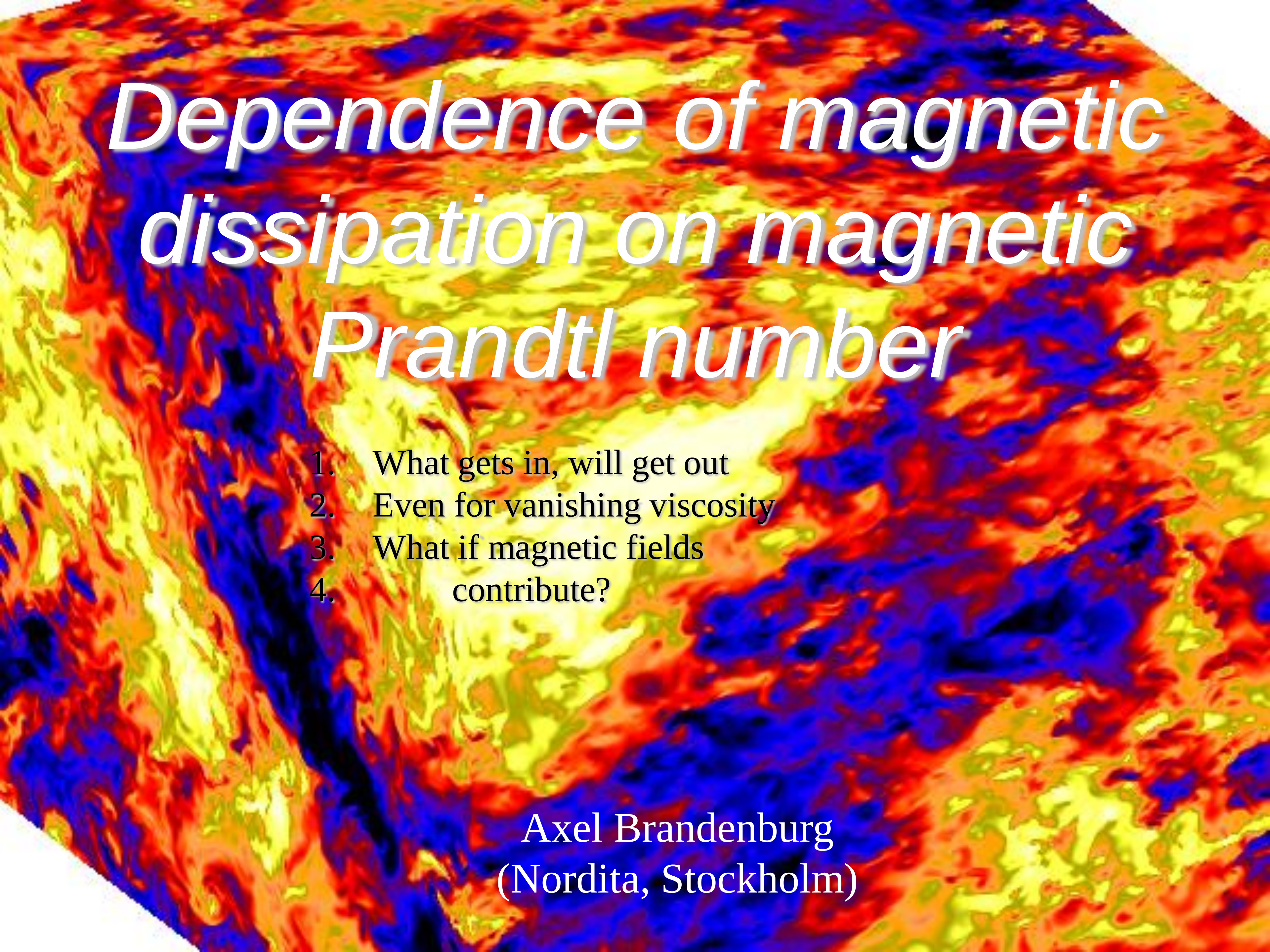


Dependence of magnetic dissipation on magnetic Prandtl number

1. What gets in, will get out
2. Even for vanishing viscosity
3. What if magnetic fields
4. contribute?

Axel Brandenburg
(Nordita, Stockholm)

Finite dissipation at vanishing viscosity

$$\frac{d}{dt}\left\langle \rho \mathbf{u}^2/2 \right\rangle = \left\langle \rho \mathbf{u}\cdot\mathbf{f} \right\rangle - \left\langle \rho \nu \boldsymbol{\omega}^2 \right\rangle$$

$$\left\langle 2\rho\nu \mathbf{S}^2 \right\rangle$$

if $\nu \rightarrow 0$ then $\omega^2 \rightarrow$ infty

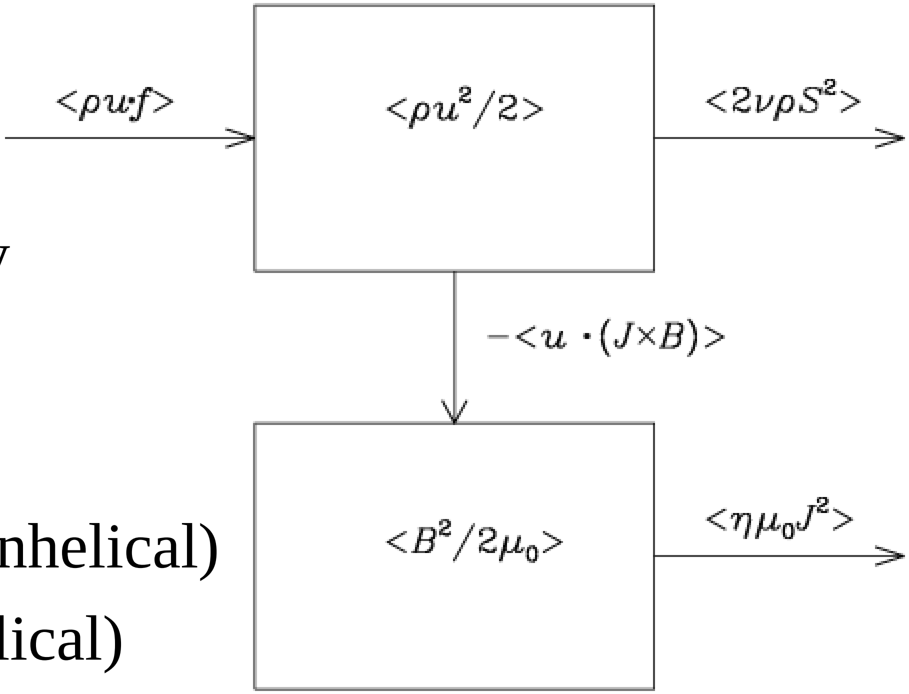
Traceless rate-of-strain tensor $S_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) - \frac{1}{3}\delta_{ij} u_{k,k}$

How is this modified by magnetic fields?

- Smaller η, more J, same dissipation
- Or: dynamo stronger, more dissipation
- Or: less dissipation $\leftarrow$

Couple to B-field

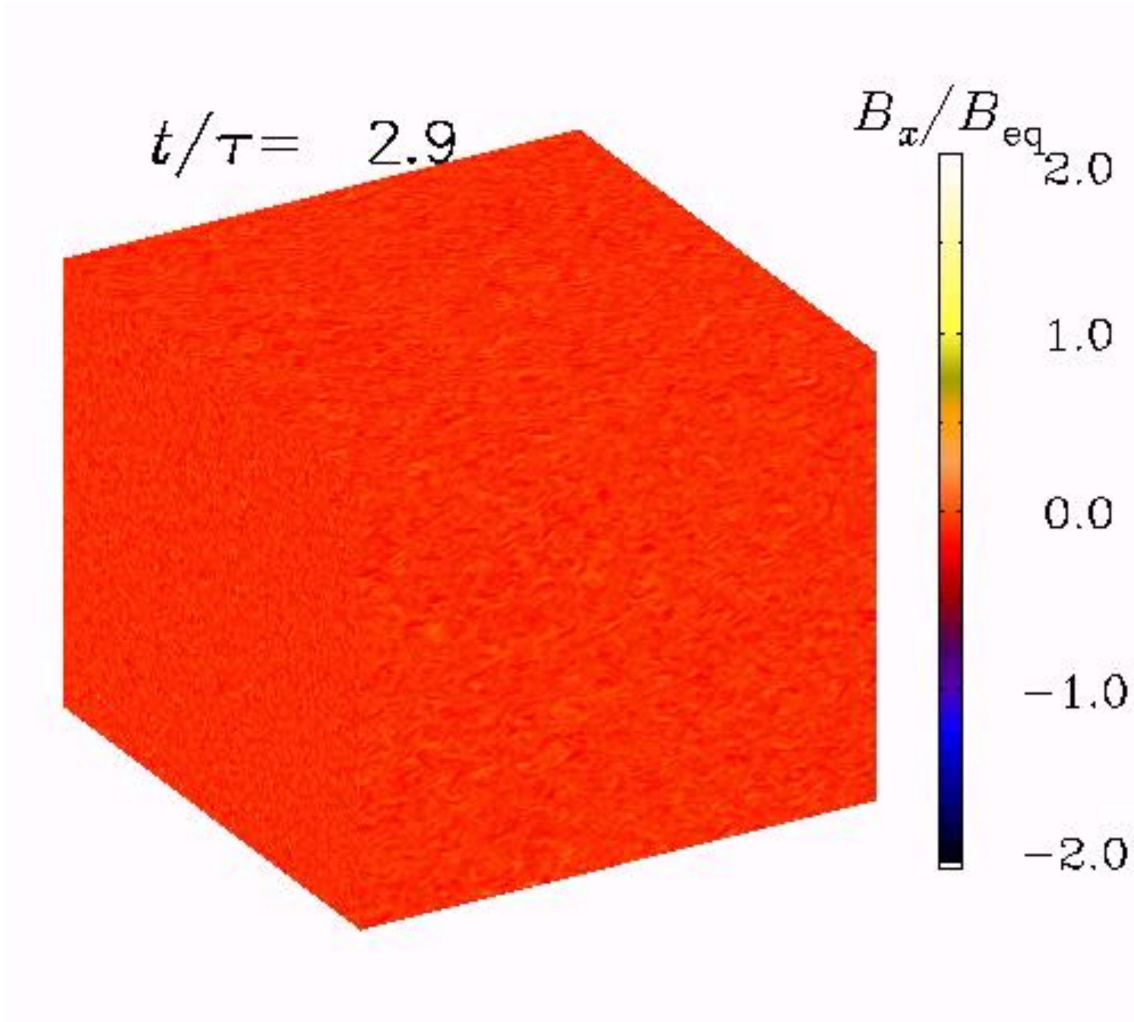
- $\langle \boldsymbol{u}.(\boldsymbol{J}\text{x}\boldsymbol{B})\rangle$ taps energy
  - Dynamo if negative
  - Self-excited if instability
  - Requires 3-D ***B***-field
- Isotropic turbulence
  - Small-scale dynamo (nonhelical)
  - Large-scale dynamo (helical)

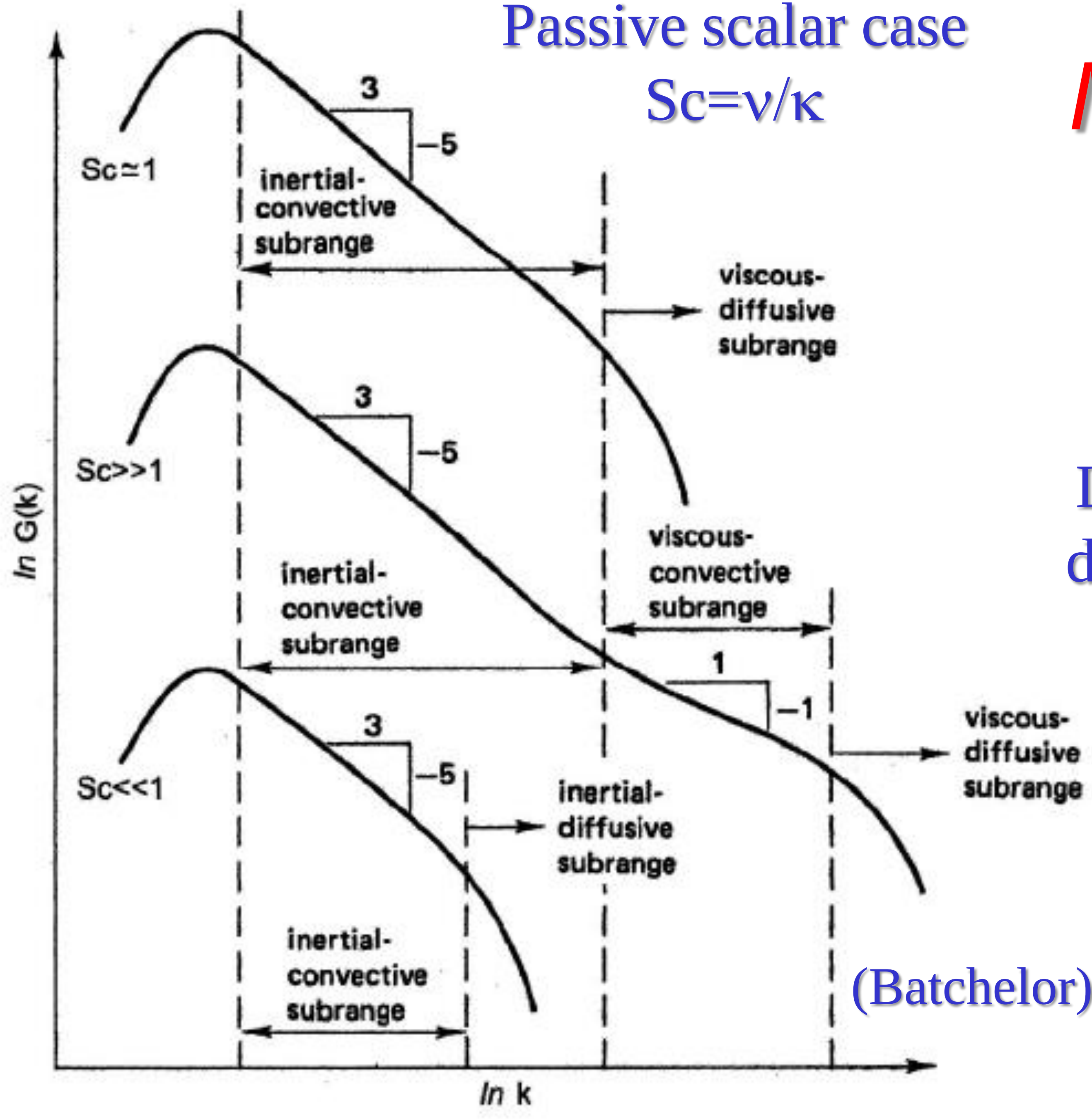


$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\rho\boldsymbol{u}^2/2\rangle = \langle p\boldsymbol{\nabla}\cdot\boldsymbol{u}\rangle + \langle\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B})\rangle + \langle\rho\boldsymbol{u}\cdot\boldsymbol{f}\rangle - \langle 2\rho\nu\mathbf{S}^2\rangle, \quad (6)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\boldsymbol{B}^2/2\mu_0\rangle = -\langle\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B})\rangle - \langle\eta\mu_0\boldsymbol{J}^2\rangle, \quad (7)$$

Inverse cascade





Passive scalar case

$Sc=\nu/\kappa$

Motivation of LES modeling

Location of cutoff does not matter for inertial range

(Batchelor)

Order of limits

$\nu \rightarrow 0$

$\eta \rightarrow 0$

unimportant

Coupling to B-field

- Magnetic dissipation depends on work term
  - Independent of microphysics?
  - Basic assumption of LES and iLES
  - e.g., hyperdiffusion (Galsgaard & Nordlund 1996)
- Not confirmed by DNS
  - Ratio scales with ν/η
  - Either ν or η fixed
  - What if replace Spitzer by Hall?

PHYSICAL REVIEW E **76**, 026316 (2007)

Inverse cascades and α effect at a low magnetic Prandtl number

P. D. Mininni
NCAR, P.O. Box 3000, Boulder, Colorado 80307-3000, USA
(Received 13 February 2007; revised manuscript received 25 June 2007; published 24 August 2007

Dynamo action in a fully helical Arn'old-Beltrami-Childress flow is studied using both direct num simulations and subgrid modeling. Sufficient scale separation is given in order to allow for large-scale netic energy buildup. Growth of magnetic energy obtains down to a magnetic Prandtl number $P_M=R$ close to 0.005, where R_V and R_M are the kinetic and magnetic Reynolds numbers. The critical mag Reynolds number for dynamo action R_M^c seems to saturate at values close to 20. Detailed studies c dependence of the amplitude of the saturated magnetic energy with P_M are presented. When P_M is decre numerical experiments are conducted with either R_V or R_M kept constant. In the former case, the rat magnetic to kinetic energy saturates to a value slightly below unity as P_M decreases. Examination of e spectra and structures in real space reveals that quenching of the velocity by a large-scale magnetic field place, with an inverse cascade of magnetic helicity and a force-free field at large scale in the saturated re

DOI: 10.1103/PhysRevE.76.026316 PACS number(s): 47.65.−d, 47.27.E−, 91.25.Cw, 95.3

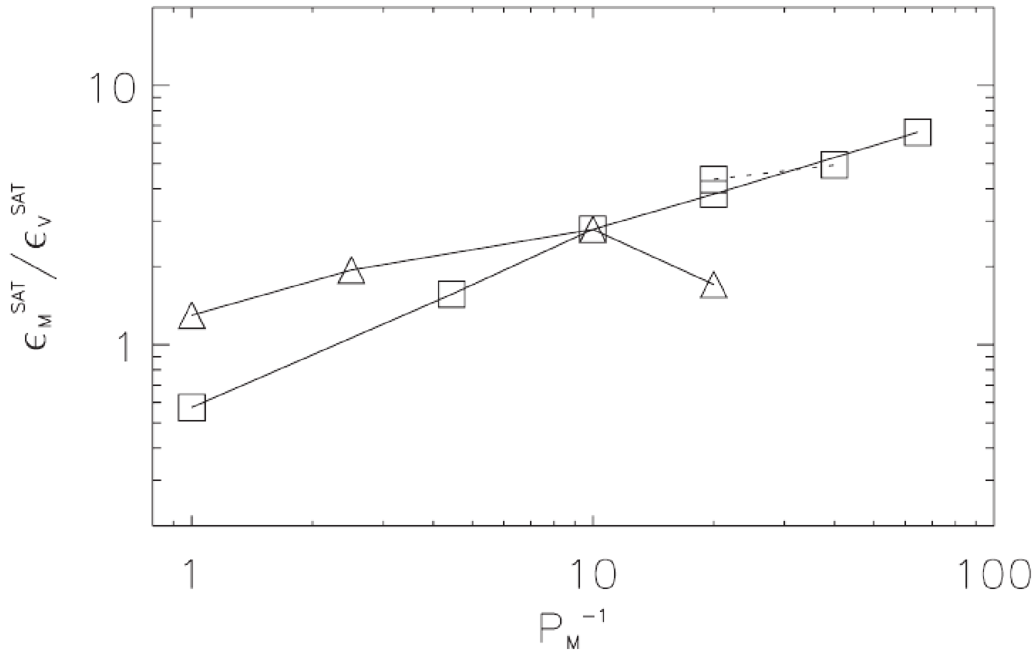


Figure 8 shows the ratio of the magnetic energy dissipation rate ϵ_M to the kinetic energy dissipation rate ϵ_V in the saturated state for the same runs as in Fig. 7. At constant R_V, for small values of P_M, a drop in the ratio is observed as the value of R_M gets closer to the threshold. On the other hand, at constant R_M, more and more energy is dissipated by Ohmic dissipation as P_M is decreased.

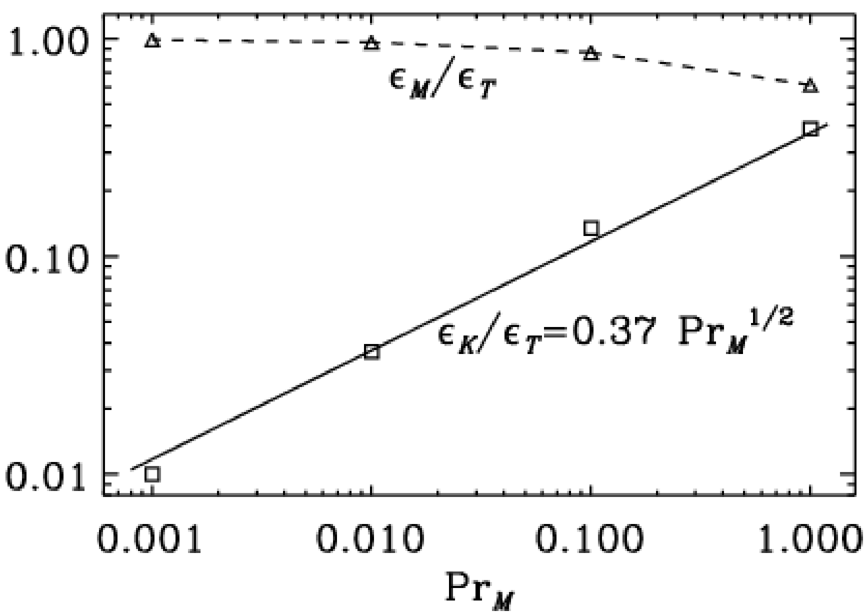
ν smaller, $\nu\omega^2$ smaller, $\eta \mathbf{J}^2$ larger

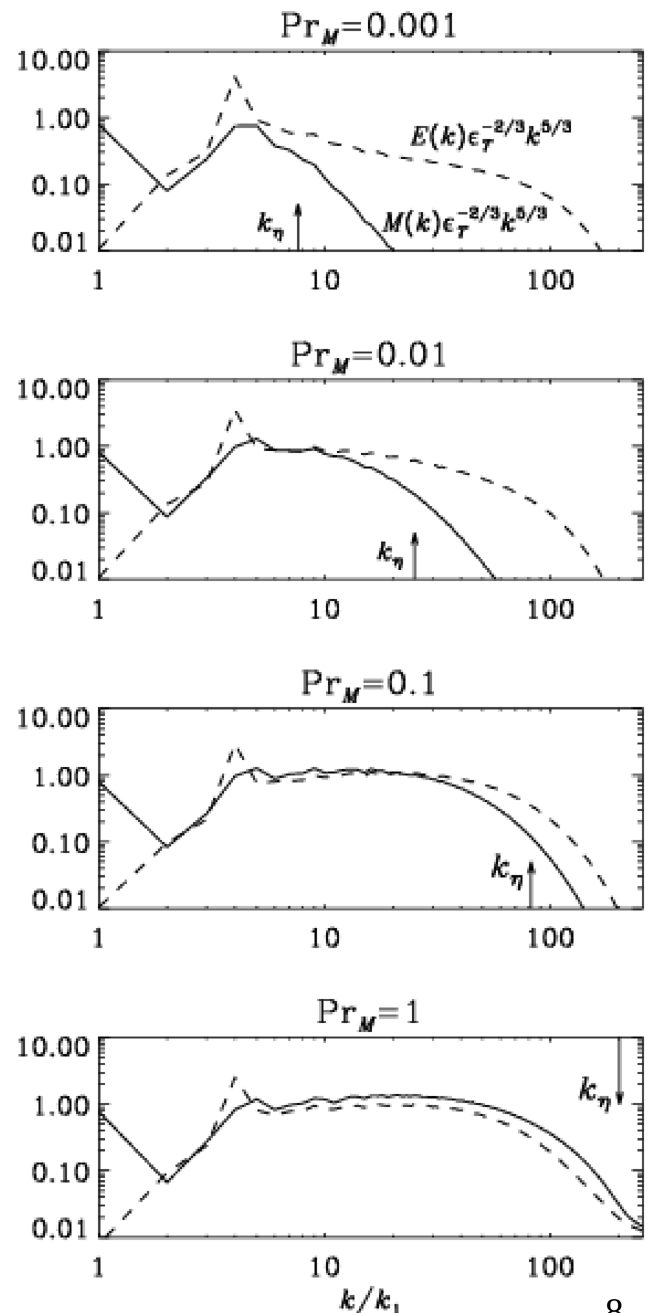
Numerically difficult?

- Energy dissipation via Joule
- Viscous dissipation weak
- Can increase Re *substantially*!

$Pr_M = \nu/\eta$

Brandenburg (2009, ApJ)





Isothermal MHD

$$\frac{D \ln \rho}{Dt} = -\boldsymbol{\nabla} \cdot \boldsymbol{u},$$

$$\frac{D\boldsymbol{u}}{Dt} = -c_{\rm s}^2 \boldsymbol{\nabla} \ln \rho - 2\boldsymbol{\Omega} \times \boldsymbol{u} + \boldsymbol{f} + \rho^{-1} \left[\boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{\nabla} \cdot (2\nu\rho\mathsf{S}) \right],$$

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u} \times \boldsymbol{B} - \eta\mu_0 \boldsymbol{J},$$

Forcing: helical or nonhelical

where $\mathrm{D}/\mathrm{D}t = \partial/\partial t + \boldsymbol{u} \cdot \boldsymbol{\nabla}$ is the advective derivative, $\boldsymbol{u}$ is the velocity, $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ is the magnetic field, $\boldsymbol{A}$ is the magnetic vector potential, $\boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B}/\mu_0$ is the current density, μ_0 is the vacuum permeability, and

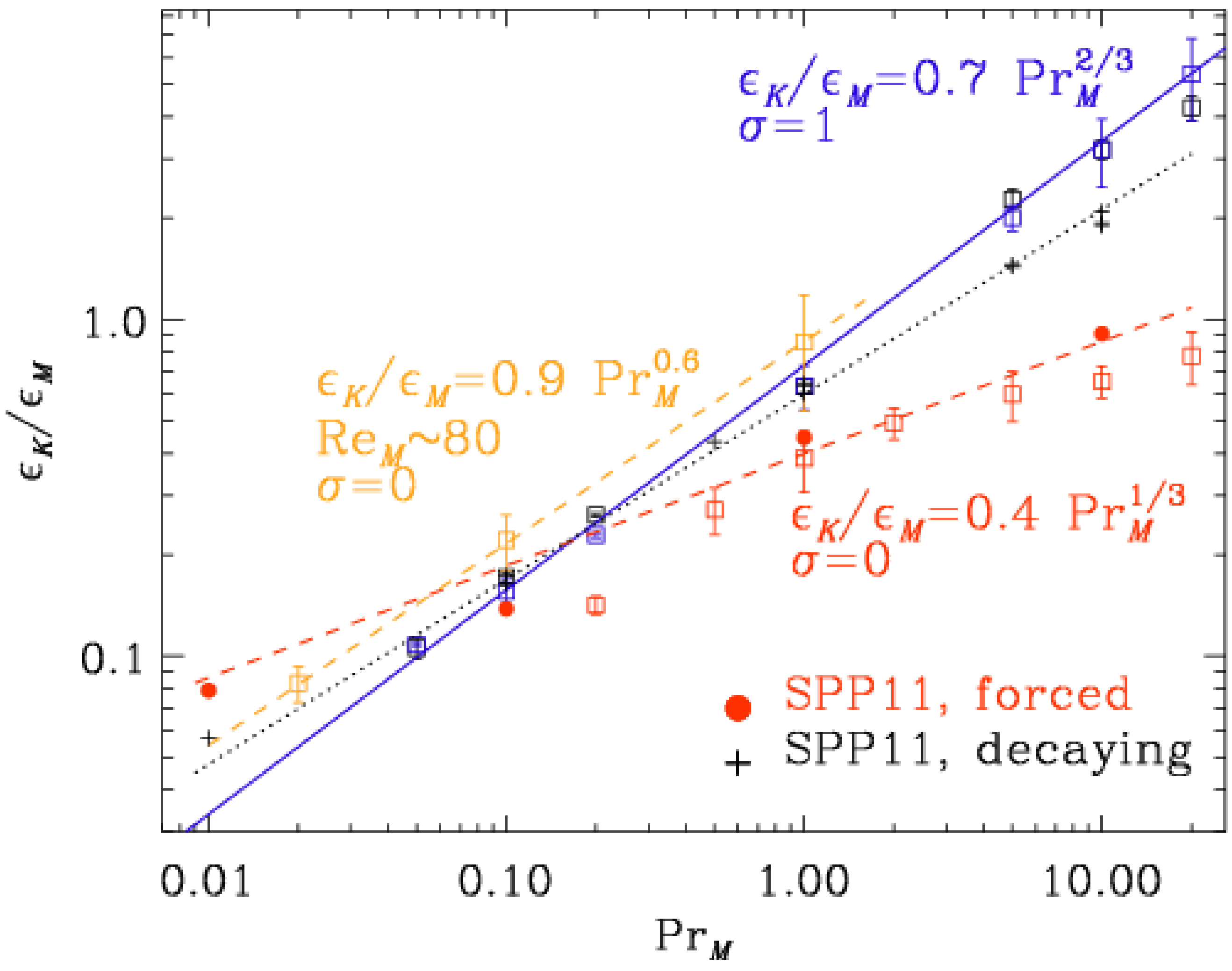
$$\mathsf{S}_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}) - \tfrac{1}{3}\delta_{ij}\boldsymbol{\nabla} \cdot \boldsymbol{u} \qquad (4)$$

is the traceless rate of strain tensor. It is useful to note that

$$\rho^{-1}\boldsymbol{\nabla} \cdot (2\rho\mathsf{S}) = \tfrac{4}{3}\boldsymbol{\nabla}\boldsymbol{\nabla} \cdot \boldsymbol{u} - \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{u} + \mathsf{S} \cdot \boldsymbol{\nabla} \ln \rho, \quad (5)$$

where we call attention to the presence of the $4/3$ factor that

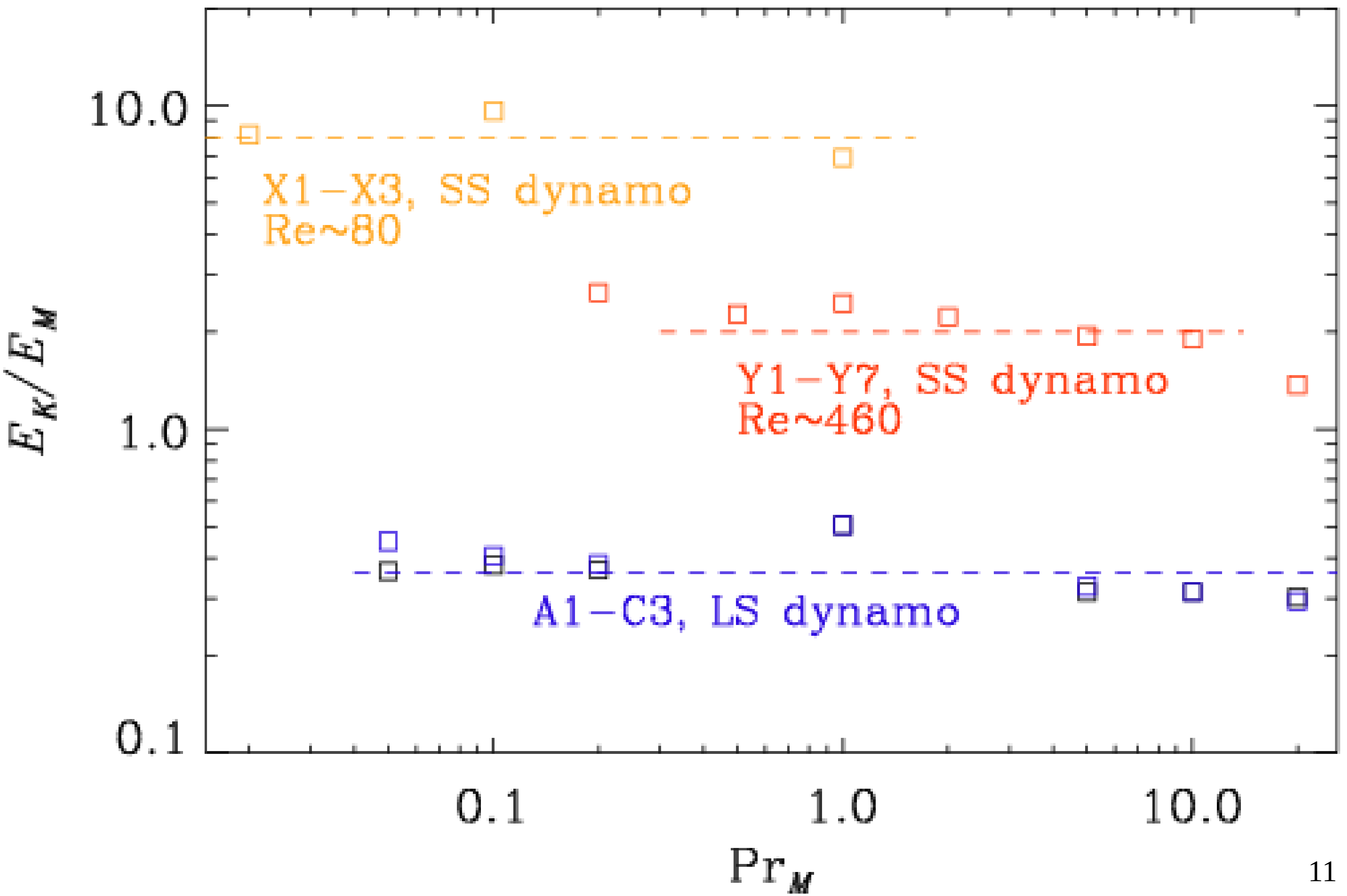
Pr_M dependence of dissipation ratio



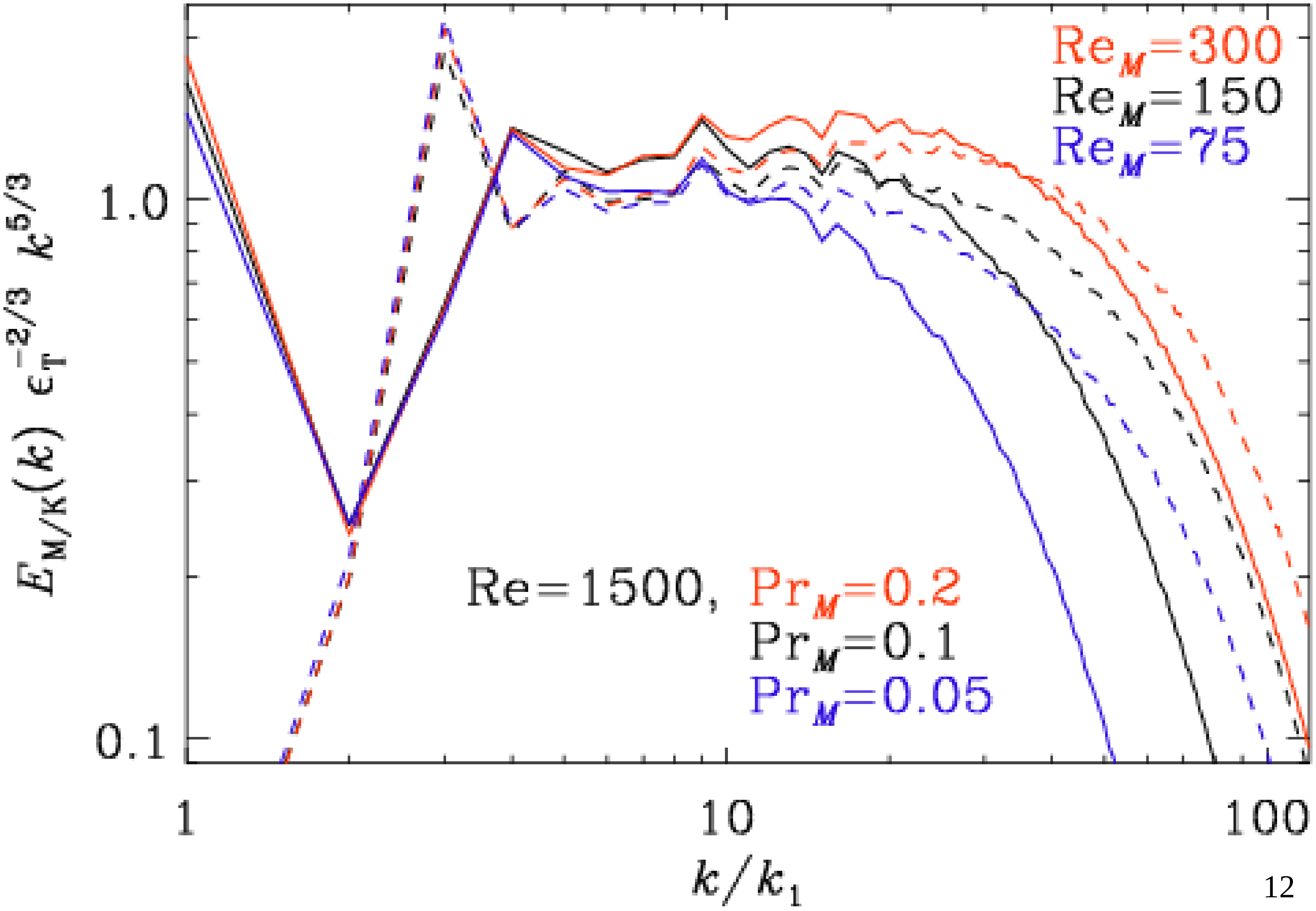
Brandenburg (2014, ApJ, 791, 12)

SPP = Sahoo, Perlekar, Pandi (2011)

Energy ratio nearly unchanged



Inertial range → compensated spectra



2-D MHD (Tran et al, JFM 2013)

this linear behaviour turns out to be fully accessible to numerical simulations. Indeed, the results reported in § 4 show a nearly linear decrease of $\nu\|\omega\|^2$ with ν even for moderately small Pm. Second, we have

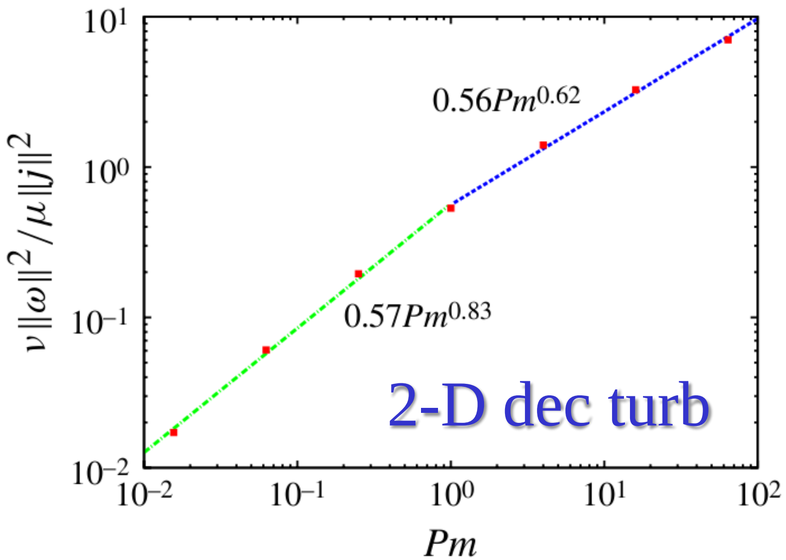
$$\int_0^t \|\nabla j\|^2 \, d\tau < \infty. \tag{3.10}$$

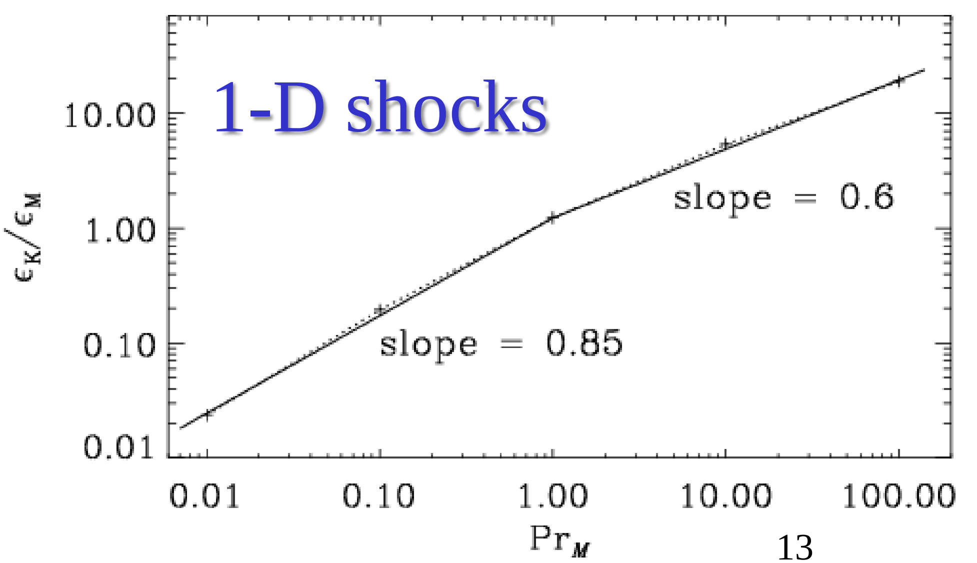
In conclusion, we see that for solution regularity beyond $t = T$ of the 2D MHD system with $Pm = \infty$, it is sufficient to require either

$$\frac{\|j\|_\infty}{\|j\|} \leqslant C(T-t)^{-\alpha} \quad \text{or} \quad \frac{\|\nabla u\|_\infty}{\|\omega\|} \leqslant C(T-t)^{-\alpha} \tag{3.39}$$

for $\alpha < 1/2$.

η smaller,
ν/η larger,
J^2 finite,
$\eta J^2 \rightarrow 0$
$\nu\omega^2/\eta J^2$
keeps incr.

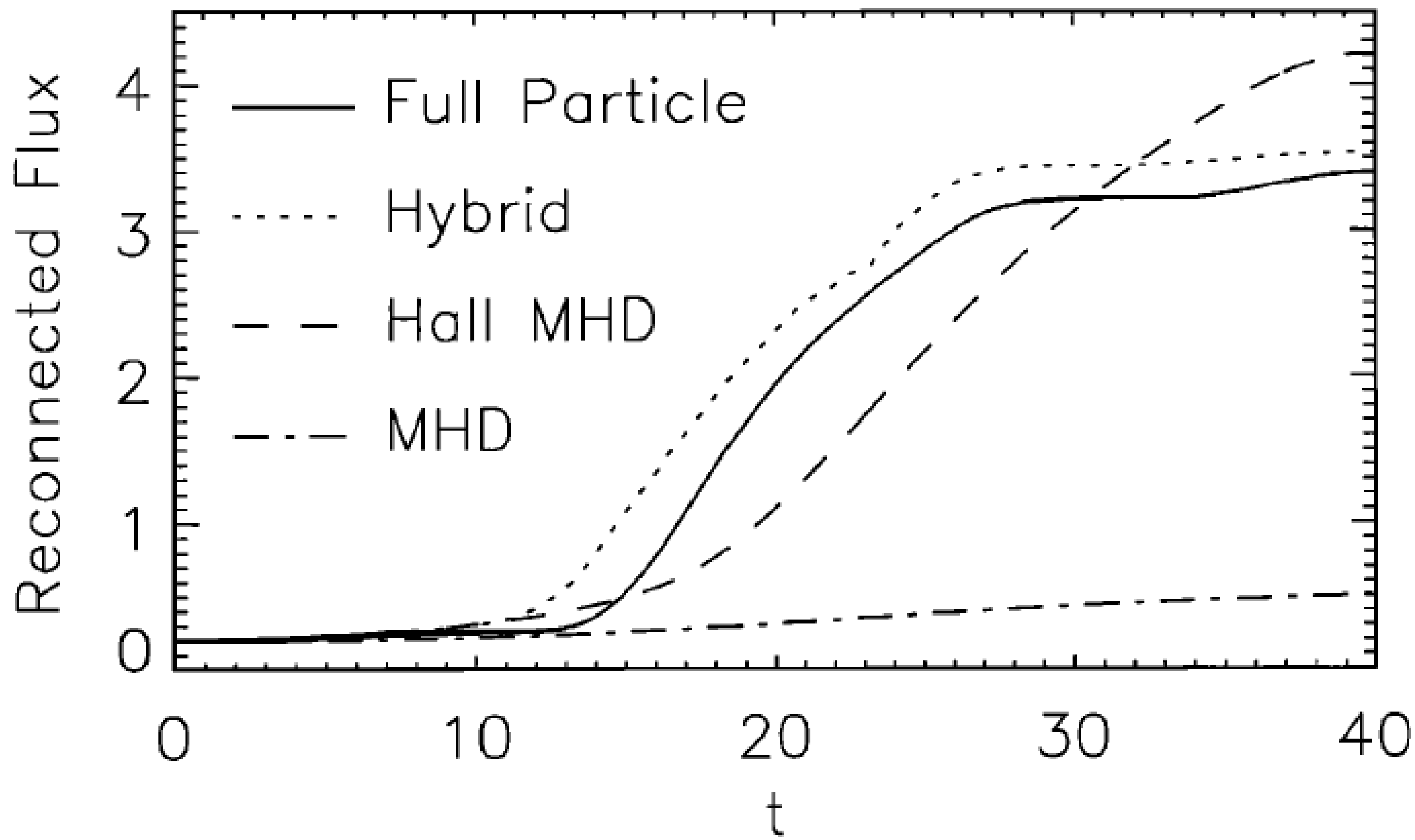




Brandenburg (2014, ApJ, 791, 12)

Geospace Environmental Modeling (GEM) Magnetic Reconnection Challenge

J. Birn,[1] J. F. Drake,[2] M. A. Shay,[2] B. N. Rogers,[2] R. E. Denton,[2] M. Hesse,[3] M. Kuznetsova,[3] Z. W. Ma,[4] A. Bhattacharjee,[4] A. Otto,[5] and P. L. Pritchett,[6]



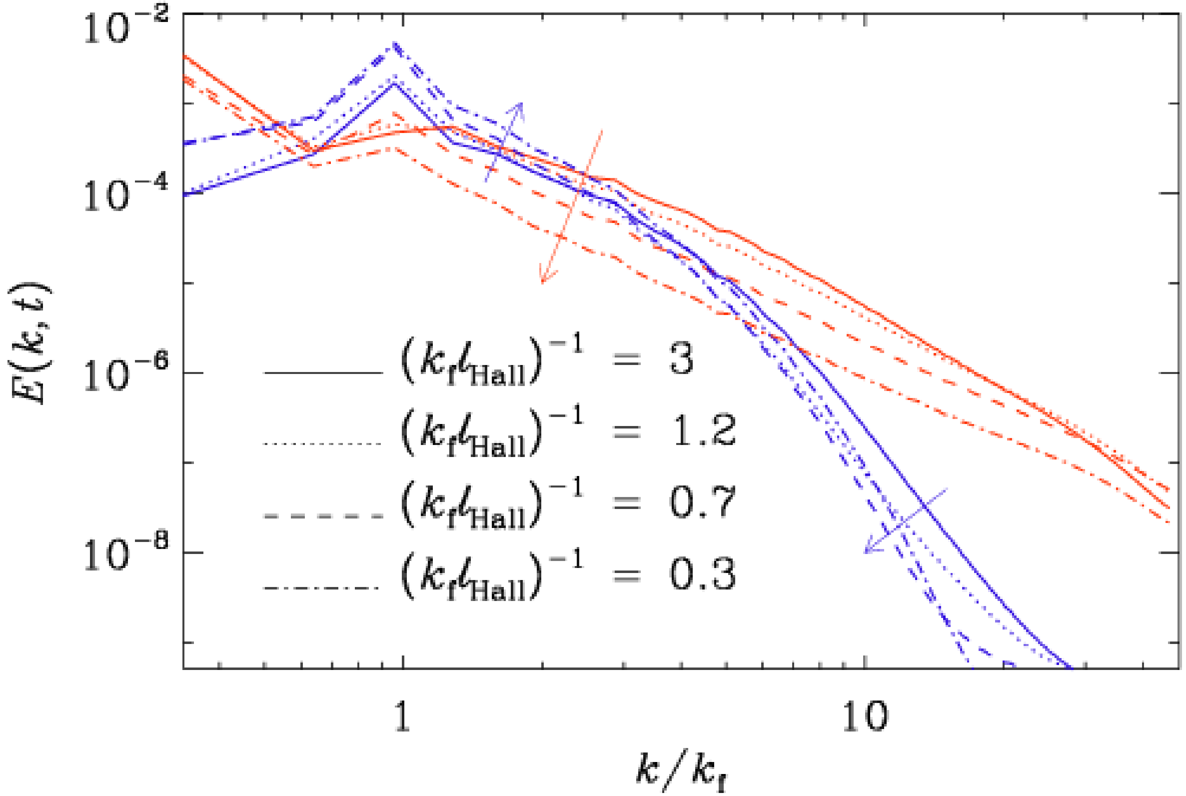
The key conclusion of this project is that the Hall effect is the critical factor which must be included to model collisionless magnetic reconnection. When the

Hall MHD

- How does it affect dissipation ratio?
- Does it "replace" ohmic diffusion somehow
- Does it affect the dynamo
  - Backscatter from magnetic to kinetic energy (Mininni, Alexakis, Pouquet 2006)
  - Nature of MHD Alfven waves changed

Hall MHD simulations

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{u} \times \mathbf{B} - \ell_{\mathrm{Hall}} \mathbf{J} \times \mathbf{B} - \eta \mu_0 \mathbf{J} \right)$$

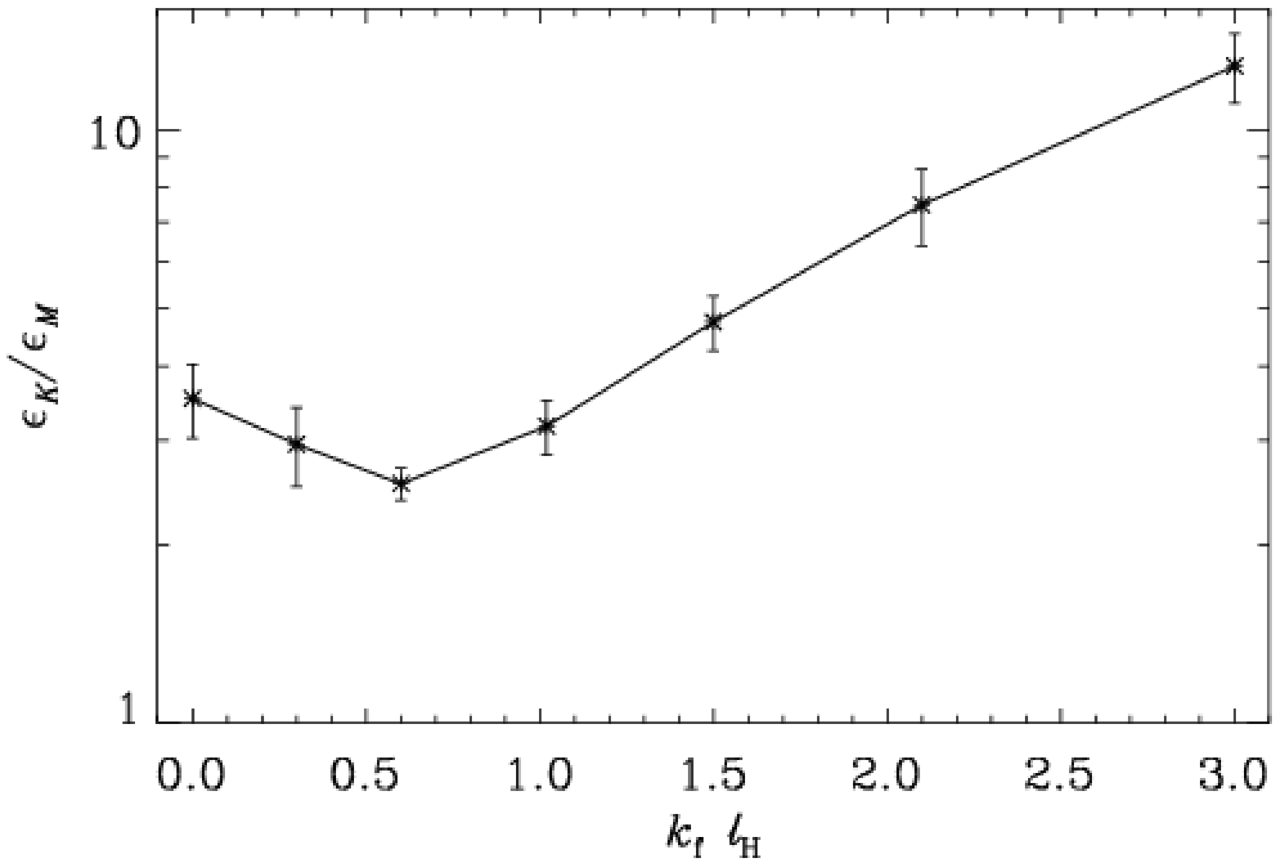


288^3 resol.

Makes dynamo weaker → less magnetic dissipation

Hall MHD simulations

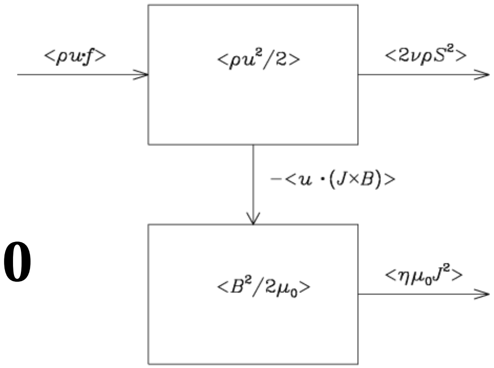
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{u} \times \mathbf{B} - \ell_{\mathrm{Hall}} \mathbf{J} \times \mathbf{B} - \eta \mu_0 \mathbf{J} \right)$$

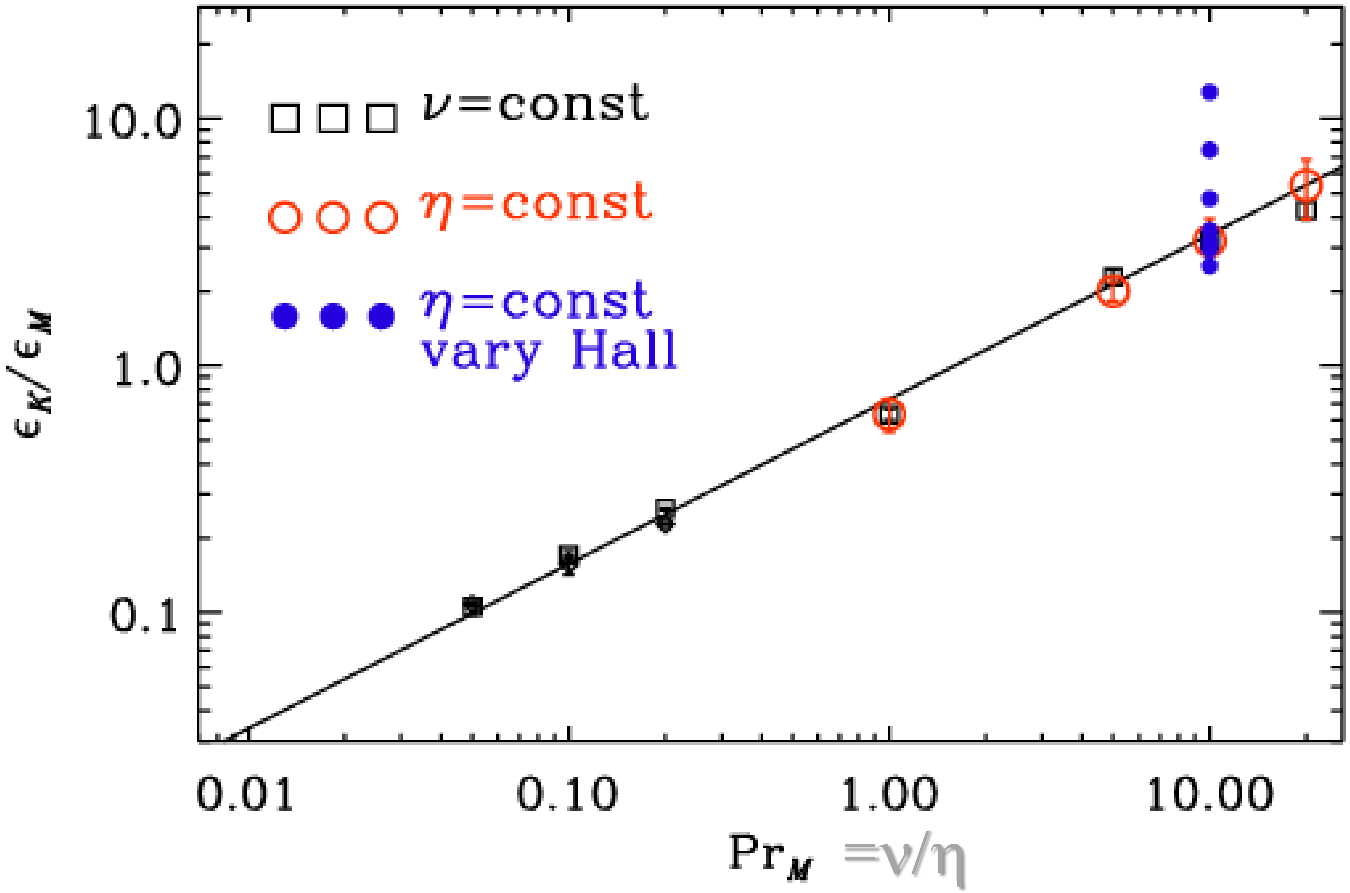


E_M weaker, ϵ_M weaker, ϵ_M/ϵ_M larger

Hall effect

Makes dynamo weaker $\mathbf{J}\cdot\left(\ell_{\mathrm{Hall}}\mathbf{J}\times\mathbf{B}\right)=\mathbf{0}$



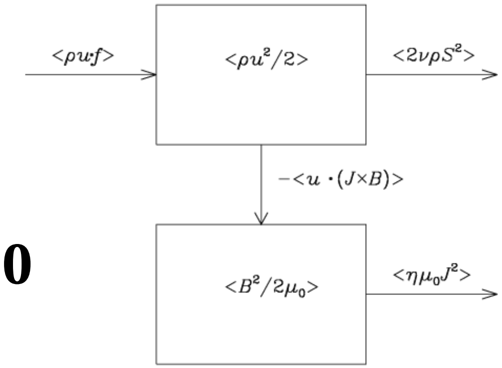


Conclusions

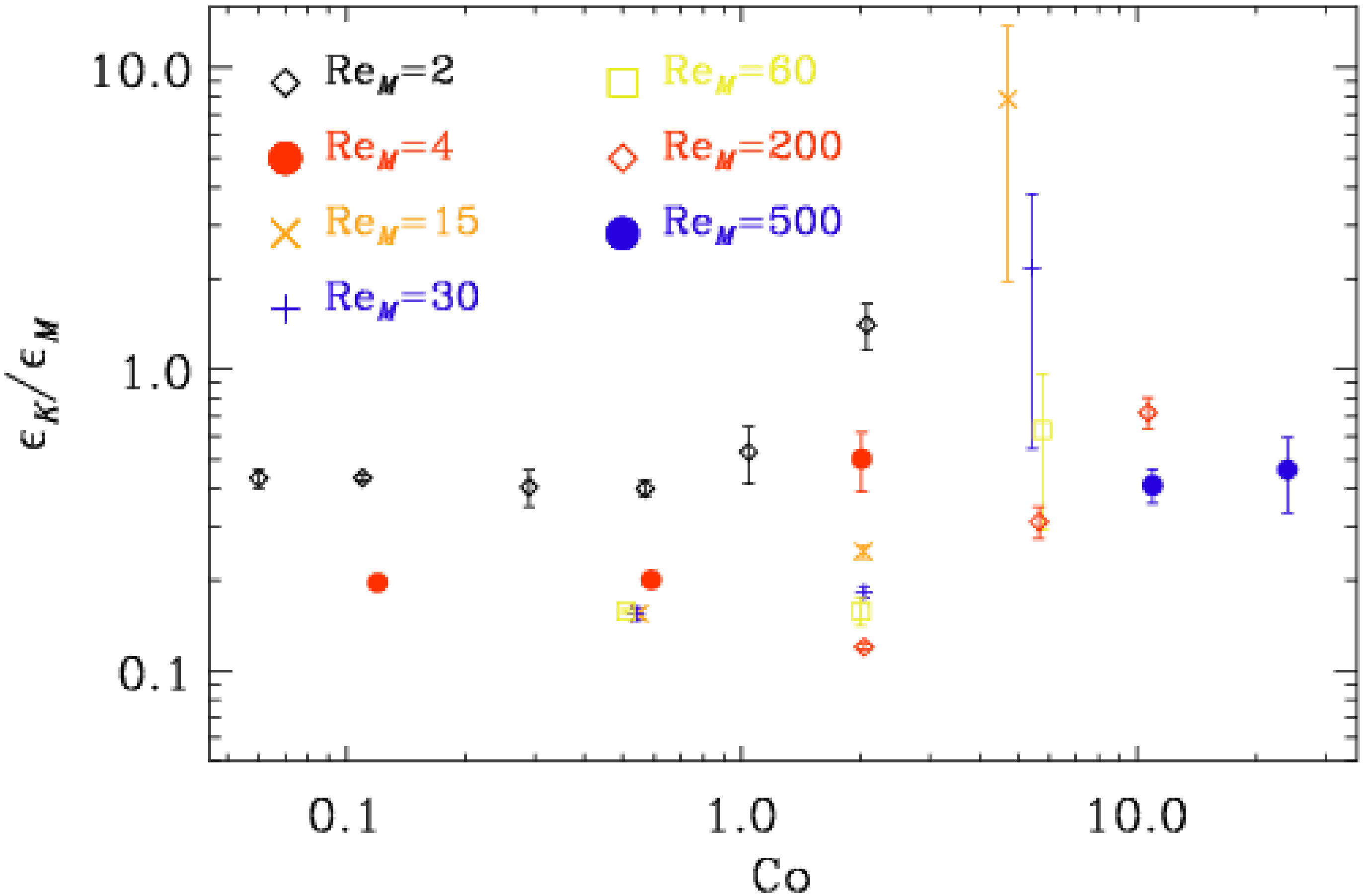
- Dissipation ratio scales with Pr_M
  - Both for $Pr_M > 1$ and < 1
  - SS dynamo scaling shallower (nonuniversal)
- Qualitatively reproduced with MHD shocks
- $\langle \mathbf{u}.(\mathbf{J} \times \mathbf{B}) \rangle$ determined by microphysics!?
  - Hall does affect dynamo, if $1/\ell_{Hall}$ subinertial
  - Questions about LES or iLES

Run	$\nu k_1/c_s$	$\eta k_1/c_s$	Re	Re_M	Co	u_{rms}/c_s	b_{rms}/c_s	ϵ_K/ϵ_T	ϵ_M/ϵ_T	C_ϵ	k_ν/k_1	k_η/k_1	res.
RA1	1×10^{-3}	1×10^{-2}	30	3	0.1	0.090	0.066	0.30	0.70	1.71	17	4	16^3
RA2	1×10^{-3}	1×10^{-2}	30	3	0.1	0.089	0.064	0.30	0.70	1.68	17	4	16^3
RA3	1×10^{-3}	1×10^{-2}	29	3	0.3	0.088	0.065	0.29	0.71	1.66	17	4	16^3
RA4	1×10^{-3}	1×10^{-2}	29	3	0.6	0.088	0.065	0.29	0.71	1.63	17	4	16^3
RA5	1×10^{-3}	1×10^{-2}	32	3	1.0	0.096	0.063	0.35	0.65	1.31	18	4	16^3
RA6	1×10^{-3}	1×10^{-2}	40	4	2.1	0.121	0.053	0.58	0.42	0.77	21	3	16^3
RB1	5×10^{-4}	5×10^{-3}	56	6	0.1	0.084	0.092	0.16	0.84	2.10	25	7	32^3
RB2	5×10^{-4}	5×10^{-3}	57	6	0.6	0.085	0.094	0.17	0.83	2.03	25	7	32^3
RB3	5×10^{-4}	5×10^{-3}	83	8	2.0	0.124	0.073	0.34	0.67	0.62	30	6	32^3
RC1	2×10^{-4}	2×10^{-3}	150	15	0.6	0.090	0.117	0.13	0.87	1.74	48	14	64^3
RC2	2×10^{-4}	2×10^{-3}	205	21	2.0	0.123	0.100	0.20	0.80	0.66	52	13	64^3
RC3	2×10^{-4}	2×10^{-3}	353	35	4.7	0.212	0.019	0.89	0.11	0.07	65	7	64^3
RD1	1×10^{-4}	1×10^{-3}	310	31	0.5	0.093	0.119	0.13	0.87	1.56	80	23	128^3
RD2	1×10^{-4}	1×10^{-3}	410	41	2.0	0.123	0.127	0.15	0.85	0.69	83	23	128^3
RD3	1×10^{-4}	1×10^{-3}	613	61	5.4	0.184	0.037	0.69	0.31	0.12	105	16	128^3
RE1	5×10^{-5}	5×10^{-4}	647	65	0.5	0.097	0.123	0.14	0.86	1.44	137	39	256^3
RE2	5×10^{-5}	5×10^{-4}	833	83	2.0	0.125	0.134	0.14	0.86	0.69	138	39	256^3
RE3	5×10^{-5}	5×10^{-4}	1160	116	5.8	0.174	0.099	0.39	0.61	0.17	160	32	256^3
RF1	2×10^{-5}	2×10^{-4}	2033	203	2.0	0.122	0.116	0.11	0.89	0.59	243	74	256^3
RF2	2×10^{-5}	2×10^{-4}	2950	295	5.6	0.177	0.106	0.24	0.76	0.14	272	65	256^3
RF3	2×10^{-5}	2×10^{-4}	3917	392	10.6	0.235	0.084	0.42	0.58	0.06	318	62	256^3
RG1	1×10^{-5}	1×10^{-4}	7600	760	10.9	0.228	0.091	0.29	0.71	0.07	501	111	512^3
RG2	1×10^{-5}	1×10^{-4}	6933	693	24.0	0.208	0.089	0.32	0.68	0.09	496	107	512^3

Effect of rotation



No effect if supercritical $\mathbf{u}\cdot\left(2\mathbf{\Omega}\times\mathbf{u}\right)=0$



Passive scalar

Specifically, the equations considered by Ohkitani & Dowker (2010) are

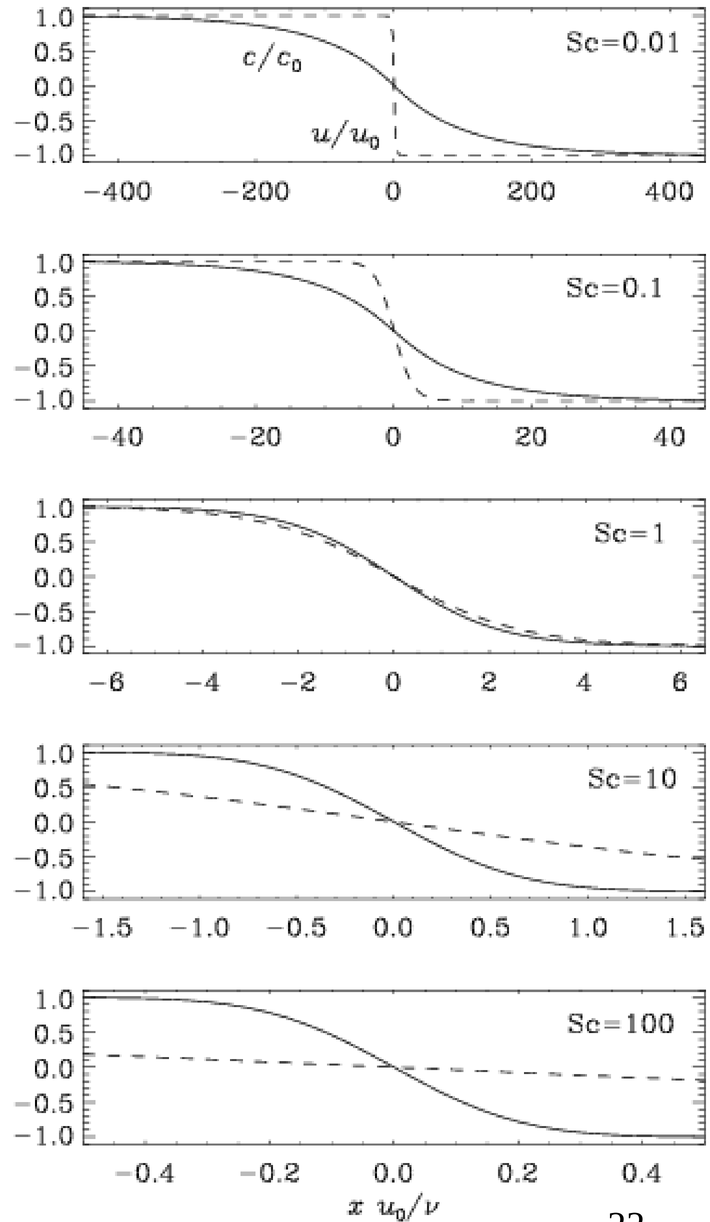
$$\partial u/\partial t = -uu' + \tilde{\nu}u'', \tag{22}$$
$$\partial c/\partial t = -uc' + \kappa c'', \tag{23}$$

where primes denotes differentiation with respect to x. The solution to Equation (22) decouples and possesses a shock. In a frame of reference moving with the shock, the solution is stationary and given by

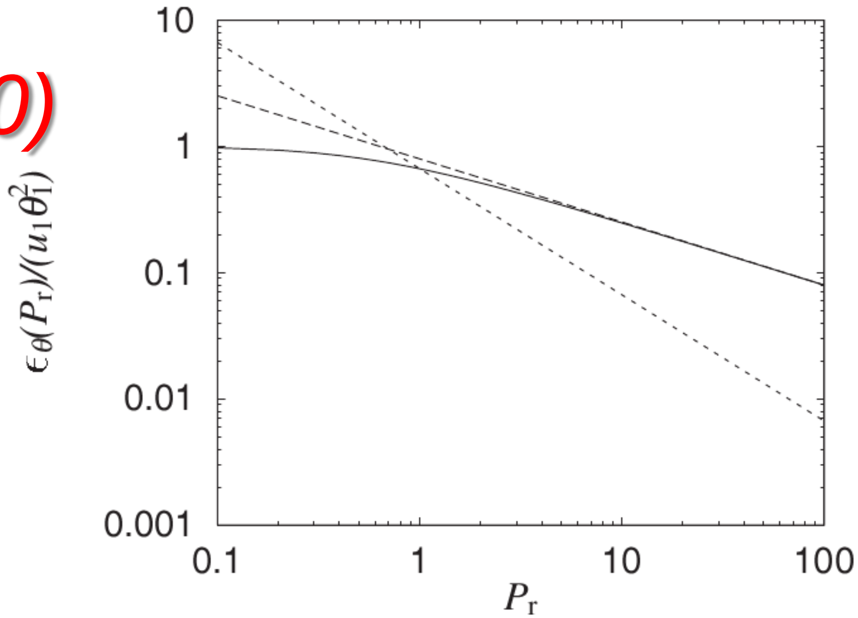
$$u(x) = -u_0 \tanh x/w, \tag{24}$$

where u_0 is the velocity jump and $w_u = 2\tilde{\nu}/u_0$ is the width of the shock with $\tilde{\nu} = 4\nu/3$ being a rescaled viscosity. These equations can be obtained from the hydrodynamic version (i.e., $\boldsymbol{B} = \mathbf{0}$) of Equation (2) after setting $c_s = 0$, so the density gradient does not enter, so we can ignore Equation (1) and put $\rho = 1$. The $4/3$ factor in the expression for $\tilde{\nu}$ comes from the fact that, owing to compressibility, the viscous acceleration term includes a $\frac{1}{3}\boldsymbol{\nabla\nabla}\cdot\boldsymbol{u}$ term in addition to the usual $\nu\nabla^2\boldsymbol{u}$ term; see Equation (5) for a corresponding reformulation of the dissipation terms. The viscous dissipation $\epsilon_K = \tilde{\nu}\int(u')^2\,dx/L$ is then, using $\partial u/\partial x \propto 1/\cosh^2(x/w)$,

$$\epsilon_K = \tilde{\nu}\frac{w}{L}\int\frac{dx/w}{\cosh^4(x/w)} = \frac{4}{3}\frac{\tilde{\nu}u_0^2}{wL} = \frac{2}{3}\frac{u_0^3}{L}, \tag{25}$$



Ohkitani & Dowker (2010)



III. DISSIPATION RATE OF A PASSIVE SCALAR

By (5), the dissipation rate of passive scalar variance ϵ_θ is evaluated as follows:

$$\epsilon_\theta = \kappa\int_{-\infty}^{\infty}\left(\frac{\partial\theta}{\partial x}\right)^2 dx = \kappa\tilde{c}'^2\int_{-\infty}^{\infty}\left[\cosh\frac{u_1}{2\nu}(X+c)\right]^{-4P_r} dX = \kappa\frac{(u_1\theta_1)^2}{4\nu^2 I_{P_r}(\infty)^2}\frac{2\nu}{u_1}\int_{-\infty}^{\infty}\cosh^{-4P_r}(\xi)d$$

For $P_r = n$, we obtain an exact expression

$$\epsilon_\theta = u_1\theta_1^2\frac{\{(2n)!\}^4}{(4n)!(n!)^4}.$$

[For more general real-valued $P_r = \alpha$, we have $I_\alpha(\infty) = \sqrt{\pi}/2\Gamma(\alpha)/\Gamma(1/2+\alpha)$ and thus $\epsilon_\theta = u_1\theta_1^2 2/\sqrt{\pi}\Gamma(2\alpha)\Gamma(1/2+\alpha)^2/\alpha\Gamma(\alpha)^2\Gamma(1/2+2\alpha)$, where $\Gamma(\alpha)$ is the gamma function.]

By Stirling's formula $n! \simeq \sqrt{2\pi n}n^n e^{-n}$ for $n \gg 1$, we deduce that

$$I_{n+1}(\infty) \simeq \frac{1}{2}\sqrt{\frac{\pi}{n}}.$$

Therefore, the dissipation rate of θ in the limit of large P_r is

$$\epsilon_\theta \simeq u_1\theta_1^2\sqrt{\frac{2}{\pi P_r}} \quad \text{as} \quad P_r \to \infty, \tag{9}$$

which decays as $P_r^{-1/2}$ with P_r. Even in this simple one-dimensional (1D) model, the problem of dissipation anomaly is subtle in that ϵ_θ does depend on P_r in a nontrivial fashion.

MHD model

$$\partial u/\partial t = -uu' - bb' + \tilde{\nu}u'',$$
$$\partial b/\partial t = -ub' - bu' + \eta b'',$$

$$\partial u/\partial x = (u^2 + b^2 - u_0^2)/2\tilde{\nu},$$
$$\partial b/\partial x = ub/\eta.$$