SGS Modeling for Fast MHD Magnetic Reconnection

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Magnetic Reconnection in the Corona, Turbulent Approach

- Corona realm of MHD and reconnection a key process
- Diffusive timescale $t_{diff} = L^2/\eta$
- Resistive MHD (Spitzer 1962) not fast enough
  for solar flare:
  $t_{diff} \cong 3 * 10^8$years,
  $t_{obs} \cong 100s$
- Turbulent diffusivity assumed
  $\Rightarrow$ Model to account for turbulence

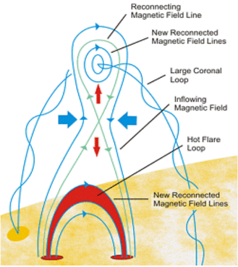
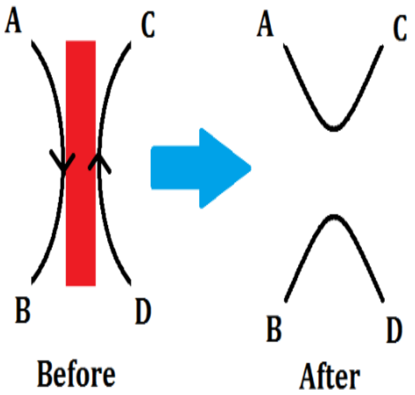


Figure : [http://science.nasa.gov]

Magnetic Reconnection

- Magnetic Reconnection
  - Change in field lines topology and connectivity
  - Magnetic energy converted to heat and kinetic energy
- Reconnection Rate
  - Speed at which magnetic field lines are carried towards the 'X' point
  - Ratio inflow to outflow



Models: Different Approach

Induction equation, resistive MHD

$$\partial_t \boldsymbol{B} = \nabla \times (\boldsymbol{U} \times \boldsymbol{B}) - \nabla \times (\eta \boldsymbol{J})$$

- Mean field turbulent model
- Reynolds Averaged Navier-Stokes (RANS)

  $f = F + f'$, $F \equiv \langle f \rangle$
- $\partial_t \boldsymbol{B} = \nabla \times (\boldsymbol{U} \times \boldsymbol{B})$
  $-\nabla \times (\eta \boldsymbol{J})$
  $+\nabla \times (\langle \boldsymbol{u'} \times \boldsymbol{b'} \rangle)$

- Non-linear SGS model
- Filtering:

  $\overline{x} = G * x$, $\widetilde{y} = \overline{y\rho}/\overline{\rho}$
- $\partial_t \overline{\boldsymbol{B}} = \nabla \times (\widetilde{\boldsymbol{U}} \times \overline{\boldsymbol{B}})$
  $-\nabla \times (\eta \overline{\boldsymbol{J}})$
  $+\nabla \times (\overline{\boldsymbol{U} \times \boldsymbol{B}} - \widetilde{\boldsymbol{U}} \times \overline{\boldsymbol{B}})$

$\beta - \gamma$ Model

$$\text{(Yokoi 2013):}\ \boldsymbol{E_M} = -\beta \boldsymbol{J} + \gamma \boldsymbol{\Omega}$$

- $\beta = C_\beta \tau_t K$
- $K = \frac{1}{2} \left\langle \boldsymbol{u}'^2 + \boldsymbol{b}'^2 \right\rangle$
- K: turbulent energy
- $\gamma = C_\gamma \tau_t W$
- $W = \left\langle \boldsymbol{u}' \cdot \boldsymbol{b}' \right\rangle$
- W: turbulent cross-helicity

τ_t: turbulent timescale
System closed by evolution equations

MHD Equations, RANS Model

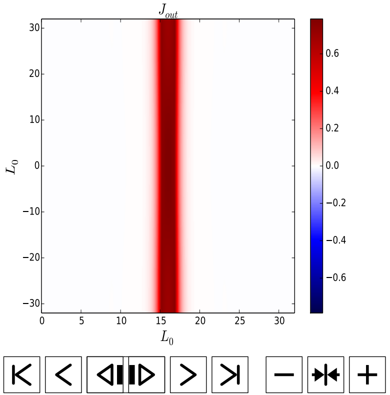
$$\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\boldsymbol{\nabla}\cdot(\rho \boldsymbol{U}) + \chi \nabla^2 \rho \\
\frac{\partial \rho \boldsymbol{U}}{\partial t} &= -\nabla \cdot \left[\rho \boldsymbol{U} \otimes \boldsymbol{U} + \frac{1}{2}(p + B^2)\boldsymbol{I} - \boldsymbol{B} \otimes \boldsymbol{B}\right] + \chi \nabla^2 (\rho \boldsymbol{U}) \\
\frac{\partial \boldsymbol{B}}{\partial t} &= \nabla \times (\boldsymbol{U} \times \boldsymbol{B}) - (\nabla(\eta + \beta)) \times \boldsymbol{J} + (\eta + \beta)\nabla^2 \boldsymbol{B} \\
&\quad + \nabla \times (\gamma \sqrt{\rho}\Omega) \\
\frac{\partial h}{\partial t} &= -\nabla \cdot (h\boldsymbol{U}) + \frac{\gamma - 1}{\gamma h^{\gamma - 1}}(\eta \boldsymbol{J}^2 + \frac{\rho K}{\tau_t}) + \chi \nabla^2 h \\
\frac{\partial K}{\partial t} &= -\boldsymbol{U} \cdot \nabla K + C_\beta \tau_t K \frac{\boldsymbol{J}^2}{\rho} - C_\gamma \tau_t W \frac{\boldsymbol{\Omega} \cdot \boldsymbol{J}}{\sqrt{\rho}} + \frac{\boldsymbol{B}}{\rho} \cdot \nabla W - \frac{K}{\tau_t} \\
\frac{\partial W}{\partial t} &= -\boldsymbol{U} \cdot \nabla W + C_\beta \tau_t K \frac{\boldsymbol{\Omega} \cdot \boldsymbol{J}}{\sqrt{\rho}} - C_\gamma \tau_t W \boldsymbol{\Omega}^2 + \frac{\boldsymbol{B}}{\sqrt{\rho}} \cdot \nabla K - C_W \frac{W}{\tau_t}
\end{aligned}$$

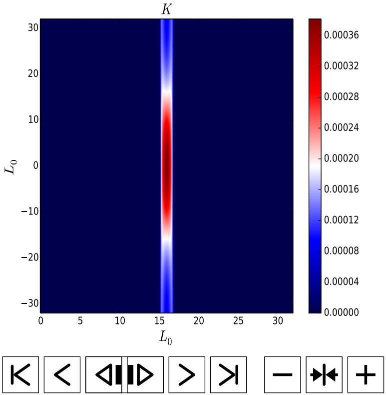
Current Sheet Equilibrium Tested

- Harris equilibrium
  - Pressure equilibrium across the current sheet
  - Not realistic for the Solar Corona

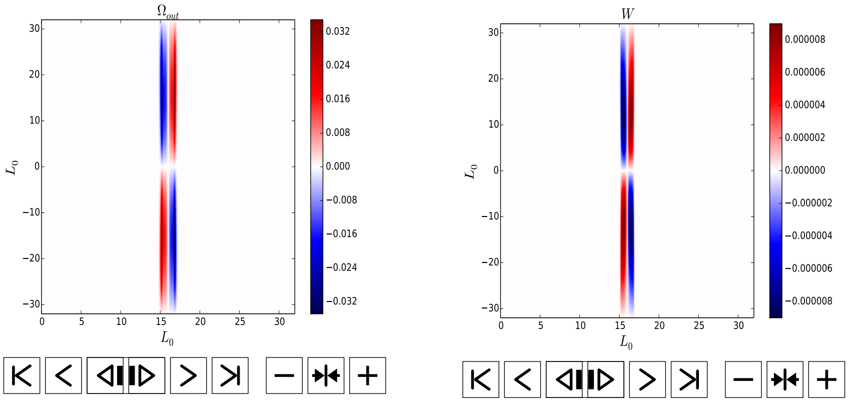
- Force free equilibrium with out of plane guide field b_g
  - No initial Lorentz force: $\boldsymbol{J} \times \boldsymbol{B} = 0$
  - More realistic for the Solar Corona

Mean Current Density and Turbulent Energy

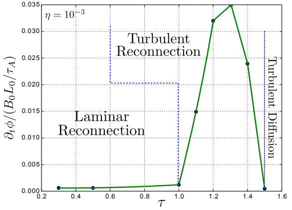


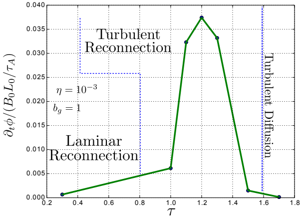


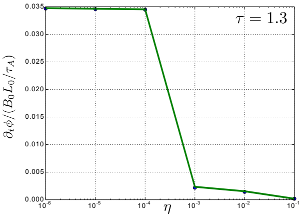
Mean Vorticity and Turbulent Cross-Helicity

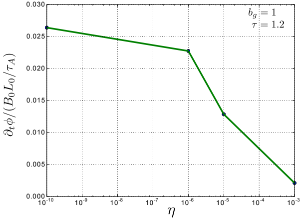


RANS Reconnection Regimes, Harris and Force Free

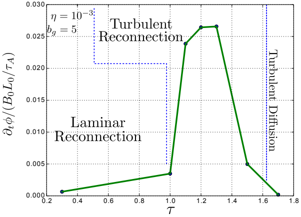


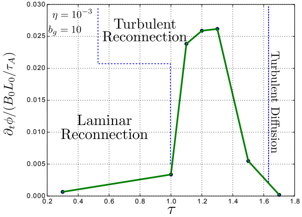


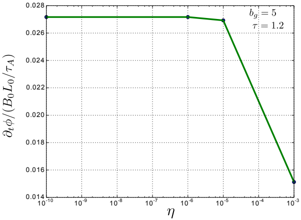


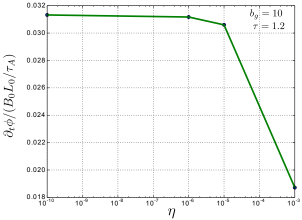


RANS Reconnection Regimes, Force Free with Guide Field

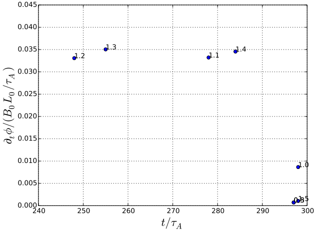


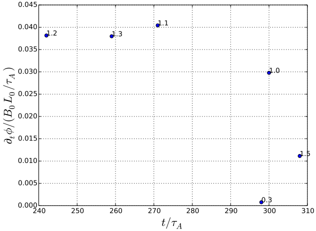


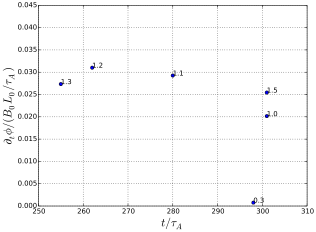


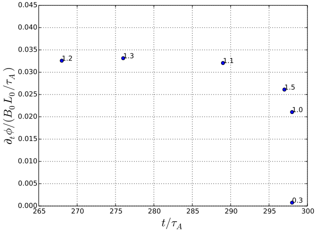


Maximum Reconnection Rate









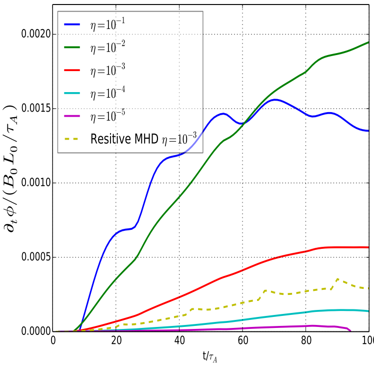
Modified MHD Equations, LES Filtering

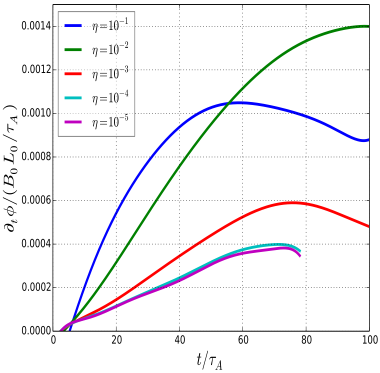
Momentum and induction equation (Grete & Vlaykov 2014)

$$\begin{aligned}\frac{\partial \rho \boldsymbol{u}}{\partial t} &= -\nabla\cdot\Bigg[\rho\boldsymbol{u}\otimes\boldsymbol{u} + \frac{1}{2}(p+B^2)\boldsymbol{I} - \boldsymbol{B}\otimes\boldsymbol{B} \\ &\quad \tau^{u,\star}_{SGS} - \tau^{b,\star}_{SGS} + \frac{2}{3}\left(E^u_{SGS} - E^b_{SGS}\right)\boldsymbol{I} + \frac{1}{2}E^b_{SGS}\boldsymbol{I}\Bigg]\end{aligned}$$

$$\begin{aligned}\boldsymbol{E} &= -\boldsymbol{u}\times\boldsymbol{B} + \eta\boldsymbol{J} - \boldsymbol{E}_M \\ \boldsymbol{E}_M &= \epsilon_{ijk}\Delta^2\left(C_1 u_{j,n} b_{k,n}\right)\end{aligned}$$

Non-Linear Model





Dimensional Analysis of the Reconnection Rate

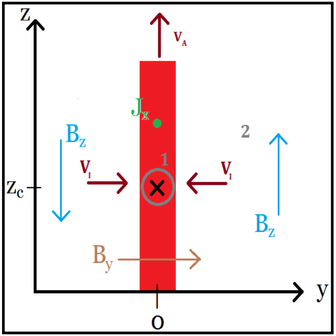


Figure : Current Sheet

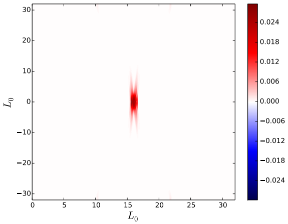
- Reconnection rate:

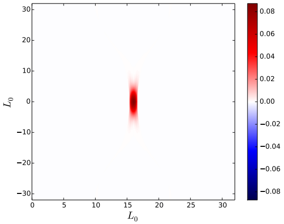
$$M_0^2 = \left(\frac{V_i}{V_o}\right)^2 = \eta$$

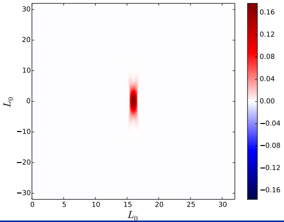
$$M_{MF}^2 = \eta + \beta(1 + \frac{|\gamma|}{\beta}\sqrt{\eta})$$

$$M_{NL}^2 = \eta\left(\frac{1}{1-C_1}\right)$$

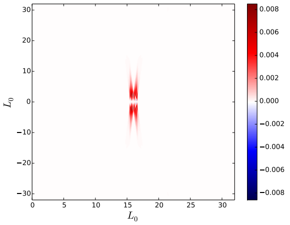
$\alpha = KJ \cdot B$

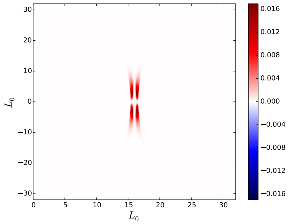


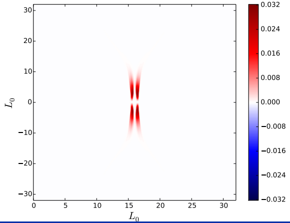




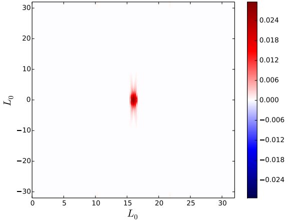
$\alpha = W\mathbf{\Omega} \cdot \mathbf{B}$

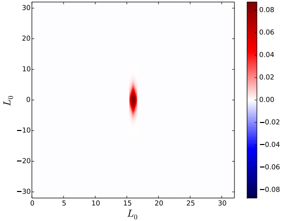


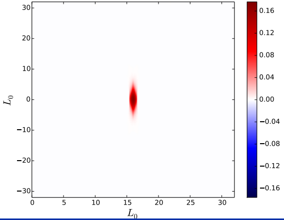




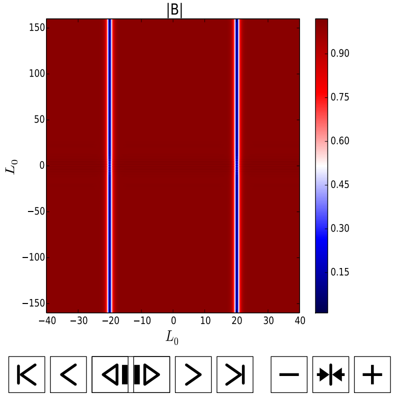
$\alpha = (KJ - W\Omega) \cdot B$

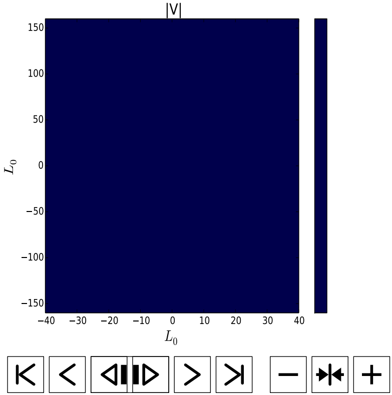






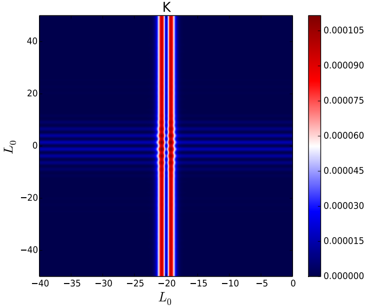
Magnetic Field and Velocity

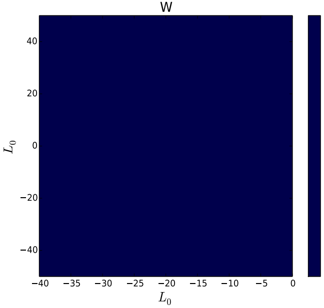




Turbulent Energy and Cross-Helicity

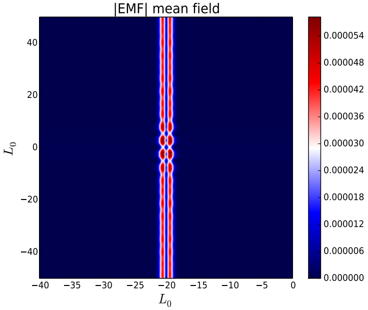
$$K = \left\langle v'^2 + b'^2 \right\rangle \qquad W = \left\langle v' \cdot b' \right\rangle$$



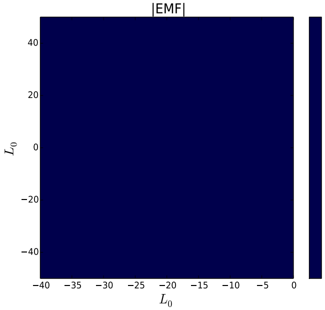


Electromotive Force

$$EMF = \beta \boldsymbol{J} - \gamma \boldsymbol{\Omega}$$



$$EMF = \langle v' \times b' \rangle$$



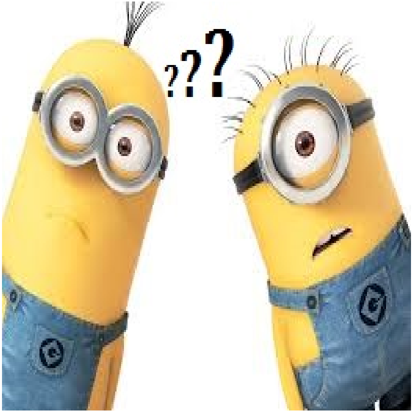
Conclusions

- Mean filed turbulent model
  - Regime controlled by τ
  - Small η enhance the process
  - Filtering confirms the model
- Non-linear SGS model
  - Enhance reconnection rate, not efficient at small η
  - Reconnection rate behaves like resistive MHD
  - Non-linear closure of β related term to be added

Outlook

- Mean field model
  1. 3D simulations and α related term
- Non-linear Model
  1. Adding a β related term
  2. 3D simulations

Questions



Acknowledgment

Thank you for listening