Magnetic Reconnection in Plasmas
NORDITA, 13 August 2015

Turbulence modeling approach to fast reconnection

Nobumitsu YOKOI
Institute of Industrial Science, University of Tokyo

Main collaborators:
Masahiro HOSHINO, Katsuaki HIGASHIMORI
Jörg BÜCHNER, Fabien WIDMER

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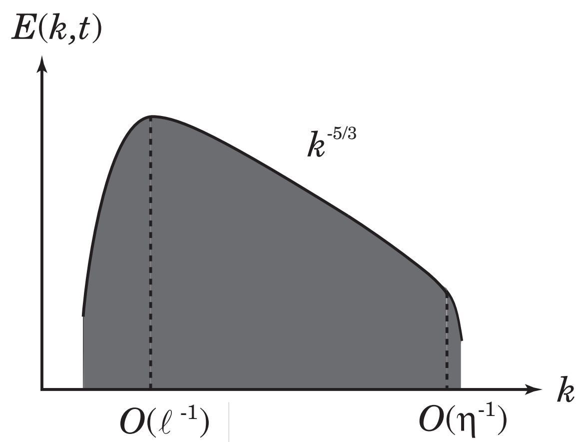
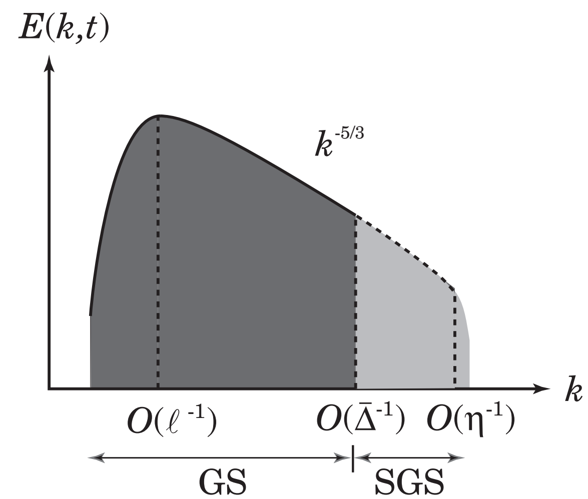
I. Turbulence modeling

Turbulence modeling

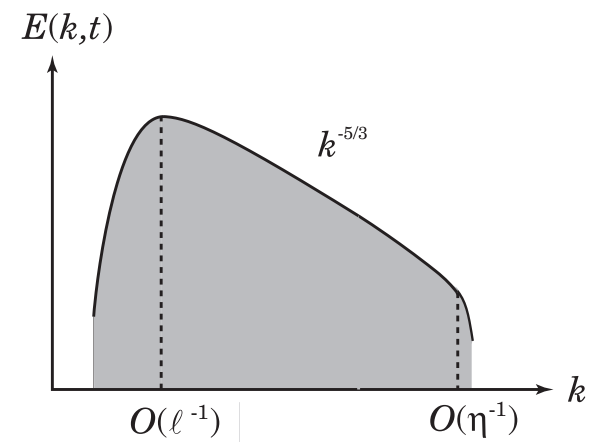
Direct Numerical Simulation (DNS)



Large Eddy Simulation (LES)



Reynolds-Averaged Navier–Stokes (RANS)



: Resolved

: Unresolved (modelled)

- Mean–fluctuation interaction
- Transport enhancement and suppression

Mean and fluctuation

$$u'^{\alpha}(\mathbf{r};t) = \int \hat{u}'^{\alpha}(\mathbf{k};t) \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{r}} d\mathbf{k}$$

$$f = F + f', \quad F = \langle f \rangle$$

$$\frac{\partial \hat{u}'^{\alpha}(\mathbf{k};t)}{\partial t} = ik_a \iint d\mathbf{p} d\mathbf{q} \delta(\mathbf{k}-\mathbf{p}-\mathbf{q}) \hat{u}'^{a}(\mathbf{p};t) \hat{u}'^{\alpha}(\mathbf{q};t) + ik^{\alpha}\hat{p}(\mathbf{k};t) - \nu k^2 \hat{u}'^{\alpha}(\mathbf{k};t)$$

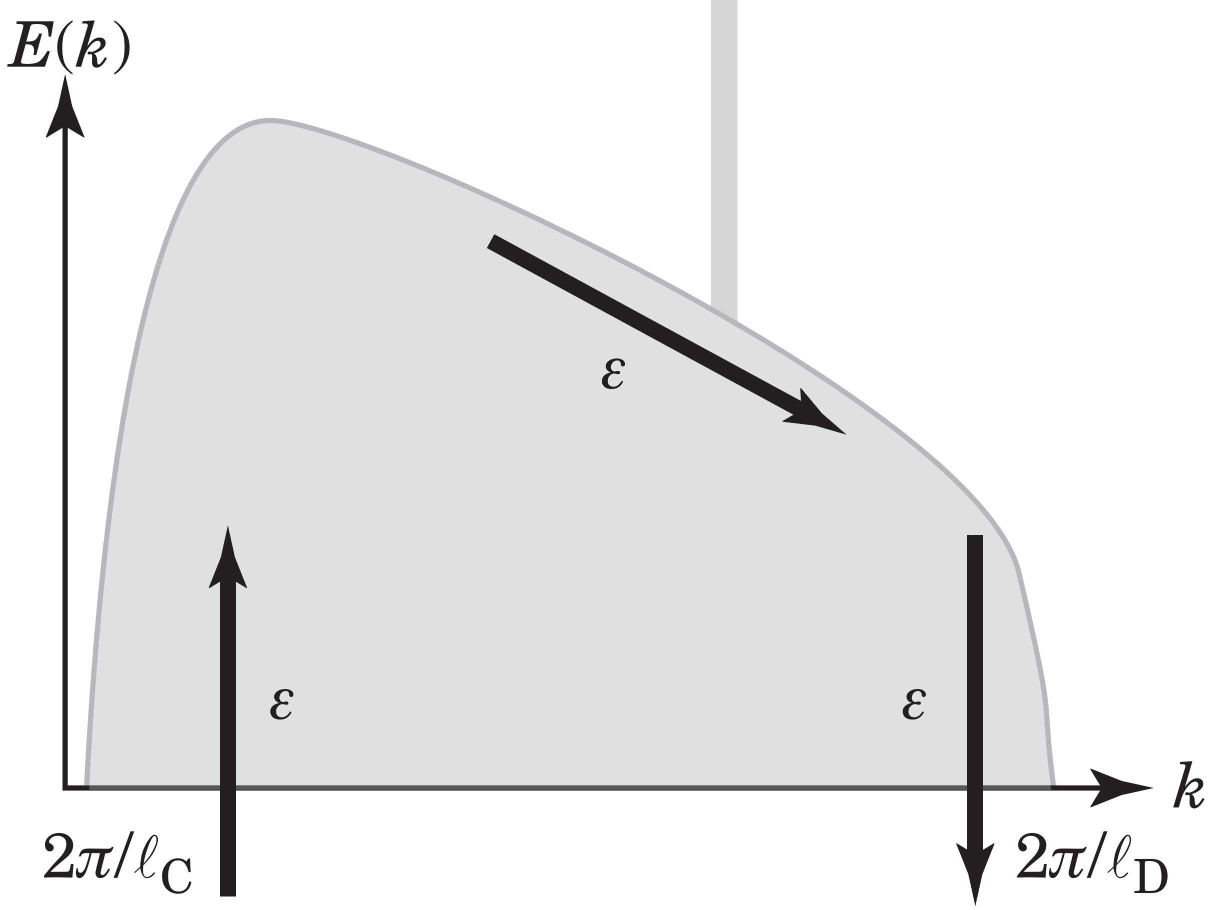
Mean fields

$\overline{\rho}$

$\mathbf{U}$

$\mathbf{B}$

described by



One-point turbulent statistical quantities

$$K \equiv K_u + K_b = \frac{1}{2}\left\langle \mathbf{u}'^2 \right\rangle + \frac{1}{2}\left\langle \mathbf{b}'^2 \right\rangle / (\mu_0 \overline{\rho})$$

$$\varepsilon \equiv \nu \left\langle \left(\frac{\partial u'^a}{\partial x^b} \right)^2 \right\rangle + \eta \left\langle \left(\frac{\partial b'^a}{\partial x^b} \right)^2 \right\rangle / (\mu_0 \overline{\rho})$$

$$W \equiv \langle \mathbf{u}' \cdot \mathbf{b}' \rangle / (\mu_0 \overline{\rho})^{1/2}$$

$$\varepsilon_W \equiv (\nu + \eta) \left\langle \frac{\partial u'^a}{\partial x^b} \frac{\partial b'^a}{\partial x^b} \right\rangle / (\mu_0 \overline{\rho})^{1/2}$$

$$K_{\mathrm{R}} \equiv K_u - K_b = \frac{1}{2}\left\langle \mathbf{u}'^2 \right\rangle - \frac{1}{2}\left\langle \mathbf{b}'^2 \right\rangle / (\mu_0 \overline{\rho})$$

$$H \equiv -H_u + H_b = -\langle \mathbf{u}' \cdot \boldsymbol{\omega}' \rangle + \langle \mathbf{b}' \cdot \mathbf{j}' \rangle / \overline{\rho}$$

$$K_{\rho} \equiv \left\langle \rho'^2 \right\rangle$$

Transport coefficients

An example: Turbulent magnetic diffusivity β

- Parameters $\quad \beta = \beta_0$

- Mixing length $\quad \beta = v\ell$

- Turbulence energy $\quad \beta = \tau K \qquad K = \frac{1}{2}\langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle$

- Propagators
$$\beta = \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \times \left[Q_{uu}(k, \mathbf{x}; \tau, \tau_1, t) + Q_{bb}(k, \mathbf{x}; \tau, \tau_1, t) \right]$$

- Transport equations
$$\frac{\partial K}{\partial t} + (\mathbf{U} \cdot \nabla) K = -\langle \mathbf{u}'\mathbf{u}' - \mathbf{b}'\mathbf{b}' \rangle \cdot \nabla \mathbf{U} - \langle \mathbf{u}' \times \mathbf{b}' \rangle \cdot \nabla \times \mathbf{B} + \cdots$$

Mean–fluctuation interactions

Turbulence $\longrightarrow$ Effective transport in large-scale fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \nabla \times \langle \mathbf{u}' \times \mathbf{b}' \rangle + \eta \nabla^2 \mathbf{B}$$

$$\langle \mathbf{u}' \times \mathbf{b}' \rangle = -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \boldsymbol{\Omega}$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\boldsymbol{\Omega} = \nabla \times \mathbf{U}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) - \nabla \times [(\eta + \underline{\beta) \nabla \times \mathbf{B}}] + \nabla \times \underline{(\alpha \mathbf{B} + \gamma \boldsymbol{\Omega})}$$

Large-scale inhomogeneities $\longrightarrow$ Turbulence properties

Turbulent energy

$$\left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle / 2 = -\langle \mathbf{u}'\mathbf{u}' - \mathbf{b}'\mathbf{b}' \rangle \cdot \nabla \mathbf{U} - \langle \mathbf{u}' \times \mathbf{b}' \rangle \cdot \nabla \times \mathbf{B} - \varepsilon_K + \cdots$$

Turbulent cross helicity

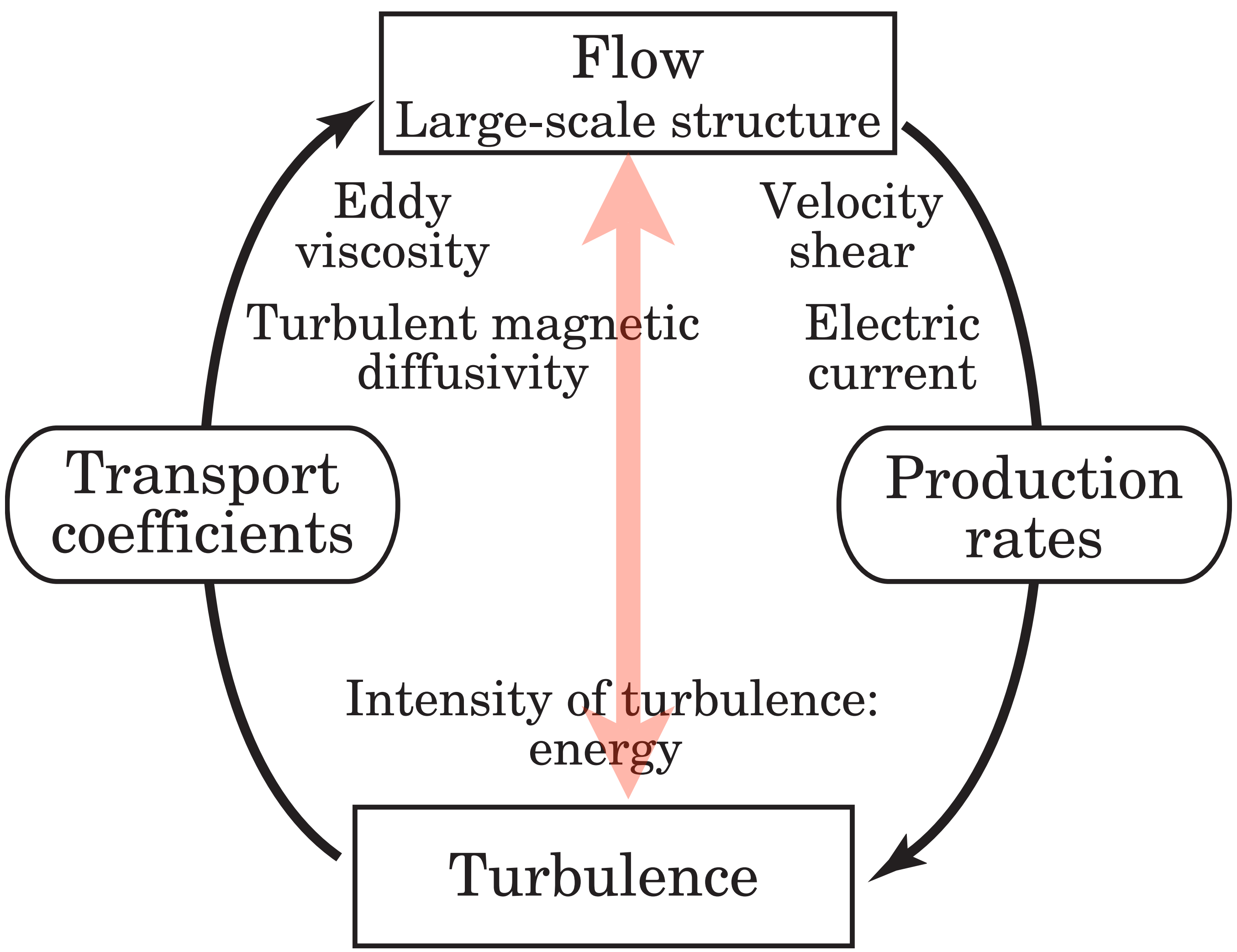
$$\left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \langle \mathbf{u}' \cdot \mathbf{b}' \rangle = -\langle \mathbf{u}'\mathbf{u}' - \mathbf{b}'\mathbf{b}' \rangle \cdot \nabla \mathbf{B} - \langle \mathbf{u}' \times \mathbf{b}' \rangle \cdot \nabla \times \mathbf{U} - \varepsilon_W + \cdots$$

Turbulent residual helicity

$$\left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \langle \mathbf{b}' \cdot \mathbf{j}' - \mathbf{u}' \cdot \boldsymbol{\omega} \rangle = -\langle \mathbf{u}'\mathbf{u}' - \mathbf{b}'\mathbf{b}' \rangle \cdot \nabla \boldsymbol{\Omega} - \frac{1}{\tau \beta} \langle \mathbf{u}' \times \mathbf{b}' \rangle \cdot \mathbf{B} - \varepsilon_H + \cdots$$

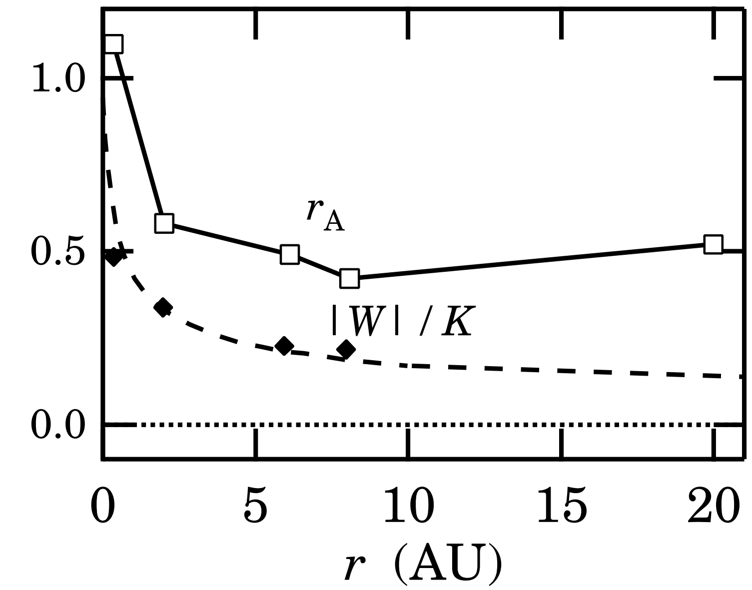
$$\mathbf{j}' = \nabla \times \mathbf{b}', \quad \boldsymbol{\omega}' = \nabla \times \mathbf{u}'$$

- Mean–Fluctuation interactions



Solar-wind turbulence

Alfvén ratio $$r_{\rm A} \equiv \frac{\langle \mathbf{u}'^2 \rangle}{\langle \mathbf{b}'^2 \rangle} \simeq 0.5$$



spatial evolutions of cross correlation and Alfvén ratio (Roberts *et al*. 1990)

Turbulent residual energy

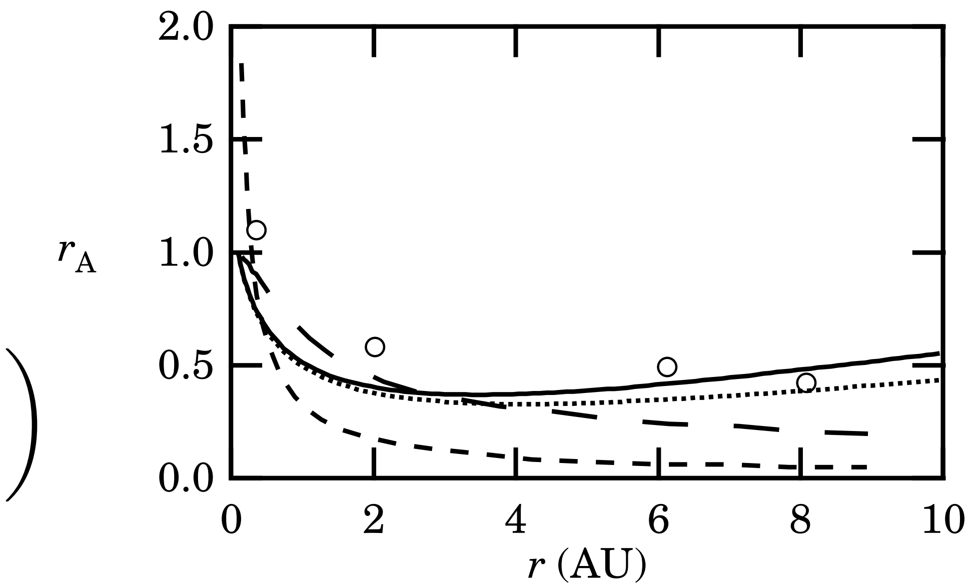
(Yokoi 2006, Yokoi & Hamba 2007)

$$\boxed{K_{\rm R} = \langle \mathbf{u}'^2 - \mathbf{b}'^2 \rangle / 2}$$

$$\frac{\partial K_{\rm R}}{\partial t} = -\left(\mathbf{U} \cdot \nabla \right) K_{\rm R} - \frac{1}{6} \left(K + 3K_{\rm R} \right) \nabla \cdot \mathbf{U}$$
$$- \frac{1}{3} W \nabla \cdot \mathbf{B} + \frac{1}{2} \nu_{\rm R} \boldsymbol{\mathcal{S}}^2 - \frac{1}{2} \nu_{\rm R} \boldsymbol{\mathcal{M}}^2$$
$$- C_{\varepsilon {\rm R}} \left(1 + \frac{C_{r1}}{C_{\varepsilon {\rm R}}} \frac{\mathbf{B}^2}{K} \right) \frac{\varepsilon}{K} K_{\rm R} + \frac{1}{\bar{\rho}} \nabla \cdot \left(\frac{\nu_{\rm K}}{\sigma_{\rm R}} \bar{\rho} \nabla K_{\rm R} \right)$$

with $$\nu_{\rm K} = C_\nu K / \varepsilon, \quad \nu_{\rm R} = (W/K) \nu_{\rm K}$$

$$K = \langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle / 2, \qquad W = \langle \mathbf{u}' \cdot \mathbf{b}' \rangle$$



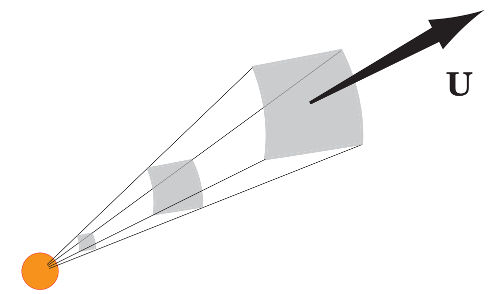
cf. Zhou & Matthaeus (1990)
Tu & Marsch (1993)

Expansion effects

(Yokoi et al., JoT, 2008; Yokoi, JoT, 2011)

$$K_{\mathrm{R}} = \langle \mathbf{u}'^2 - \mathbf{b}'^2 \rangle /2, \qquad K = \langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle /2$$

$$\frac{DK_{\mathrm{R}}}{Dt} = -\frac{1}{6}\left(3K_{\mathrm{R}} + K\right)\nabla \cdot \mathbf{U} + \cdots$$



$\nabla \cdot \mathbf{U} > 0$

expanding flow

Equipartition near the Sun $\quad K_{\mathrm{R}} = 0$

$$\frac{DK_{\mathrm{R}}}{Dt} \simeq -\frac{1}{6}K\nabla \cdot \mathbf{U} < 0$$

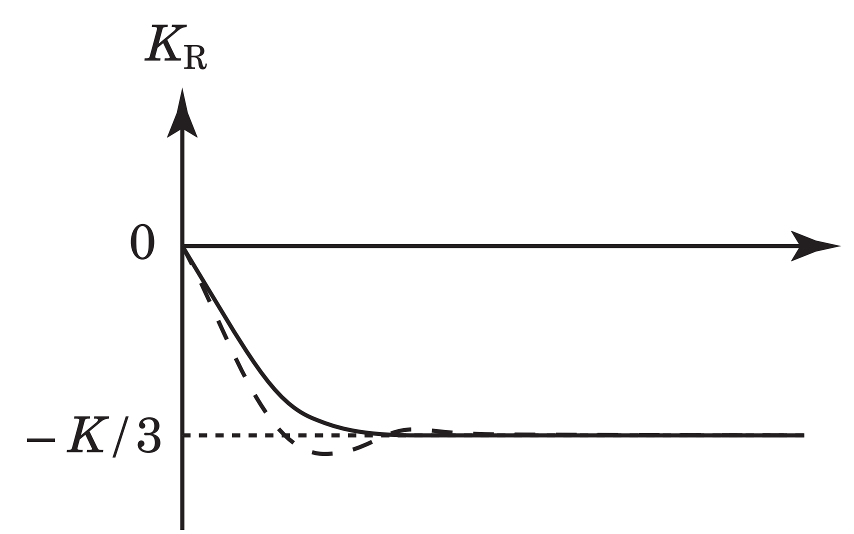
Production of negative K_{R} due to the expansion effect

$$\frac{DK_{\mathrm{R}}}{Dt} \simeq -\frac{1}{6}(K + 3K_{\mathrm{R}})\nabla \cdot \mathbf{U} < 0 \quad \text{for} \quad |K_{\mathrm{R}}| < K/3$$

$K_{\mathrm{R}} \searrow$

$$\frac{DK_{\mathrm{R}}}{Dt} \simeq -\frac{1}{6}(K + 3K_{\mathrm{R}})\nabla \cdot \mathbf{U} > 0 \quad \text{for} \quad |K_{\mathrm{R}}| > K/3$$

$K_{\mathrm{R}} \nearrow$



Stationary solution far from the Sun

$$-\frac{1}{6}\left(3K_{\mathrm{R}} + K\right)\nabla \cdot \mathbf{U} \simeq 0$$

$$r_{\mathrm{A}} = \frac{\langle \mathbf{u}'^2 \rangle}{\langle \mathbf{b}'^2 \rangle} = \frac{1}{2}$$

$$\leftrightarrow \quad \frac{K_{\mathrm{R}}}{K} = \frac{\langle \mathbf{u}'^2 - \mathbf{b}'^2 \rangle}{\langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle} = \frac{1-2}{1+2} = -\frac{1}{3}$$

Value stationary in space can be interpreted as a stationary solution of the turbulence model

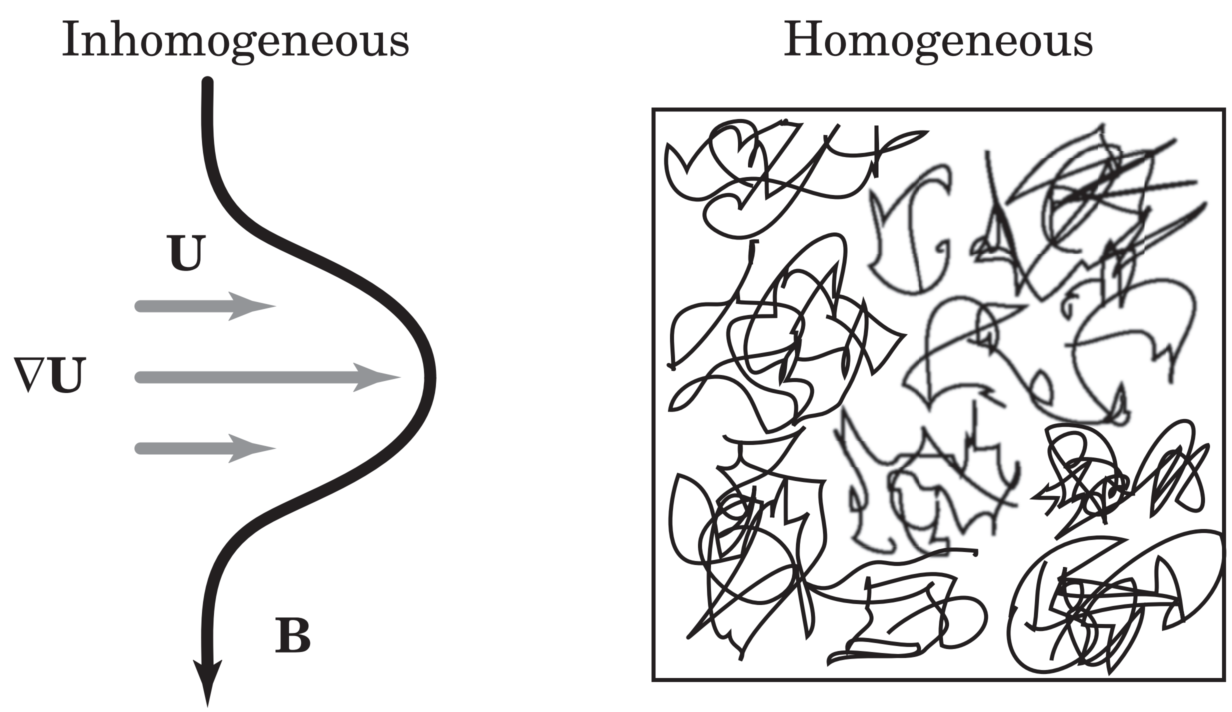
II. Flow inhomogeneities

Mean field

$$\mathbf{b} = \mathbf{B} + \mathbf{b}', \quad \mathbf{B} = \langle \mathbf{b} \rangle$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \nabla \times \langle \mathbf{u}' \times \mathbf{b}' \rangle + \eta \nabla^2 \mathbf{B}$$

$(\mathbf{B} \cdot \nabla)\mathbf{U}$ $\longrightarrow$ differential rotation, "Ω effect"



Turbulence

$$\mathbf{U} = \mathbf{U}_0(\text{constant}) \ \text{ or } \ \mathbf{0}$$

$$\frac{\partial \mathbf{u}'}{\partial t} + (\mathbf{U} \cdot \nabla)\mathbf{u}' = (\mathbf{B} \cdot \nabla)\mathbf{b}' + (\mathbf{b}' \cdot \nabla)\mathbf{B} \cancel{- (\mathbf{u}' \cdot \nabla)\mathbf{U}} + \cdots$$

$$\frac{\partial \mathbf{b}'}{\partial t} + (\mathbf{U} \cdot \nabla)\mathbf{b}' = (\mathbf{B} \cdot \nabla)\mathbf{u}' - (\mathbf{u}' \cdot \nabla)\mathbf{B} \cancel{+ (\mathbf{b}' \cdot \nabla)\mathbf{U}} + \cdots$$

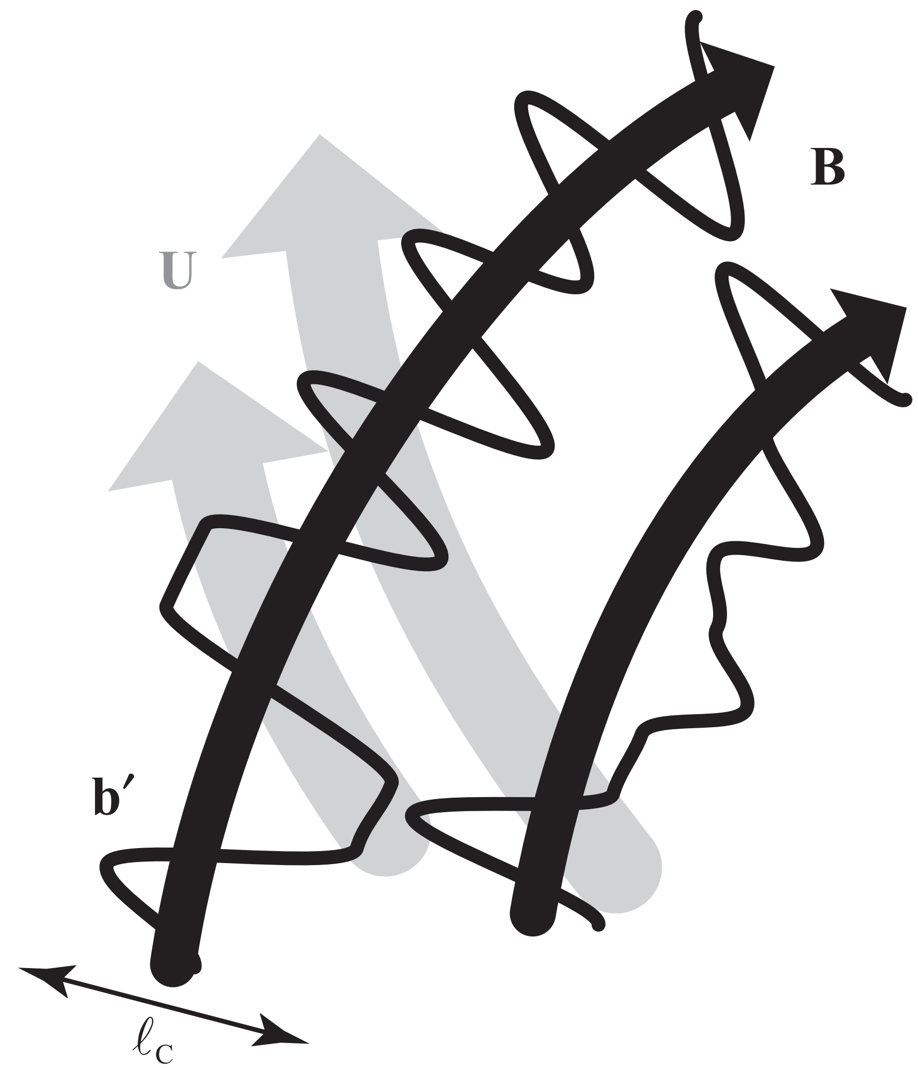
$$\longrightarrow \quad \langle \mathbf{u}' \times \mathbf{b}' \rangle^\alpha = \alpha^{\alpha a} B^a + \beta^{\alpha a b} \frac{\partial B^a}{\partial x^b} + \cdots$$

$$\frac{\partial \mathbf{u}'}{\partial t} + (\mathbf{U}\cdot\nabla)\mathbf{u}' = (\mathbf{B}\cdot\nabla)\mathbf{b}' + (\mathbf{b}'\cdot\nabla)\mathbf{B} - (\mathbf{u}'\cdot\nabla)\mathbf{U} + \cdots$$

$$\frac{\partial \mathbf{b}'}{\partial t} + (\mathbf{U}\cdot\nabla)\mathbf{b}' = (\mathbf{B}\cdot\nabla)\mathbf{u}' - (\mathbf{u}'\cdot\nabla)\mathbf{B} + (\mathbf{b}'\cdot\nabla)\mathbf{U} + \cdots$$

Turbulent electromotive force $\langle \mathbf{u}' \times \mathbf{b}' \rangle$

$$\frac{\partial}{\partial t}\langle \mathbf{u}' \times \mathbf{b}' \rangle = \left\langle \frac{\partial \mathbf{u}'}{\partial t} \times \mathbf{b}' \right\rangle + \left\langle \mathbf{u}' \times \frac{\partial \mathbf{b}'}{\partial t} \right\rangle = \cdots$$



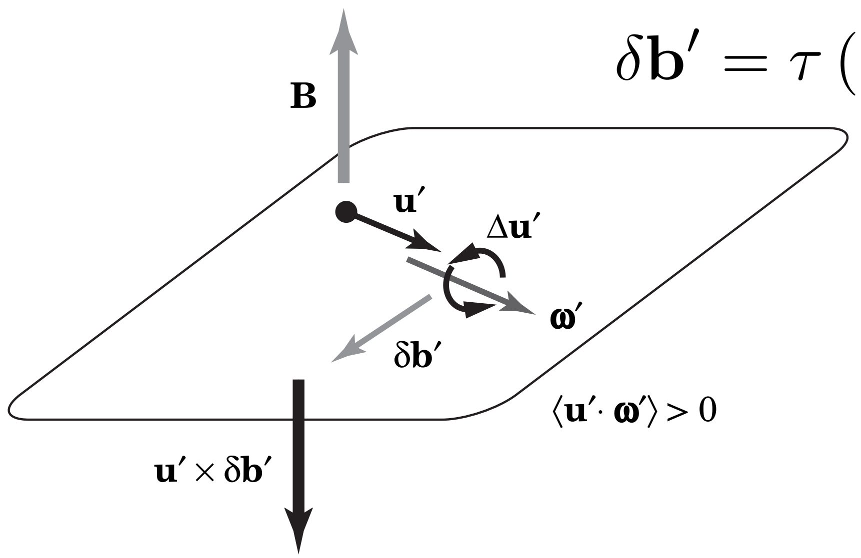
$$\begin{aligned}
&\langle \mathbf{u}' \times [(\mathbf{b}'\cdot\nabla)\mathbf{U}] + [(\mathbf{u}'\cdot\nabla)\mathbf{U}] \times \mathbf{b}' \rangle^{\alpha} \\
&\quad = \epsilon^{\alpha ab}\langle u'^{a} b'^{c}\rangle \frac{\partial U^b}{\partial x^c} - \epsilon^{\alpha ba}\langle b'^{a} u'^{c}\rangle \frac{\partial U^b}{\partial x^c} \\
&\quad = \left(\langle u'^{a} b'^{c}\rangle + \langle u'^{c} b'^{a}\rangle\right) \epsilon^{\alpha ab} \frac{\partial U^b}{\partial x^c}
\end{aligned}$$

$$\longrightarrow \qquad \langle \mathbf{u}' \times \mathbf{b}' \rangle = \cdots + \tau \underbrace{\langle \mathbf{u}' \cdot \mathbf{b}' \rangle}_{\text{cross helicity}} \nabla \times \mathbf{U} + \cdots$$

α and cross-helicity effect

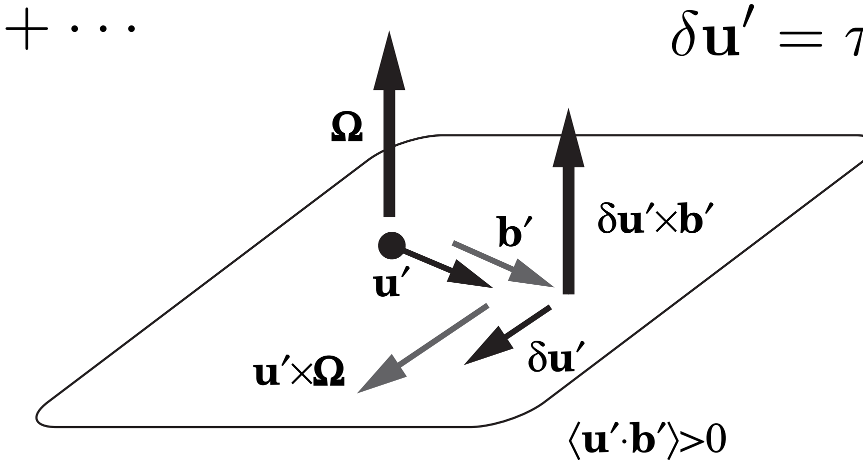
(Yokoi, GAFD **107**, 114, 2013)

$$\delta \mathbf{b}' = \tau \left(\mathbf{B} \cdot \nabla \right) \mathbf{u}' + \cdots$$



kinetic helicity effect

$$\delta \mathbf{u}' = \tau \mathbf{u}' \times \boldsymbol{\Omega} + \cdots$$



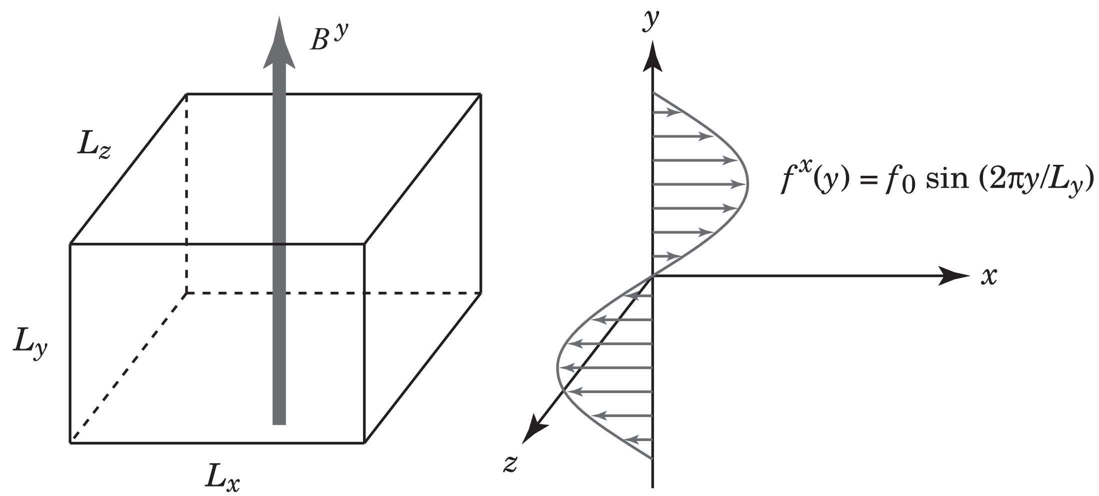
mean vorticity
$\boldsymbol{\Omega} = \nabla \times \mathbf{U}$

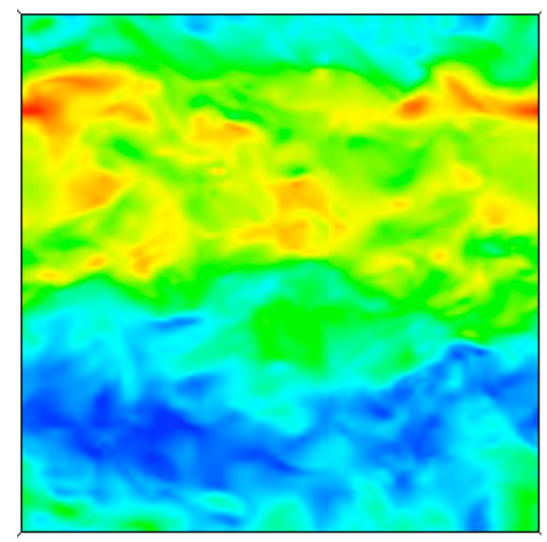
cross-helicity effect

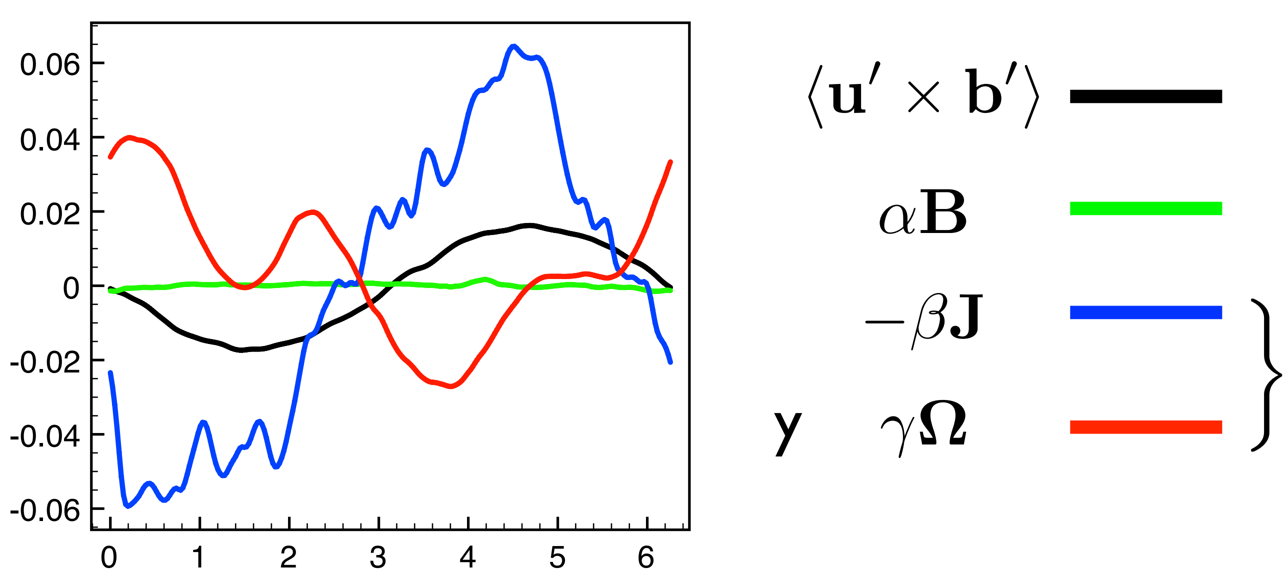
$$\langle \mathbf{u}' \times \mathbf{b}' \rangle = \overbrace{\alpha \mathbf{B} - \beta \nabla \times}^{\alpha \text{ dynamo}} \underbrace{\mathbf{B} + \gamma \nabla \times \mathbf{U}}_{\text{cross-helicity dynamo}}$$

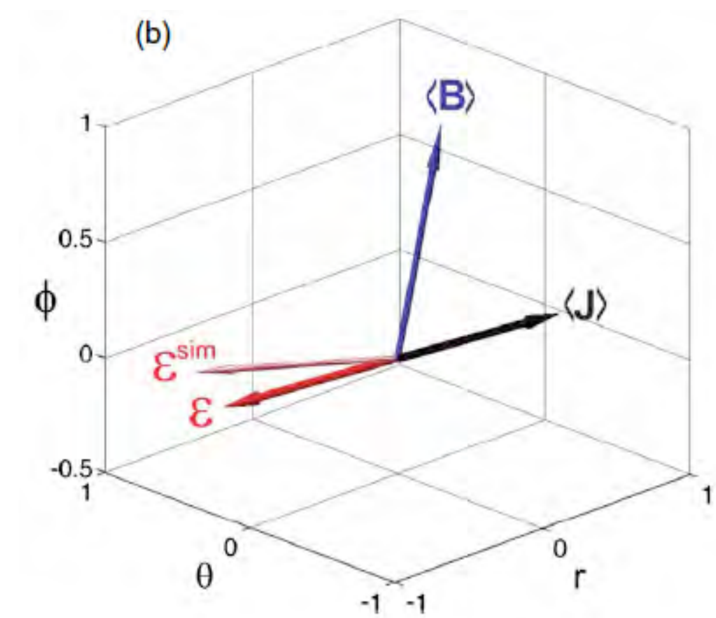
Cross helicity in EMF

DNS of electromotive force in Kolmogorov flow (Yokoi & Balarac, 2011)





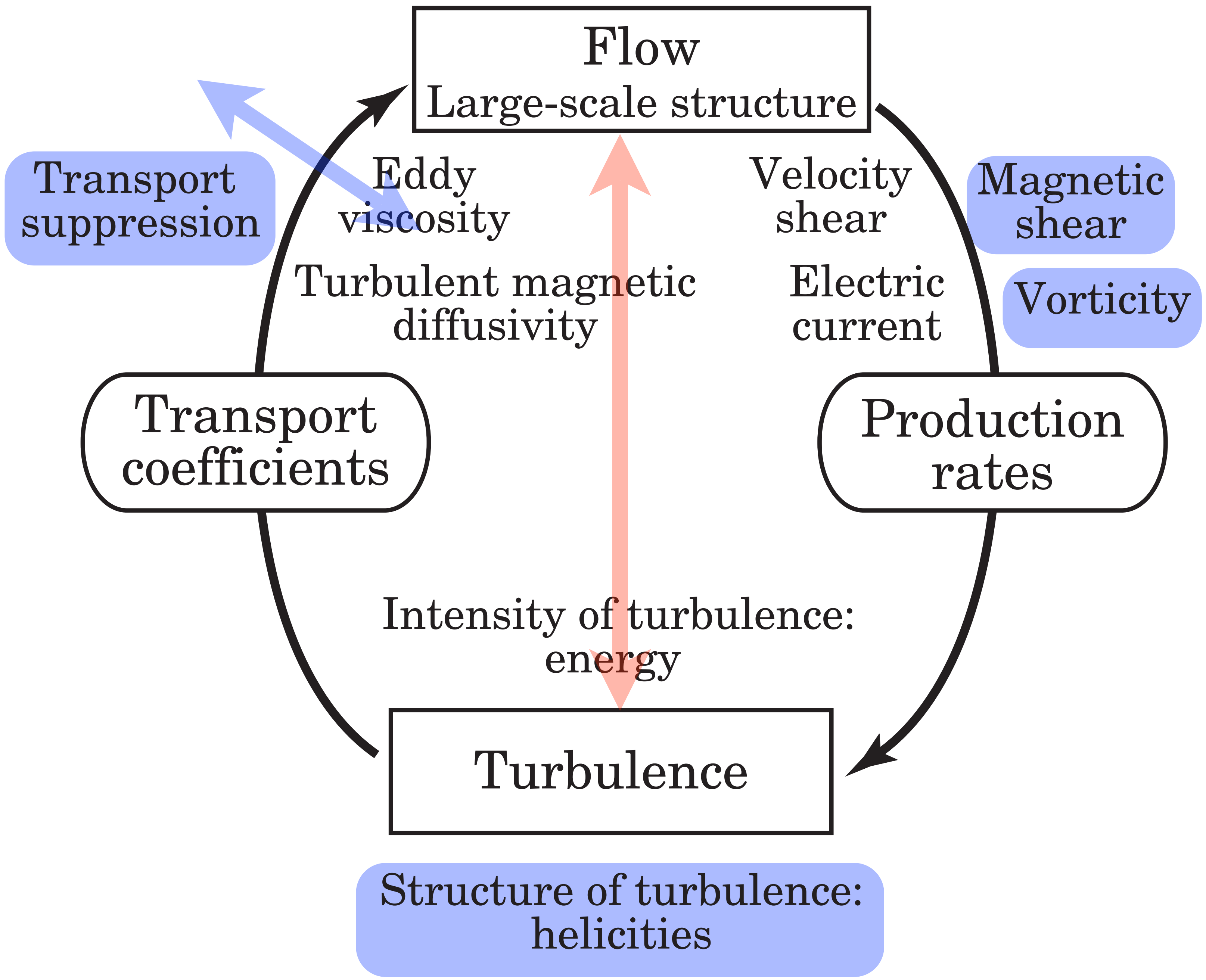




(Rahbarnia, et al. ApJ, 2012)

- Mean–Fluctuation interaction

- Transport Enhancement–Suppression



Turbulence effects

Yoshizawa, PoF 1990
Yokoi, GAFD 2013

Reynolds (+ turbulent Maxwell) stress

$$\mathcal{R}_{\rm D}^{\alpha\beta} \equiv \left\langle u'^\alpha u'^\beta - b'^\alpha b'^\beta \right\rangle_{\rm D}$$

$$= -\nu_{\rm K} \mathcal{S}_{\rm D}^{\alpha\beta} + \nu_{\rm M} \mathcal{M}_{\rm D}^{\alpha\beta} + [\boldsymbol{\Gamma\Omega}]_{\rm D}^{\alpha\beta}$$

Enhancement Suppression

$$\mathcal{S}^{\alpha\beta} = \frac{\partial U^\beta}{\partial x^\alpha} + \frac{\partial U^\alpha}{\partial x^\beta}$$

$$\mathcal{M}^{\alpha\beta} = \frac{\partial B^\beta}{\partial x^\alpha} + \frac{\partial B^\alpha}{\partial x^\beta}$$

$$\boldsymbol{\Omega} = \nabla \times \mathbf{U}$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

Turbulent electromotive force

$$\mathbf{E}_{\rm M} \equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle$$

$$= -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \boldsymbol{\Omega}$$

Enhancement Suppression

$$\alpha = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[-H_{uu}(k, \mathbf{x}; \tau, \tau_1, t) + H_{bb}(k, \mathbf{x}; \tau, \tau_1, t) \right] \qquad \alpha \propto \tau H$$

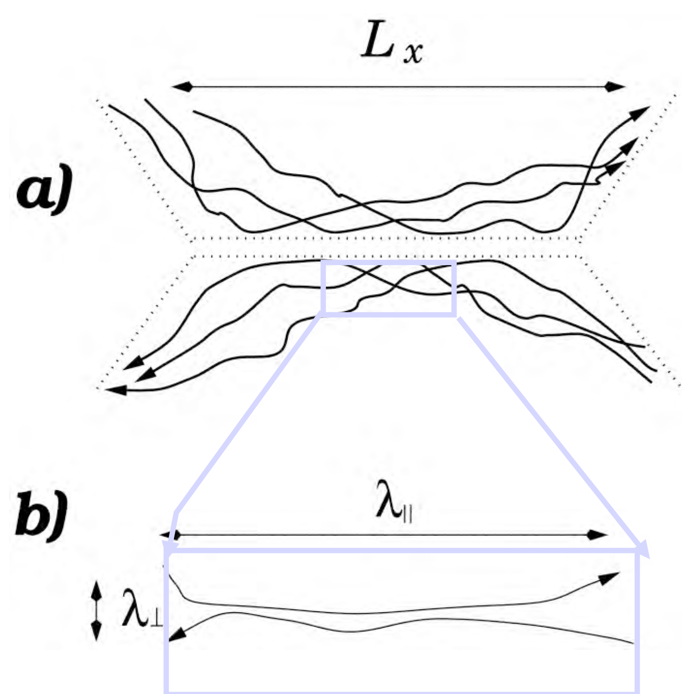
$$\beta = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[Q_{uu}(k, \mathbf{x}; \tau, \tau_1, t) + Q_{bb}(k, \mathbf{x}; \tau, \tau_1, t) \right] = \frac{5}{7} \nu_{\rm K} \qquad \beta \propto \tau K$$

$$\gamma = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[Q_{ub}(k, \mathbf{x}; \tau, \tau_1, t) + Q_{bu}(k, \mathbf{x}; \tau, \tau_1, t) \right] = \frac{5}{7} \nu_{\rm M} \qquad \gamma \propto \tau W$$

$$\boldsymbol{\Gamma} = \frac{1}{15} \int d\mathbf{k}\ k^{-2} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \nabla H_{uu}(k, \mathbf{x}; \tau, \tau_1, t) \qquad \boldsymbol{\Gamma} \propto \tau \ell^2 \nabla H$$

III. Magnetic reconnection

Turbulent reconnection



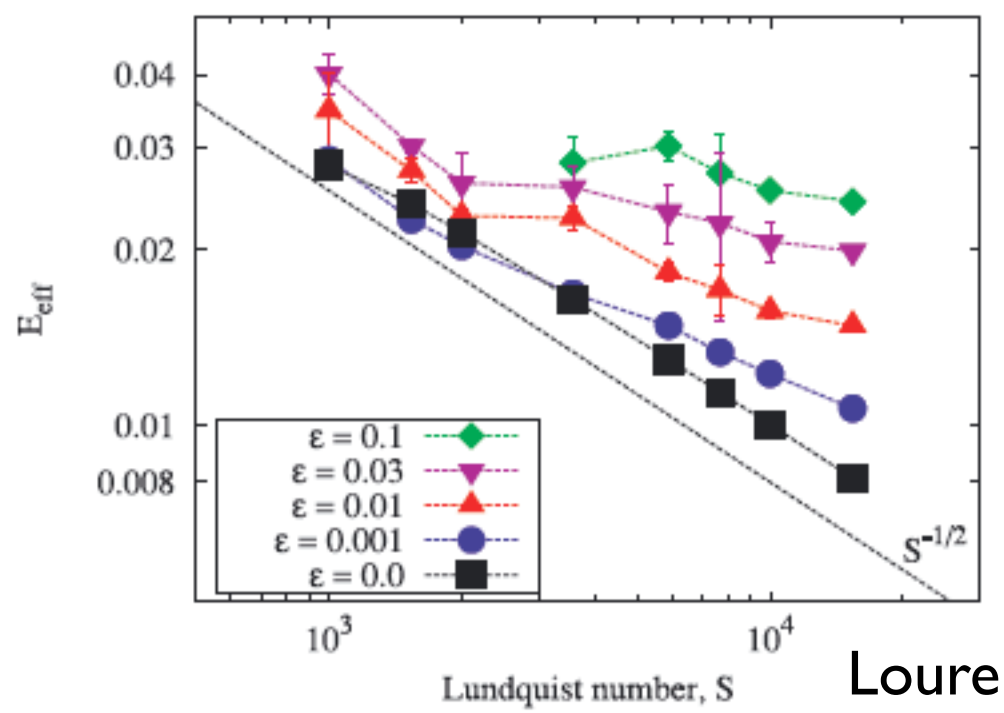
Matthaeus & Lamkin (1985)

Lazarian & Vishniac (1999)

$$M_{\rm in} = \frac{U_{\rm in}}{V_{\rm Ain}} \leq M_{\rm turb}$$

$M_{\rm turb}$: large-scale magnetic Mach number of turbulence

Eyink, et al. (2011) Lagrangian trajectory



Loureiro, et al. (2009)

Numerical simulations

Kowal, et al. (2009)

Servidio, et al. (2009)

Loureiro, et al. (2009)

Lapenta & Lazarian (2011)

. . .

Karimabadi & Lazarian (2013)

Inhomogeneous turbulence in reconnection

Yokoi & Hoshino, PoP 2011
Higashimori, Yokoi & Hoshino, PRL 2013
Yokoi, Higashimori & Hoshino, PoP 2013
Widmer, et al. in preparation

β : Energy

$$\left(\frac{\partial}{\partial t} + \mathbf{U}\cdot\nabla\right)\langle \mathbf{u}'^2 + \mathbf{b}'^2\rangle/2 = \cdots - \langle \mathbf{u}' \times \mathbf{b}'\rangle \cdot \mathbf{J} - \varepsilon_K + \cdots$$
$$= \cdots \boxed{+ \beta \mathbf{J}^2} - \varepsilon_K + \cdots$$

γ : Cross helicity

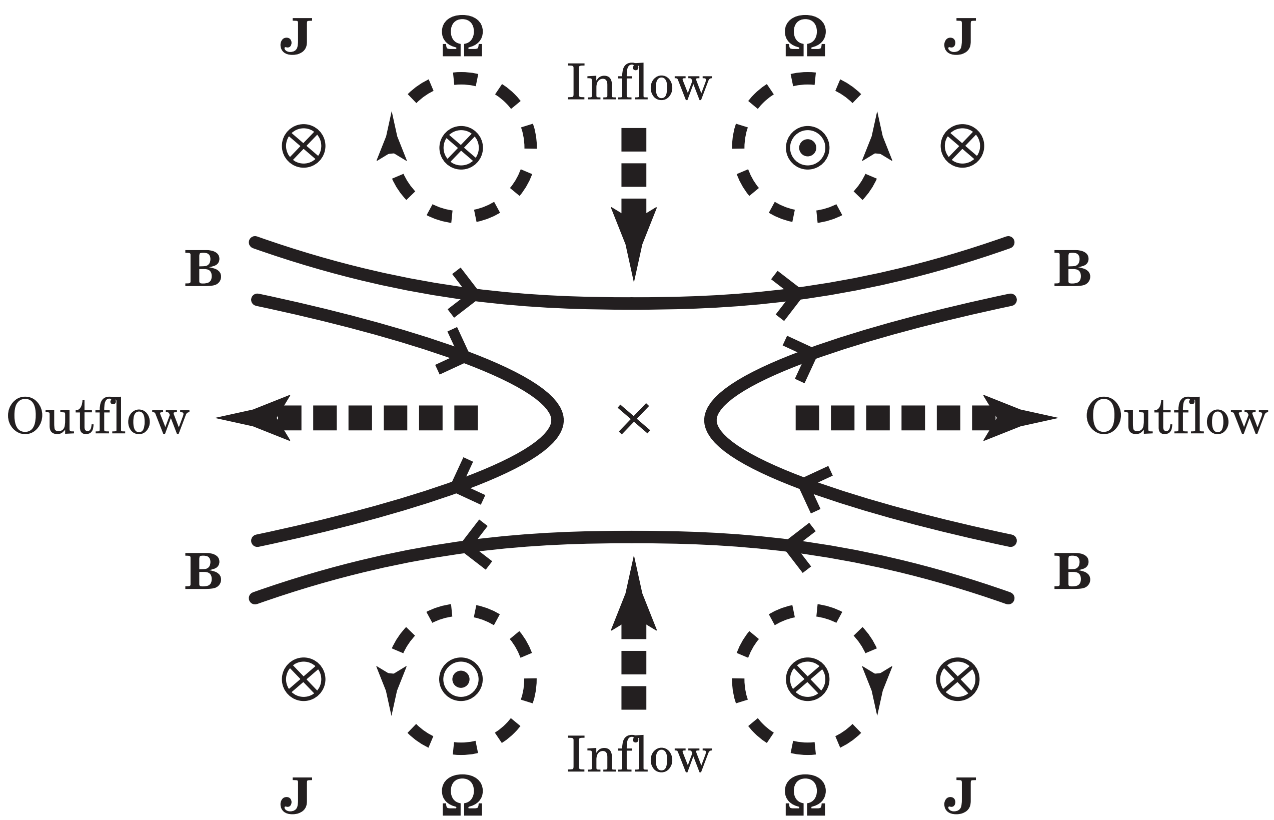
$$\left(\frac{\partial}{\partial t} + \mathbf{U}\cdot\nabla\right)\langle \mathbf{u}' \cdot \mathbf{b}'\rangle = \cdots - \langle \mathbf{u}' \times \mathbf{b}'\rangle \cdot \boldsymbol{\Omega} - \varepsilon_W + \cdots$$
$$= \cdots \boxed{+ \beta \mathbf{J}\cdot\boldsymbol{\Omega}} - \varepsilon_W + \cdots$$

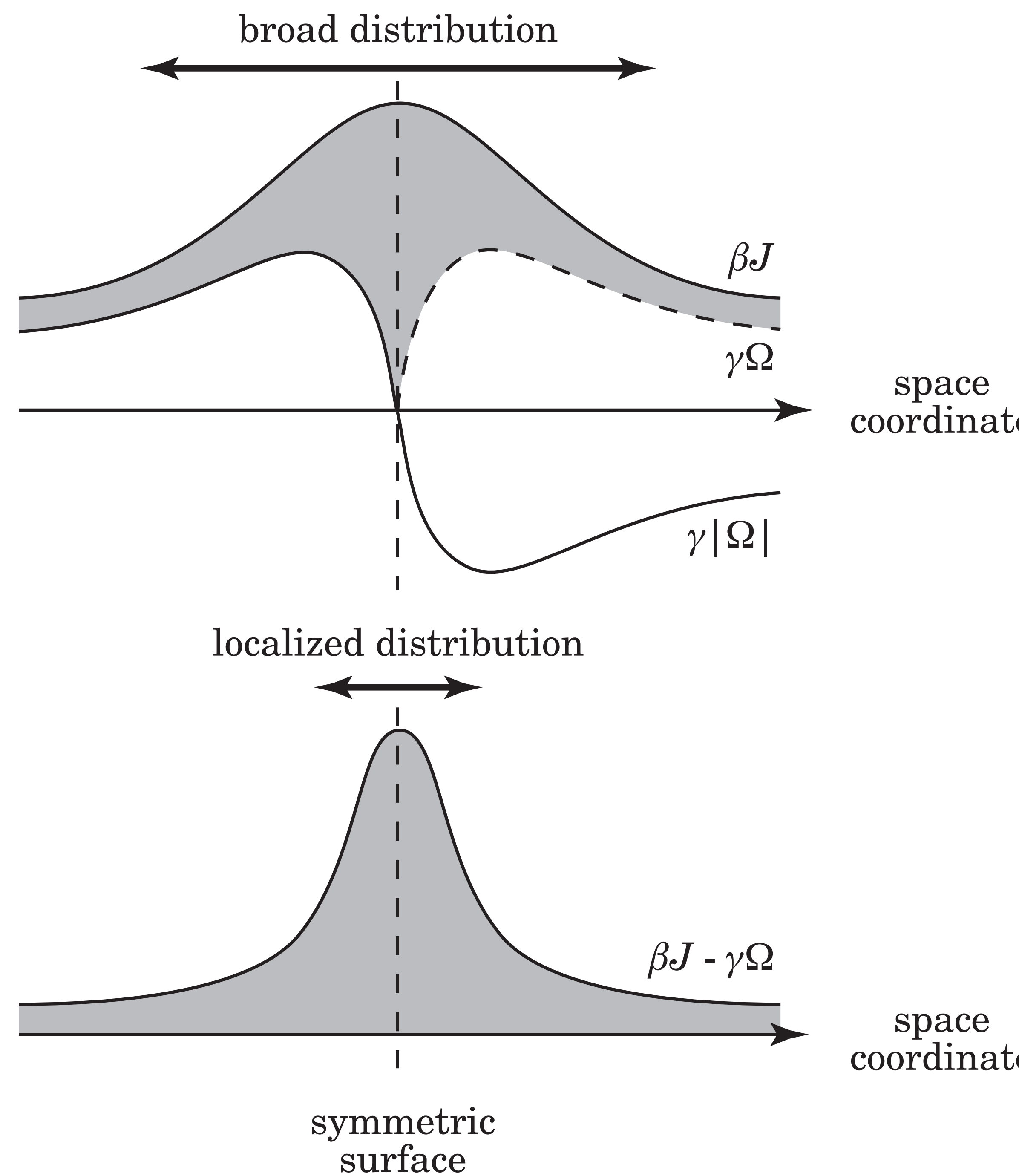
α : Residual helicity

$$\left(\frac{\partial}{\partial t} + \mathbf{U}\cdot\nabla\right)\langle \mathbf{b}' \cdot \mathbf{j}' - \mathbf{u}' \cdot \boldsymbol{\omega}'\rangle = \cdots - \langle \mathbf{u}' \times \mathbf{b}'\rangle \cdot \mathbf{B} - \varepsilon_H + \cdots$$
$$= \cdots \boxed{+ \frac{1}{\tau}\mathbf{B}\cdot\mathbf{J}} - \varepsilon_H + \cdots$$

$$\mathbf{E}_\mathrm{M} \equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle$$
$$= \boxed{-\beta \mathbf{J}} \boxed{+ \alpha \mathbf{B} + \gamma \boldsymbol{\Omega}}$$

Enhancement Suppression





γ : Cross helicity

$$\left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \langle \mathbf{u}' \cdot \mathbf{b}' \rangle$$
$$= \cdots \boxed{+ \beta \mathbf{J} \cdot \boldsymbol{\Omega}} - \varepsilon_W + \cdots$$

Mean momentum equation

$$\frac{\partial \mathbf{U}}{\partial t} = \mathbf{U} \times \boldsymbol{\Omega} + \mathbf{J} \times \mathbf{B} \underline{- \nabla \cdot \boldsymbol{\mathcal{R}}} + \mathbf{F} - \nabla \left(P + \frac{1}{2}\mathbf{U}^2 + \left\langle \frac{1}{2}\mathbf{b}'^2 \right\rangle \right)$$

Turbulence

$$\mathbf{J} = \sigma \left(\mathbf{E} + \mathbf{U} \times \mathbf{B} + \underline{\mathbf{E}_{\mathrm{M}}} \right)$$

Turbulence

$$\begin{cases} \mathcal{R}^{\alpha\beta} = \frac{2}{3} K_{\mathrm{R}} \delta^{\alpha\beta} - \nu_{\mathrm{K}} \mathcal{S}^{\alpha\beta} + \nu_{\mathrm{M}} \mathcal{M}^{\alpha\beta} \\ \mathbf{E}_{\mathrm{M}} = -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \boldsymbol{\Omega} \end{cases}$$

Mean Lorentz force $\mathbf{J} \times \mathbf{B} = \frac{1}{\beta} (\mathbf{U} \times \mathbf{B}) \times \mathbf{B} + \frac{\gamma}{\beta} \boldsymbol{\Omega} \times \mathbf{B} - \frac{1}{\beta} \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla \varphi \right) \times \mathbf{B}$

$$\mathbf{U} = \mathbf{U}_0 + \delta \mathbf{U}, \quad \boldsymbol{\Omega} = \boldsymbol{\Omega}_0 + \delta \boldsymbol{\Omega}$$

Reference $\frac{\partial \boldsymbol{\Omega}_0}{\partial t} = \nabla \times \left[\mathbf{U}_0 \times \boldsymbol{\Omega}_0 + \nu_{\mathrm{K}} \nabla^2 \mathbf{U}_0 + \mathbf{F} - \frac{1}{\beta} \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla \varphi \right) \times \mathbf{B} \right]$

Modulation $\frac{\partial \delta \boldsymbol{\Omega}}{\partial t} = \nabla \times \left[\left(\delta \mathbf{U} - \frac{\gamma}{\beta} \mathbf{B} \right) \times \boldsymbol{\Omega}_0 + \nu_{\mathrm{K}} \nabla^2 \left(\delta \mathbf{U} - \frac{\gamma}{\beta} \mathbf{B} \right) \right]$

$$\longrightarrow \quad \delta \mathbf{U} = \frac{\gamma}{\beta} \mathbf{B} = C_\gamma \frac{W}{K} \mathbf{B}$$

$$\frac{|W|}{K} = \frac{|\langle \mathbf{u}' \cdot \mathbf{b}' \rangle|}{\langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle / 2} \leq 1$$

Basic equations to be solved

Mean fields

4th-order Runge-Kutta scheme in time
4th-order centered difference in space

$$\frac{\partial \overline{\rho}}{\partial t} = -\nabla \cdot (\overline{\rho} \mathbf{U})$$

$$\frac{\partial}{\partial t} (\overline{\rho} \mathbf{U}) = -\nabla \cdot \left[\overline{\rho} \mathbf{U}\mathbf{U} - \mathbf{B}\mathbf{B} + \left(p + \frac{1}{2} \mathbf{B}^2 \right) \boldsymbol{\mathcal{I}} \right]$$

$$\frac{\partial}{\partial t} \left(\frac{p}{\gamma - 1} + \frac{1}{2} \overline{\rho} \mathbf{U}^2 + \frac{1}{2} \mathbf{B}^2 \right) = -\nabla \cdot \left[\left(\frac{\gamma}{\gamma - 1} p + \frac{1}{2} \overline{\rho} \mathbf{U}^2 \right) \mathbf{U} + \mathbf{E} \times \mathbf{B} \right]$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

Effective transport due to the turbulent motions

$$\mathbf{E} = -\mathbf{U} \times \mathbf{B} + \eta \mathbf{J} + \beta \mathbf{J} - \gamma \boldsymbol{\Omega} = -\mathbf{U} \times \mathbf{B} + \eta \mathbf{J} + \tau \left(K \mathbf{J} - W \boldsymbol{\Omega} \right)$$

$$\beta = \tau K$$

$$\gamma = \tau W$$

Turbulent quantities

Turbulence production due to large-scale inhomogeneities

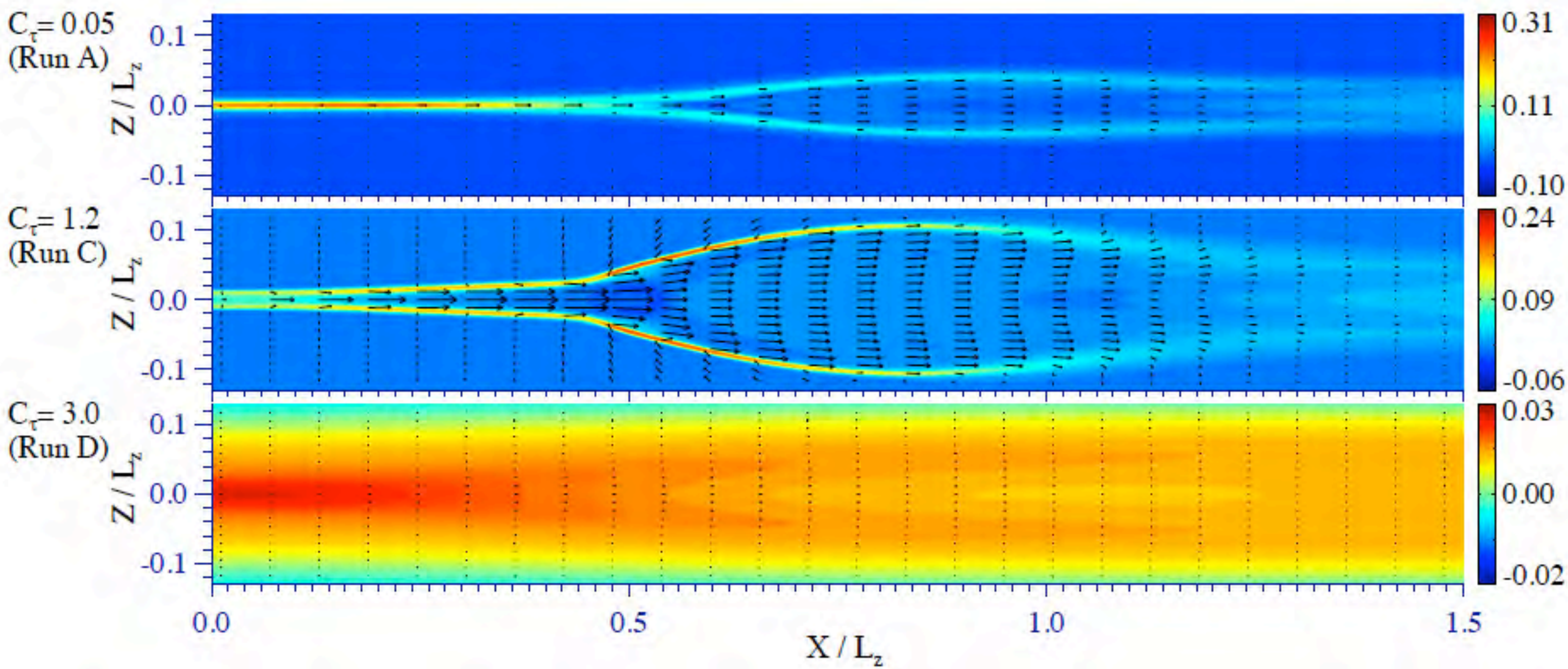
Turbulent energy

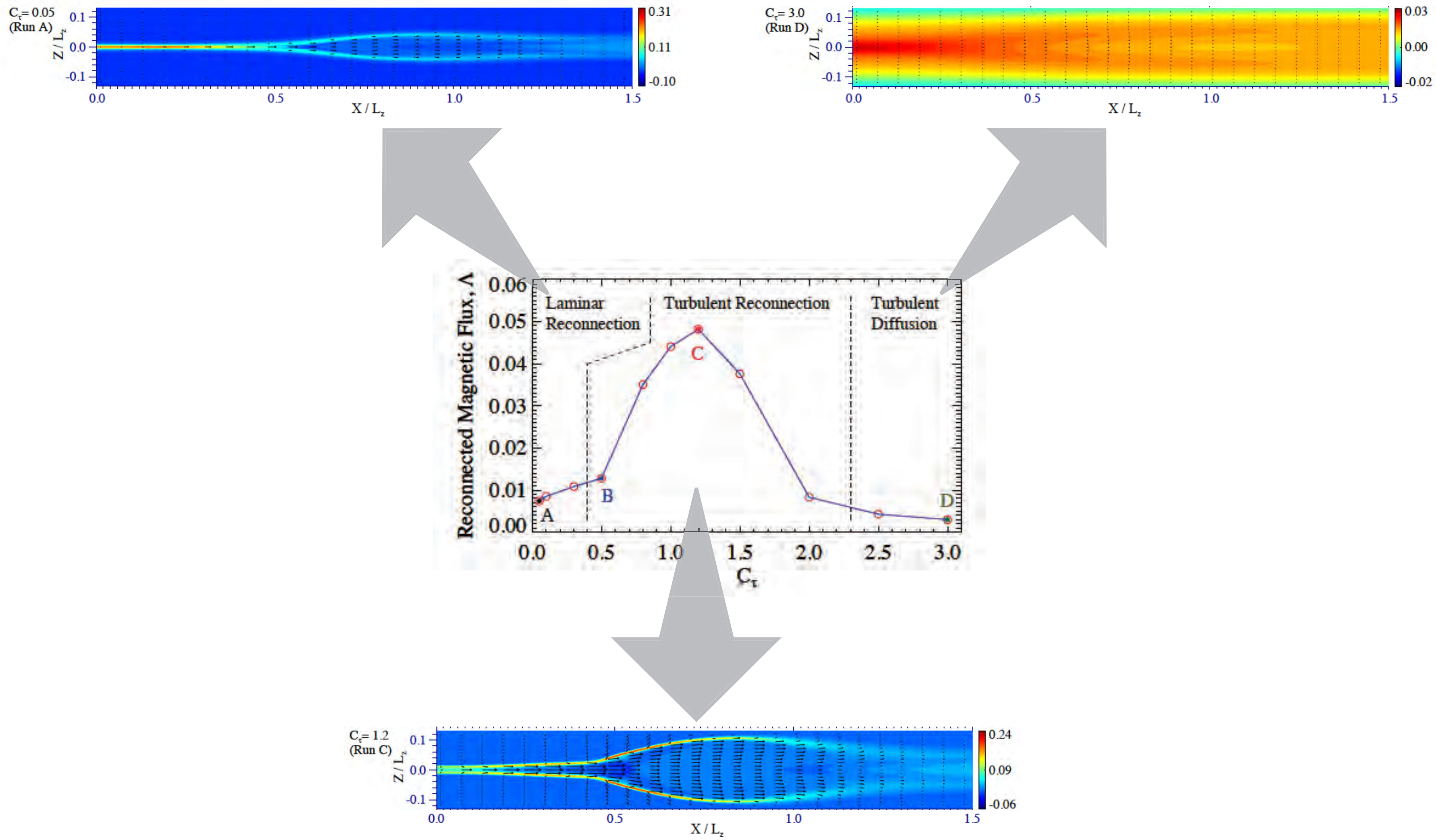
$$\frac{\partial K}{\partial t} = -\mathbf{U} \cdot \nabla K + \tau K \mathbf{J}^2 - \tau W \boldsymbol{\Omega} \cdot \mathbf{J} + \mathbf{B} \cdot \nabla W - \frac{K}{\tau}$$

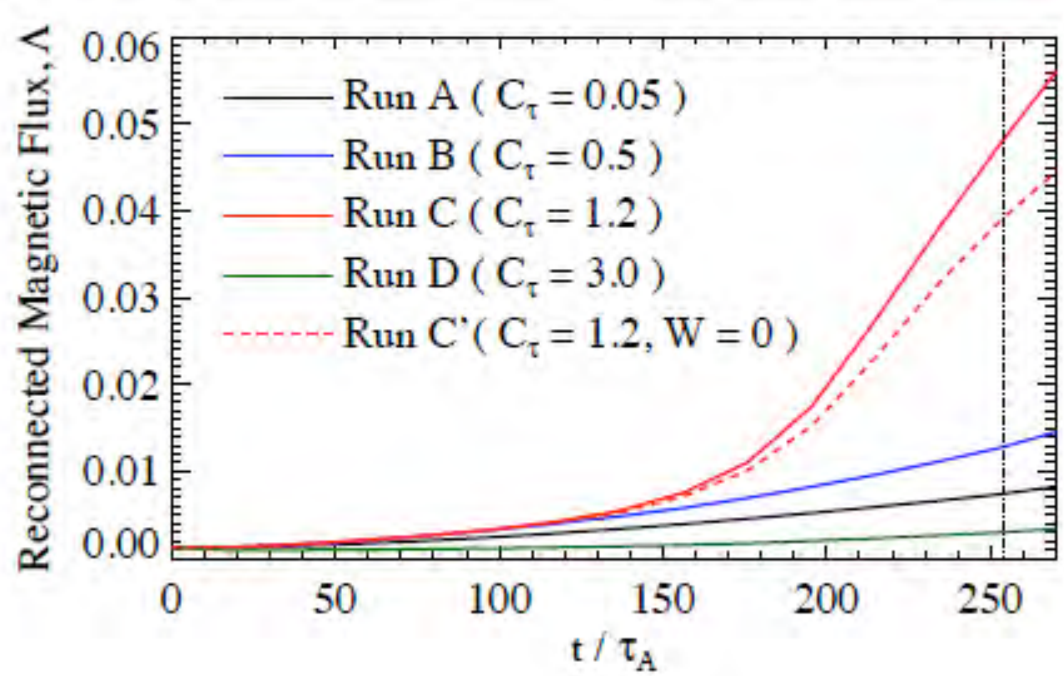
Turbulent cross helicity

$$\frac{\partial W}{\partial t} = -\mathbf{U} \cdot \nabla W + \tau K \boldsymbol{\Omega} \cdot \mathbf{J} - \tau W \boldsymbol{\Omega}^2 + \mathbf{B} \cdot \nabla K - C_W \frac{W}{\tau}$$

Electric-current and flow structures





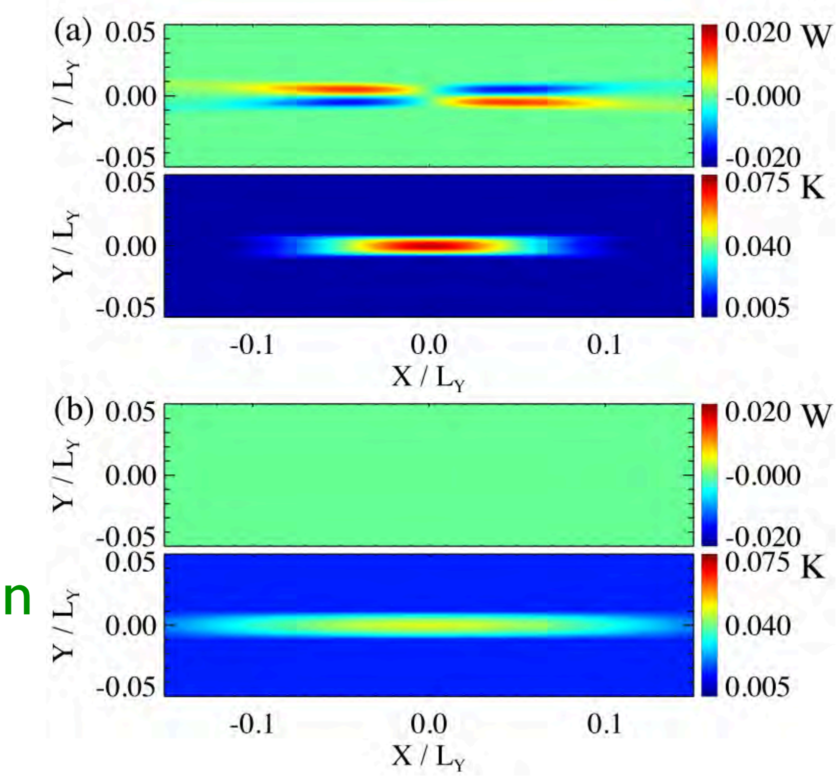


Turbulent reconnection

Turbulent reconnection without cross helicity

Laminar reconnection

Turbulent magnetic diffusion (too much turbulent)



IV. Summary

Summary

Turbulence modeling (based on an analytical statistical theory):

A powerful tool for investigating a realistic (strongly nonlinear and inhomogeneous) turbulence

Even in the framework of MHD:

Large-scale fast reconnection can be obtained and sustained if the turbulence effects are included with a proper model

Turbulence:

Self-generated by inhomogeneities of the large-scale fields without resorting to the external turbulence injection

Turbulent magnetic diffusivity:

- Localized turbulent diffusivity promotes magnetic reconnection
- Too much fluctuation level everywhere leads to just a turbulent magnetic diffusion

Breakage of symmetry (intrinsic to magnetic reconnection):

In addition to the turbulent energy (intensity information), some other turbulent quantities (structural information) may contribute to enhance magnetic reconnection by localizing the effective transport

IV. Density-variance effect

Several points to be examined further

- Mean-field approaches

  Validation from large-eddy, direct numerical, and kinetic simulations

- Model expressions

  Descriptors of turbulence

  Several helicities (other than cross helicity), dissipation rates

- Shock–turbulence interaction

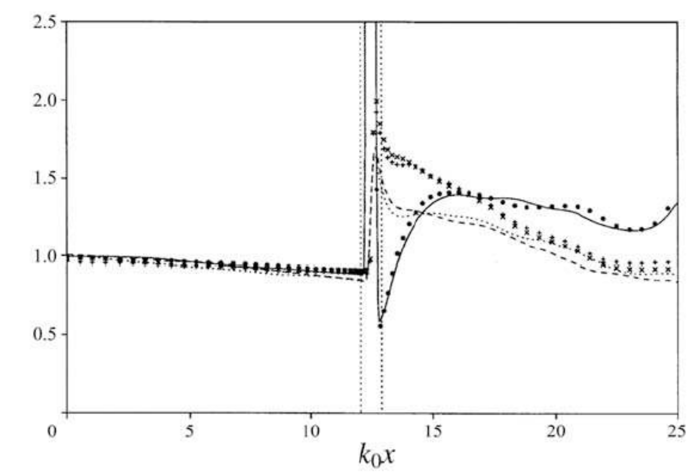
  A very strong density variation

  $\longrightarrow$ Density fluctuation (density variance)

  $\longrightarrow$ Large-scale vortical structures, turbulence structures

Shock–turbulence interaction

Anisotropy of turbulent intensities

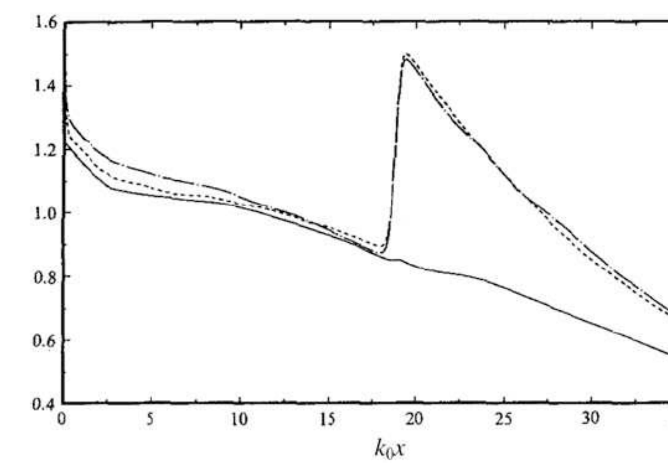


(Lee, Lele & Moin, 1997)

Transverse vortical structures

Vortex generation mechanisms

Baroclinicity, helicities

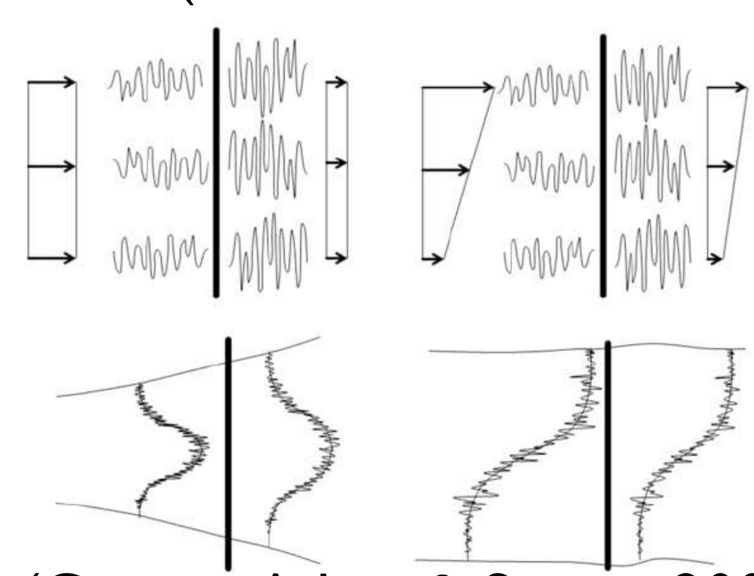


(Lee, Lele & Moin, 1993)

Turbulence generation

Turbulence transport

Transport enhancement or suppression



(Garnier, Adam & Sagaut, 2009)

Fundamental equations

Mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

Momentum

$$\frac{\partial}{\partial t}\rho u^{\alpha} + \frac{\partial}{\partial x^{a}}\rho u^{a} u^{\alpha}$$

$$= -\left(\gamma_0 - 1\right) \frac{\partial}{\partial x^{\alpha}}\rho q + \frac{\partial}{\partial x^{\alpha}}\mu s^{a\alpha} + (\mathbf{j} \times \mathbf{b})^{\alpha}$$

Internal energy

$$\frac{\partial}{\partial t}\rho q + \nabla \cdot (\rho \mathbf{u} q) = \nabla \cdot (\kappa \nabla \theta) - p \nabla \cdot \mathbf{u} + \phi$$

Magnetic field

$$\frac{\partial \mathbf{b}}{\partial t} = -\nabla \times \mathbf{e}$$

Ohm's law

$$\mathbf{j} = \frac{1}{\mu_0} \nabla \times \mathbf{b} = \sigma \left(\mathbf{e} + \mathbf{u} \times \mathbf{b}\right)$$

Eq. of state

$$p = R \rho \theta = \left(\gamma_0 - 1\right) \rho q$$

$$q = C_V(\theta) \theta$$

Physical interpretations

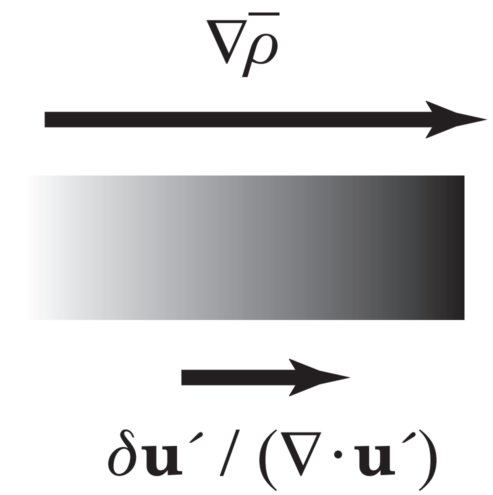
Mean density variation

$$\langle \mathbf{u}' \times \mathbf{b}' \rangle / \mu_0 = -\eta_\rho \nabla \overline{\rho} \times \mathbf{B} + \cdots$$

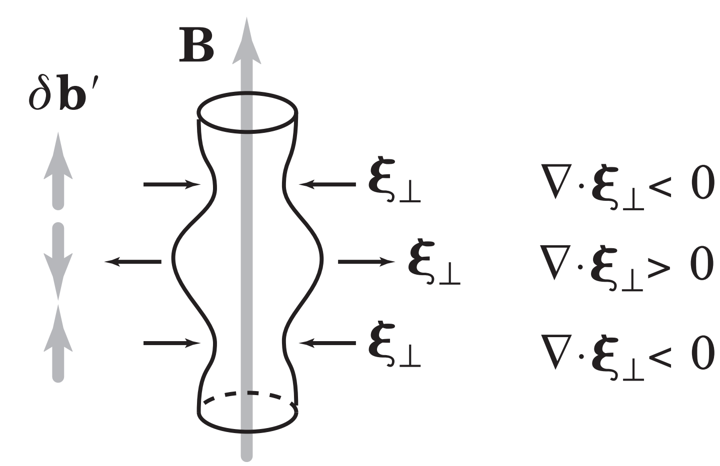
Simplest expressions for the density and internal-energy fluctuations

$$\rho' = -\tau_\rho \overline{\rho} \nabla \cdot \mathbf{u}' \qquad\qquad q' = -(\gamma_{\rm s} - 1)\tau_q Q \nabla \cdot \mathbf{u}'$$

$$\begin{aligned}\frac{\partial \mathbf{u}'}{\partial t} &= \cdots - (\gamma_{\rm s} - 1)\frac{q'}{\overline{\rho}} \nabla \overline{\rho} + \cdots \\ &= \cdots + (\gamma_{\rm s} - 1)^2 \tau_q \frac{Q}{\overline{\rho}} (\nabla \cdot \mathbf{u}') \nabla \overline{\rho} + \cdots\end{aligned}$$

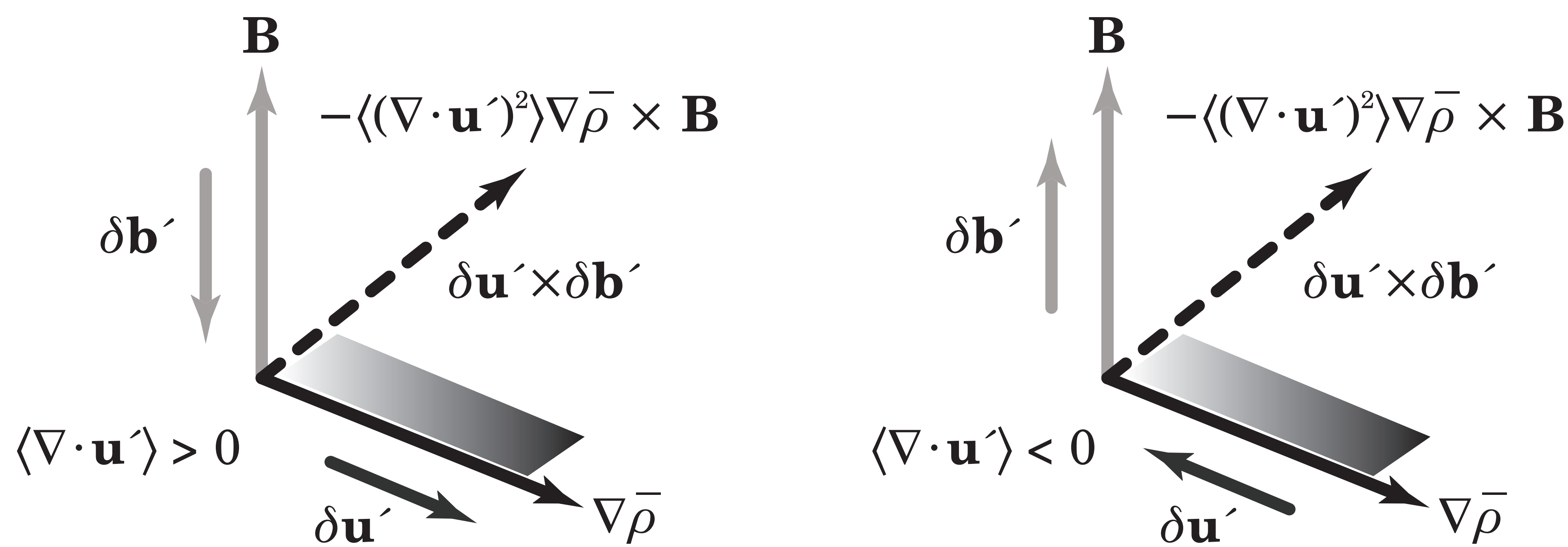


$$\frac{\partial \mathbf{b}'}{\partial t} = \cdots - (\nabla \cdot \mathbf{u}')\mathbf{B} + \cdots$$



$$\frac{\partial}{\partial t}\langle \mathbf{u}' \times \mathbf{b}' \rangle \simeq \cdots + (\gamma_{\rm s} - 1)\frac{1}{\overline{\rho}}\langle q' \nabla \cdot \mathbf{u}' \rangle \nabla \overline{\rho} \times \mathbf{B} + \cdots$$

$$= \cdots - (\gamma_{\rm s} - 1)\tau_q \langle (\nabla \cdot \mathbf{u}')^2 \rangle \frac{Q}{\overline{\rho}} \nabla \overline{\rho} \times \mathbf{B} + \cdots$$



Irrespective of the sign of dilatation, the electromotive force is generated in the direction of **B** x ∇ρ

Energy generation due to the mean density variation

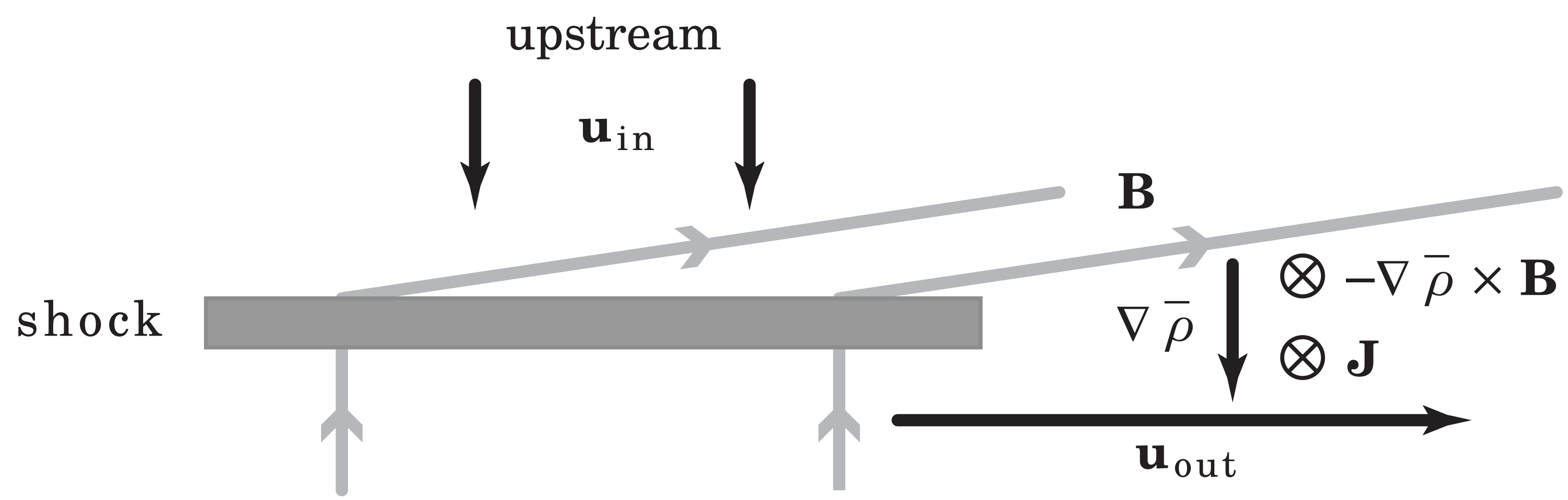
$$\frac{D}{Dt}\frac{1}{2}\left\langle \mathbf{u}'^2 \right\rangle = \boxed{-\frac{1}{2\overline{\rho}}\left\langle \mathbf{u}' \times \mathbf{b}' \right\rangle \cdot \mathbf{J}} - \left\langle u'^a u'^b \right\rangle \frac{\partial U^a}{\partial x^b} + \frac{1}{2\mu_0 \overline{\rho}} \left\langle u'^a b'^b \right\rangle \left(\frac{\partial B^b}{\partial x^a} + \frac{\partial B^a}{\partial x^b} \right)$$

$$- (\gamma_{\rm s} - 1) \frac{1}{\overline{\rho}} \left(\left\langle \rho' \mathbf{u}' \right\rangle \cdot \nabla Q + \left\langle q' \mathbf{u}' \right\rangle \cdot \nabla \overline{\rho} \right) - \frac{1}{\overline{\rho}} \left\langle \rho' \mathbf{u}' \right\rangle \cdot \frac{D\mathbf{U}}{Dt} - \varepsilon_u + T_u$$

where

$$\left\langle \mathbf{u}' \times \mathbf{b}' \right\rangle / \mu_0 = \boxed{-\eta_\rho \nabla \overline{\rho} \times \mathbf{B}} + \cdots$$

Note that $\eta_\rho \propto K_\rho = \left\langle \rho'^2 \right\rangle$



In the slow shocks in magnetic reconnection, the turbulent electromotive force due to the mean density variation enhances the turbulence generation in the foreshock (upstream) region but no effects in the aftershock (downstream) region.