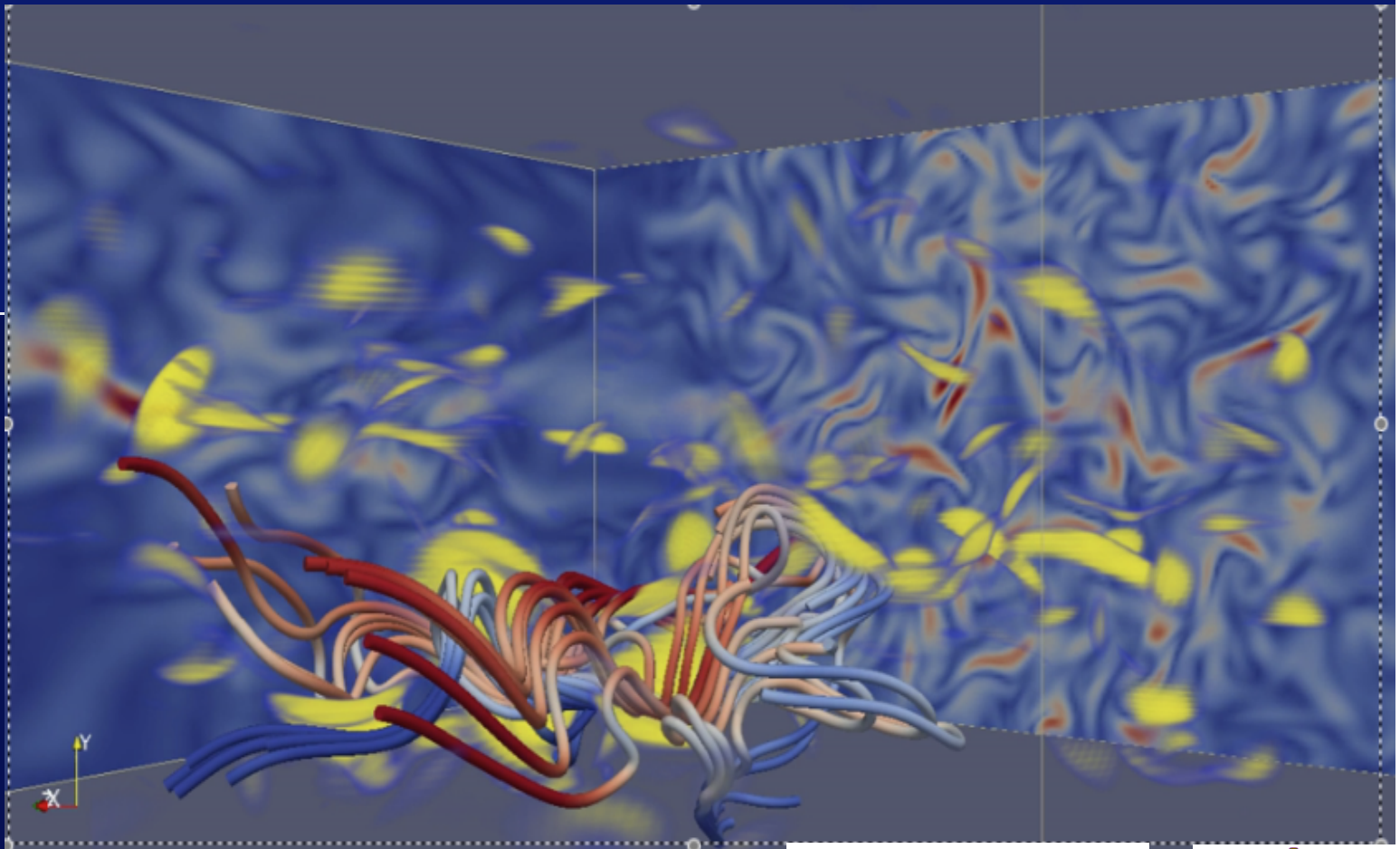


# Non-relativistic and relativistic turbulent reconnection



**Alex Lazarian**

*Special Thanks to G. Eyink, G. Kowal, M. Takamoto, E. Vishniac*



Alexander von Humboldt  
Stiftung/Foundation



THE UNIVERSITY  
of  
**WISCONSIN**  
MADISON

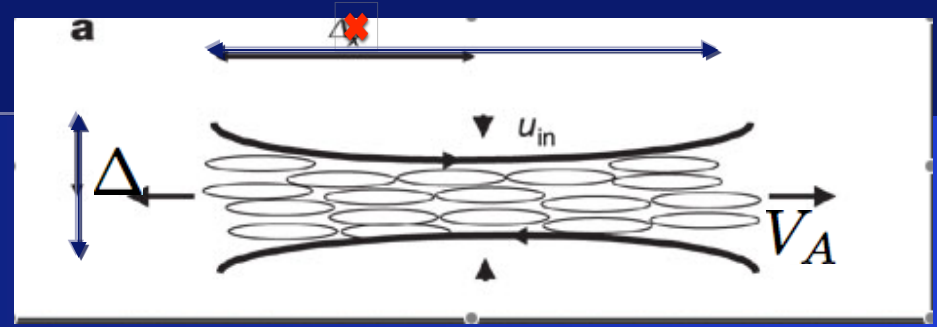
# Plasmoids/tearing is a transient regime transferring to fully turbulent reconnection in 3D

$$S = \frac{LV_A}{\eta}$$

$$Re = \frac{\Delta V_A}{\nu}$$

$$\Delta = L \frac{V_{rec}}{V_A}, \text{ i.e. } \Delta \propto S$$

$$S \rightarrow \infty \text{ means } Re \rightarrow \infty$$




Sweet-Parker happened to be a transient reconnection up to  $S=10^4$ . After that tearing happens. Fast reconnection means that the outflow thickness  $\Delta$  grows in proportion to  $S$ . Thus the Reynolds number  $Re = \frac{\Delta V_A}{\nu}$  of the outflow grows as  $S$ . This entails to the transition to turbulent regime.

Turbulence is known to suppress the instabilities and therefore one expects tearing to be suppressed. If turbulence does not make reconnection fast then  $\Delta$  will stop growing after a critical  $Re$  is achieved. Thus reconnection would not be fast and would scale as  $1/S$ .

Many phenomena require reconnection larger than the 0.01 or even 0.1 of  $V_A$ . Tearing cannot provide this!

# Ubiquitous turbulence controls non-relativistic and relativistic magnetic reconnection

- 
- A vertical white line is positioned on the left side of the slide. Three spheres are placed along this line: a large blue sphere at the top, and two smaller gold spheres below it. The gold spheres are positioned next to the first two list items.
- Turbulent reconnection in incompressible fluids
  - Relativistic turbulent reconnection
  - Turbulent self-sustained reconnection

# Ubiquitous non-relativistic and relativistic turbulence controls magnetic reconnection

- Turbulence
- Relativistic
- Turbulence



uids



# **Turbulence was considered in terms of reconnection, with interesting possibilities discussed**

**Microturbulence affects the effective resistivity by inducing anomalous effect**

**Some papers which attempted to go beyond this:**

**Speizer (1970) --- effect of line stochasticity in collisionless plasmas**

**Jacobs & Moses (1984) --- inclusion of electron diffusion perpendicular mean B**

**Strauss (1985), Bhattacharjee & Hameiri (1986) --- hyperresistivity**

**Matthaeus & Lamkin (1985) --- numerical studies of 2D turbulent reconnection**

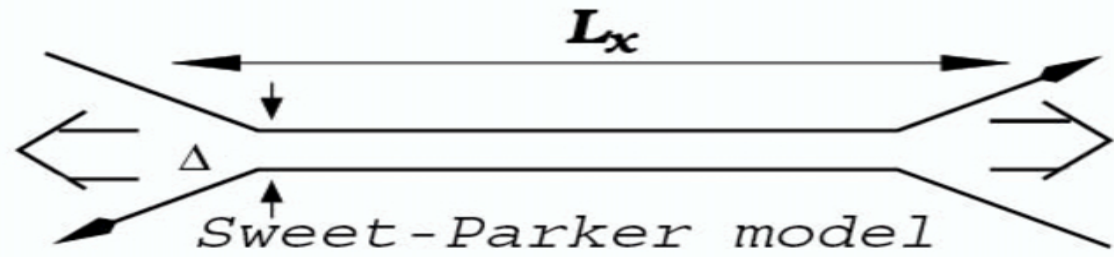
**On the contrary, Kim & Diamond (2001) conclude that turbulence makes any reconnection slow, irrespective of the local reconnection rate**

**Boozer (2013) claim about 3D requirement for fast reconnection**

# LV99 model extends Sweet-Parker model for turbulent astrophysical plasmas and makes reconnection fast

## Turbulent reconnection:

Outflow is determined by field wandering.



$$V_{rec} = V_A \frac{\Delta}{L_x}$$

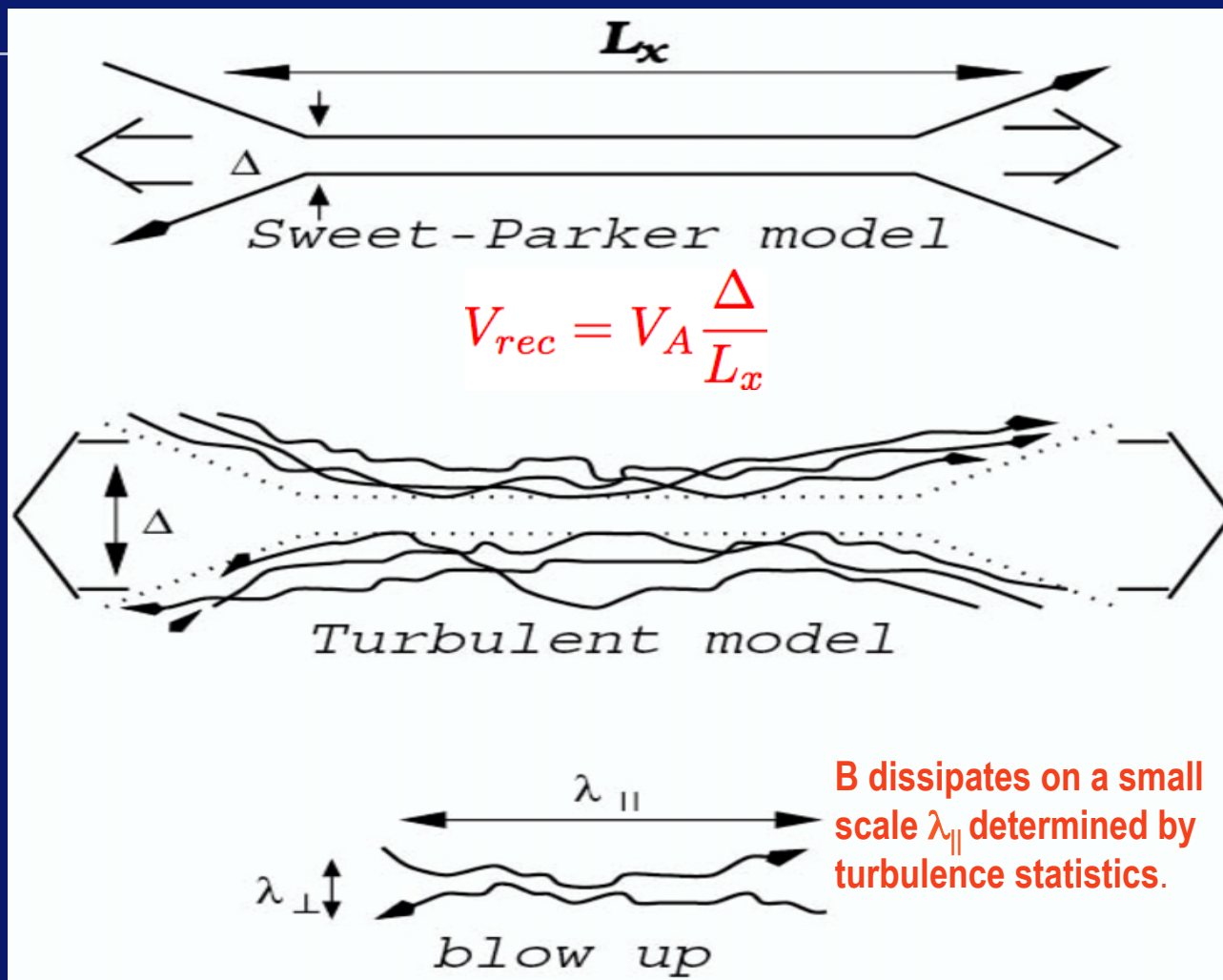
AL & Vishniac (1999)

henceforth referred to as LV99

# LV99 model extends Sweet-Parker model for turbulent astrophysical plasmas and makes reconnection fast

## Turbulent reconnection:

Outflow is determined by field wandering.



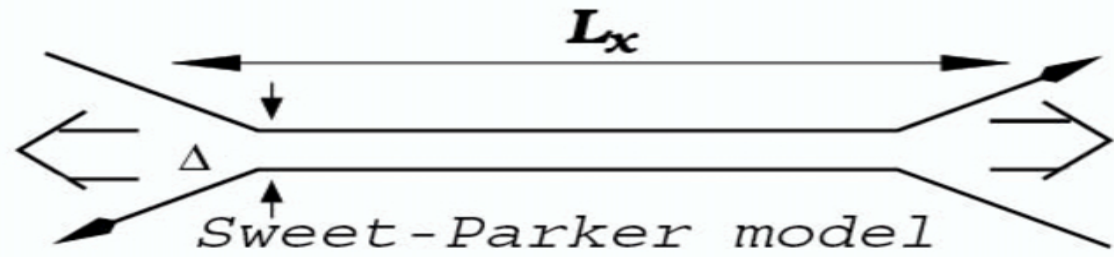
**AL & Vishniac (1999)**

henceforth referred to as LV99

# LV99 model extends Sweet-Parker model for turbulent astrophysical plasmas and makes reconnection fast

## Turbulent reconnection:

Outflow is determined by field wandering.



Without turbulence:

molecular diffusion coefficient  $D \sim 10^{-5} \text{ cm}^2/\text{sec}$   
( $\leftarrow$  It's for small molecules in water.)

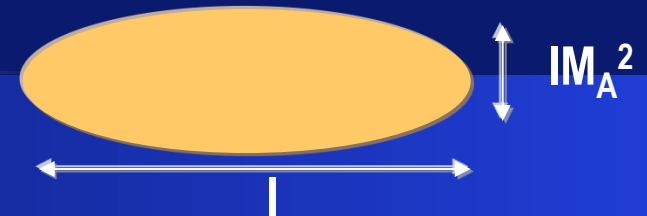
$\rightarrow$  Mixing time  $\sim (\text{size of the cup})^2/D \sim 10^7 \text{ sec} \sim 0.3 \text{ year} !$

**AL & Vishniac (1999)**

henceforth referred to as LV99

# The outflow region is determined by wandering magnetic field lines

For subAlfvenic turbulence eddies are elongated



If injection scale  $l$  is less than  $L_x$  then the magnetic field lines undergo random walk with  $IM_A^2$  and outflow thickness is

$$\Delta = (L_x/l)^{1/2} l M_A^2 = (L_x l)^{1/2} (v_l/V_A)^2$$

Mass conservation

$$\rho_{in} V_{rec} L_x = \rho_s v_s \Delta$$

Which gives

$$V_{rec} = (\rho_s/\rho_{in}) (l/L_x)^{1/2} (v_l/V_A)^2$$

For incompressible fluid LV99 provide:

$$V_{rec} = \min[(L_x/l)^{1/2}, (l/L_x)^{1/2}] (v_l/V_A)^2$$

# Eyink, AL & Vishniac 2011 related LV99 to the well-known concept of Richardson diffusion



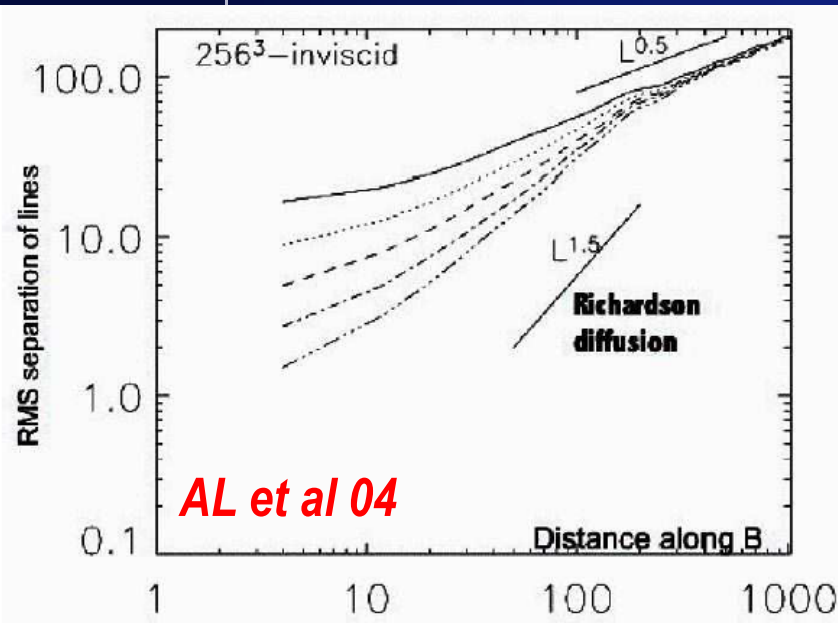
$$\langle |\mathbf{x}_1(t) - \mathbf{x}_2(t)|^2 \rangle \sim t^3$$

Richardson's law

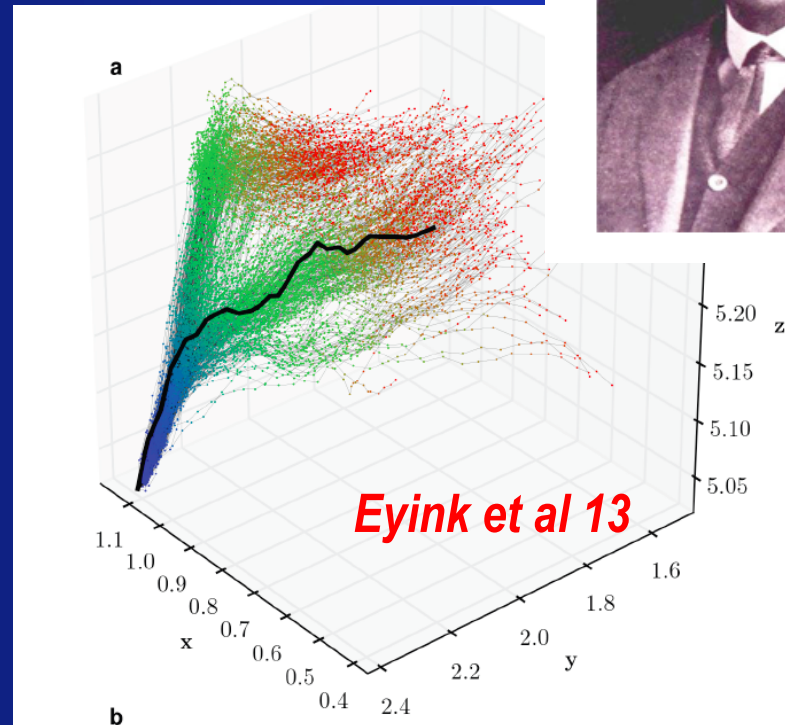


# Eyink, AL & Vishniac 2011 related LV99 to the well-known concept of Richardson diffusion

## *Richardson diffusion measured in MHD*



*Diffusion in space*

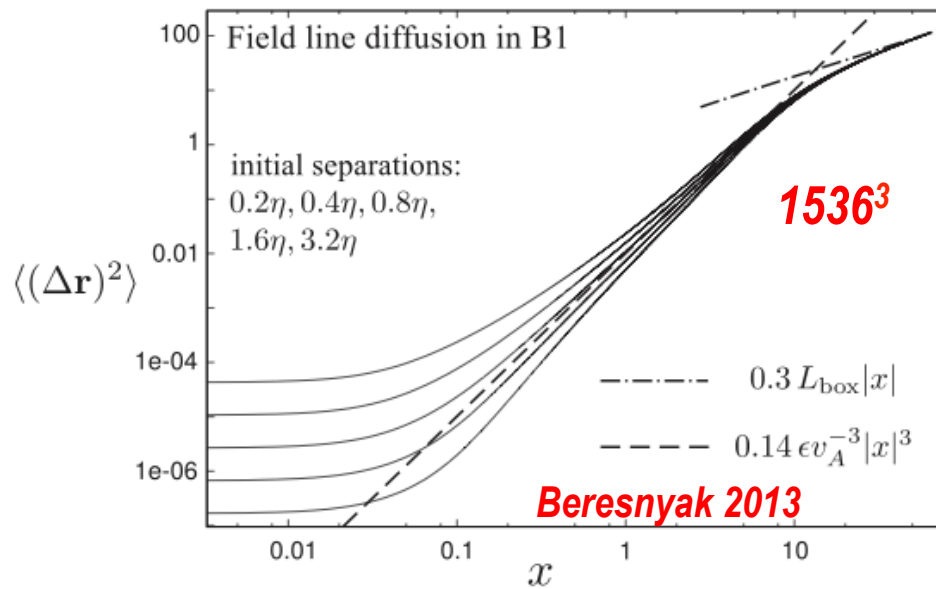


*Diffusion in time*

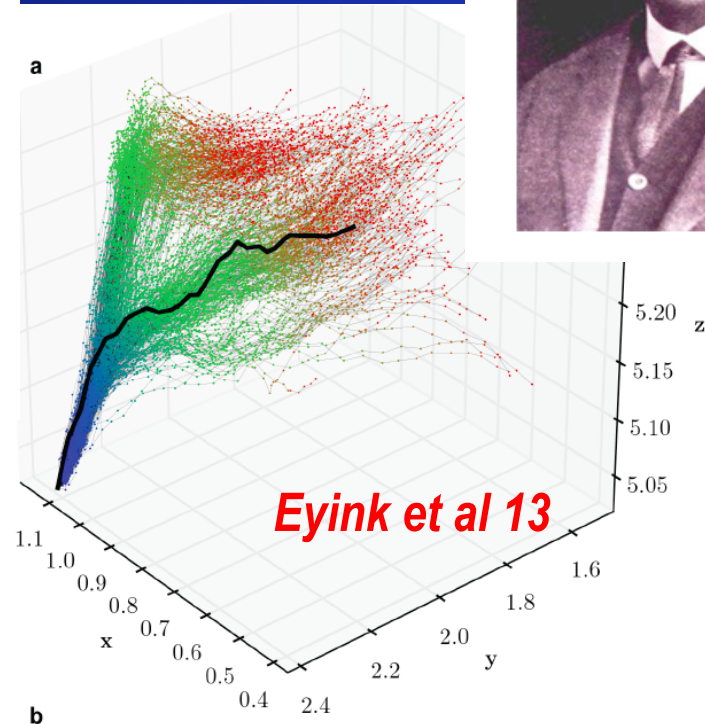


# Eyink, AL & Vishniac 2011 related LV99 to the well-known concept of Richardson diffusion

## Richardson diffusion measured in MHD



Diffusion in space



Diffusion in time



# New theoretical study of Eyink derives LV99 relations, formulates Generalized Ohm's Law and shows that effects of turbulence dominate those of plasma microphysics

## Turbulent General Magnetic Reconnection

G. L. Eyink

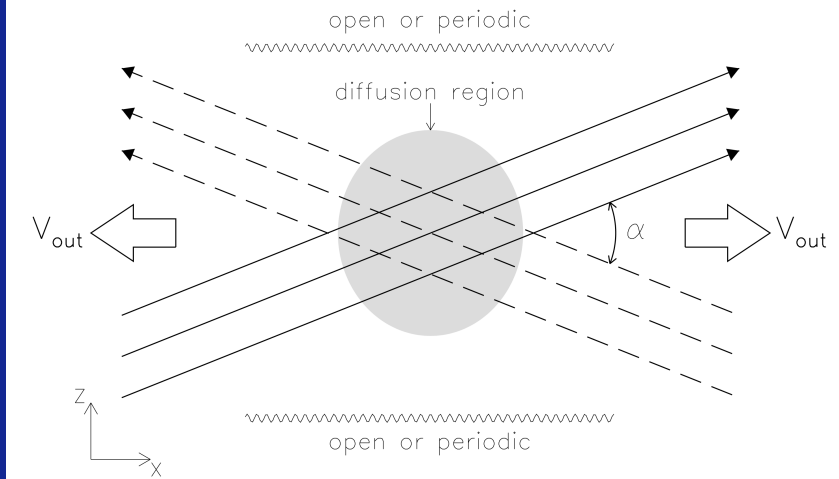
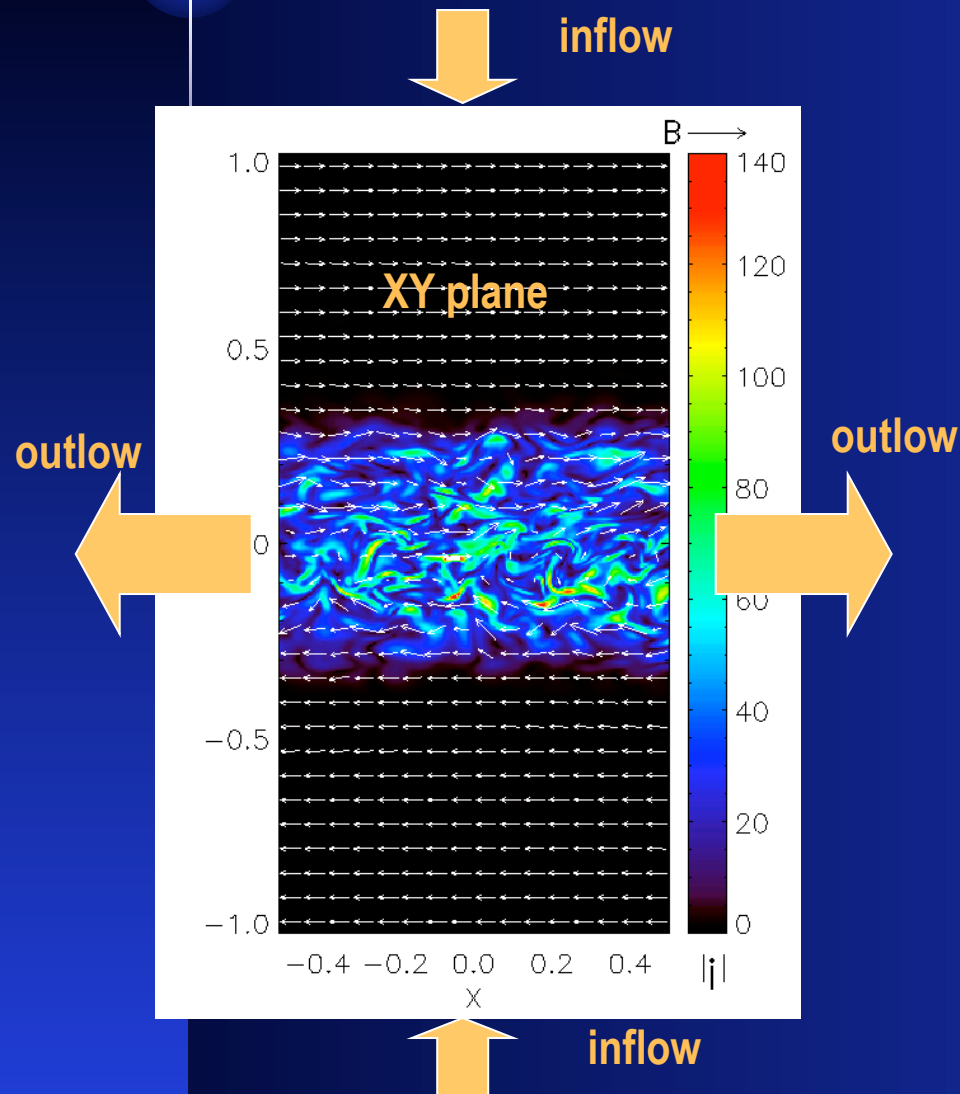
*Department of Applied Mathematics & Statistics and Department of Physics & Astronomy, The Johns Hopkins University, Baltimore, MD 21218*

### ABSTRACT

Plasma flows with an MHD-like turbulent inertial range, such as the solar wind, vitiate many assumptions of standard theories of magnetic reconnection. In particular, the “roughness” of turbulent velocity and magnetic fields implies that magnetic field-lines are nowhere “frozen-in” in the usual sense. This situation demands an essential generalization of the so-called “General Magnetic Reconnection” (GMR) theory. Following ideas of Axford and Lazarian & Vishniac, we identify magnetic field-lines by “tagging” them with plasma fluid elements and then determine their slip-velocity relative to the plasma fluid by integrating in arc-length along the wandering field-lines. The main new concept introduced here is the *slip-velocity source vector*, which gives the rate of development of slip-velocity per unit arc-length of field line. The slip-source vector is the ratio of the curl of the non-ideal electric field  $\mathbf{R}$  in the Generalized Ohm's Law and the

# NUMERICAL TESTING:

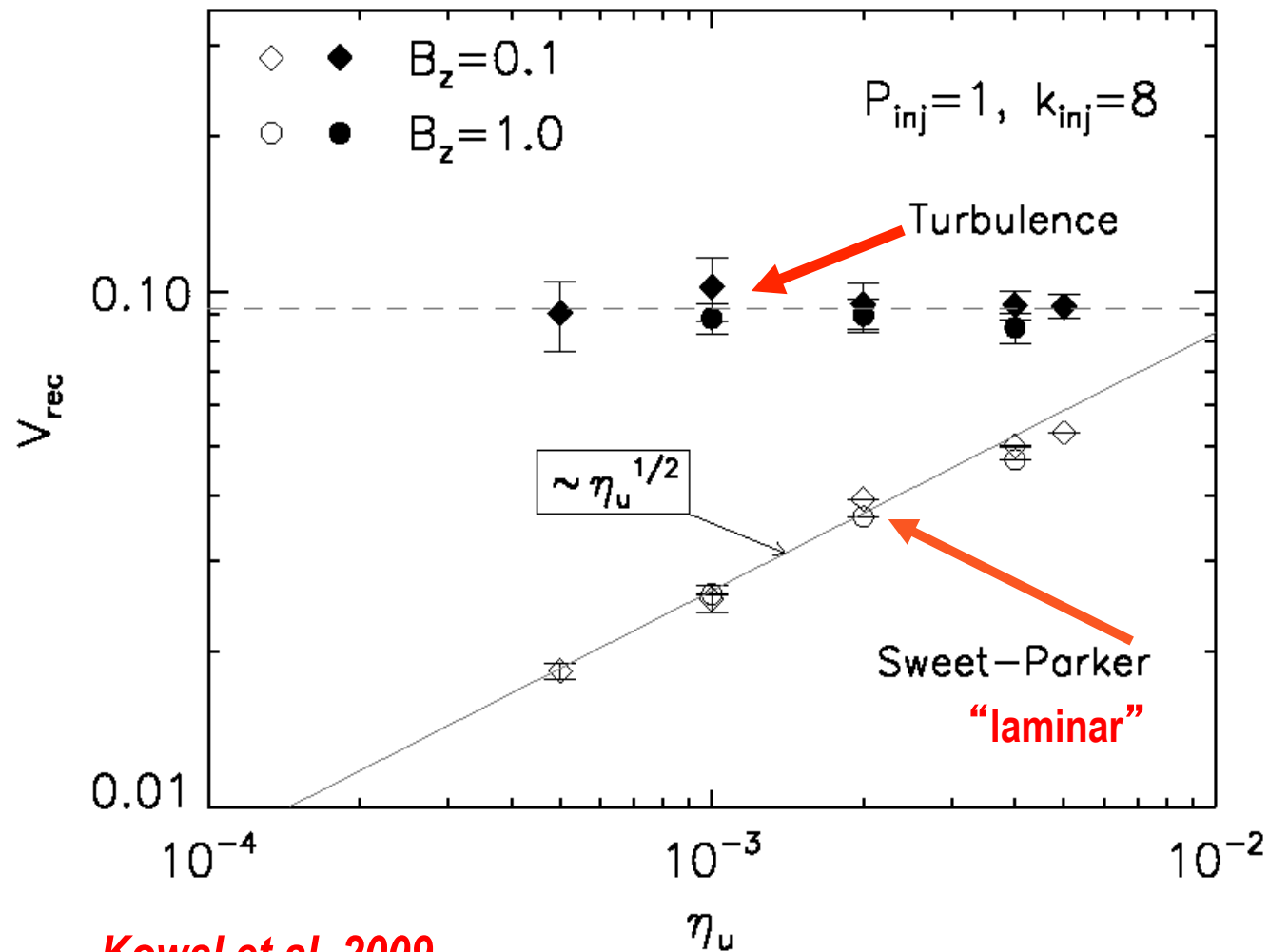
## All calculations are 3D with non-zero guide field



**Magnetic fluxes intersect at an angle**

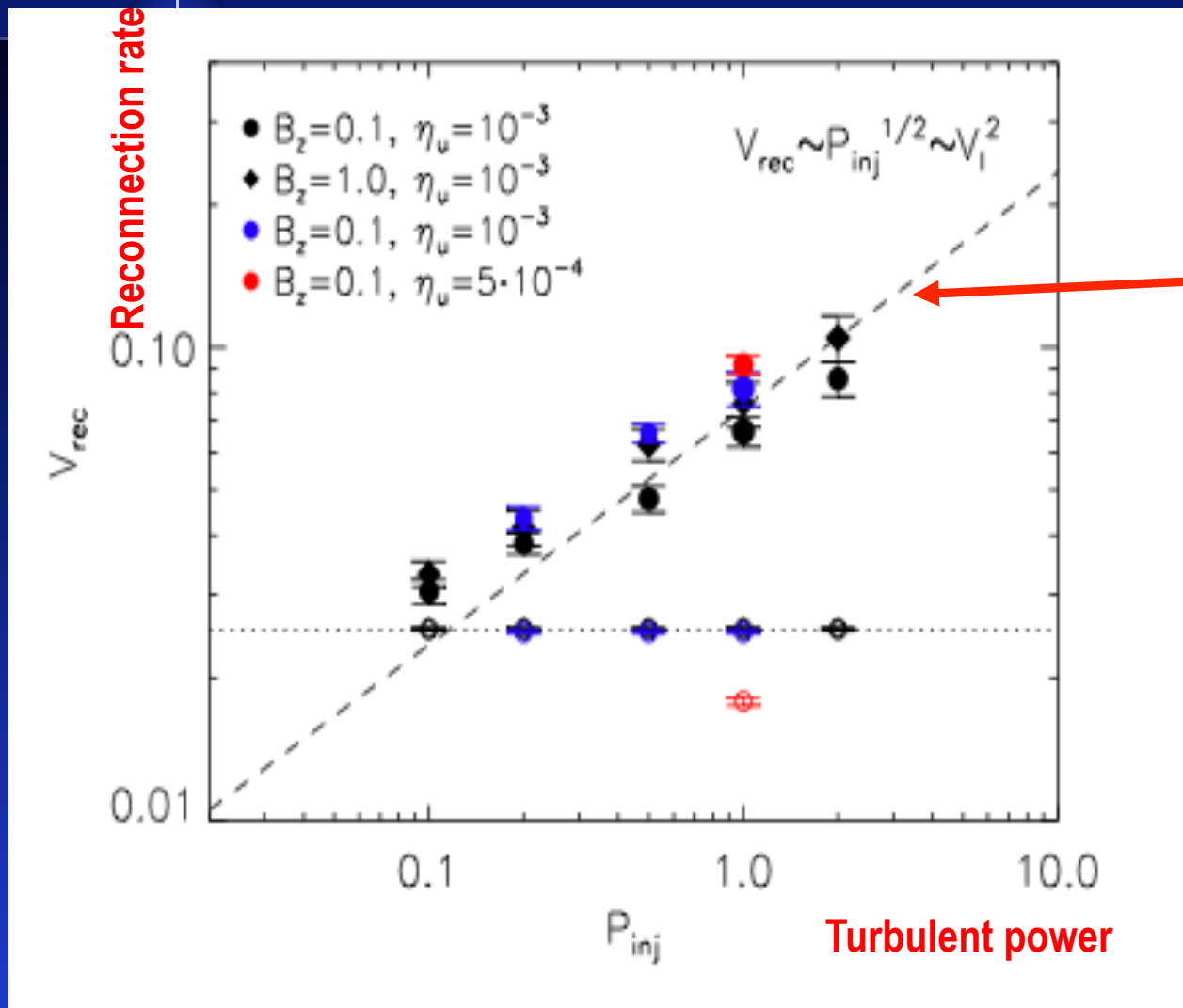
Driving of turbulence:  $r_d=0.4$ ,  $h_d=0.4$  in box units.  
Inflow is not driven.

# Reconnection is Fast: speed does not depend on Ohmic resistivity!



*Kowal et al. 2009*

# The reconnection rate increases with input power of turbulence



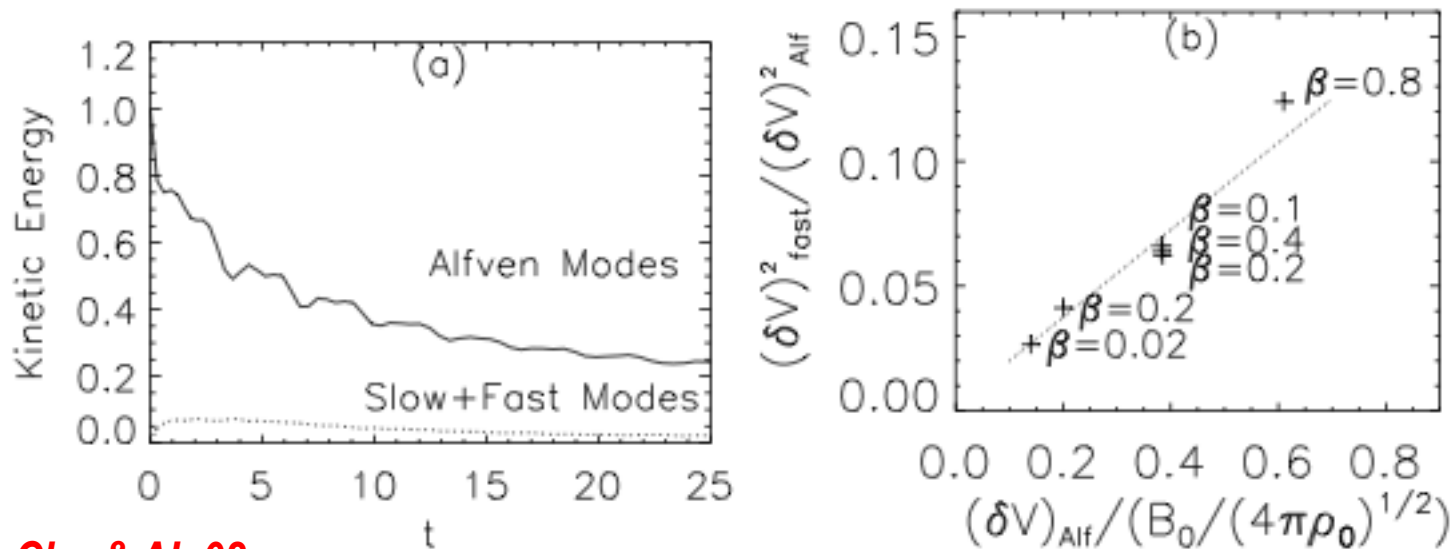
LV99 prediction is  
 $V_{\text{rec}} \sim P_{\text{inj}}^{1/2}$

**BACK to THEORY.** In compressible media  $V_{rec}$  changes with plasma density and energy in Alfvénic component

*For compressible media with injection scale  $l$  less than  $L_x$  LV99 expressions can be generalized:*

$$V_{rec} = v_s (\rho_s / \rho_{in}) (l / L_x)^{1/2} \frac{(v_{total}^2 - v_{comp}^2)}{V_{Alf}^2}$$

# Generation of compressible modes happens for incompressible driving



Cho & AL 02

$$\frac{v_{comp}^2}{V_{Alf}^2} \propto \frac{v_{inj}}{V_{Alf}}$$

Thus

$$\epsilon = \frac{v_l^4}{lV_{Alf}} = \frac{v_{inj}^2}{t_{inj}} \longrightarrow v_{inj} \propto v_l^2$$

$$v_{total}^2 - v_{comp}^2 \approx v_l^2 (1 - C_2(v_{inj}/V_{Alf})) \propto v_{inj} (1 - C_2(v_{inj}/V_{Alf}))$$



# TOWARDS RELATIVISTIC RECONNECTION: Good correspondence between turbulence in relativistic and non-relativistic limits

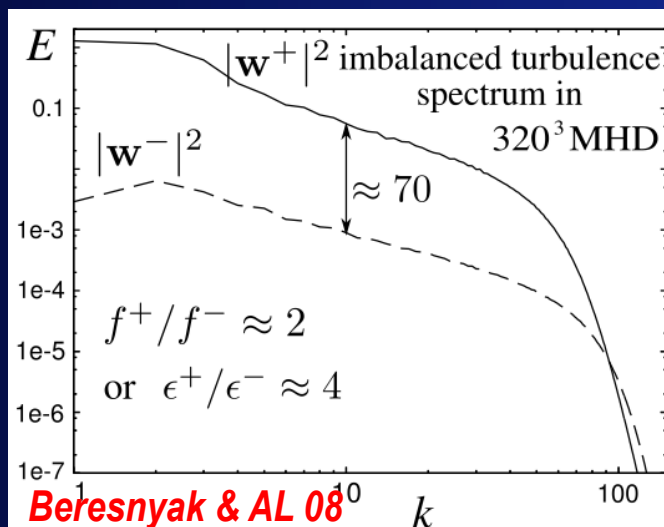
## Non-relativistic

Balanced  
Turbulence

Theory: Goldreich & Sridhar 95  
Numerics: Cho & Vishniac 00  
Maron & Goldreich 01, Cho & AL 02

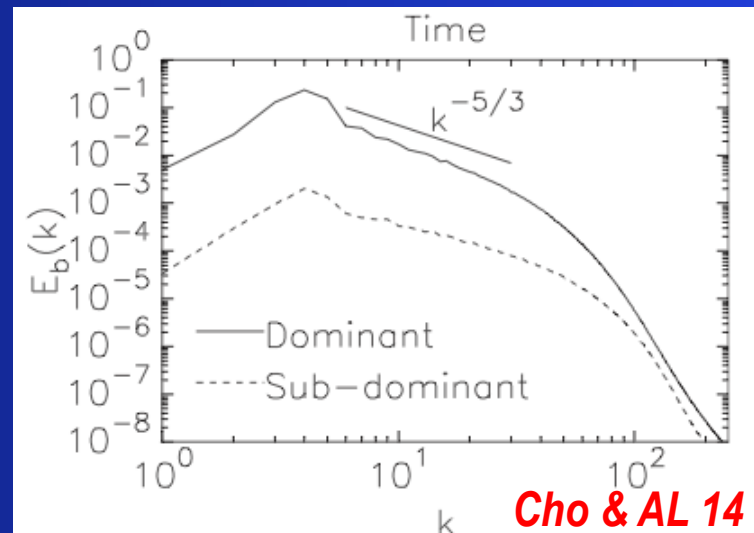
Imbalanced  
Turbulence

Theory: Beresnyak & AL 08



## Relativistic

Theory: Thompson & Blaes 98  
Numerics: Cho 05



# Good correspondence between turbulence in relativistic and non-relativistic limits

## Non-relativistic

Balanced  
Turbulence

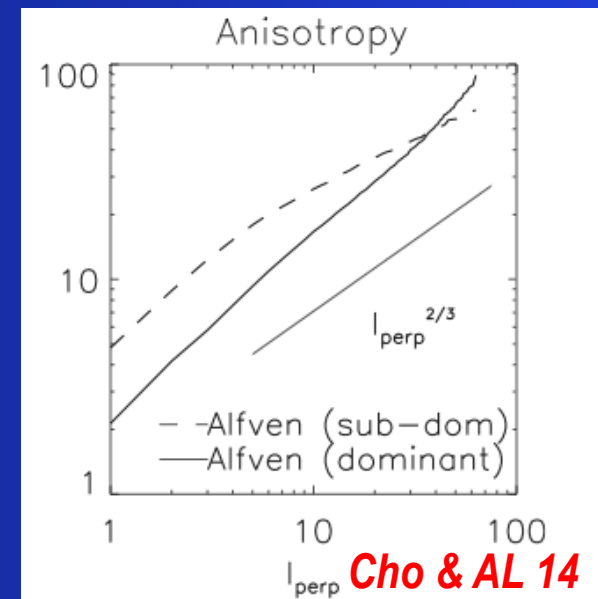
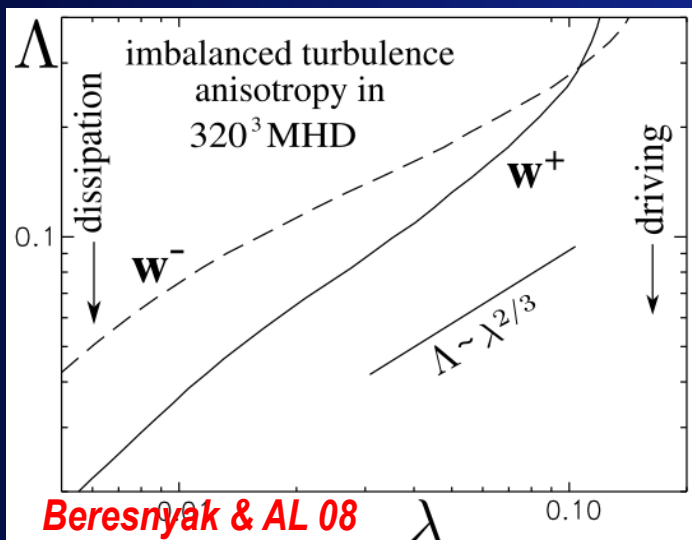
Theory: Goldreich & Sridhar 95  
Numerics: Cho & Vishniac 00  
Maron & Goldreich 01, Cho & AL 02

## Relativistic

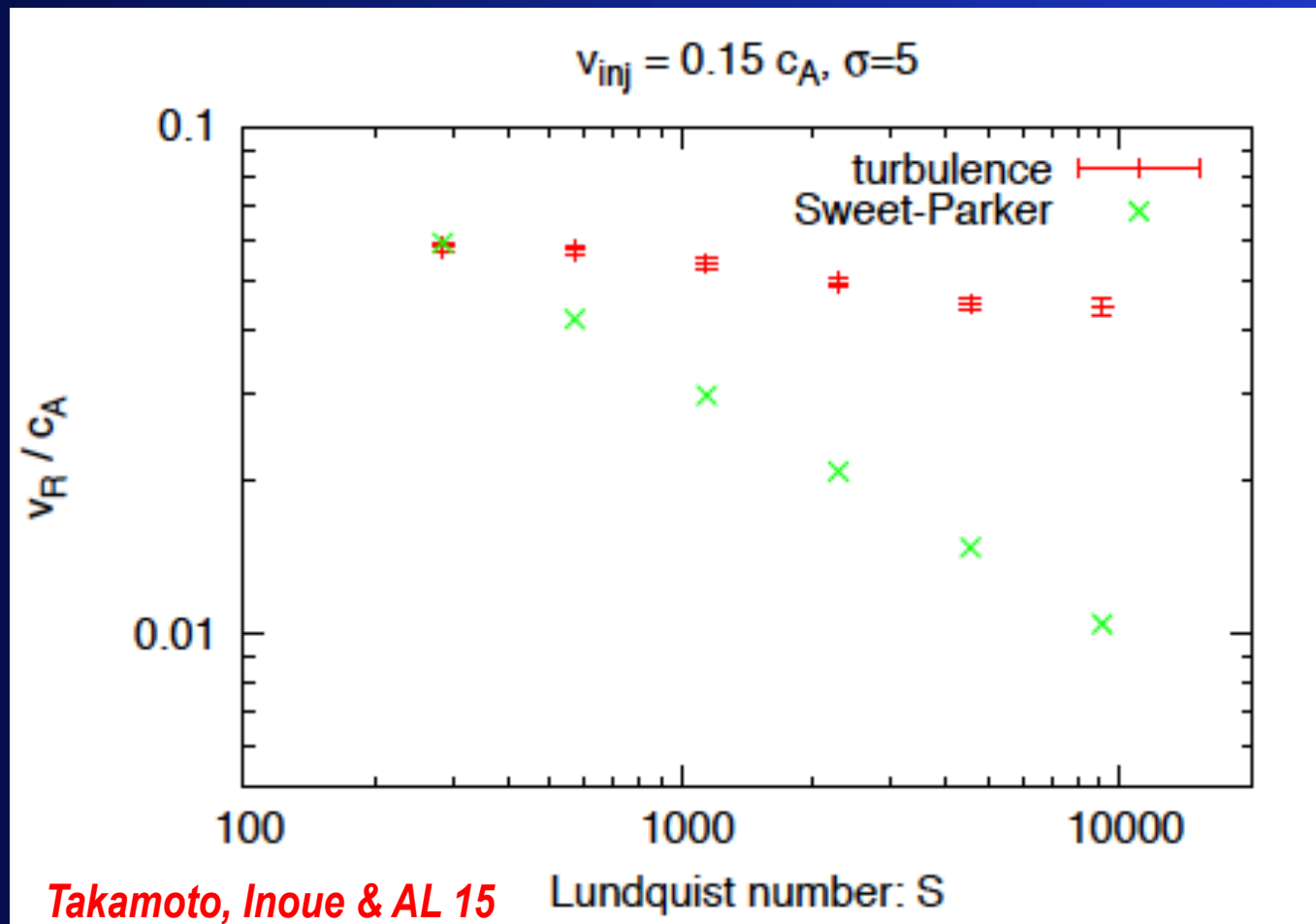
Theory: Thompson & Blaes 98  
Numerics: Cho 05

Imbalanced  
Turbulence

Theory: Beresnyak & AL 08



Simulations show that the relativistic turbulent reconnection is fast, i.e. does not depend on  $S$



# Will LV99 expression be applicable to relativistic reconnection?

*For compressible media with injection scale  $l$  less than  $L_x$  LV99 expression:*

$$V_{rec} = v_s (\rho_s / \rho_{in}) (l / L_x)^{1/2} \frac{(v_{total}^2 - v_{comp}^2)}{c_A^2}$$

*changes*

*changes*

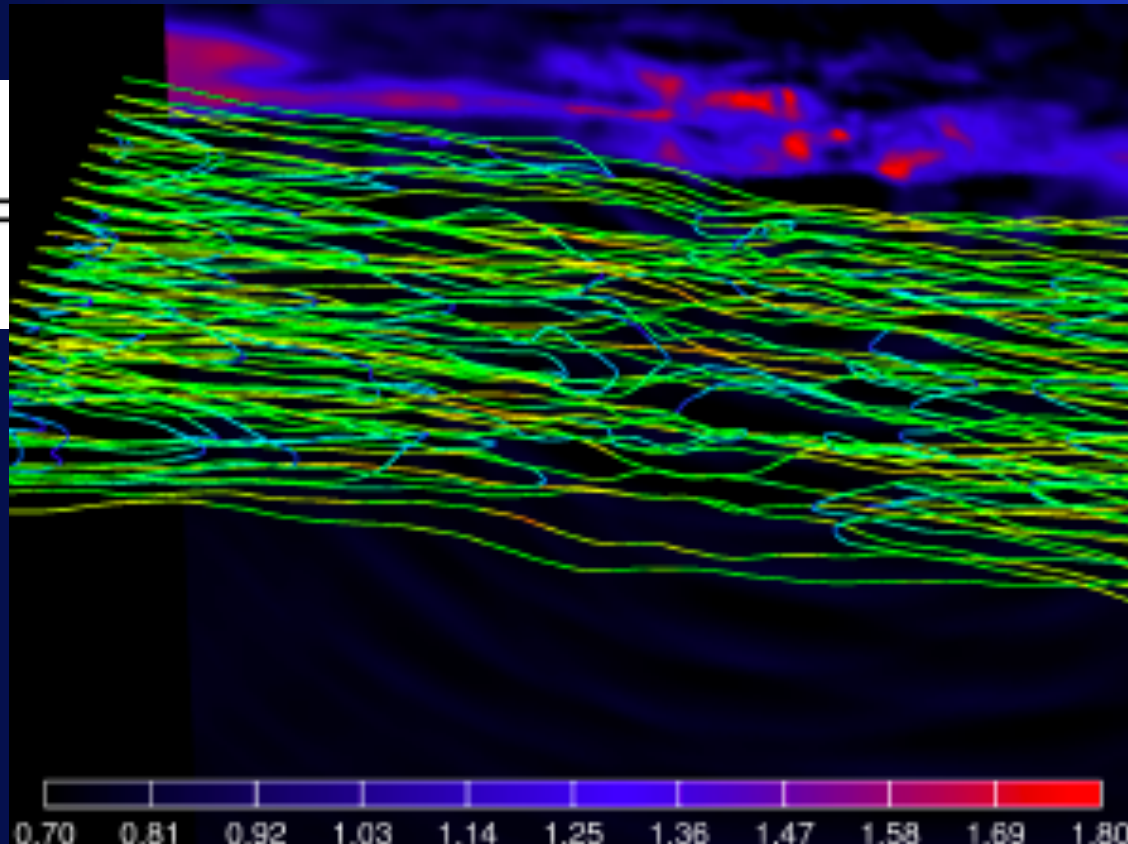
# Will LV99 expression be applicable to relativistic reconnection?

*For compressible media with injection scale  $l$  less than  $L_x$  LV99 expression:*

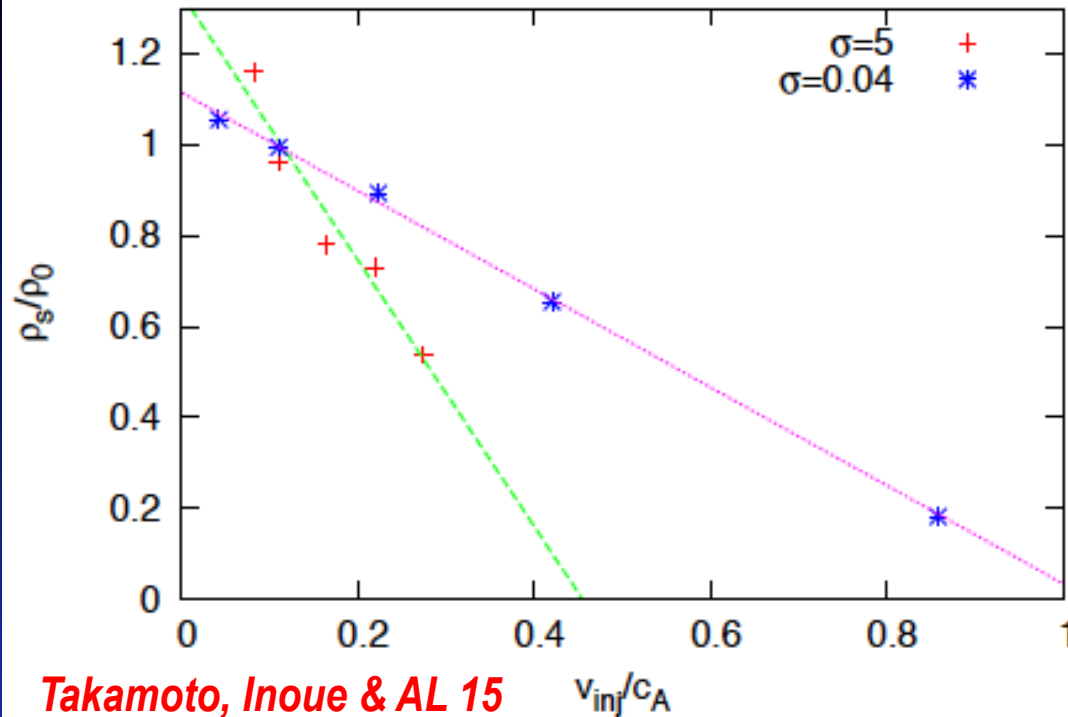
$$V_{rec} =$$

$$\frac{(1 - v_{comp}^2)}{c_A^2}$$

anges



# Density in reconnection region decreases with increasing turbulent velocity



*The density decrease decreases reconnection speed*

*Conservation of energy flux*

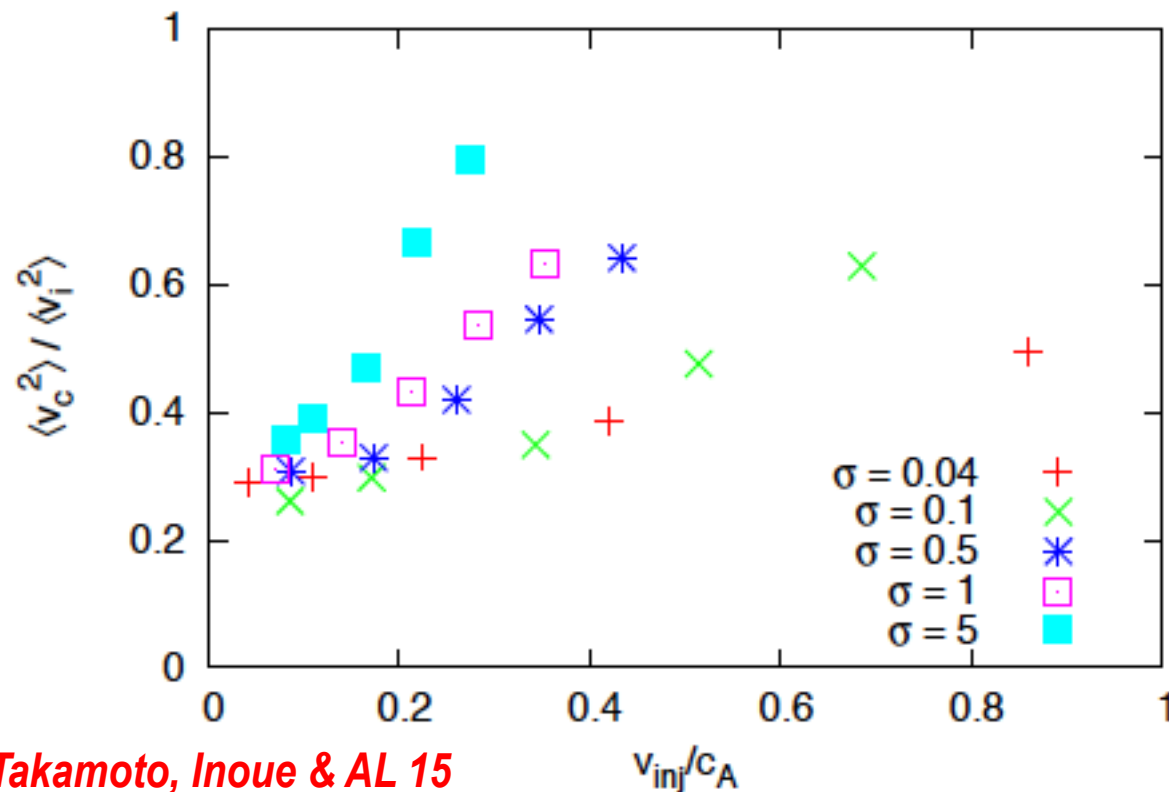
$$\begin{aligned} \rho_{in} c^2 (1 + \sigma) v_{in} L + \rho_{in} (1 + 2\sigma) \epsilon_{inj} l_x l_z \\ = \left( \rho_s h_s c^2 \gamma_s^2 + \frac{B_s^2}{4\pi} \right) v_s \delta. \end{aligned}$$



$$\rho_s / \rho_{in} = \alpha (1 - \beta v_{inj})$$

# The transfer of energy to compressible modes also decreases reconnection speed

$$v_{total}^2 - v_{comp}^2 \approx v_l^2 [1 - C_2(v_{inj}/c_A)]$$



Takamoto, Inoue & AL 15

With

$$\epsilon = \frac{v_l^4}{l c_A} = \frac{v_{inj}^2}{t_{inj}}$$

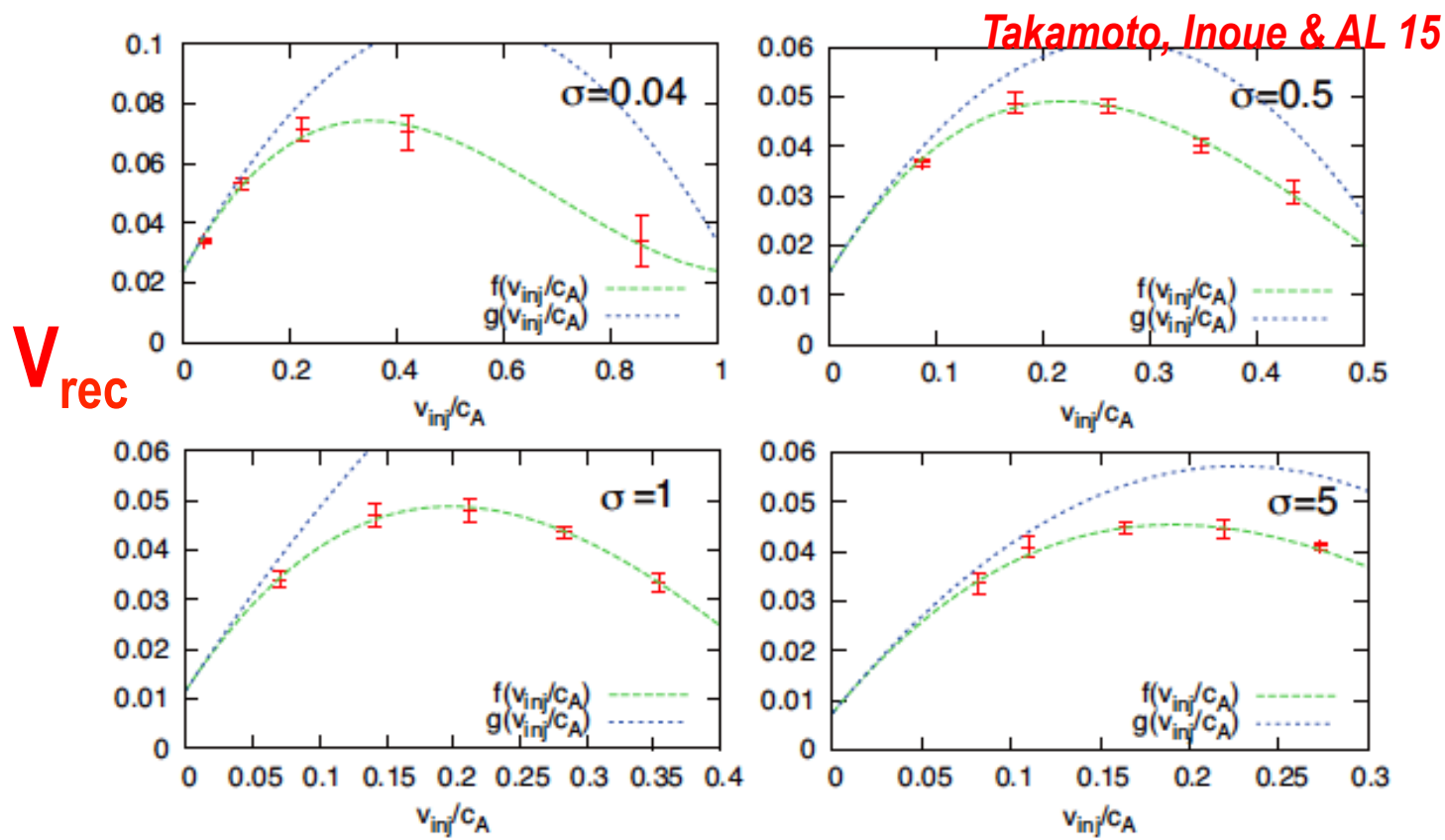
$$v_{inj} \propto v_l^2$$

And squared Alfvénic component

$$\propto v_{inj} [1 - C_2(v_{inj}/c_A)]$$



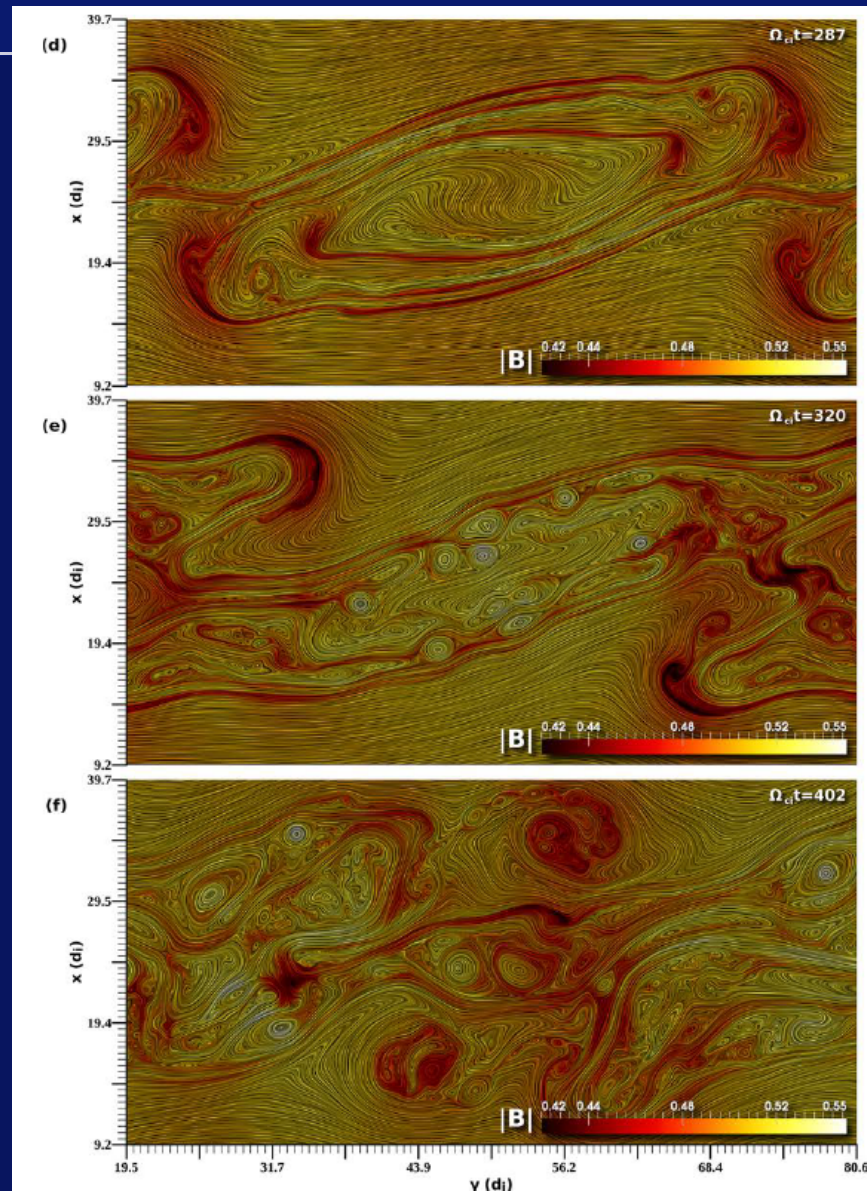
# Relativistic simulations agree well with compressible turbulent reconnection prediction



$$V_{rec} \approx 0.3 c_A (\rho_s / \rho_{in}) (l / L_x)^{1/2} \frac{v_{inj} (1 - C_2 v_{inj} / c_A)}{c_A}$$

Max reconnection  
 $\sim 0.3 c_A$

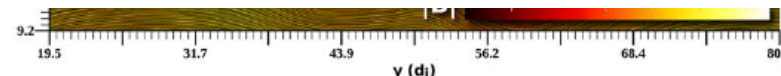
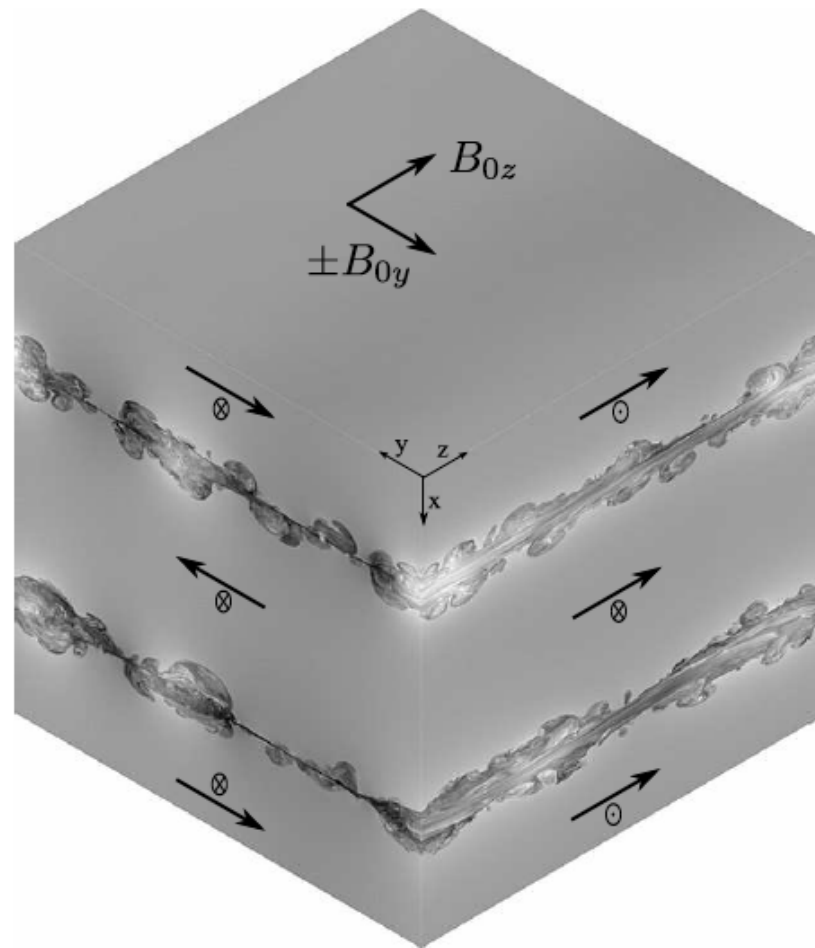
# TURBULENCE INDUCED BY RECONNECTION: Reconnection in 3D PIC simulations results development of turbulence



*Karimabadi et al. 13*

# Reconnection in 3D PIC simulations results development of turbulence

*Development of MHD turbulence is observed in 3D reconnection simulations by Beresnyak 13*

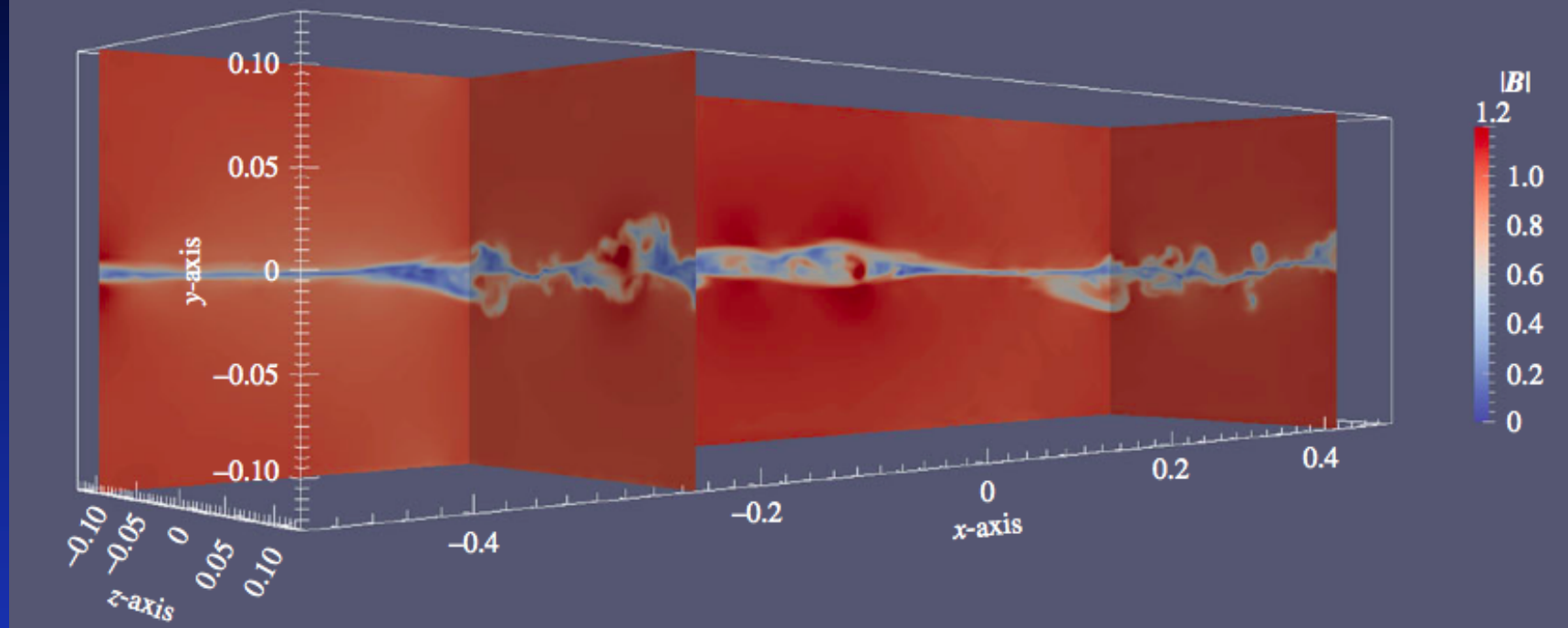


Also Oishi et al. 15 confirmed LV99 predictions

*Karimabadi et al. 13*

# Simulations demonstrate the development of turbulence through Kelvin-Helmholtz instability

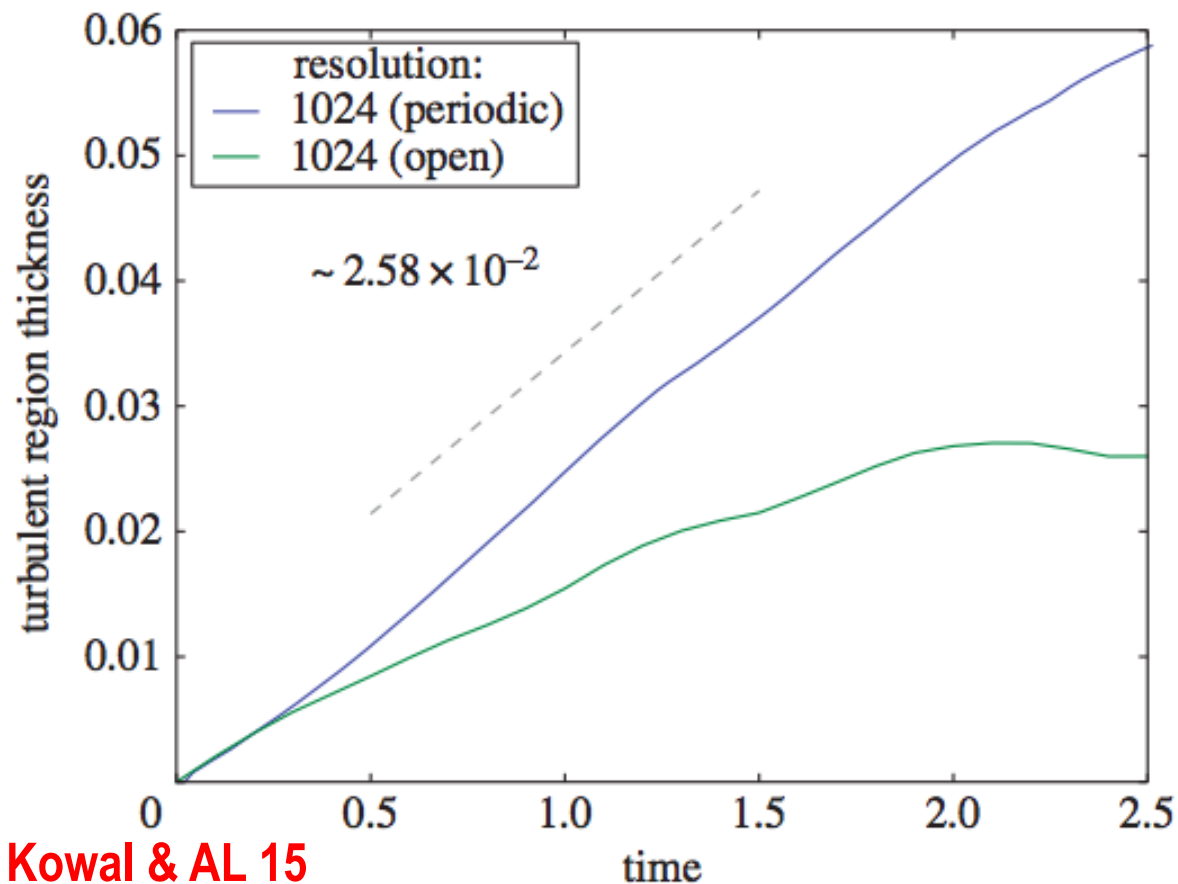
Kowal, AL & Falceta-Gonsavles 15



$$V_{\Delta} \approx (C_K r_A)^{3/4} V_{Ay} \beta^{1/2}$$

Expected reconnection rate,  $C_K$  is Kolmogorov constant,  $r_A$  is magnetization

# Self-sustained reconnection exhibits rates consistent with the predictions of LV99 theory



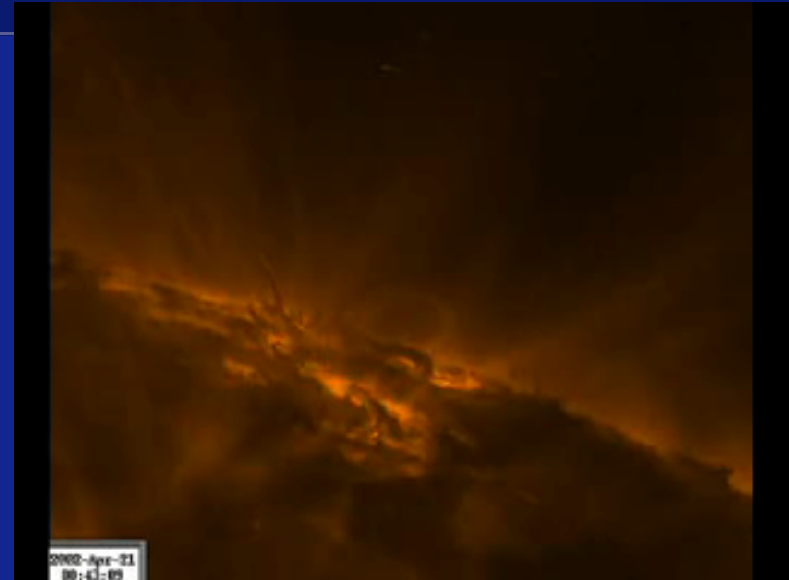
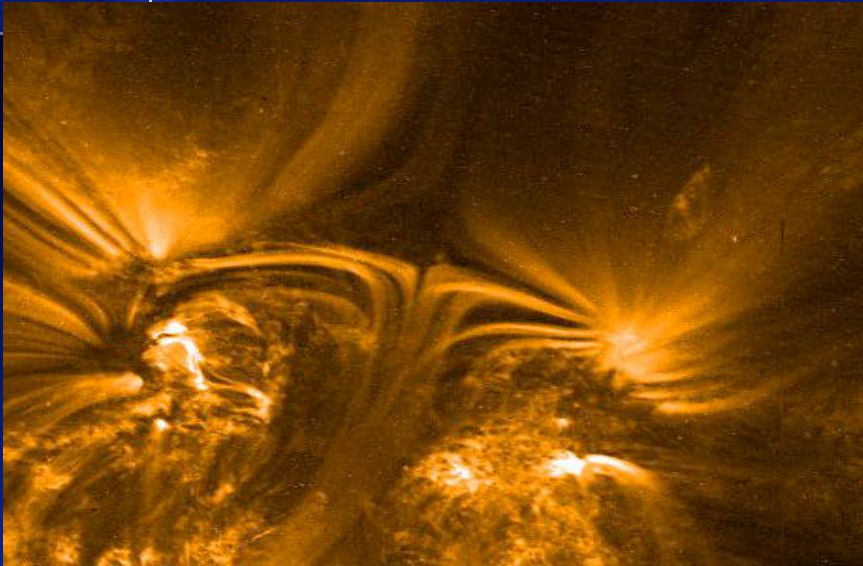
Kowal & AL 15

$$V_{\text{REC}} \sim 0.02 V_A$$

Linear growth of the layer with periodic boundaries is explained in AL et al. 15  
Slower growth and saturation are predicted with open boundary conditions



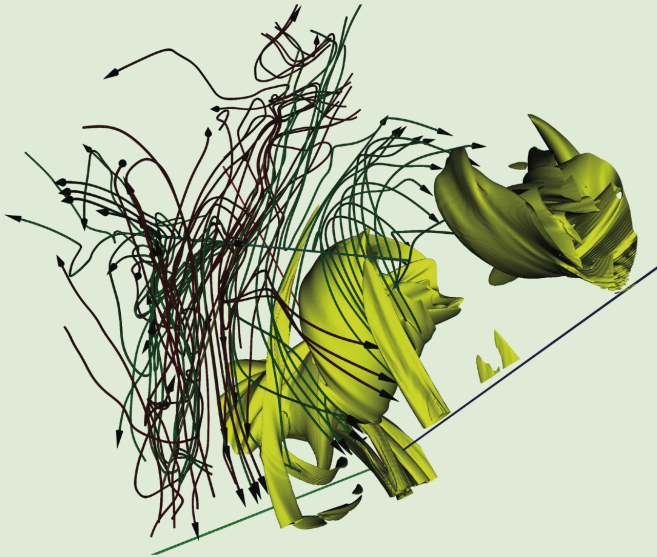
# Turbulent reconnection gains support from Solar flare observations



1. Solar flares can only be explained if magnetic reconnection can be initially slow (to accumulate flux) and then fast (to explain flares). Level of turbulence can do this (LV99)
2. Thick current layers predicted by LV99 have been observed in Solar flares (Ciaravella, & Raymond 2008).
3. Predicted by LV99 triggering of magnetic reconnection by Alfvén waves was observed by Sych et al. (2009).
4. Reconnection is fast in collisional and collisionless plasmas (Shibata et al. 2012)

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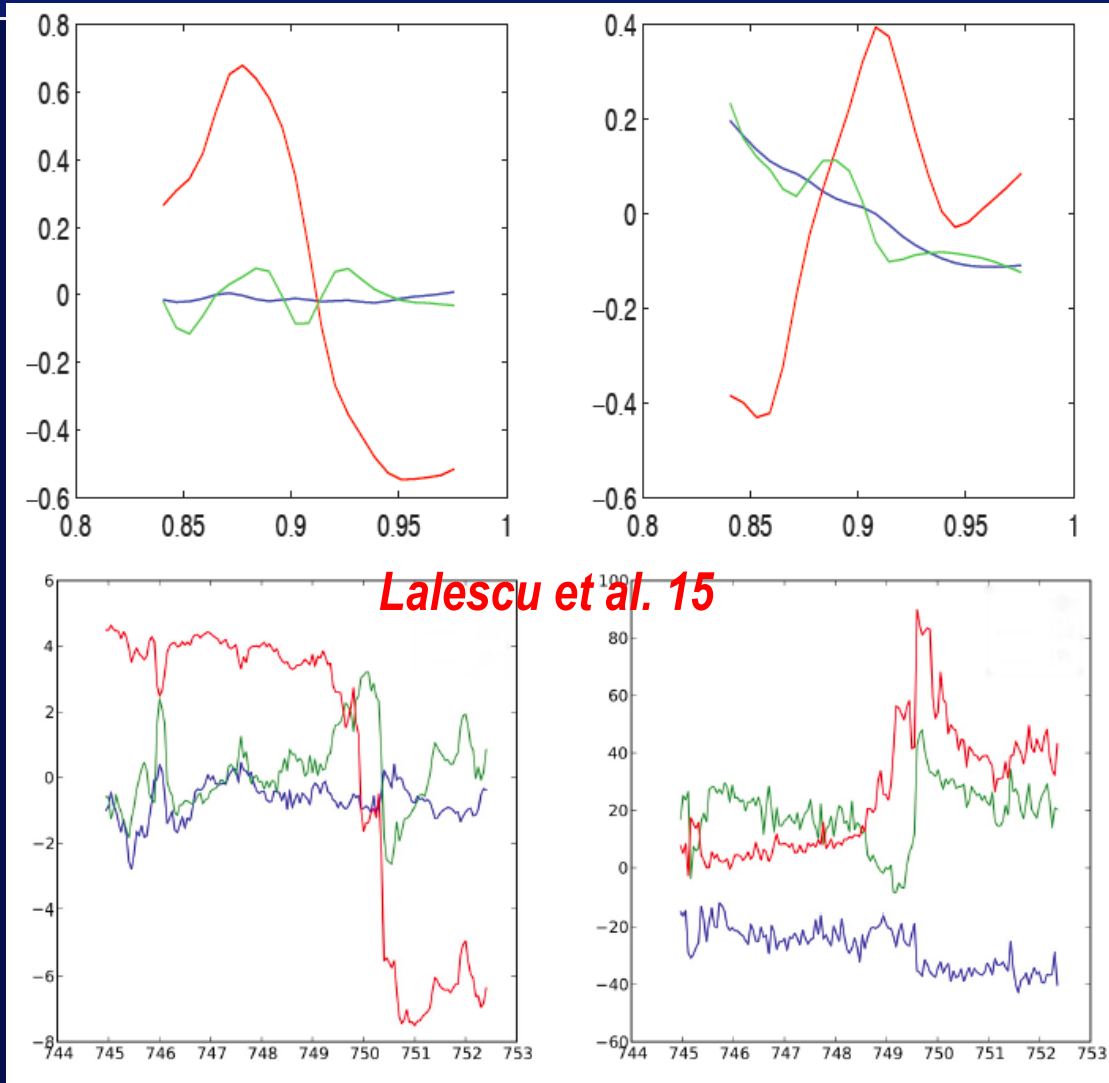
Volume 115, Number 2

The complex structure of magnetic reconnection similar to one in solar wind is revealed in simulations of MHD turbulence

Lalescu et al. 2015



# Turbulent reconnection is consistent with Solar wind measurements (cf. Karimabadi & AL 14)

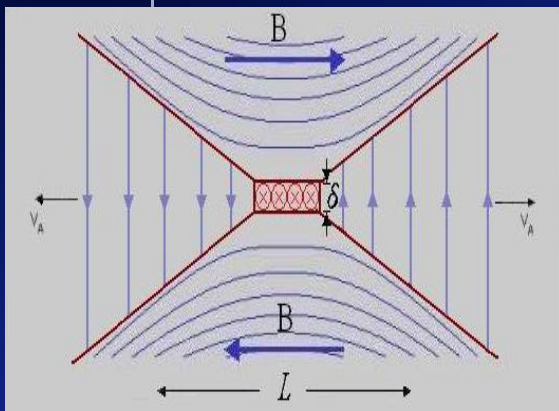


*MHD turbulence  
data set events*

*Solar wind  
reconnection  
events*

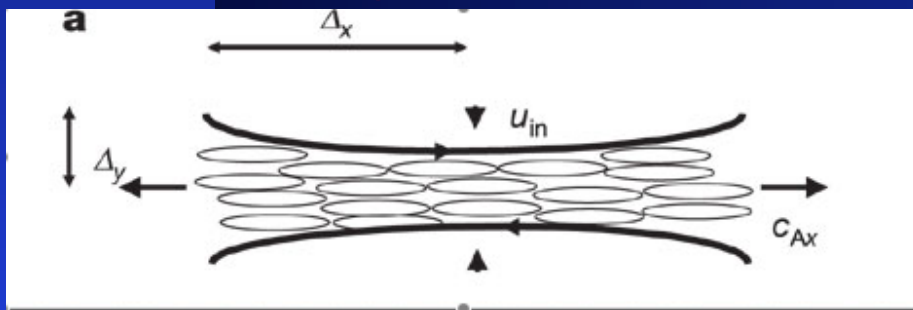
# Convergence between the plasma-based reconnection and turbulent model is evident!

## Paradigm 1999



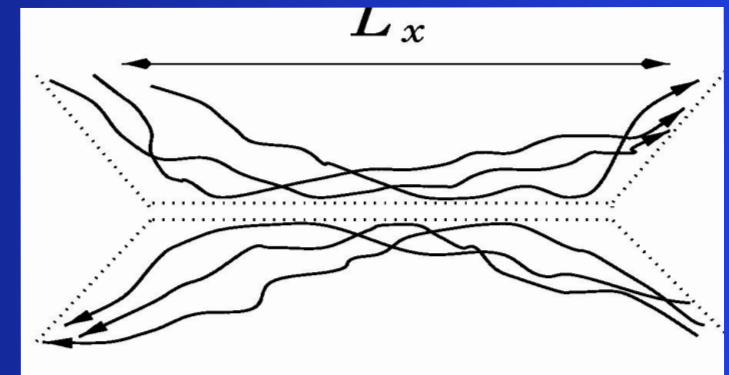
Hall effect is **required**

## Paradigm 2015



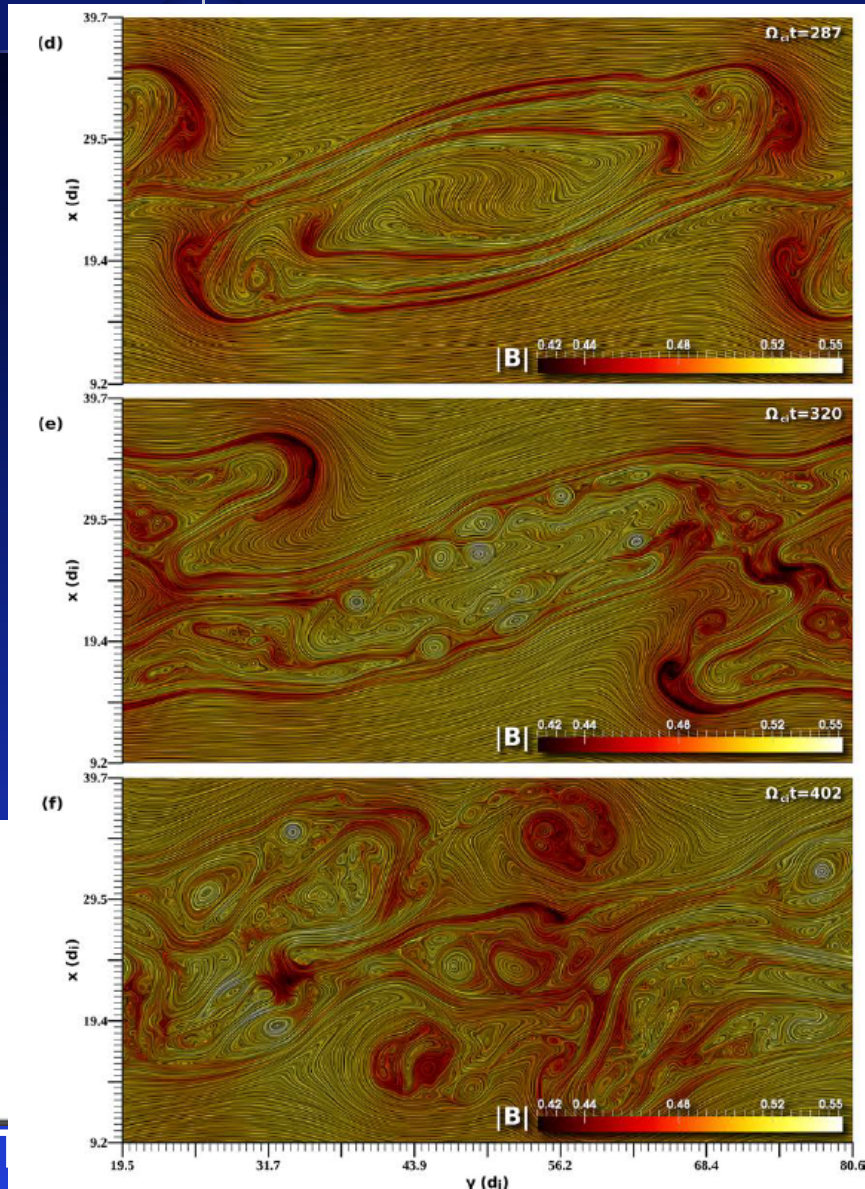
Tearing reconnection (Hall effect is **not required** )

## LV99 model

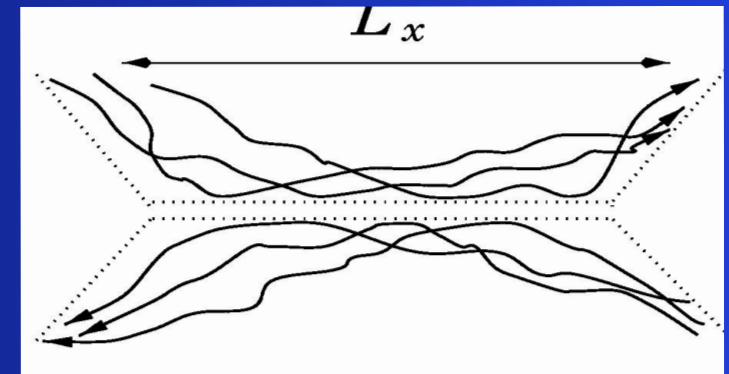


Hall effect is **not required**  
(Fully 3D, turbulence)

# Convergence between the plasma-based reconnection and turbulent model is evident!



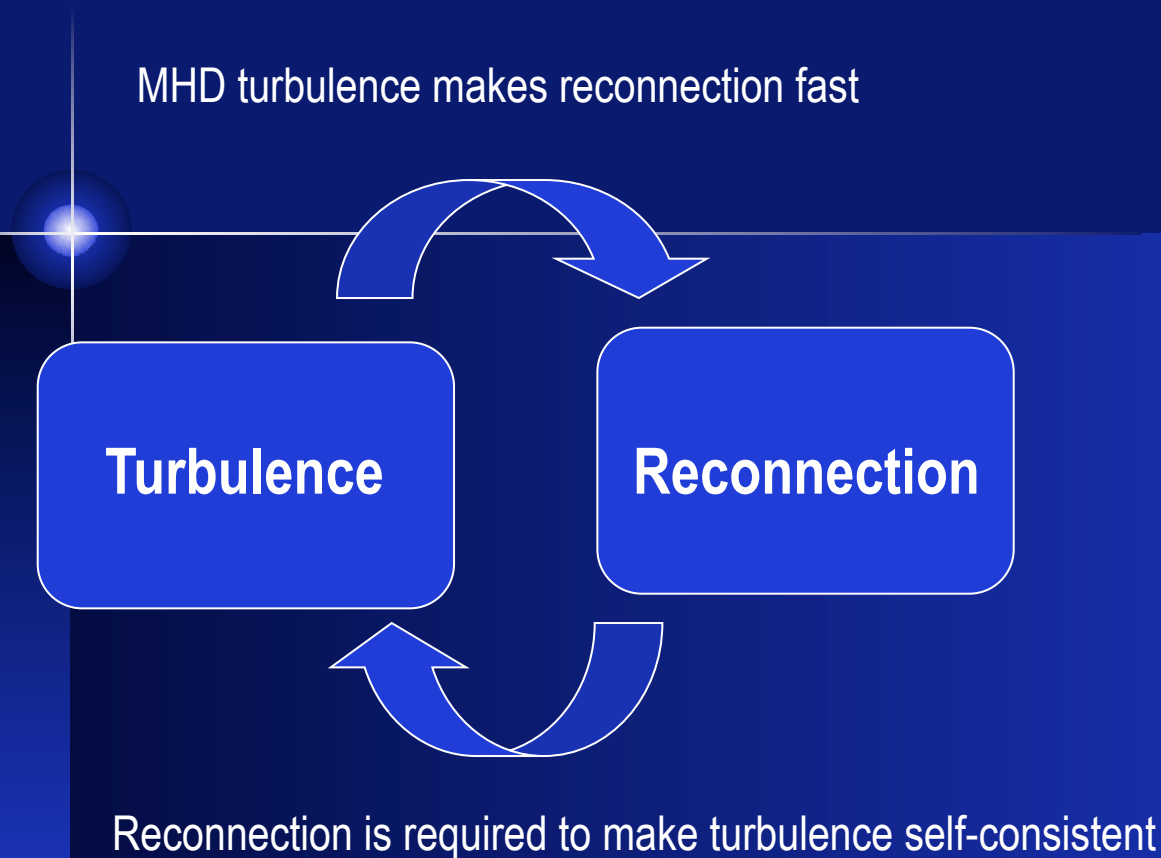
LV99 model



Hall effect is **not required**  
(Fully 3D, turbulence)

3D simulations without turbulence  
show transfer to turbulent state (e.g.  
Karimabadi 2012)

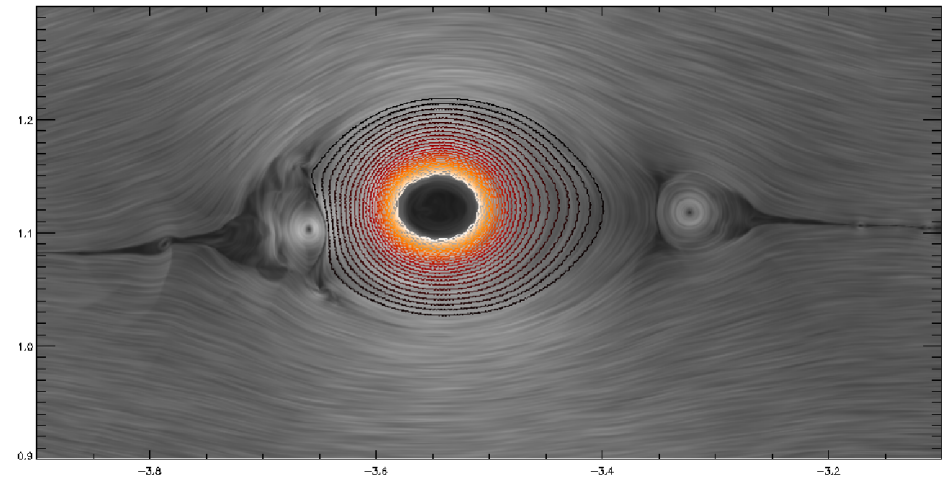
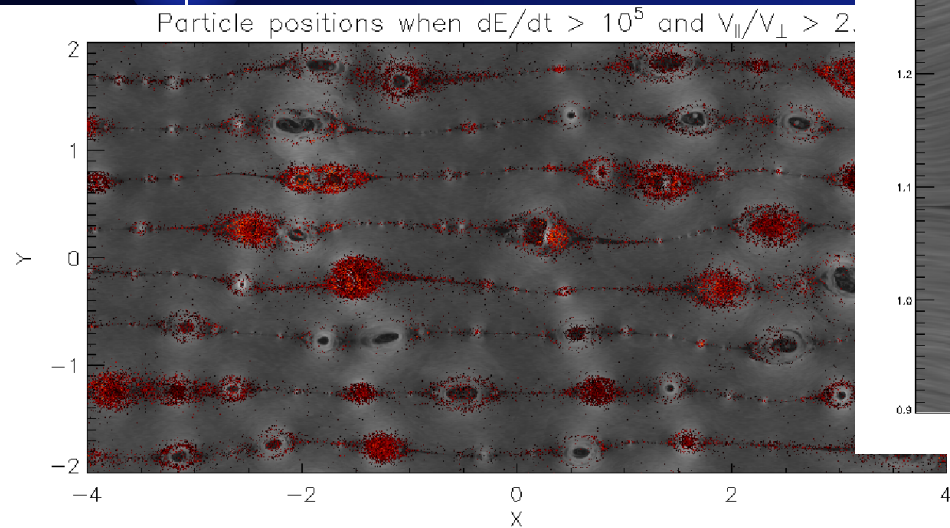
**Turbulence and fast astrophysical reconnection are interconnected. Relativistic and non-relativistic cases are similar.**



**EXTRA SLIDES FOLLOW**



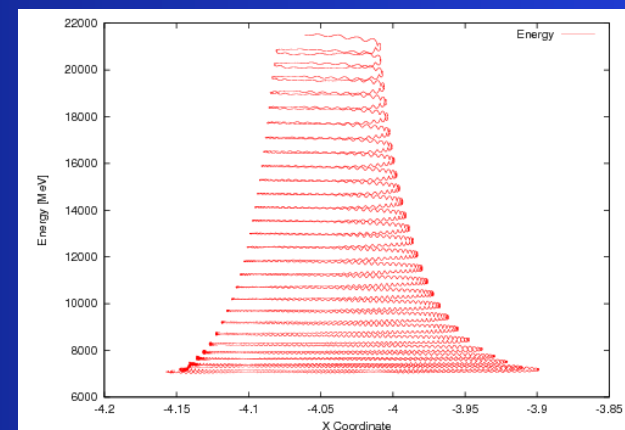
# MHD calculations reproduce 2D PIC calculations by Drake et al and go beyond



Multiple reconnection layers are used to produce volume reconnection.

Kowal, Lazarian, de Gouveia dal Pino 2011

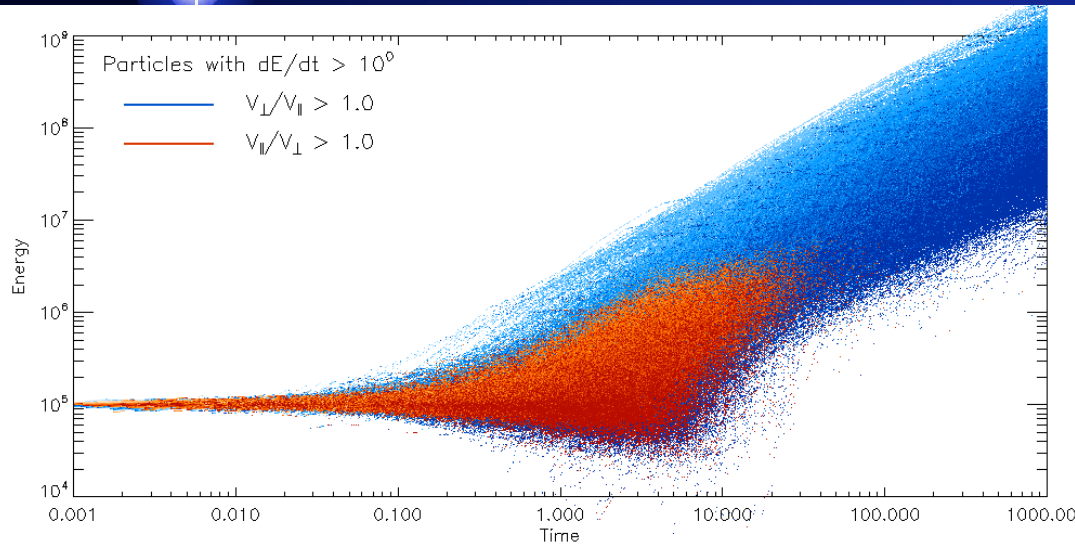
Zoom in into trajectories



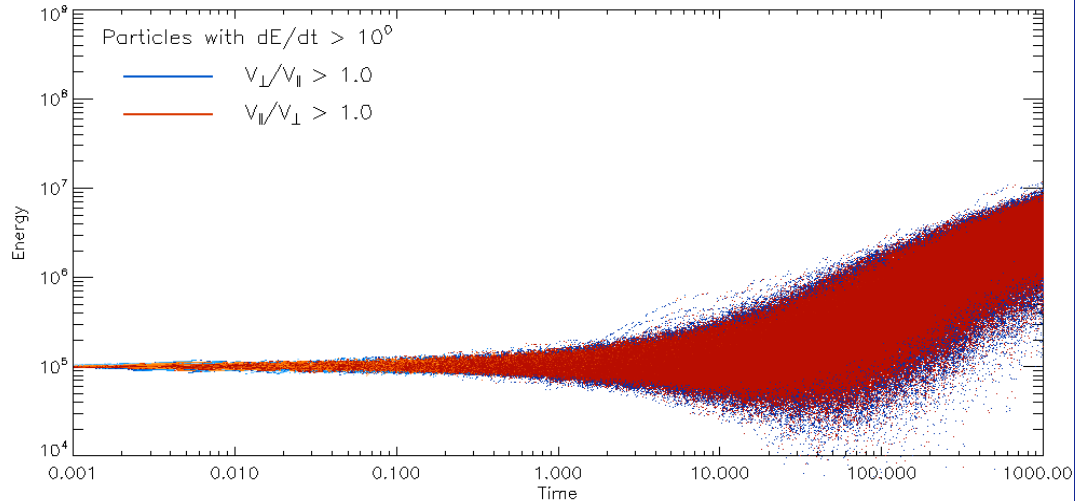
Regular energy increase



# 2D and 3D reconnection accelerates particles very differently: Loops and spirals behave differently!



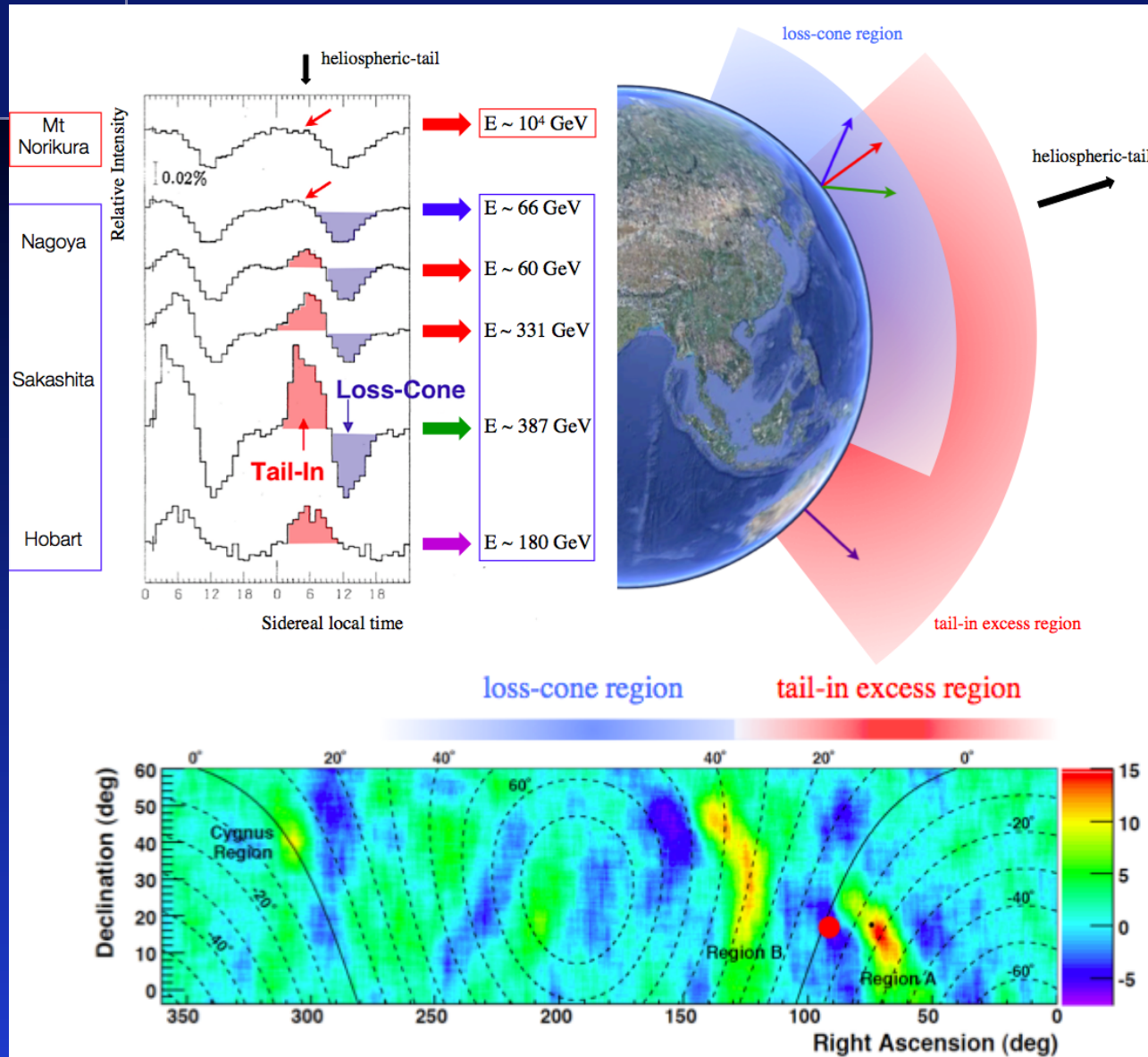
Perpendicular acceleration gets important for 2D at longer integration times



Parallel momentum mostly increases for the acceleration in 3D

Kowal, Lazarian, de Gouveial dal Pino 2010

# Excess of cosmic rays is observed in the tail in region

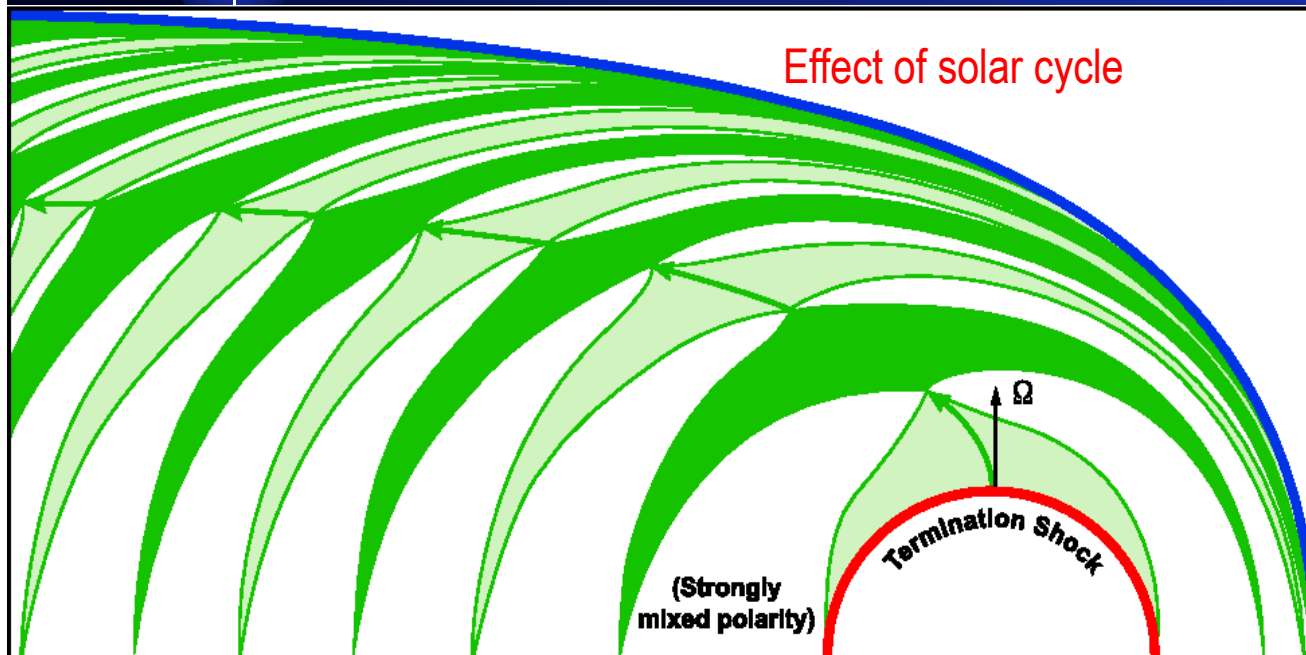


Low energy tail-in anisotropy

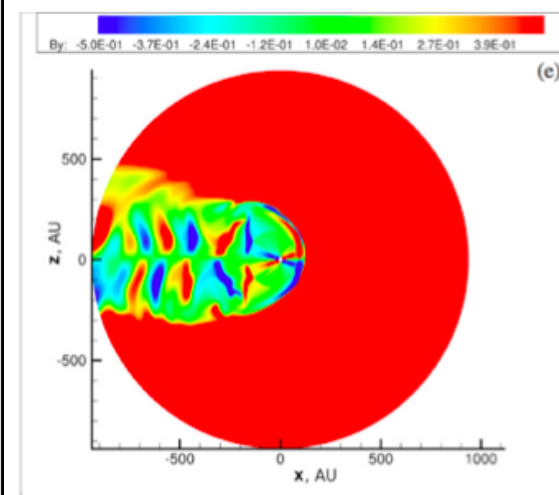
1-10TeV from Milagro, TibetIII, AGRO-YBJ and ICECUBE



# MILAGRO data: Magnetic reconnection expected in magnetotail can explain both the TeV and lower energy excess observed



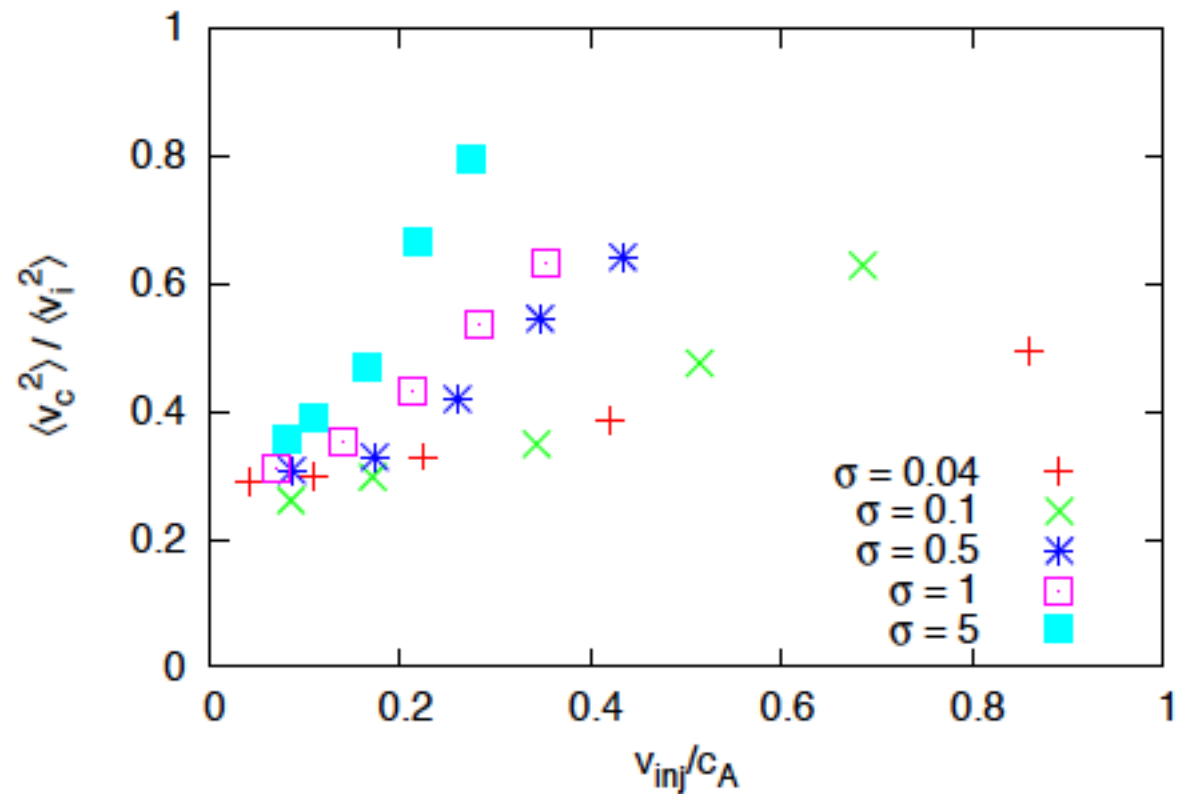
Pogorelov et al., ApJ, 696, 1478, 2009



Lazarian & Desiatii 2010

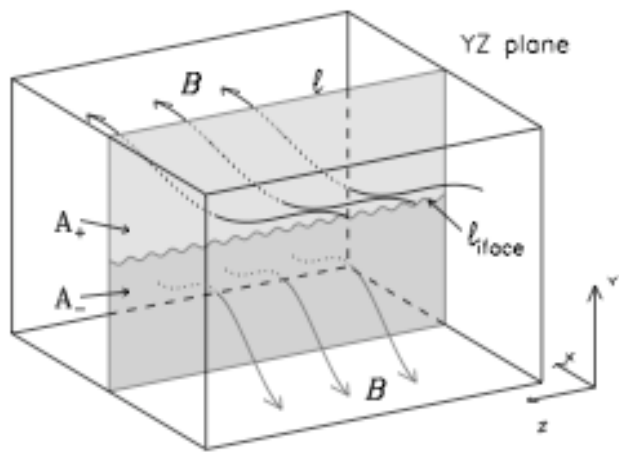
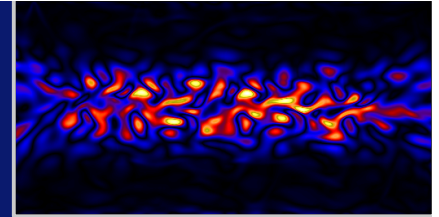
# Compressible modes drain energy from Alfvénic ones

$$\frac{v_{comp}^2}{c_A^2} \propto \frac{v_{inj}}{c_A}$$



$$v_{total}^2 - v_{comp}^2 \approx v_l^2 (1 - C_2(v_{inj}/c_A)) \propto v_{inj} (1 - C_2(v_{inj}/c_A))$$

We used both an intuitive measure,  $V_{\text{inflow}}$ , and a new measure of reconnection



$$\partial_t \Phi = - \oint \mathbf{E} \cdot d\mathbf{l} = \oint (\mathbf{v} \times \mathbf{B} - \eta \mathbf{j}) \cdot d\mathbf{l}$$

$$\partial_t \Phi_+ - \partial_t \Phi_- = \partial_t \int |B_x| dA,$$

$$\partial_t \int |B_x| dS = \oint \vec{E} \cdot d\vec{l}_+ - \oint \vec{E} \cdot d\vec{l}_- = \oint \text{sign}(B_x) \vec{E} \cdot d\vec{l} + \int 2 \vec{E} \cdot d\vec{l}_{\text{interface}}$$

$$\int 2 \vec{E} \cdot d\vec{l}_{\text{interface}} \equiv -2 V_{\text{rec}} |B_{x,\infty}| L_z$$

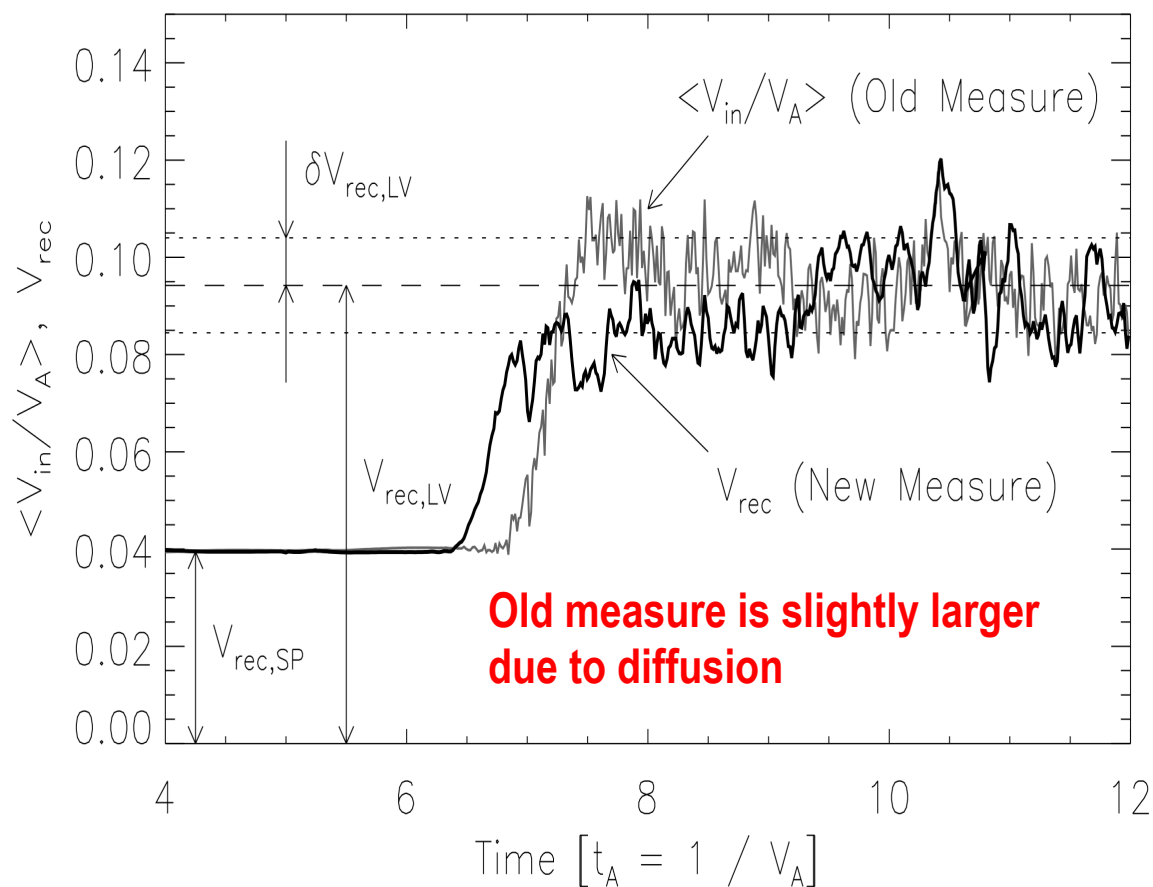
Asymptotic absolute value of  $B_x$

New measure:

$$V_{\text{rec}} = -\frac{1}{2 |B_{x,\infty}| L_z} \left[ \partial_t \int |B_x| dA - \oint \text{sign}(B_x) \vec{E} \cdot d\vec{l} \right]$$

# Calculations using the new measure are consistent with those using the intuitive one

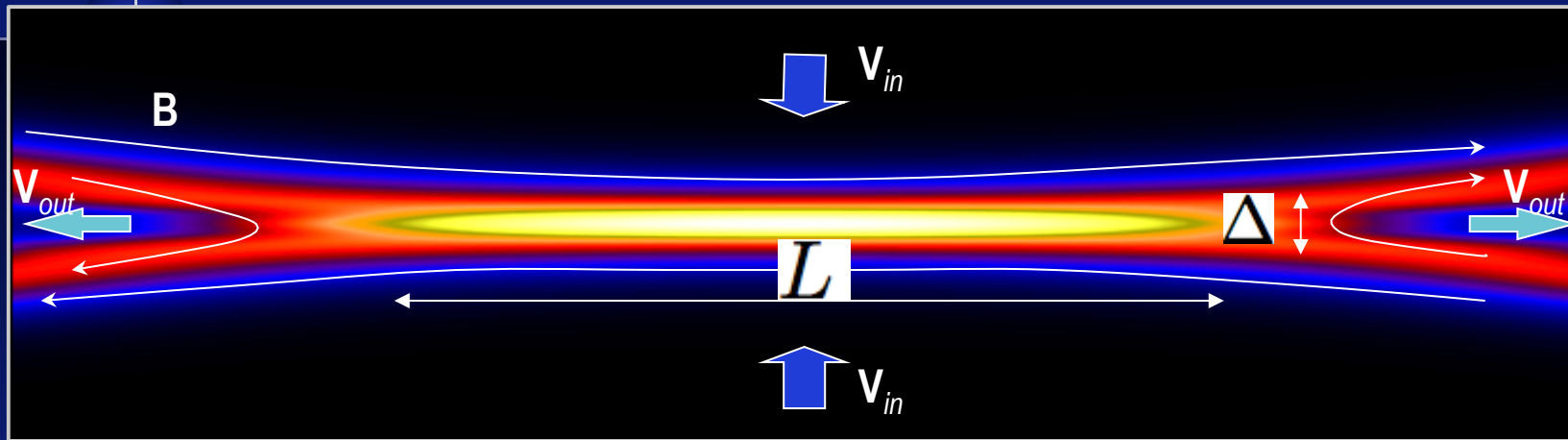
## Stochastic reconnection



Intuitive, “old” measure is the measure of the influx of magnetic field

New measure probes the annihilation of the flux

# Turbulence is expected to change Sweet-Parker reconnection and its tearing extension



1. *Magnetic field lines get not straight.*
2. *Tearing instability gets suppressed when the eddy turnover rate is larger than the instability rate.*
3. *The outflow gets inevitably turbulent for sufficiently large Re numbers of the outflow*

$$R_{\Delta} = \frac{\Delta V_A}{\nu}$$

- a. *if turbulence suppresses instability, then  $\Delta$  gets constant and reconnection rate start dropping as  $1/S$*
- b. *Turbulence induces a transfer to a new regime of reconnection*