MEASURES OF *DIFFUSION REGIONS* APPLIED TO 2D PIC RECONNECTION SIMULATIONS

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A central goal of MMS is to study *diffusion regions* during magnetic reconnection.

- ***"Diffusion region"* is ambiguous**
  - Many physically different *measures*
  - Electrons, ions or MHD?
- **Measures of *diffusion regions*** *(Goldman, et al, Sp. Sc. Rev, 2015)*
  - Generalized Ohm's laws *(fluid moments)*
    - *"Slippage"* of particle motion from field line motions
    - Conservation of magnetic flux
    - Work
  - Agyrotropy (kinetic or fluid)
  - Magnetic null or $\beta >> 1$
- **What do measures reveal when applied to 2D PIC?**

Generalized Ohm's law for each electron or ion *fluid* ⇒ measures of e or i diffusion region

$$\{\mathbf{E}+\mathbf{u}_s\times\mathbf{B}\}=\mathbf{R}_s \quad (\text{kinetic}) \quad \mathrm{s}=\mathrm{e},\mathrm{i}$$

velocity slippage: $\{\ \}\left(\times\mathbf{B}/B^2\right) = \mathbf{u}_{s\perp}-\mathbf{E}\times\mathbf{B}/B^2,\ \mathbf{E}_{\parallel}=\mathbf{R}_{s\parallel}$

magnetic flux: $\nabla\times\{\ \}=-\partial_t\mathbf{B}+\nabla\times\left(\mathbf{u}_s\times\mathbf{B}\right)$

work: $\mathbf{J}_s\cdot\{\ \}=\mathbf{J}_s\cdot\mathbf{E}$

Ideal fluid: $\mathbf{R}_s=0$ (no reconnection)

$\boxed{\mathbf{E}=-\mathbf{u}_s\times\mathbf{B}\text{ convective field}}$

$\mathbf{u}_{s\perp}=\mathbf{E}\times\mathbf{B}/B^2$ field lines frozen-in to fluid, $\mathbf{E}_{\parallel}=0$

$\partial_t\mathbf{B}=\nabla\times\left(\mathbf{u}_s\times\mathbf{B}\right)$ flux conserv. (when integrated)

$\mathbf{J}_s\cdot\mathbf{E}=0$ no work done by electric field on e/i-fluid

Resistive fluid: $\mathbf{R}_s=\eta_s\mathbf{J}_s$

Momentum loss

$\partial_t\mathbf{B}=\nabla\times\left(\mathbf{u}_s\times\mathbf{B}\right)-\eta\nabla^2\mathbf{B}$ magnetic *diffusion*

$\mathbf{J}_s\cdot\mathbf{E}=\eta_s J_s^2$ work causes *e or i Joule heating*

Language of diffusion/resistive (non-ideal) regions

R_e in *collisionless kinetic plasma*

- Separate Vlasov eqs. for electrons and ions ⇒ separate electron/ion fluid momentum eqns
  - $\mathbf{R}_e$ (or $\mathbf{R}_i$) can be calculated from PIC *simulation*

$$\mathbf{E} + \frac{\mathbf{u}_e}{c} \times \mathbf{B} = \mathbf{R}_e \equiv \left[\frac{1}{ne} \nabla \cdot \mathbf{P}_e \right]_{pressure} + \left[\frac{-m_e}{e} \frac{D\mathbf{u}_e}{Dt} \right]_{inertial}$$

Hall physics for R_i leads to separate electron and ion diffusion regions

$$\mathbf{E} + \mathbf{u}_e \times \mathbf{B} = \mathbf{R}_e \quad \text{Add } \mathrm{u}_i - u_e = \frac{\mathrm{J}}{ne} \text{ to both sides}$$

$$\mathbf{E} + \mathbf{u}_i \times \mathbf{B} = \left[\frac{\mathbf{J}}{ne} \times \mathbf{B}\right]_{Hall} + \mathbf{R}_e$$

"Hall" ➔ $$\boxed{\mathbf{R}_e = 0, \quad \mathbf{E}_H = \left[\frac{\mathbf{J}}{nec} \times \mathbf{B}\right]_{Hall}}$$

Observers use $E_H \neq [JxB/ne]_{Hall}$ as evidence that *electrons* are NOT ideal. ($R_e \neq 0$)

MHD generalized Ohm's law

$$\boxed{\begin{array}{l}\mathbf{E}+\mathbf{u}_e\times\mathbf{B}=\mathbf{R}_e\\ \mathbf{E}+\mathbf{u}_i\times\mathbf{B}=\mathbf{R}_i\end{array}} \rightarrow \boxed{\mathbf{E}+\mathbf{U}\times\mathbf{B}=\mathbf{R}_{MHD}} \equiv \frac{m_e\mathbf{R}_e+M_i\mathbf{R}_i}{M_i+m_e}$$

$$\mathbf{U}\equiv\frac{m_e\mathbf{u}_e+M_i\mathbf{u}_i}{m_e+M_i}$$

Work criteria for diffusion regions. Do **convective E-fields** perform work?

- $\mathbf{E}_{conv}$ **= -u**$_e$x **B**, of *electron fluid* does *no work* on $\mathbf{J}_e$

$$\mathbf{J}_e \cdot (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) = \boxed{\mathbf{J}_e \cdot \mathbf{E}}, \quad \text{because } \mathbf{u}_e \cdot (\mathbf{u}_e \times \mathbf{B}) = 0$$

- $\mathbf{E}_{conv}$ **= -u**$_i$x **B**, of *ion fluid* does *no work* on $\mathbf{J}_i$

$$\mathbf{J}_i \cdot (\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) = \boxed{\mathbf{J}_i \cdot \mathbf{E}}, \quad \text{because } \mathbf{u}_i \cdot (\mathbf{u}_i \times \mathbf{B}) = 0$$

- Convective field, $\mathbf{E}_{conv}$ **= -U**x **B**, of ideal ***MHD*** **fluid** **<u>does</u>** **work** on total (MHD) current, **J**

$$\boxed{\mathbf{J} \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}) \neq \mathbf{J} \cdot \mathbf{E}}$$

Work measure of diffusion region in MHD fluid is related to Joule heating and to D_e

Frame-invariant measure: $D_e = \mathbf{J}\cdot\left[\mathbf{E} + \mathbf{u}_e \times \mathbf{B}\right] - \rho \mathbf{u}_e \cdot \mathbf{E} \neq 0$ (Zenitani)

Quasineutral: $D_e \equiv \mathbf{J}\cdot\left(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}\right)$

Define $W' = -\mathbf{J}\cdot\left(\mathbf{u}_e \times \mathbf{B}\right) \quad = -\mathbf{J}\cdot\left(\mathbf{u}_i \times \mathbf{B}\right) \leftarrow$ from $\mathbf{u}_e = \mathbf{u}_i - \mathbf{J}/en$

Physical form of W′ — work done by MHD-convective field on **J**, etc

$$\boxed{W' = -\mathbf{J}\cdot\left(\mathbf{U} \times \mathbf{B}\right),\ \ \mathrm{D}_e = \mathbf{J}\cdot\left[\mathbf{E} + \mathbf{U} \times \mathbf{B}\right]}$$

For **resistive** MHD: $\mathbf{R}_{MHD} = \eta_{MHD}\mathbf{J}$, $\boxed{\mathrm{D}_e = \eta_{MHD} J^2}$ Joule heating

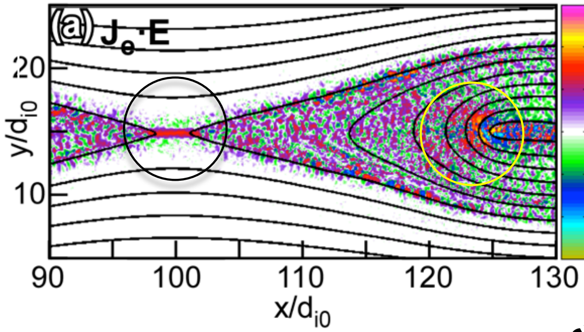
Also note that: $W' = \mathbf{U}\cdot\left(\mathbf{J} \times \mathbf{B}\right)$

Application of diffusion-region measures in 2D tail reconnection *simulations*

- Initiated by Harris sheet + perturbation
  - artificial onset but good agreement with tail measurements

- Small guide field $B_g=0.1B_0$
  - **not** antiparallel reconnection but not inconsistent with tail measurement

Work on *electron* fluid

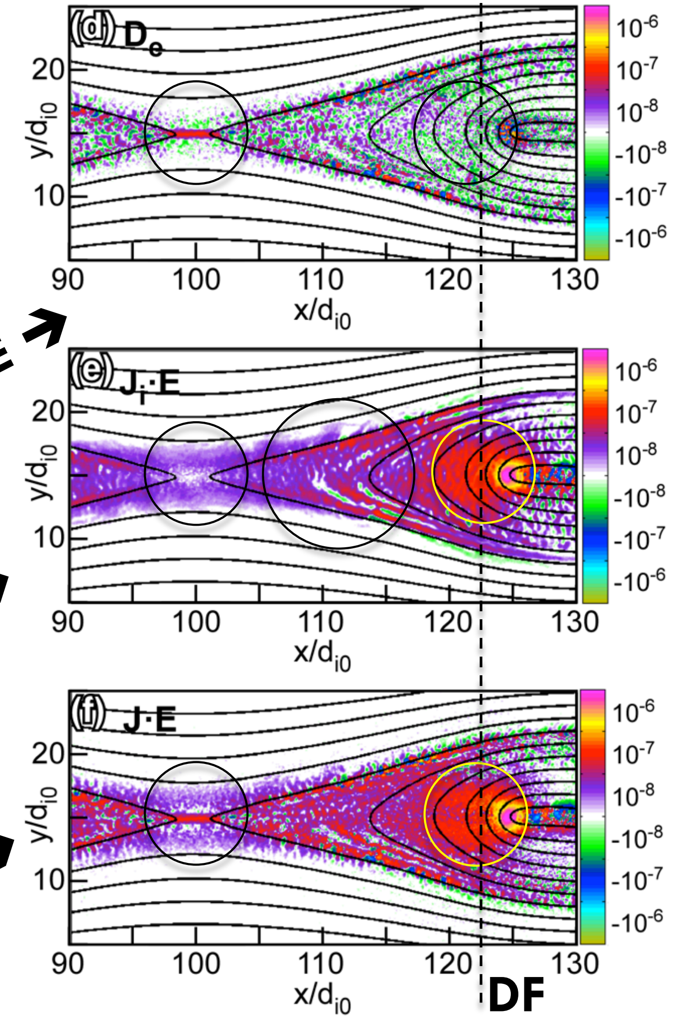


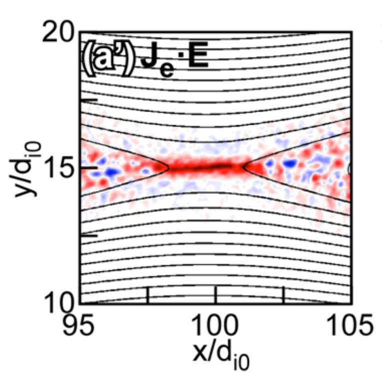
Work measures of diffusion regions in PIC reconnection

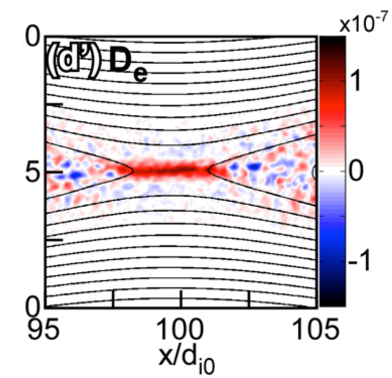
Work on *MHD fluid* by non-convective E →

Work on *ion fluid* →

Work on *MHD* fluid by total E →







Flux non-conservation measure of diffusion region in fluid, s

$$\nabla\times[\mathbf{E}+\mathbf{u}_s\times\mathbf{B}] = \nabla\times\mathbf{R}_s \neq 0$$

Magnitude of at least one component must differ significantly from zero to violate flux conservation

Even if no slippage ($\mathbf{E}_\perp+\mathbf{u}_s\times\mathbf{B} = 0$) can still **violate flux conservation** if

$$\nabla\times[\mathbf{E}+\mathbf{u}_s\times\mathbf{B}] = \nabla\times\mathbf{E}_{||} \neq 0$$

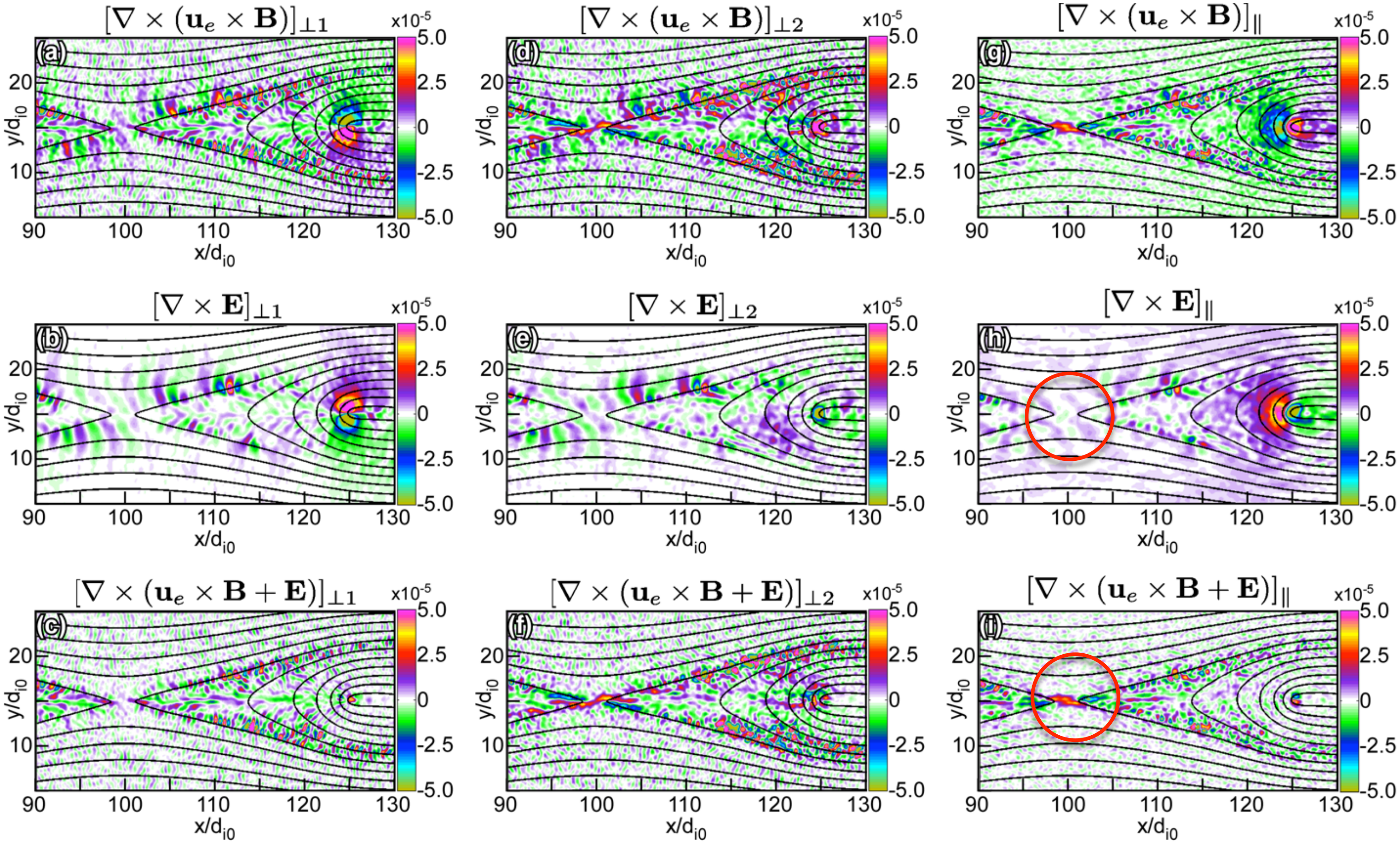
Flux non-conservation measure of *electron* diff. region in PIC reconnection simulation

in-plane perp component | other perp component | parallel component

$C_2 \equiv \nabla \times (\boldsymbol{u}_e \times \boldsymbol{B})$

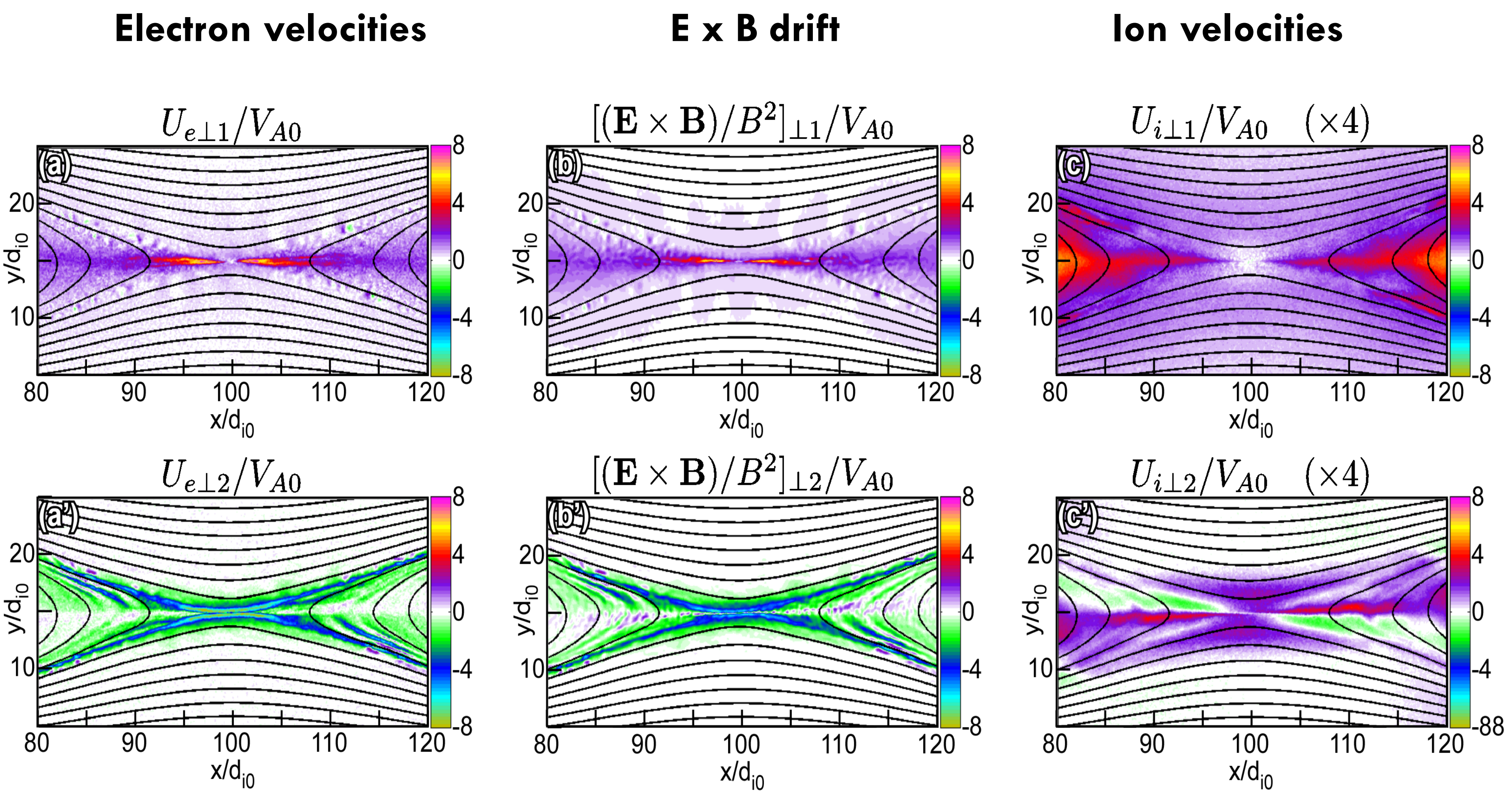
$C_1 \equiv \nabla \times \boldsymbol{E}$

$C = C_1 + C_2 = \nabla \times (\boldsymbol{E} + \boldsymbol{u}_e \times \boldsymbol{B})$

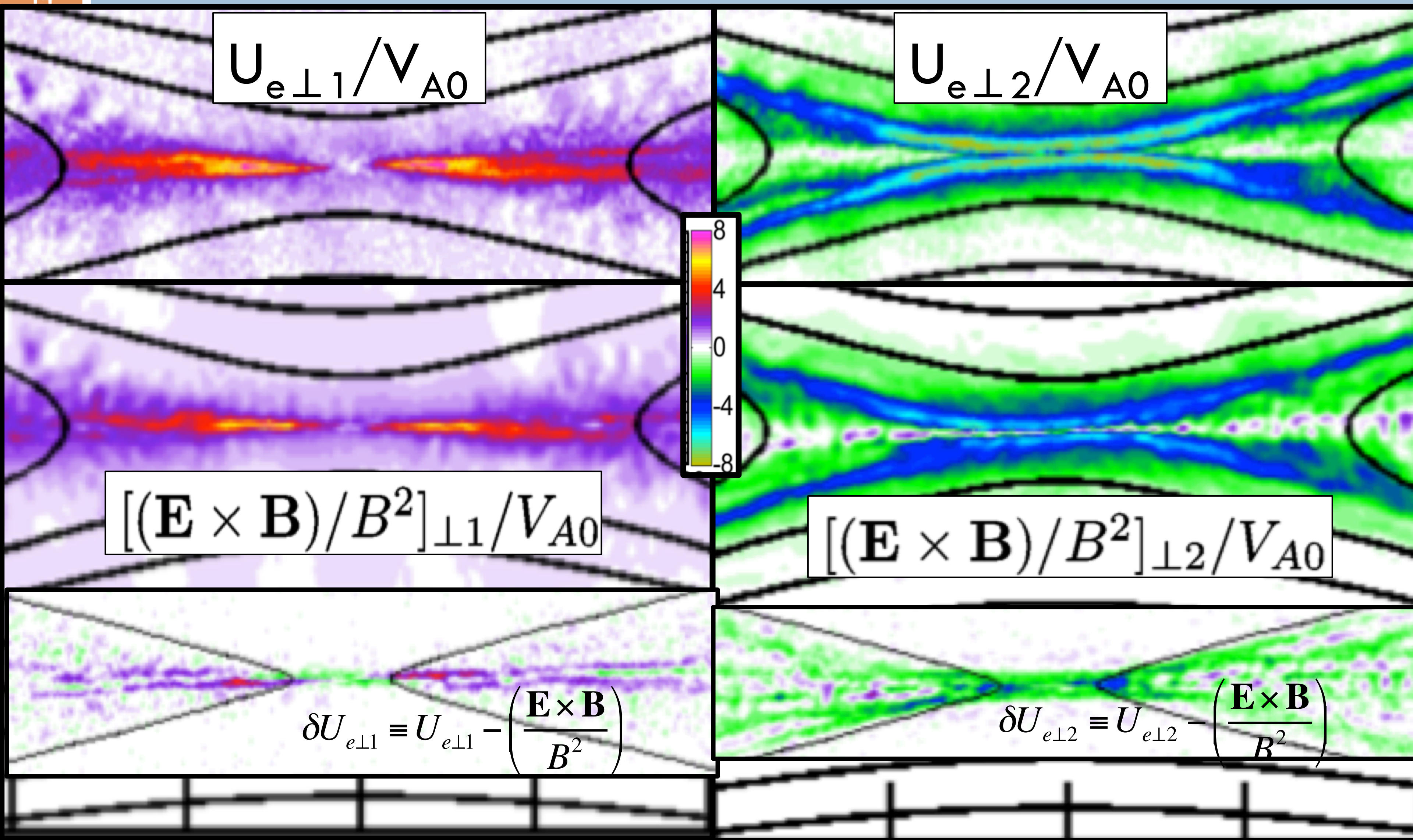


$C \neq 0$ if
$|C_{1j}| \ll |C_{2j}|$,
$|C_{1j}| \gg |C_{2j}|$

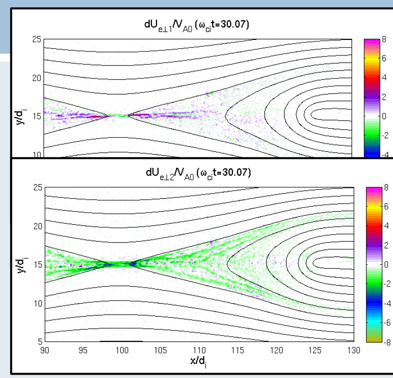
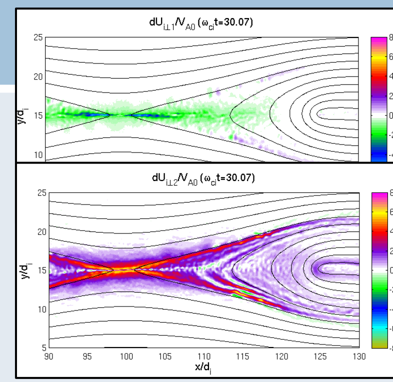
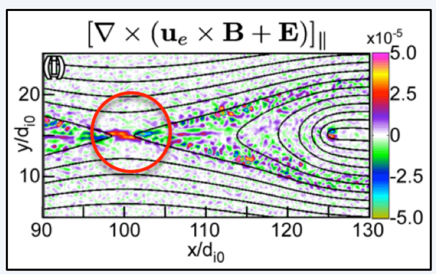
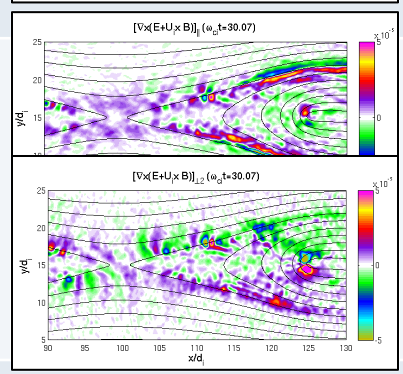
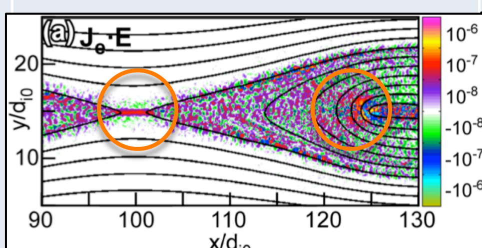
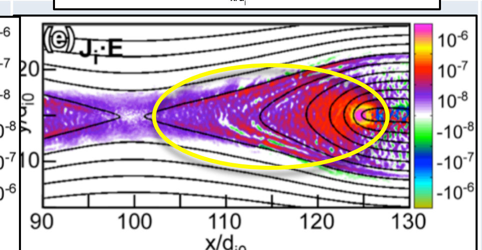
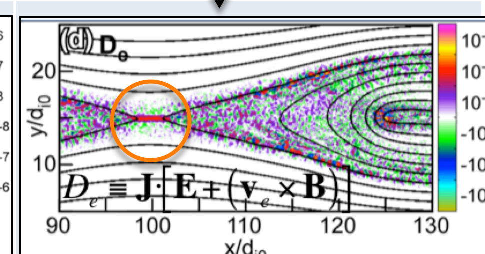
What do electron and ion slippage measures of diffusion regions look like in simulation?



More detail: *Slippage* measure of electron diffusion region around x-line



Simulation diffusion regions found by different fluid measures

Measures of diffusion region	Electron fluid	Ion fluid	
Slippage (for $\mathbf{E}_{\|\|}$=0) $$u_{s\perp 1,2}-\left(\frac{\mathbf{E}\times\mathbf{B}}{B^2}\right)_{\perp 1,2}\neq 0$$			
Non-conservation of magnetic flux $$\left[\nabla\times\left(\mathbf{E}+\mathbf{u}_s\times\mathbf{B}\right)\right]_j\neq 0$$			**MHD fluid** ↓
Work on fluid current $$\mathbf{J}_s\cdot\mathbf{E}\neq 0,\quad s=e,\,i,$$ $$\mathbf{J}\cdot\left(\mathbf{E}+\mathbf{U}_{\mathrm{MHD}}\times\mathbf{B}\right)\neq 0$$			

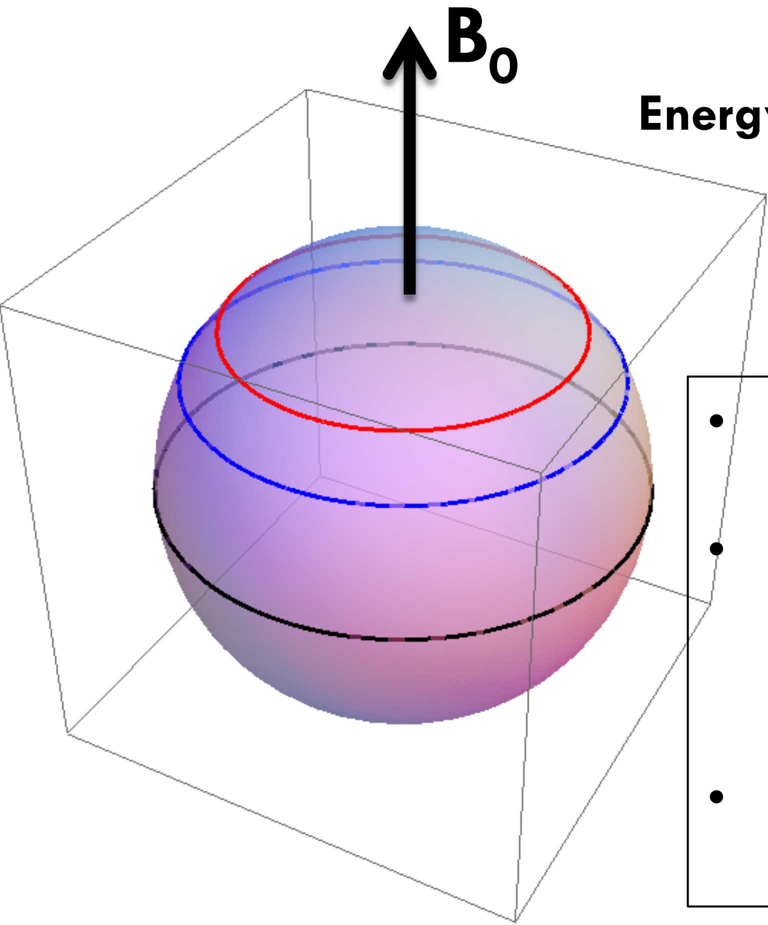
Agyrotropy in velocity distribution

$$f_e(|\mathbf{v}|, \theta, \phi)$$

Energy

Pitch angle

Gyro-angle



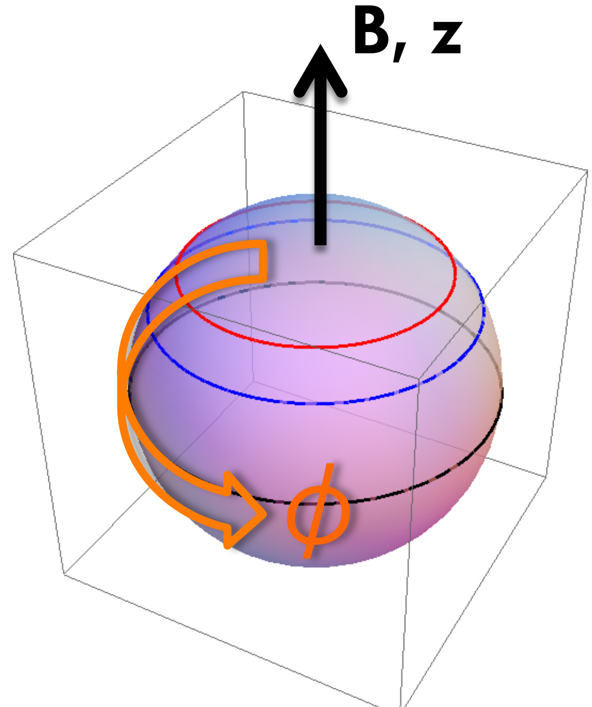
- **Complete gyrotropy:** $f_e = f_e(\phi)$ at all energies and pitch angles in frame with <**v**> = 0
- **Complete agyrotropy:** measure of diffusion-region
  - $f_e \neq f_e(\phi)$ at *all* pitch angles and energies
  - *At fluid* level
    - Pressure tensor with 3 different temperatures
- ***Kinetic (partial) gyrotropy/agyrotropy***
  - can occur at certain energies and pitch angles

Kinetic agyrotropy requires spatial inhomogeneity scale length, L $\lesssim$ **electron gyroradius**

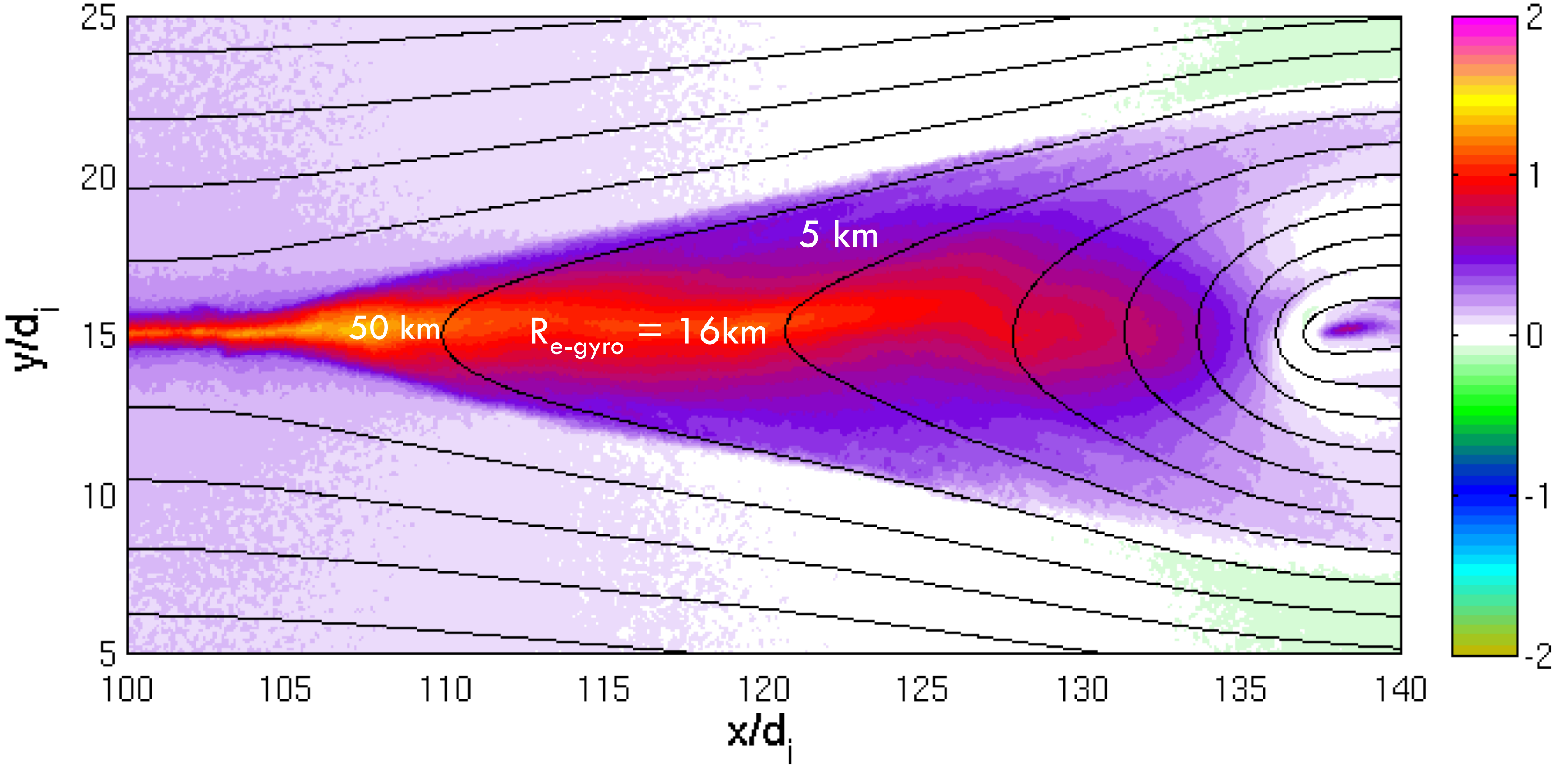
- L $\lesssim$ $R_{e\text{-}gyro}$
- Vlasov:
  - Stationary inhomogeneous agyrotropic equilibrium $f_e(\mathbf{v})$ obeys

$$\boxed{\left[v_y \partial_y + \Omega_e \partial_\phi\right] f = 0}$$

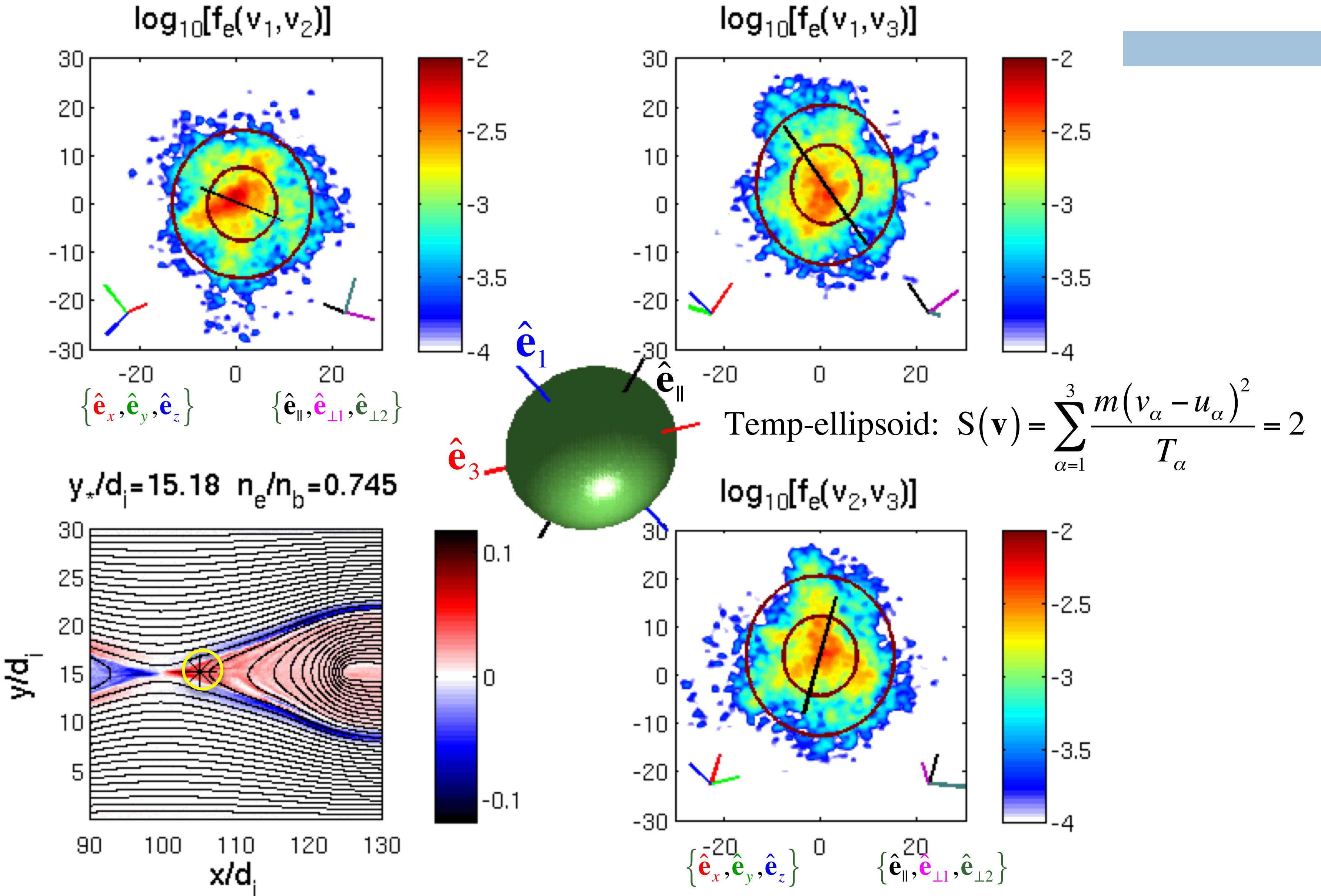
  - $v_y/(\Omega_e L_y) \gtrsim 1$
  - Fluid-moment: 3 different temps in diagonalized pressure tensor.
    - 3D kinetic distribution (FPI on MMS) better!



Thermal electron gyroradius, $R_{e\text{-}gyro}$ (sim. with $B_g = 0.1B_0$)



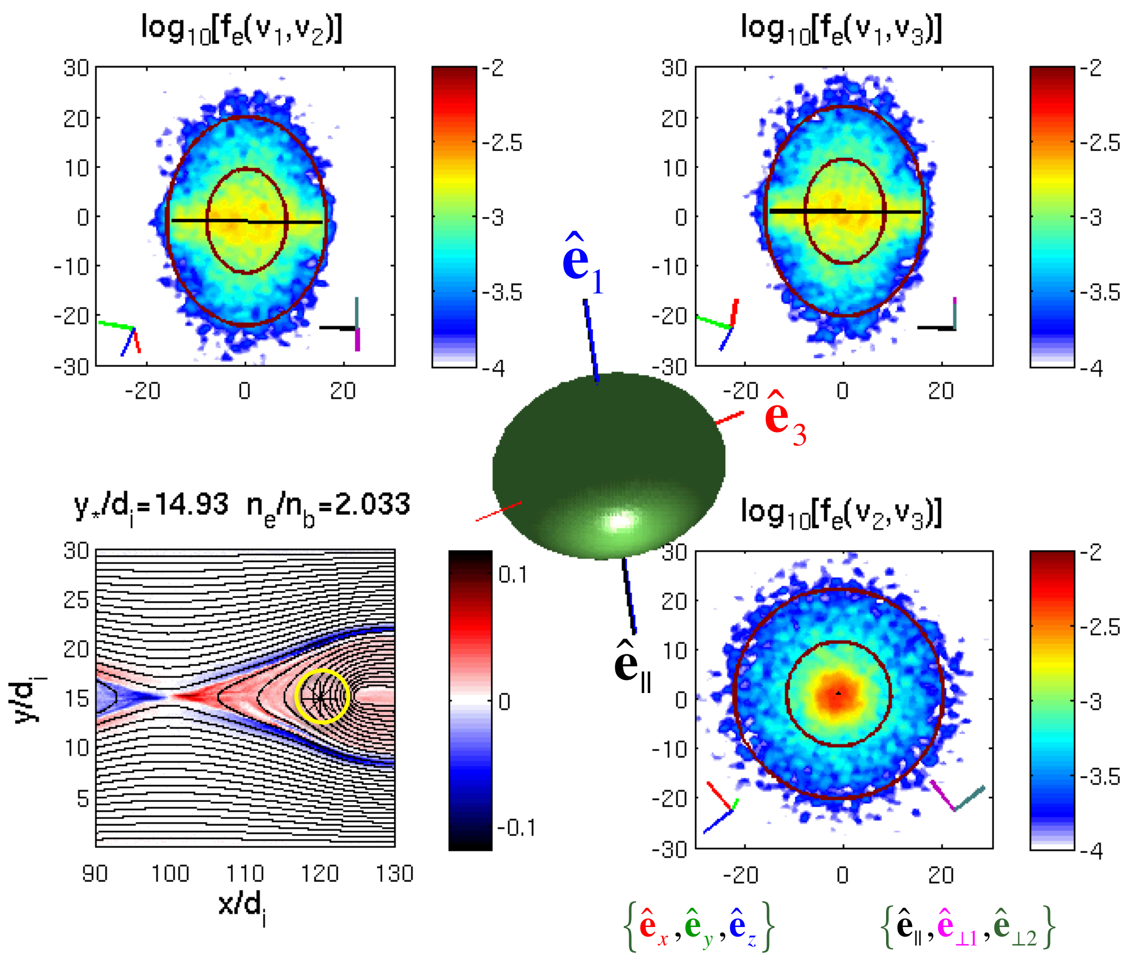
Agyrotropy in electron diffusion region



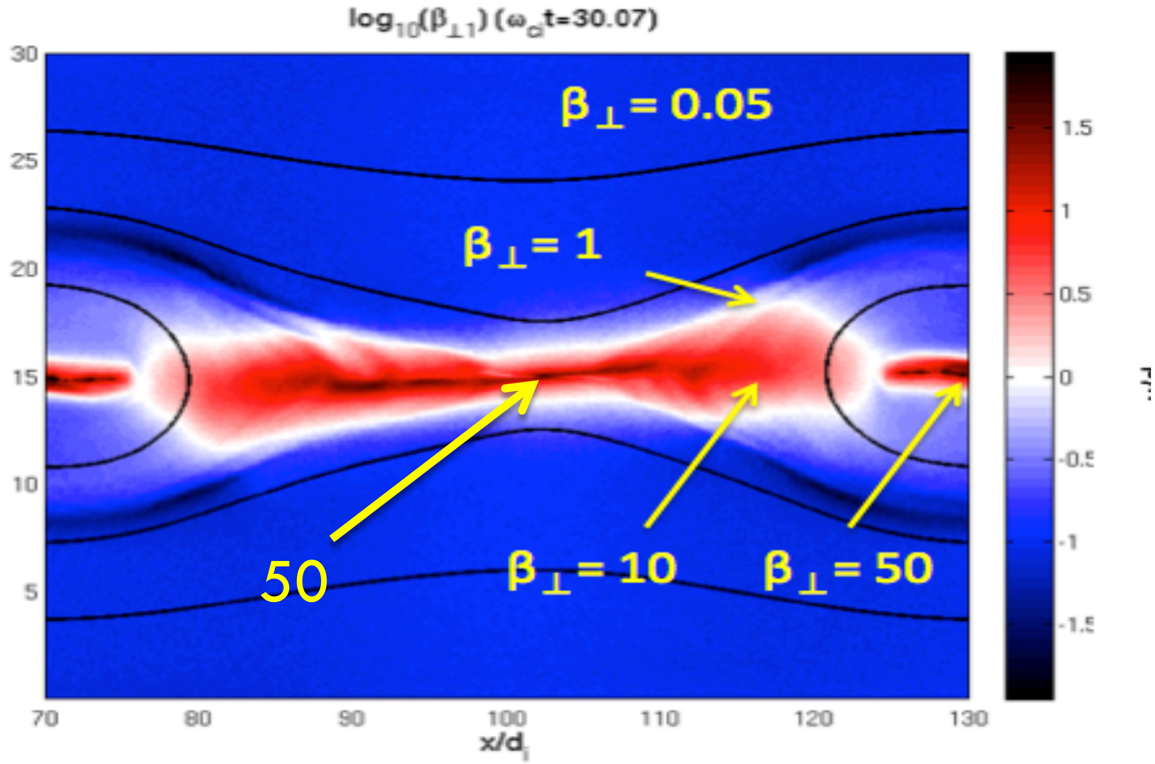
$$\text{Temp-ellipsoid:} \quad S(\mathbf{v}) = \sum_{\alpha=1}^{3} \frac{m\left(v_\alpha - u_\alpha\right)^2}{T_\alpha} = 2$$

Gyrotropy on axis behind DF and elsewhere in exhaust

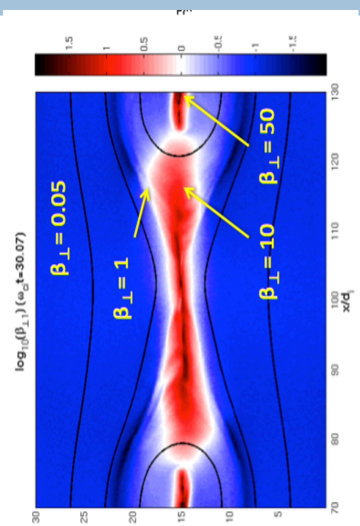
- Magnetic field is direction $\mathbf{e}_1$ so isotropy is in v_2, v_3



Large β (magnetic near-nulls) measure of electron diffusion regions in 2D simulations $(B_g=0.1B_0)$

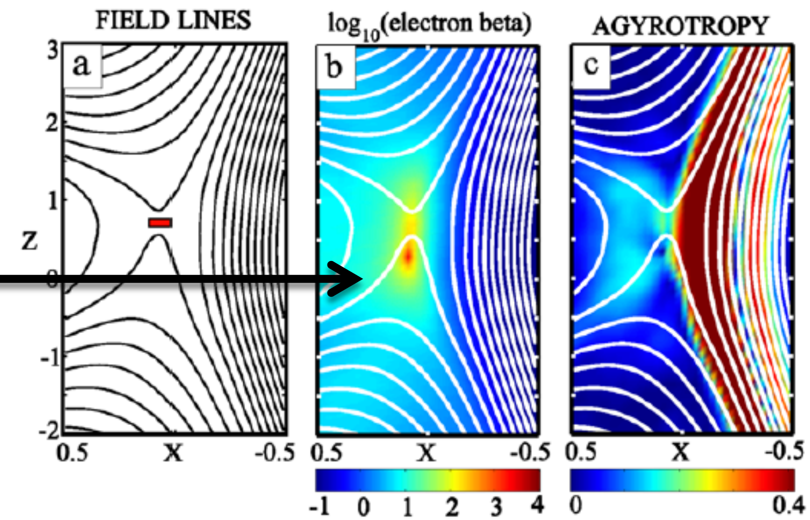


Diffusion region measures in magnetopause simulations (Pritchett and Mozer)

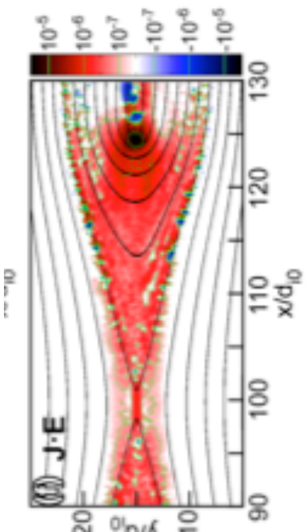


MP simu-lations→

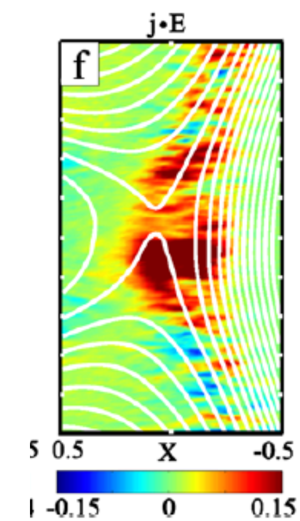
β



←Tail simula-tions



j·E



Summary

- Generalized Ohm's laws and consequences for diffusion regions
  - Slippage, flux and work signatures of diffusion regions
    - "Diffusion region" makes implicit reference to a **particular fluid**
    - Electron, ion . MHD fluids can be ideal. resistive or collisionless
- Diffusion regions in 2D PIC reconnection simulations
  - Non-ideal fluid measures include *slippage*, *flux nonconservation*, *work*
    - Diffusion regions near *fronts* seen in ***work*** in ion/electron fluids
- Agyrotropy large in electron diffusion region.
  - Kinetic better than moments.
    - Need high-res 3D distributions (FPI on MMS)
- Large β >> 1 in diffusion regions (magnetic nulls & near-nulls)
- Future: 3D simulations, diffusion regions to be studied by MMS