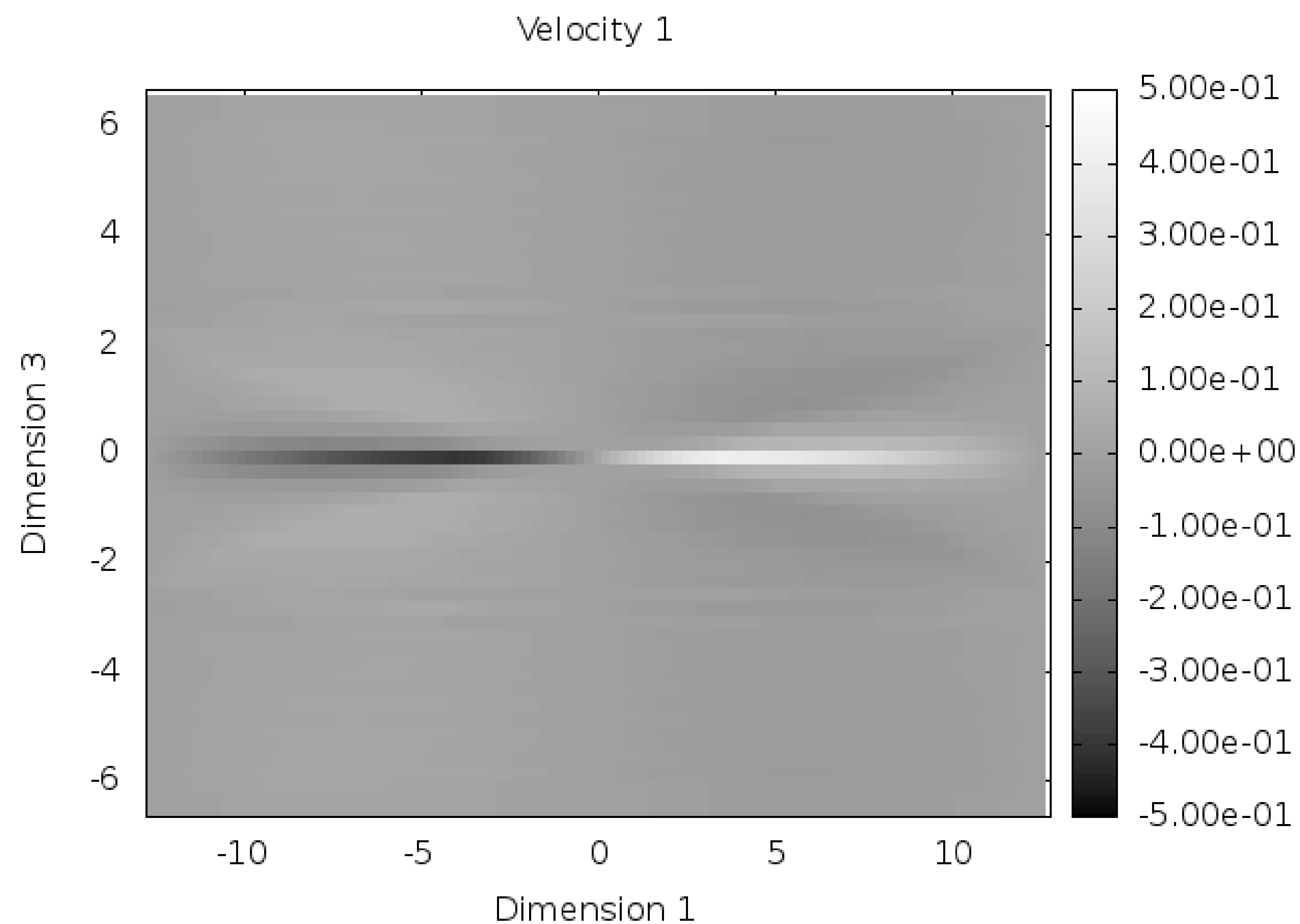
Reconnection part 2

All mass in fluid, final state at 15

Out of plane current



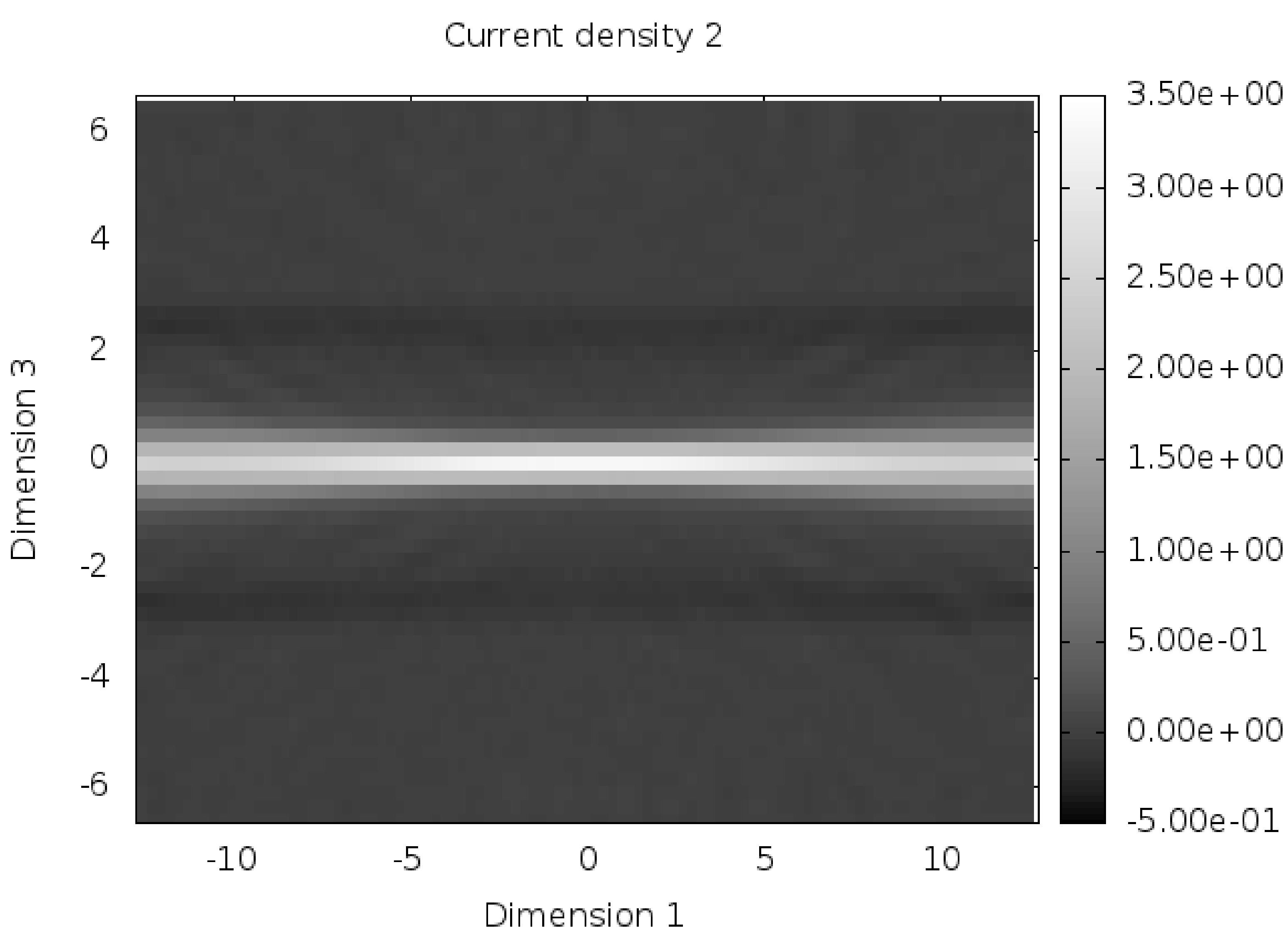
Horizontal velocity



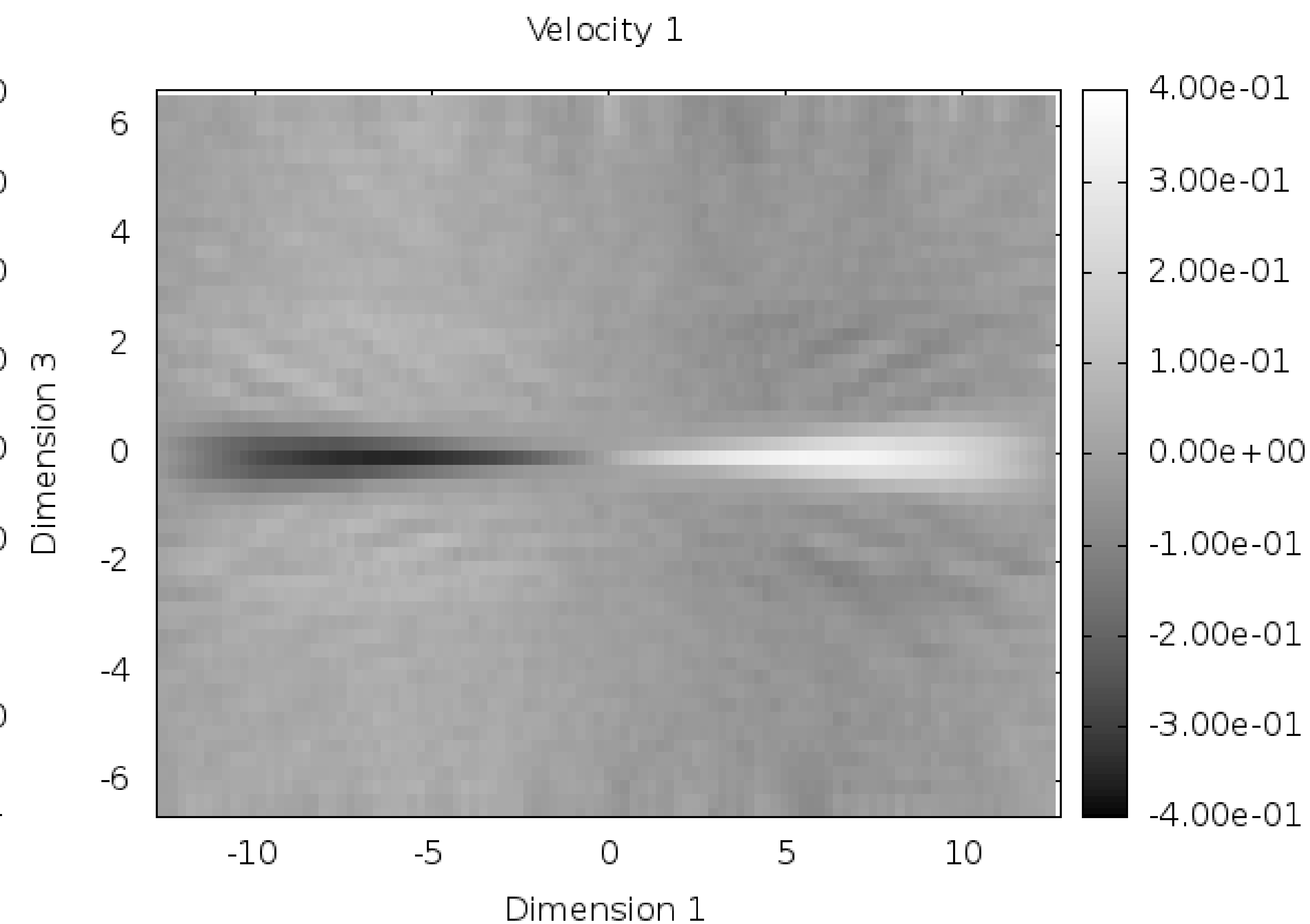
Reconnection part 3

- Z < -3 && Z > 3: all mass in fluid
- -3 < Z < 3: fraction of mass in particles (X % * 800 / cell)

Out of plane current at 5 for 25% particles



Horizontal velocity

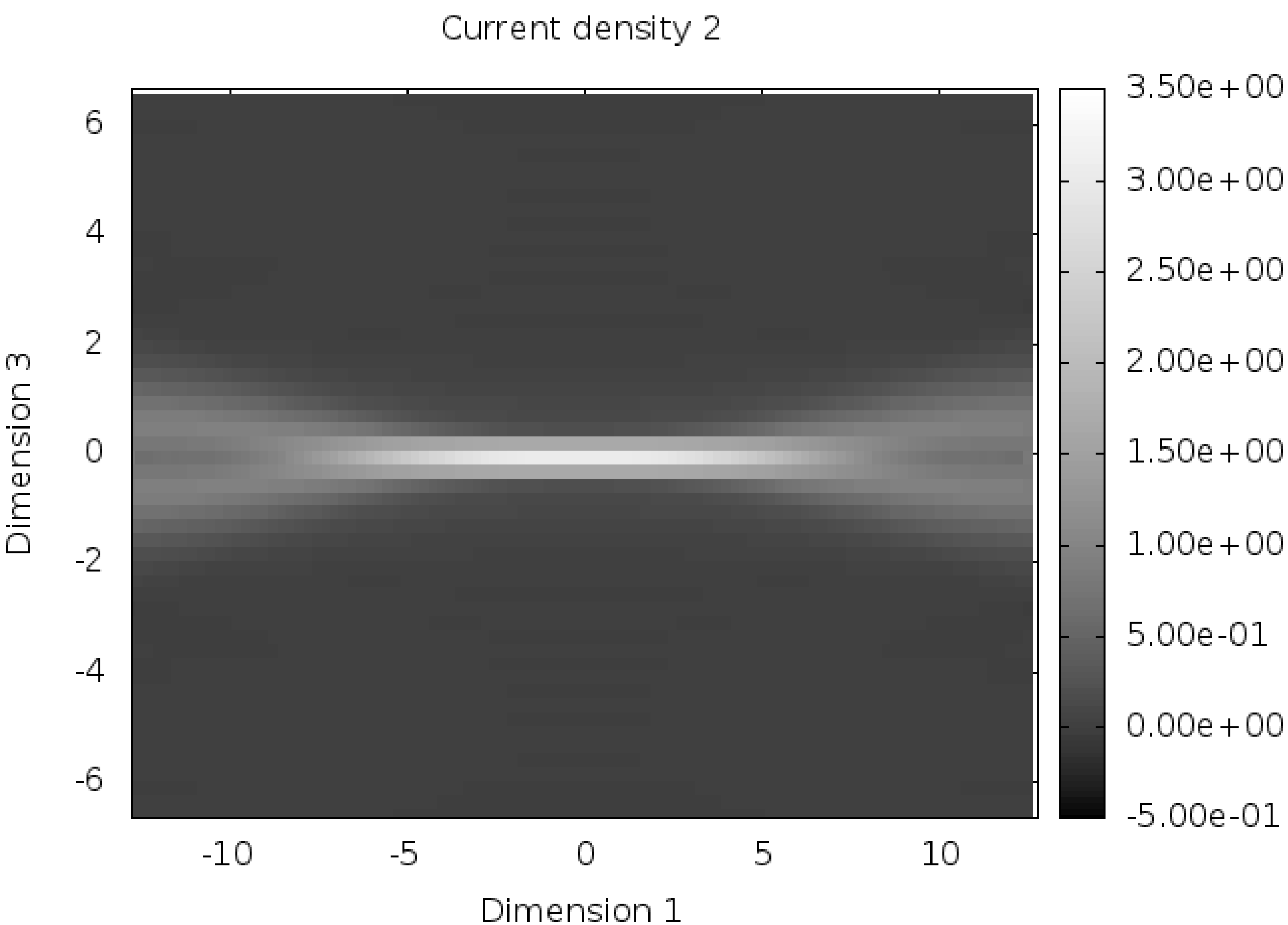


Waves seem to result from interaction of all-fluid and fluid-particle regions

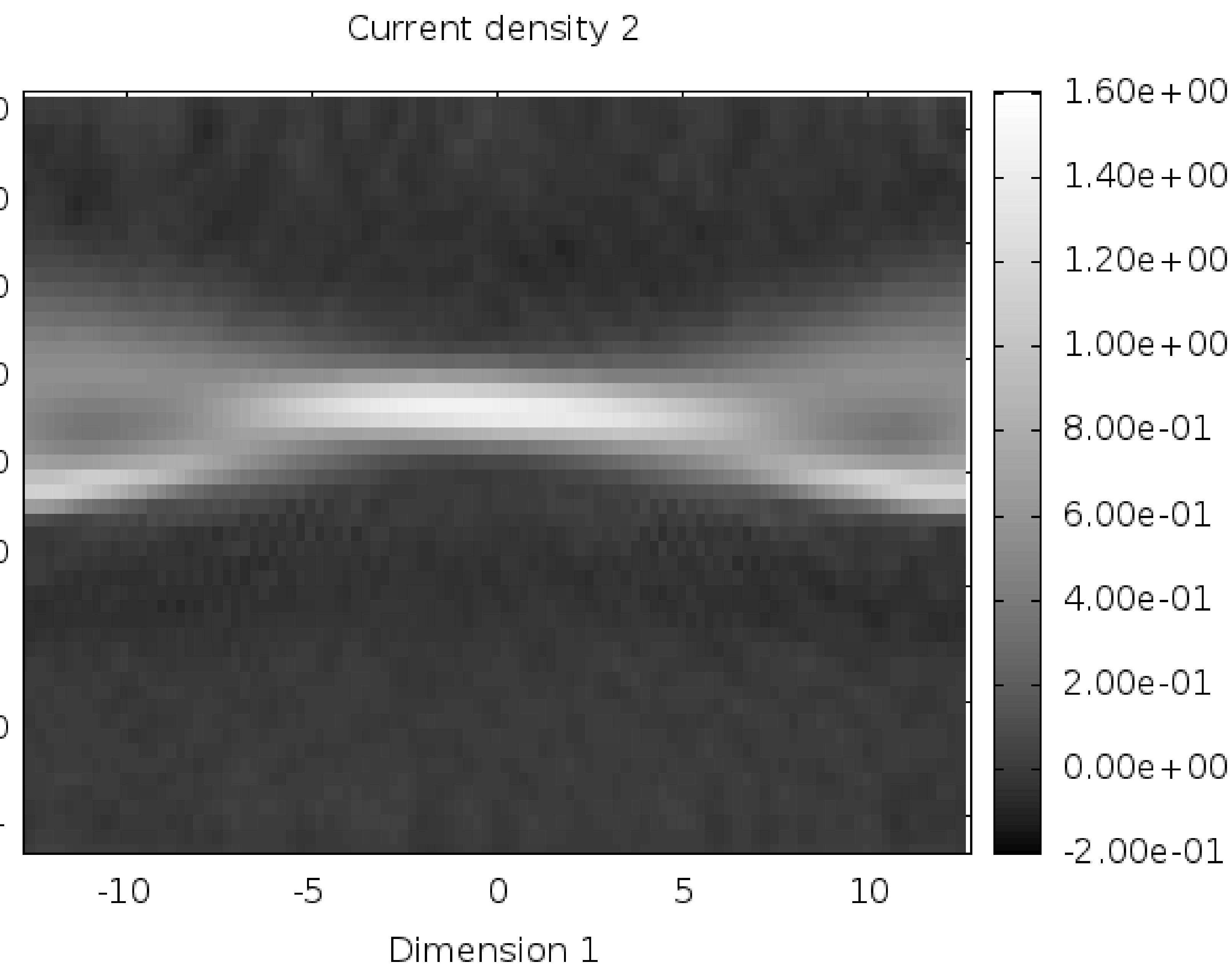
Reconnection part 4

- Z < -3 && Z > 3: all mass in fluid
- -3 < Z < 3: fraction of mass in particles

Out of plane current, fluid only



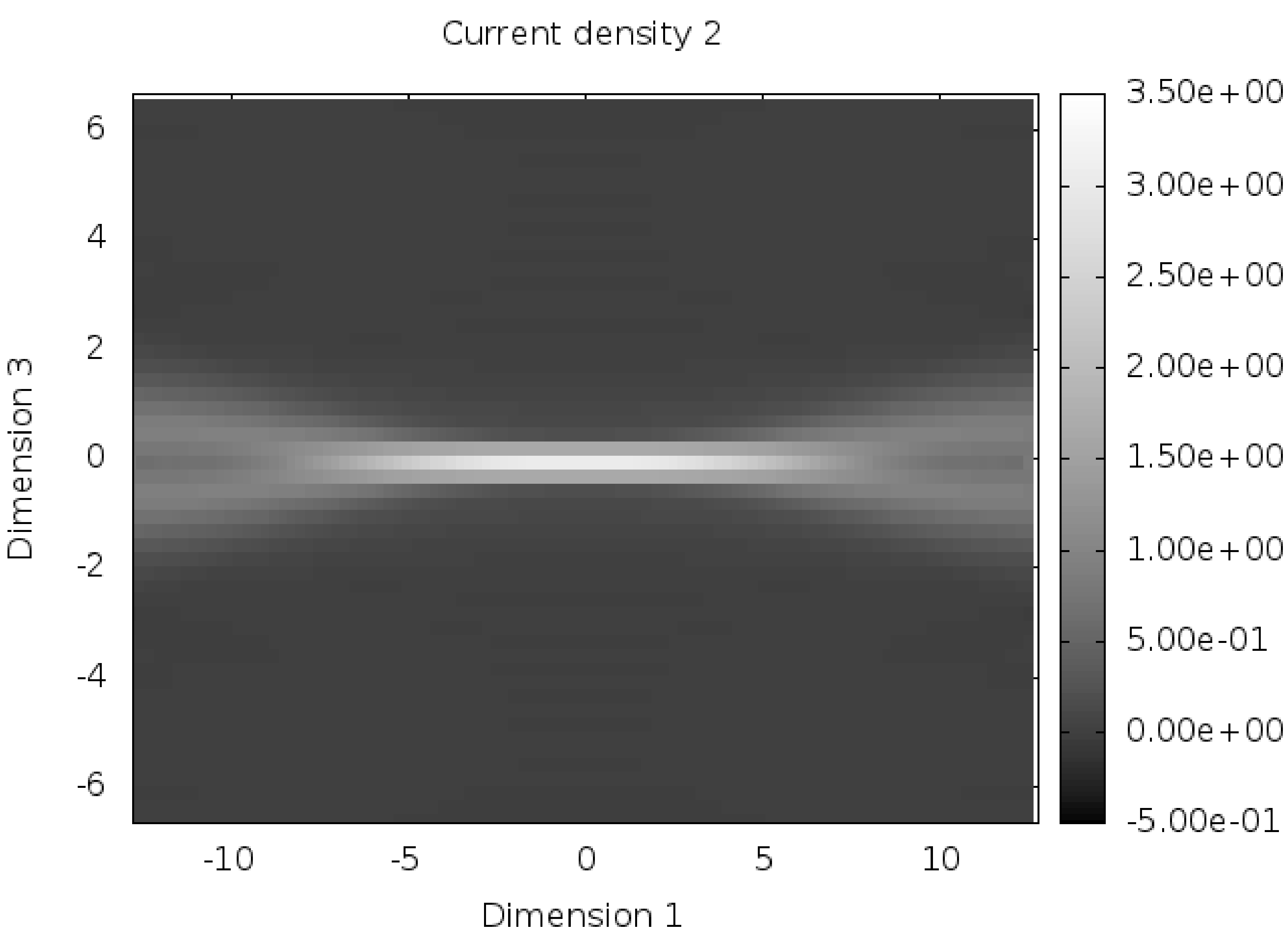
Out of plane current at 15 for 25% particles



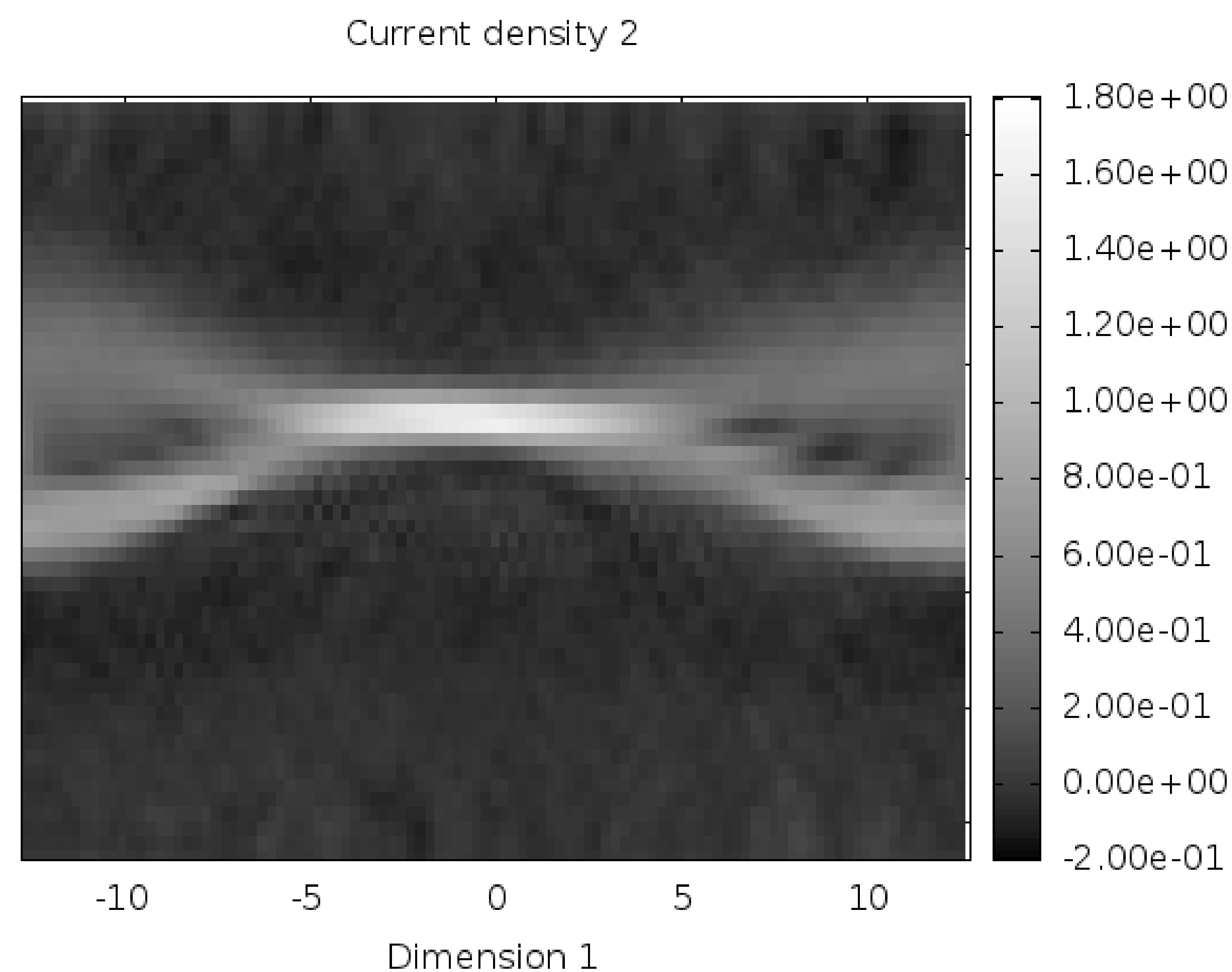
Reconnection part 5

- Z < -3 && Z > 3: all mass in fluid
- -3 < Z < 3: fraction of mass in particles

Out of plane current, fluid only



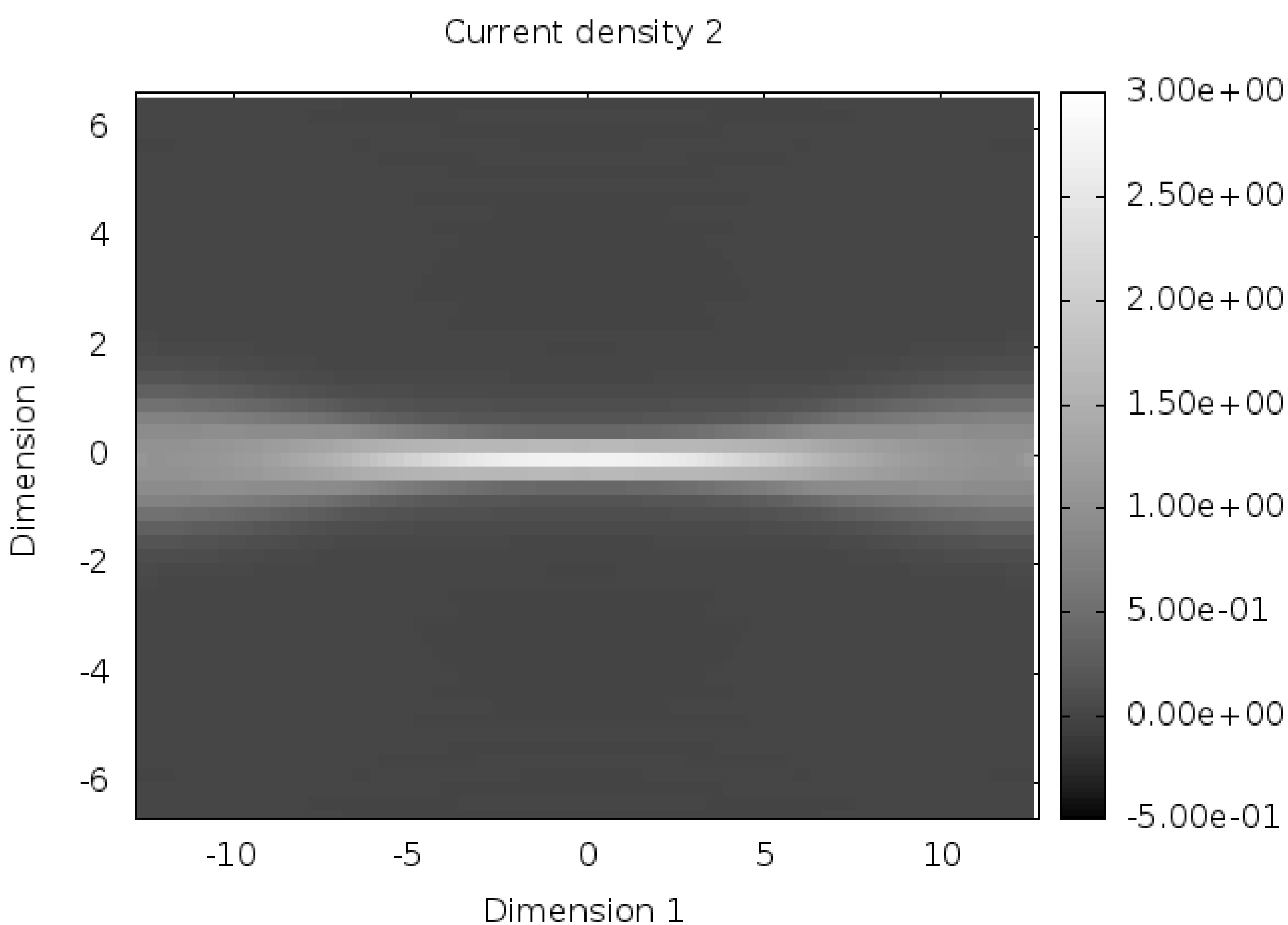
Out of plane current at 15 for 50% particles



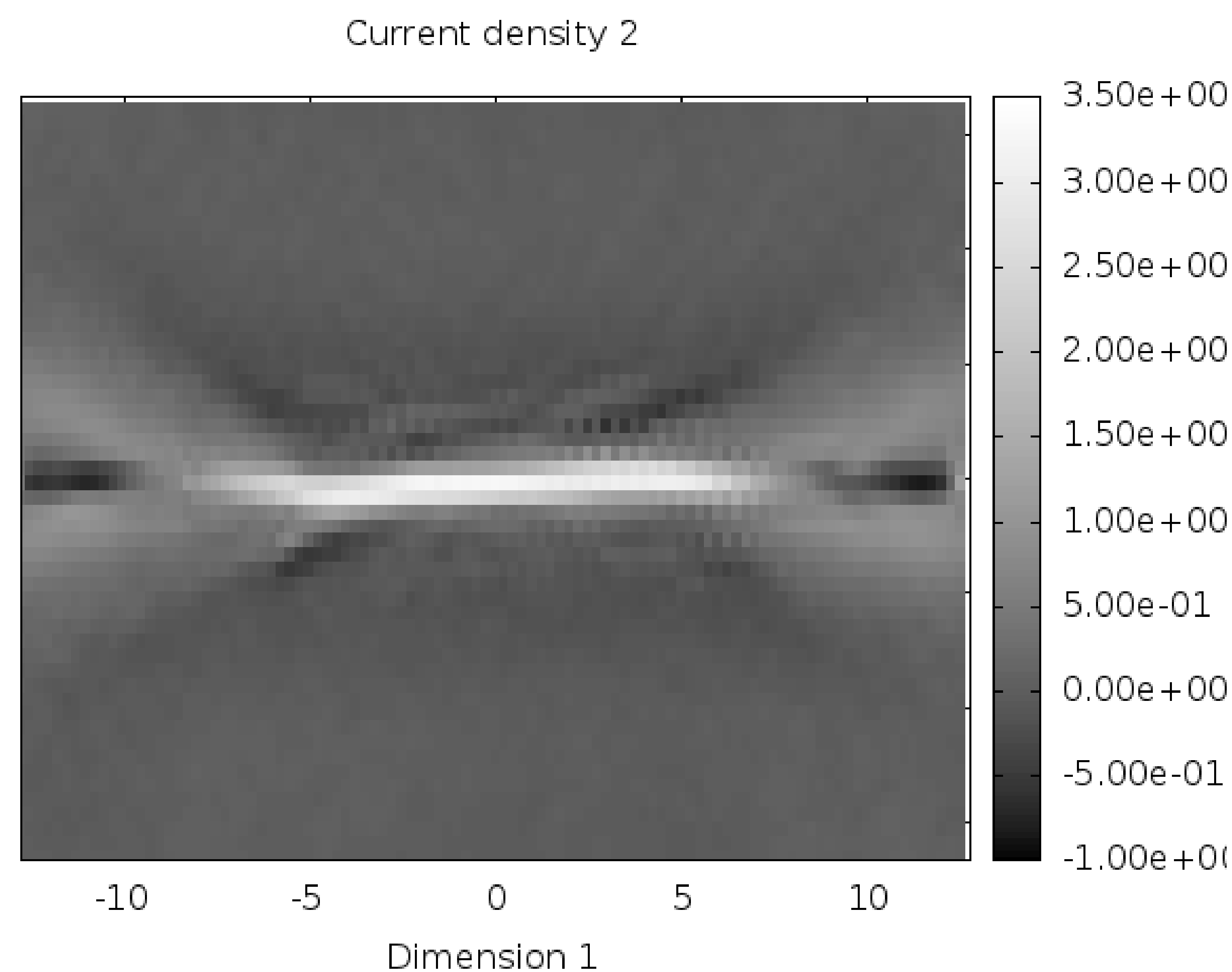
Reconnection part 6

- Z < -3 && Z > 3: all mass in fluid
- -3 < Z < 3: fraction of mass in particles

Out of plane current at 10, fluid only



Out of plane current at 10 for 75% particles



Conclusions

- New method for including kinetic effects of ions into an MHD model
  - Allows a smooth transition between fluid and kinetic physics
- Source code freely available at https://github.com/nasailja/pamhd for anyone to download, use, study, discuss, modify and redistribute
  - Process of making the model available through CCMC is ongoing
- Tests show that:
  - Effect of particle noise can be reduced by using fluid as background
  - Behavior of system seems reasonable when plasma mass divided into both particles and fluid, ”linear” type transition in case of two-stream instability
  - Differences in numerical diffusion of fluid and particles can lead to undesired effects
    - Could be solved by a sharp stationary boundary between fluid & particles as currently used in many models

Example flux calculation

Density flux calculation when using HLL MHD solver, A is area of interface, dt is length of time step, b is maximum signal speed in a cell on negative or positive side of interface, b+ > 0, b- < 0

$$F_{\rho -}(i-1/2) = \rho(i-1)\frac{b_+(i-1/2)}{b_+(i-1/2)-b_-(i-1/2)}[v_x(i-1)-b_-(i-1/2)]A\Delta t$$

$$F_{\rho +}(i-1/2) = \rho(i)\frac{b_-(i-1/2)}{b_-(i-1/2)-b_+(i-1/2)}[v_x(i)-b_+(i-1/2)]A\Delta t$$

$$F_{\rho -}(i+1/2) = \rho(i)\frac{b_+(i+1/2)}{b_+(i+1/2)-b_-(i+1/2)}[v_x(i)-b_-(i+1/2)]A\Delta t$$

$$F_{\rho +}(i+1/2) = \rho(i+1)\frac{b_-(i+1/2)}{b_-(i+1/2)-b_+(i+1/2)}[v_x(i+1)-b_+(i+1/2)]A\Delta t$$