Turbulent Reconnection after Formation of Bipolar Structures from Stratified Helical Dynamos



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Active regions and Sunspots





Active regions



Solar convection



Problems

- What is the mechanism of formation of solar magnetic structures in turbulent convection zone?
- > Solar dynamo mechanism can generate only
 - weak (<< 1000G) nearly uniform large-scale magnetic field.
- > How is it possible to create strongly inhomogeneous magnetic structures from originally uniform magnetic field?

Theories of Sunspots Formation 1. Flux-Transport Dynamo

- I). The solar dynamo produces strong magnetic fields at the bottom of the convection zone at the tachocline region (Parker 1975; Spiegel & Weiss 1980; Spiegel & Zahn 1992, Schou et al. 1998), where there is a strong shear layer, that produces strong toroidal magnetic field.
- > 2) The field becomes unstable and rises upward in form of flux tubes, which reach the surface of the sun and form bipolar structures and sunspots (Caligari et al. 1995). Criticism:
- > A) However, the field in the tachocline region should be reach $10^5 G$, which is needed for a coherent flux tube to reach the surface without strong distortion (Choudhuri & Gilman 1987; D'Silva & Choudhuri 1993).
- B) The generation of such strong coherent magnetic flux tubes has not yet been seen in self-consistent dynamo simulations (Guerrero & Käpylä 2011; Nelson et al. 2011; Fan & Fang 2014).
- C) Helioseismology also does not support a deeply rooted flux tube scenario (Kosovichev & Stenflo 2008, Stenflo & Kosovichev 2012, Howe et al. 2009; Antia & Basu 2011).

2. Negative Effective Magnetic Pressure Instability (NEMPI)

Lorentz Force and Momentum Equation $\mathbf{J} imes \mathbf{B} = (\mathbf{
abla} imes \mathbf{B}) imes \mathbf{B} = -\mathbf{
abla} rac{\mathbf{B}^2}{2} + (\mathbf{B} \cdot \mathbf{
abla}) \mathbf{B} = -
abla_j \left[rac{1}{2} \mathbf{B}^2 \delta_{ij} - B_i B_j
ight].$ $\frac{\partial}{\partial t} \rho \, \mathbf{U}_i = -\nabla_j \, \boldsymbol{\Pi}_{ij}$ where $\Pi_{ij} = \rho U_i U_j + \delta_{ij} \left(p + \frac{1}{2} \mathbf{B}^2 \right) - B_i B_j - \sigma_{ij}^{\nu} (\mathbf{U}) + \dots$ $B = \bar{B} + b$ Averaged equation: $U = \overline{U} + u$, $\frac{\partial}{\partial t}\bar{\rho}\,\bar{\mathbf{U}}_i = -\nabla_j\,\bar{\boldsymbol{\Pi}}_{ij}$ where $\overline{\Pi}_{ij} = \overline{\rho} \overline{U}_i \overline{U}_j + \delta_{ij} \left(\overline{\rho} + \frac{1}{2} \overline{B}^2\right) - \overline{B}_i \overline{B}_j - \overline{\sigma}_{ij}' (\overline{U}) + \frac{1}{2} \langle \mathbf{b}^2 \rangle \, \delta_{ij} - \langle b_i b_j \rangle + \overline{\rho} \langle u_i u_j \rangle + \dots$

DNS: 3D Stratified Forced Turbulence Effective Mean Magnetic Pressure (sum of turbulent and non-turbulent contributions) $(\overline{\mathbf{B}}\cdot \nabla)\overline{\mathbf{B}} =$ $\mathbf{J} \times \mathbf{B} = (\nabla \times \mathbf{B}) \times \mathbf{B} = B = \overline{B} + b$ $\mathcal{P}_{\rm eff}(\beta) = \frac{1}{2} [1 - q_{\rm p}(\beta)] \beta^2$ versus $\beta \equiv |\overline{B}|/B_{eq}$ $\mathcal{P}_{\mathsf{eff}} = rac{1}{2} (1 - q_p) rac{ar{\mathrm{B}}^2}{\mathrm{B}_{\mathsf{eq}}^2}$ 0.06 $R_{\rm m} = 0.7$ 0.04 0.02 0.00 $\mathcal{P}_{\rm eff}$ -0.023.5 -0.04 $R_{\rm m}=6$ -0.06 $R_{m} = 11$ -0.08Effective magnetic pressure for Rm < 1 0.5 0.0 0.2 0.1 0.3 0.4is positive, and for Rm > 1, it can be 0.06 0.04 0.02 negative. 0.00 e^H -0.02 $R_{\rm m} = 70$ -0.04-0.06 $R_{m} = 38$ $R_{\rm m} = 11$ Onasi-linear theory works only -0.080.0 0.2 0.1 0.3 0.5 0.4ß $\mathsf{Rm} \ll 1$ Figure 7. Normalized effective magnetic pressure, $\mathcal{P}_{\text{eff}}(\beta)$, for low (upper panel) and higher

Figure 7. Normalized effective magnetic pressure, $\mathcal{P}_{\text{eff}}(\beta)$, for low (upper panel) and higher (lower panel) values of Re_M . The solid lines represent the fits to the data shown as dotted lines

Equation of State for Isotropic Turbulence N. Kleeorin, I. Rogachevskii and A. Ruzmaikin, Sov. Astron. Lett. 15, 274-277 (1989); Sov. Phys. JETP 70, 878-883 (1990) N. Kleeorin and I. Rogachevskii, Phys. Rev. E 50, 2716-2730 (1994)

I. Rogachevskii and N. Kleeorin, Phys. Rev. E 76, 056307 (2007)

The total turbulent pressure is reduced when magnetic fluctuations are generated

The equation of state for an isotropic turbulence

$$p_T = \frac{1}{3}W_m + \frac{2}{3}W_k$$
,

where p_T is the total (hydrodynamic plus magnetic) turbulent pressure,

 $W_m = \langle \mathbf{b}^2 \rangle / 2\mu$ is the energy density of the magnetic fluctuations,

 $W_k =
ho_0 \langle {f u}^2
angle/2$ is the kinetic energy density.



Strong reduction of Turbulent Pressure

Magnetic contribution to pressure & energy different!

$$U_{i}U_{j} - B_{i}B_{j} + \frac{1}{2}\delta_{ij}\mathbf{B}^{2}$$

$$\approx \frac{1}{3}\delta_{ij}\left(\mathbf{U}^{2} + \frac{1}{2}\mathbf{B}^{2}\right)$$

$$\approx \frac{1}{3}\delta_{ij}\left(\underbrace{\mathbf{U}^{2} + \mathbf{B}^{2}}_{\approx const} - \frac{1}{2}\mathbf{B}^{2}\right)$$



Strong reduction of Turbulent Pressure

Combining the equations:

$$p_T = \frac{1}{3}W_m + \frac{2}{3}W_k = \frac{2}{3}(W_k + W_m) - \frac{1}{3}W_m$$
, $W_k + W_m = \text{const}$,

we can express the change of turbulent pressure δp_T in terms of the change of the magnetic energy density δW_m

$$\delta \mathbf{p}_{\mathrm{T}} = -\frac{1}{3} \delta \mathbf{W}_{\mathrm{m}}$$

Therefore, the turbulent pressure is reduced when magnetic fluctuations are generated (i.e., $\delta W_m > 0$).

DNS: The Result is Robust $= \frac{1}{2}(1-q_p) \frac{\overline{B}^2}{\overline{B}^2}$ **Effect does not exist only below** Rm = 1









Fig.4 Same as Fig. 3, but from simulation (dotted line). The solid line shows a fit [Eq. (26)] with $\overline{B}_{\rm p} = 0.022 c_{\rm s0} \rho_0^{1/2}$ (corresponding to $\overline{B}_{\rm p}/B_{\rm eq} = 0.18$) and $q_{\rm p0} = 21$.

Fig. 7. Effective magnetic pressure obtained from DNS in a polytropic layer with different γ for horizontal (H, red curves) and vertical (V, blue curves) mean magnetic fields.



Figure 7. Normalized effective magnetic pressure, $P_{\text{eff}}(\beta)$, for low (upper panel) and higher (lower panel) values of Re_M . The solid lines represent the fits to the data shown as dotted lines.



Figure 2. Effective magnetic pressure as a function of the mean magnetic field from weakly stratified Runs A1–A29 with an imposed horizontal field $B_0 = B_0 \hat{x}$. The black stars, red diamonds, blue crosses, and yellow triangles denote simulations with Rm $\approx 10, 20, 50, \text{ and } 70$, respectively. We omit points near the boundaries at z/d < 0.35 and z/d > 0.65. The dashed and dotted lines correspond to approximate fits determined by Eq. (30), with $q_{p0} = 35$ and $B_p = 0.2B_{eq}$, respectively.



Figure 3. Same as Figure 2 but for Runs B1–B8 for Rm = 40–50. The solid line corresponds to a fit with $q_{\rm D0} = 70$ and $B_{\rm P} = 0.063B_{\rm eq}$

Large-Scale MHD-Instability (NEMPI)

A. Brandenburg, K. Kemel, N. Kleeorin, Dh. Mitra, and I. Rogachevskii, Astrophys. J. Lett. 740, L50 (2011); Solar Phys. 280, 321-333 (2012).

Slow growth
$$\lambda = \frac{v_A}{H_{\rho}} \left(-2 \frac{d\mathcal{P}_{eff}}{d\beta^2} \right)^{1/2} \frac{k_x}{k}$$





- Several thousand turnover times
- Or ¹/₂ a turbulent diffusive time
- Exponential growth
 → linear instability
 of an already
 turbulent state

NEMPI in DNS: 3D Forced Turbulence (Vertical Imposed Weak Magnetic Field)

All simulations are performed with the **PENCIL CODE**, that uses sixth-order explicit finite differences in space and a third-order accurate time stepping method.

$$\rho \frac{\mathrm{D}U}{\mathrm{D}t} = -c_{\mathrm{s}}^{2} \nabla \rho + \boldsymbol{J} \times \boldsymbol{B} + \rho(\boldsymbol{f} + \boldsymbol{g}) + \nabla \cdot (2\nu\rho \mathbf{S}),$$

$$\frac{\partial A}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta \nabla^2 \boldsymbol{A},$$

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla} \cdot \rho \boldsymbol{U},$$

$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{\nabla} \times \boldsymbol{A}$$

BOUNDARY CONDITIONS:

- The horizontal boundaries are periodic.
- For the velocity we apply impenetrable, stress-free conditions.
- For the magnetic field we use vertical field boundary conditions.

Rm $= \frac{u_{rms}}{\eta k_f} = 18$, 40, 95 $\mathsf{Pm} = \frac{\nu}{\eta} = \frac{1}{2}$ $k_f = 30 k_1;$ $\frac{\rho_{bot}}{1} = 535;$ ρ_{top} **BOUNDARY CONDITIONS** at the top and bottom: $U_z = 0, \ \nabla_z U_x = \nabla_z U_y = 0$ $B_x = B_y = 0,$

Formation of Magnetic Spots in DNS (Vertical Imposed Weak Magnetic Field) $256^3; 512^3; 1024^3$ PENCIL CODE $k_f = 30 k_1;$



 $Pm = \frac{\nu}{\pi} =$ $\frac{\rho_{bot}}{1} = 535;$ ρ_{top}

 $Rm = \frac{u_{rms}}{1} = 18;40;95$

BOUNDARY CONDITIONS at the top and bottom:

1. stress-free conditions $\nabla_z U_x = \nabla_z U_y = 0$ $U_{z} = 0,$ 2. vertical fie $B_x = B_y = 0,$

A. Erandenburg, N. Kleeprin and I. Rogachevskii, Astrophys. J. Lett., 776, L23 (2013)

Formation and Destruction of Bipolar Magnetic Structures

J. Warnecke, I.R. Losada, A. Brandenburg, N. Kleeorin and I. Rogachevskii, Astrophys. J. Lett., 777, L37 (2013); Astron. Astrophys., submitted (2015)

Imposed horizontal field. $k_f = 30 \, k_1;$

BOUNDARY CONDITIONS at the top and bottom:

 $U_{z} = 0, \ \nabla_{z}U_{x} = \nabla_{z}U_{y} = 0$ But $z = -\pi :$ $B_{z} = 0, \ \nabla_{z}B_{x} = \nabla_{z}B_{y} = 0$

 $B_x = B_y = 0.$

Re=40, $Pr_M = 0.06 - 1$



FIG. 5.— Time series of B^2/B_{eq0}^2 in a vertical cut through the bipolar region at x = 0. Note the y axis is shifted the see the formation of the loop.

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J. Warnecke, I.R. Losada, A. Brandenburg, N. Kleeorin and I. Rogachevskii, Astrophys. J. Lett., 777, L37 (2013); Astron. Astrophys., submitted (1015).



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Magnetic Structures

J. Warnecke, I.R. Losada, A. Brandenburg, N. Kleeorin and I. Rogachevskii, Astrophys. J. Lett., 777, L37 (2013), Astron. Astrophys., submitted (2015).

Simulations

Sunspots



FIG. 1.— Upper panel: normalized vertical magnetic field $B_z/B_{\rm eq}$ of the bipolar region at the surface (z = 0) of the simulation domain. The white lines delineate the area shown in Figure 3. Lower panel: normalized magnetic energy $B^2/B_{\rm eq}^2$ of the two regions relative to the rest of the surface. Note that we clip both color tables to increase the visualization of the structure. The field strength reaches around $B_z/B_{\rm eq} = 1.4$.



DNS in Two Forced Regions: Dynamo+NEMPI Dh. Mitra, A. Brandenburg, N. Kleeorin and I. Rogachevskii, MNRAS 445, 716 (2014). All simulations are performed with the PENCIL CODE, that uses sixth-order explicit finite differences in space and a third-order accurate time stepping method.

$$\rho \frac{\mathrm{D}U}{\mathrm{D}t} = -c_{\mathrm{s}}^2 \nabla \rho + \boldsymbol{J} \times \boldsymbol{B} + \rho(\boldsymbol{f} + \boldsymbol{g}) + \boldsymbol{\nabla} \cdot (2\nu\rho \mathbf{S}),$$

$$\frac{\partial A}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta \nabla^2 \boldsymbol{A},$$

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla} \cdot \rho \boldsymbol{U},$$

Run	z_0	$\sigma_{ m max}$	$\operatorname{Re}_{\mathrm{M}}$	$ ilde{k_{\mathrm{f}}}$	$ ilde{\lambda}$	$ au_{\mathrm{to}}$	$ au_{\mathrm{td}}$
А	2	1	17	30	0.041	0.33	900
В	-1	1	17	30	0.042	0.33	900
B /2	-1	1	17	30	0.036	0.33	900
С	$^{-2}$	1	17	30	0.045	0.33	900
D	$^{-2}$	1	17	60	0.043	0.17	1800
Е	$^{-2}$	1	170	30	0.022	0.33	900
0-02	0	0.2	17	30	0.0043	0.33	900
0-1	0	1	17	30	0.043	0.33	900

 $\sigma(z-z_0) = \frac{\sigma_{\max}}{2} \left[1 - \operatorname{erf}\left(\frac{z-z_0}{w_f}\right) \right]$

 $\frac{\rho_{bot}}{\rho_{top}} = 535; \quad \mathsf{Pm} = \frac{\nu}{\eta} = \frac{1}{2}$

 $\text{Re} \equiv u_{\text{rms}}/vk_{\text{f}},$

$256^3; 512^3;$

 $w_f = 0.08 L_z$

FORCING:

1). White-in-time random forcing with the fractional helicity:

- 2). Helical forcing: $-\pi < z < z_0$ (bottom layer);
- 3). Non-helical forcing: $z_0 < z < \pi$ (upper layer).

BOUNDARY CONDITIONS:

- The horizontal boundaries are periodic.
- 2). For the velocity we apply impenetrable, stress-free conditions.
- 3). For the magnetic field we use a) vertical field boundary conditions for z=L (at the top); b) perfect conductor for z=0 (at the bottom).

>Dh. Mitra, A. Brandenburg, N. Kleeorin and I. Rogachevskii, MNRAS 445, 716 (2014).



DNS in Two Forced Regions: Dynamo+NEMPI > Dh. Mitra, A. Brandenburg, N. Kleeorin and I. Rogachevskii, MNRAS 445, 716 (2014).



Figure 2. Vertical magnetic field at the top surface at different times (from $t/\tau_{\rm td} = 0.30$ to 0.33) from Run B. The magnetic field is normalized by $B_{\rm eq}^0$.



Figure 3. Same as Fig. 2, but at later times (from $t/\tau_{td} = 0.35$ to 0.38) and the frame is re-centered, as illustrated in Fig. 4 below.

DNS in Two Forced Regions: Dynamo+NEMPI >Dh. Mitra, A. Brandenburg, N. Kleeorin and I. Rogachevskii, MNRAS 445, 716 (2014).



Figure 5. Evolution of the vertical magnetic field at the top surface. Snapshots at different times (from $t/\tau_{td} = 0.45$ to $t/\tau_{td} = 1.67$) are plotted.

DNS in Two Forced Regions: Dynamo+NEMPI

>Dh. Mitra, A. Brandenburg, N. Kleeorin and I. Rogachevskii, MNRAS 445, 716 (2014).



Figure 8. Magnetic field structure for Run A at time $t/\tau_{\rm td} \approx 1.2$. The z component of the magnetic field, B_z is plotted at $z/H_{\rho} = 3$. The height up to which dynamo operates, $z_0/H_{\rho} = 2$, is also shown as a frame. Here magnetic field, B_z is not normalized, but in units of $\sqrt{\langle \rho(z=0) \rangle_{xy}} c_{\rm s}$. In the same units $B_{\rm eq}^0 \approx 0.1$.



Reconnection Rate and Different Regimes

1. Sweet-Parker model (Parker (1957); Sweet (1969)):

$$V_{\rm rec} = V_{\rm A} S^{-1/2}$$

2. Lazarian-Vishniac (1999)

$$V_{\rm rec} \sim V_{\rm A} M_{\rm A}^2$$

Lundquist number: $S = V_{\rm A}L/\eta$

$$M_{\rm A} = u_{\rm rms}/V_{\rm A}$$

3. Loureiro-Schekochihin-Cowley (2007)

$$V_{\rm rec} \sim 10^{-2} V_{\rm A}$$

Formation and Destruction of the Current Sheet



Formation of Current Sheet



Formation of the Current Sheet and Reconnection: Rm=130



Formation of the Current Sheet and Reconnection: Rm=130





TABLE 2 Summary of the reconnection parameters.

Run	Re_{M}	L_z^{\max}	η	$u_{ m rms}/c_{ m s}$	$V_{\rm rec}/c_{ m s}$	$V_{\rm A}/c_{ m s}$	M_A	S	$S^{-1/2}$	$V_{\rm rec}/V_{\rm A}$	M_A^2
D1	16	0.69	2×10^{-4}	0.09	0.007	0.12	0.75	414	0.049	0.058	0.56
RM1	50	0.75	5.7×10^{-5}	0.086	0.007	0.3	0.4	3947	0.016	0.023	0.16
RM2	130	0.81	2×10^{-5}	0.078	0.007	0.48	0.16	19440	0.007	0.014	0.026

Reconnection Rate and Different Regimes

1. Sweet-Parker model (Parker (1957); Sweet (1969)):

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3. Loureiro-Schekochihin-Cowley (2007)

$$V_{\rm rec} \sim 10^{-2} V_{\rm A}$$

Formation of the Current Sheet and Reconnection: Rm=16

a.

0.0

3.0

3.0



Formation of the Current Sheet and Reconnection: Rm=16



Formation of the Current Sheet and Reconnection



Summary

- Generation of magnetic fluctuations in a turbulence with large plasma beta results in a strong reduction of the large-scale magnetic pressure, so that effective magnetic pressure (sum of turbulent and non-turbulent contributions) can be negative.
 This causes excitation of negative effective magnetic pressure
 - instability (NEMPI) and formation of the large-scale bipolar magnetic structures which are reminiscent Active Regions.
- DNS of two-layer systems with a helical forcing layer demonstrate formation of bipolar structures and turbulent reconnection with the rate that is independent of magnetic Reynolds number. This is the first example of turbulent reconnection in nonlinear dynamos.

THE END